

# Assessment of dike safety within the framework of large deformation analysis with the material point method

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**ABSTRACT:** Dike infrastructure is of vital importance for the safety against flooding. The standard methodologies for the assessment of dike safety for macrostability are based on limit equilibrium methods, which result in a safety factor against shear failure. The more advanced alternative consists of using finite element method to compute the safety factor against shear failure. However, these approaches do not take into account the capacity of the dike to retain water, but are only concerned with the mechanical equilibrium of the dike's initial composition. With the recent advancements in the modelling of large deformations within geotechnical engineering, e.g. by means of the material point method, the post failure behaviour of the dike can be predicted. The material point method is a mesh-free method that has been developed to address the problem of large deformation on a continuum level. The material point method offers the possibility to redefine the concept of factor of safety against shear failure. The initial shear failure of a dike does not necessarily lead to the loss of the dike's capability to retain water. In reality, after the initial shear failure the mass of soil will move and reach a new equilibrium position. This paper, after a brief description of the material point method, presents the analysis of a progressive dike failure, where the post failure behaviour is examined and a proposal is made to redefine the concept of factor of safety.

## 1 INTRODUCTION

Slope stability of dikes is an important issue in geotechnical engineering as dikes are often the first line of defence against water flooding. Traditionally, slope stability analysis is carried out using limit equilibrium methods (LEM) or the finite element method (FEM). Limit equilibrium methods have the advantage of being computational efficient, but have many drawbacks when compared to other numerical methods. The main limitation of limit equilibrium methods regards the definition of safety, as it strongly depends on the assumptions made for the slip surface (Griffiths and Lane 1999, Li et al. 2009). Different limit equilibrium methods are distinguished by the assumptions for the slip surface. A detailed review of limit equilibrium methods is reported by Duncan (1996).

The finite element method is a more advanced

methodology that has significant advantages for the analysis of slope stability. The shape and location of the slip surface are not predefined, and are automatically found. The analysis provides insight into the initiation of failure and failure mechanism. The finite element method also allows the use of complex models for the stress-strain behaviour of the soil (Farias and Naylor 1998, Griffiths and Lane 1999, Li et al. 2009).

In limit equilibrium methods, failure is defined when the shear strength of the soil is equal to the shear strength required to guarantee the slope equilibrium. In the finite element method, failure is traditionally defined when there is a non-convergence of the solution (Griffiths and Lane 1999). Slope failure and numerical non-convergence occur simultaneously, and are accompanied by a major increase in the nodal displacements within the mesh. This leads to prob-

lems related to mesh distortion that prevent the use of the finite element method for post-failure analysis of slopes. This means that, although the finite element method is an appropriate tool to analyse slopes up to the moment of initial failure, the behaviour following this initial failure cannot be correctly described.

In order to overcome the limitations of the finite element method, mesh-free methods that address the problem of large deformation on a continuum level have been developed, such as the material point method (MPM). In the material point method the continuum material is represented by a set of Lagrangian points (material points) that move through an Eulerian background mesh. The material points contain all the properties of the continuum, such as mass, stress, strain and material parameters. Therefore, the material point method can be seen as a combination of both Lagrangian and Eulerian formulations. The problems related to mesh distortion under large deformations are circumvented, as well as the diffusion associated with the convective terms of the Eulerian approach (Sulsky et al. 1994, Sulsky et al. 1995).

The use of Lagrangian material points conserves mass and allows the use of complex history dependent stress-strain material models. The discrete equations for momentum balance are obtained on the background grid similar to the finite element method with an updated Lagrangian formulation. The material point method has been successfully used for geomechanics problems involving slope stability (e.g. Zabala and Alonso 2011, Soga et al. 2016, Fern et al. 2017).

In this paper, the material point method will be used to study the stability of a dike. After an initial validation of the method against literature results, the material point method will be applied to a typical Dutch dike profile, and the differences with the standard methodologies will be highlighted and discussed.

## 2 VERIFICATION

The material point method analyses have been performed with the software Anura3D (2017). The validation has been performed by simulating a slope previously presented by Hicks and Wong (1988). The cross section of the 3D slope geometry and mesh for the material point method analysis are presented in Figure 1. The slope is assumed to be fully saturated with water, and the analysis is undrained. The system of equations is solved explicitly in the time domain. The domain was discretised using low-order tetrahedral elements (in total 1086 active elements), with initially 4 material points in each element. The displacements are constrained in both vertical and horizontal directions for the bottom boundary, and the displacements are constrained in the horizontal direction along the vertical boundaries. The slope material has been modelled as a Mohr-Coulomb material (see Table 1).

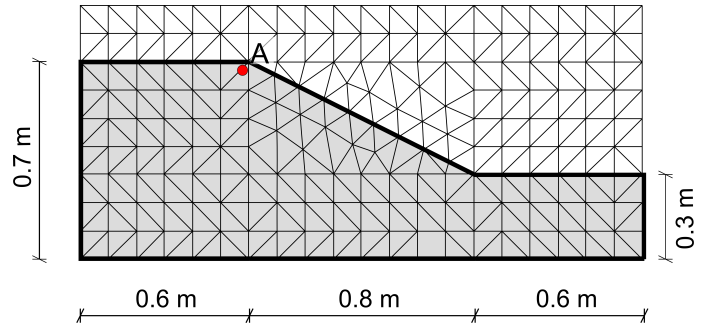


Figure 1: Cross section of 3D slope geometry and mesh (based on Hicks and Wong 1988).

Table 1: Mohr-Coulomb material parameters for the slope (based on Hicks and Wong 1988).

Parameter	Unit	Value
Volumetric weight	kN/m <sup>3</sup>	20
Young modulus	kPa	1000
Poisson ratio	-	0.3075
Poisson ratio undrained	-	0.499
Cohesion	kPa	0.6
Friction angle	°	30
Dilatancy angle	°	0

The slope factor of safety was assessed by increasing the gravity multiplier, while keeping all material parameters constant. The factor of safety is defined as the ratio between the gravity multiplier at the failure state and the initial gravity multiplier, i.e. 1g (Swan and Seo 1999, Li et al. 2009, Zheng et al. 2006).

Figure 2 shows the vertical displacement for point A (see location in Figure 1), located at the edge of the slope crest, obtained by finite element method (Hicks and Wong 1988), material point method and limit equilibrium method. The limit equilibrium analysis were performed by using Bishop's method within the D-GeoStability software (Deltares 2016).

It follows that the results provided by the material point method calculation are in agreement with the results from the finite element analysis, up to the moment of failure (gravity multiplier of  $\approx 2.3$ ), which illustrates the correctness of the material point method solution for small strain analysis. The finite element computation has the limitation of not providing any insight into the dike behaviour after the initial failure, while the material point method is able to simulate beyond this point. The limit equilibrium method provides a result that is also in agreement with the finite element analysis. This shows that when the objective is to estimate the initial factor of safety of slopes the limit equilibrium method provides a good solution with fewer computational resources.

Following the traditional design methodology it is assumed that the slope factor of safety is 2.3. However, for this factor of safety the vertical displacement of point A is only 0.08 mm (0.002% of the slope height), which means that the slope still fulfils its function of retaining high water levels. This will be further discussed in the next section.

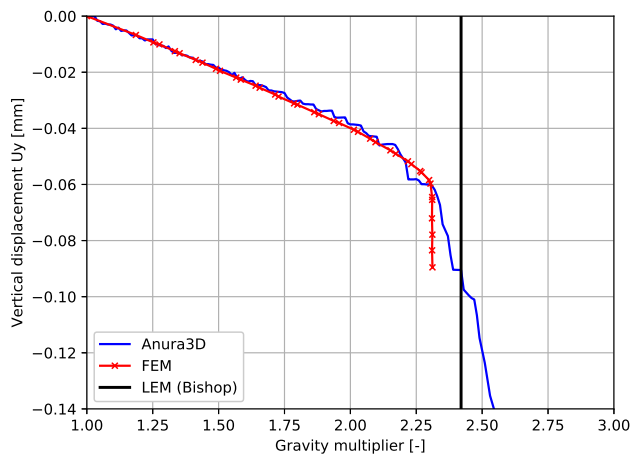


Figure 2: Comparison between FEM (Hicks & Wong 1988) MPM and LEM results for the vertical displacement of the slope crest.

### 3 DIKE ANALYSIS AND SAFETY

#### 3.1 Problem description

In order to illustrate the benefits of using the material point method for slope stability analysis, a generic dike section has been studied. The dike geometry corresponds to a typical Dutch dike section, with non-symmetric inclination of the slopes (outer slope=1/4; inner slope=1/5). The dike was considered fully saturated with the water level placed at the height of the crest. The cross section of the 3D dike geometry and mesh for the material point method analysis are presented in Figure 3.

The system of equations is solved explicitly in the time domain. The domain was discretised using low-order tetrahedral elements (in total 7017 active elements), with initially 4 material points in each element. The displacements are constrained in both vertical and horizontal directions for the bottom boundary, and the displacements are constrained for the horizontal direction along the vertical boundaries. The dike material was considered to be uniform and homogeneous, and has been modelled as a Mohr-Coulomb material (properties available in Table 2). The mesh is refined in domains where higher shear strains are expected.

The application of distributed loads in the material point method are not straight forward, as the material points can move through the mesh. To overcome this issue, the water loading was modelled by representing the water reservoir as an elastic material, with the properties presented in Table 2. Note that the water is not infiltrating into the dike.

Figure 4 shows the vertical effective stress and pore water pressure, after the initialisation of gravity loading (gravity multiplier 1g). It follows that the proposed methodology is appropriate to model the water loading.

Similar to the slope presented in the previous section, the dike safety was assessed by increasing the

Table 2: Mohr-Coulomb material parameters for the dike and water.

Material	Parameter	Unit	Value
Dike	Volumetric weight	kN/m <sup>3</sup>	20
	Young modulus	kPa	7500
	Poisson ratio	-	0.33
	Poisson ratio undrained	-	0.499
	Cohesion	kPa	1
	Friction angle	°	35
	Dilatancy angle	°	0
Water	Volumetric weight	kN/m <sup>3</sup>	10
	Young modulus	kPa	150000
	Poisson ratio	-	0
	Poisson ratio undrained	-	0.499

gravity multiplier. However, as the aim of the analysis is to analyse dike deformation, a problem arises as the dike deformation incorporates both the deformation due to shearing and the deformation due to the elastic deformation caused by the gravity increase. In order to be able to distinguish between the two, an additional one-dimensional soil column subjected to gravity increase was computed with the material point method, and its results subtracted from the dike results. The results from the one-dimensional soil column correspond entirely to the deformation caused by the gravity increase, as no shearing occurs. Therefore, the results presented in the following section are corrected, and merely concern the shear deformation.

#### 3.2 Results

Figure 5 shows the vertical displacement of several points located along the dike crest, together with the factor of safety obtained by means of the limit equilibrium method. The location of the points are indicated in Figure 3. The dike crest exhibits higher displacements at points towards the inner slope. Point 1, which is located close to the outer slope, and next to the water loading has a smaller vertical deformation.

For a gravity multiplier of 1.2, which corresponds to the factor of safety according to the limit equilibrium analysis, the crest deformation is very small. This is further illustrated in Figure 6, which presents the vertical crest displacement for low values of the gravity multiplier. For a gravity multiplier of 1.2 the maximum dike crest deformation occurs at point 5 and is smaller than 7 mm. This means that, if this dike was analysed following the standard methods, the dike would be considered to fail with a maximum displacement of 7 mm. However, for such a small deformation it can be expected that the dike still fulfils its primary function of retaining water.

Figure 7 presents the dike crest profile at different levels of the gravity multiplier. For a gravity multiplier of 1.2 the dike crest exhibits no visible displacement. At a gravity multiplier of 1.5 the dike crest starts to move, but only for the points closer to the inner slope. Only at a gravity multiplier of 3 a clear

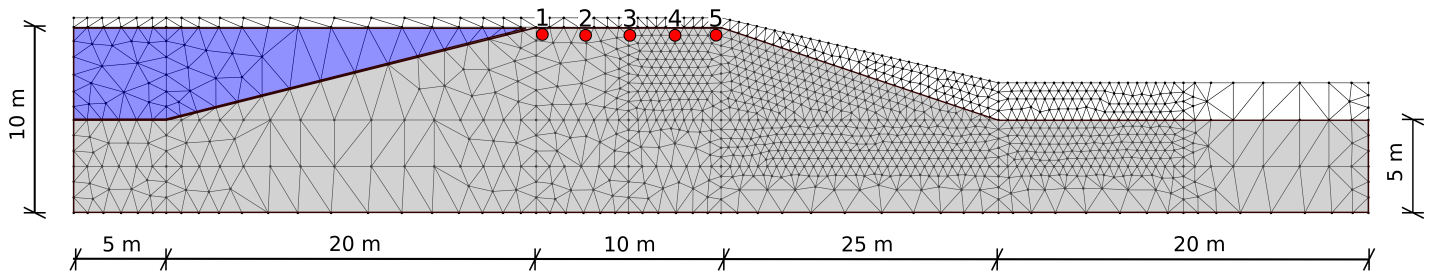
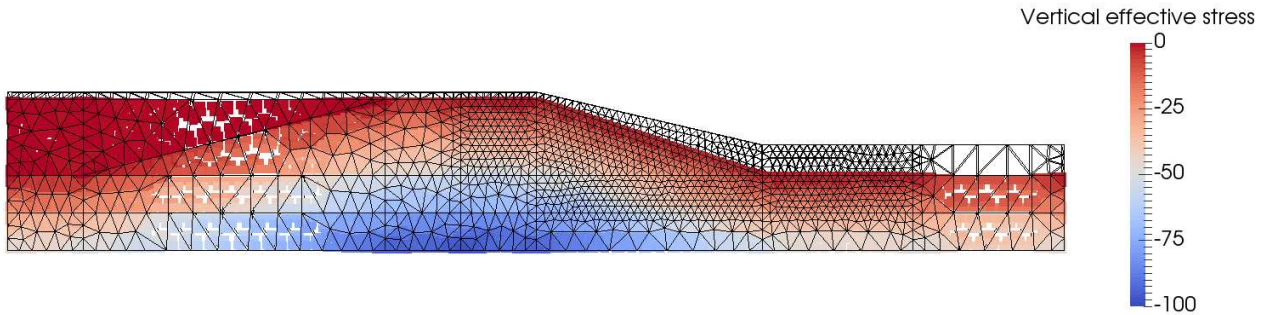
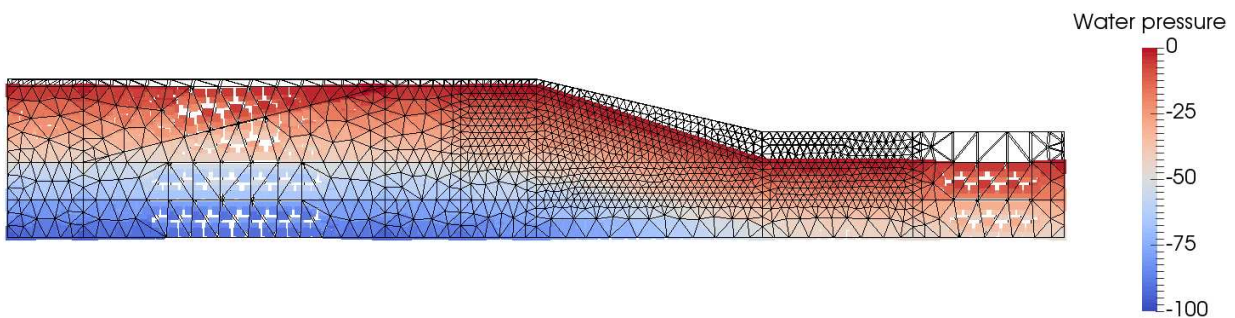


Figure 3: Cross section of dike geometry and mesh.



(a)



(b)

Figure 4: Initialisation stage: (a) vertical effective stress and (b) water pressure at gravity multiplier 1g.

movement of the entire dike crest is identified. From the figure it is also found that the dike crest not only settles, but also elongates.

The main advantage of the material point method for the analysis of dike safety is related to the possibility of analysing beyond the initial failure. As the gravity multiplier increases, the dike continuously deforms and reaches new equilibrium positions. This is because failure is progressive and not abrupt as it is assumed by the limit equilibrium methods.

From a safety point of view, if a maximum allowable displacement of the dike crest is defined, a new factor of safety can be established based on this displacement criterium. For example, for this particular dike, if it would be acceptable to have a maximum crest displacement of 25 cm, the factor of safety would be larger than 1.5, instead of 1.2 as defined from the limit equilibrium method.

The displacement and shear strain fields of the dike

are presented in Figure 8, at different gravity multipliers. The dike is found to exhibit a classic macrostability failure by movement of the inner slope. Up to a gravity multiplier of 2 no significant displacement or shear bands occur. For gravity multipliers of 2 and higher it is clear the formation of the shear band and its effects on the dike displacement are clearly visible.

### 3.3 Influence of Young modulus

The standard methodology of dike safety assessment based on limit equilibrium methods, does not take into account the dike deformation, hence the stiffness has no influence on the dike safety. However, when going towards an assessment based on displacement criteria, the stiffness is of importance, as it is fundamental to correctly model the displacement field. Therefore an analysis on the Young modulus is performed in the range between 2500 and 15000 kPa.

Figure 9 presents the results of the dike crest dis-

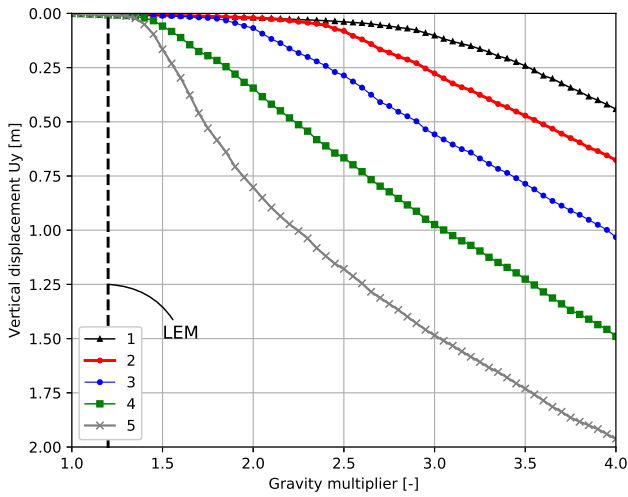


Figure 5: Vertical displacement for several points along the dike crest.

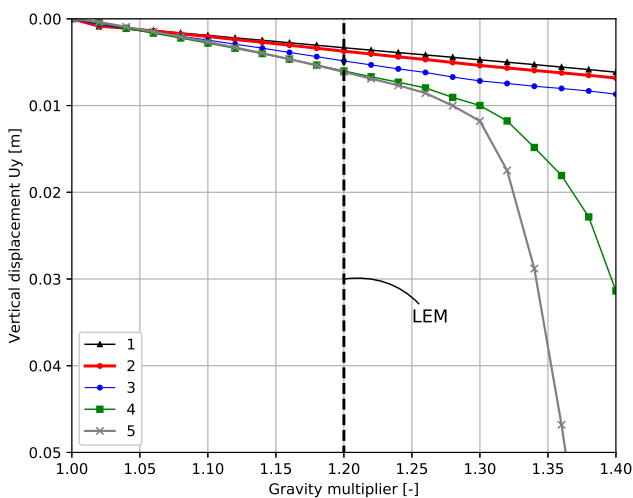


Figure 6: Vertical displacement for several points along the dike crest for low gravity multipliers.

placement for several cases. It is clear the effect of the Young modulus on the dike crest displacement results. For low gravity multipliers (Figure 9a and 9b), the deformation pattern is the same for all Young moduli, while the lower the Young modulus the larger the crest deformation is.

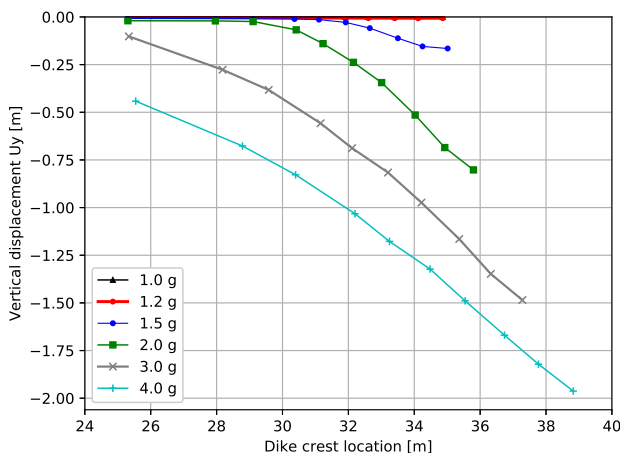


Figure 7: Dike crest profile for different values of gravity multiplier.

As the gravity multiplier increases the deformation pattern changes. At a gravity multiplier of 2 (Figure 9c) the displacement at the outer slope is larger for the lower Young modulus, however at the inner slope, the displacement is larger for the larger Young moduli. An inflection point is found halfway along the dike crest, where the influence of the Young modulus on the displacement is inverted. This is likely caused by stress redistributions caused by the large deformations, and illustrates the complexity of large deformation analysis. For the higher gravity multipliers (9d, and 9e), this effect is enhanced, whereby higher crest displacements occur for higher Young modulus.

These results illustrate the importance of correctly estimating the Young modulus when performing more advanced computations by means of the material point method. Also, it shows that as the design shifts towards displacement criteria, more soil investigation is needed in order to be able to correctly parametrise the material models.

#### 4 CONCLUSIONS

This paper presented the validation of the Anura3D material point method software, and dike safety analysis based on displacement criteria.

The standard methodology for the assessment of dike safety is based on limit equilibrium methods, which do not take into account a deformation criterion. Based on material point method calculations it has been shown that, for the factor of safety corresponding to the limit equilibrium method, the dike crest displacements are very small. This means, that the factor of safety does not imply that the dike fails its requirement of retaining water, but just that an initial failure took place. After this initial failure the dike stabilises in a new equilibrium position.

The recent developments of the material point method enable to look beyond this initial failure. From the analysed dike section, it has been shown that the dike continues deforming with increasing gravity, and that the dike failure is progressive. Profiles for the dike crest deformation were computed, which can be used to assess the dike safety, by combining it with criteria for the maximum allowed displacement of the dike crest. Moving towards a displacement based safety assessment of dikes can have a positive impact for existing dikes, as it will provide a better prediction of their safety, as well as for dikes to be constructed, as it will allow for more economical designs.

The analyses presented in this paper can be considered as a first step in a transition towards a safety assessment based on displacement criteria. Future work concerns the definition of the maximum allowed crest displacement and this must be established within the probabilistic framework of dike safety. The transition towards displacement criteria is likely to require more and different soil investigation, in order to determine the stiffness parameters reliably.

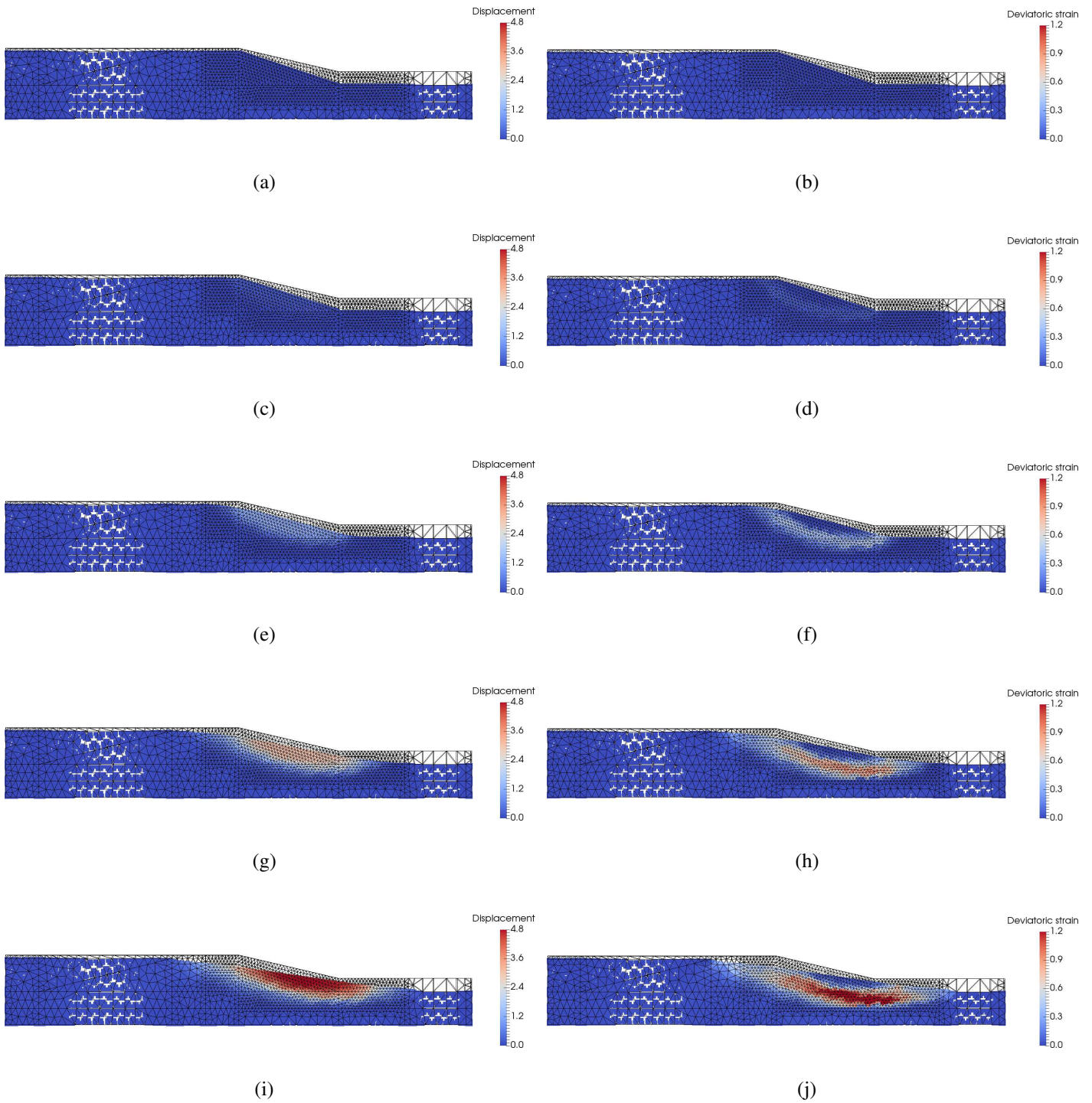
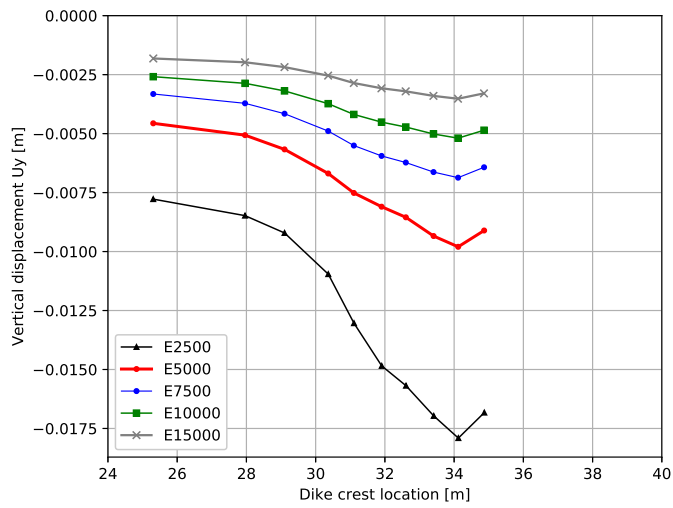
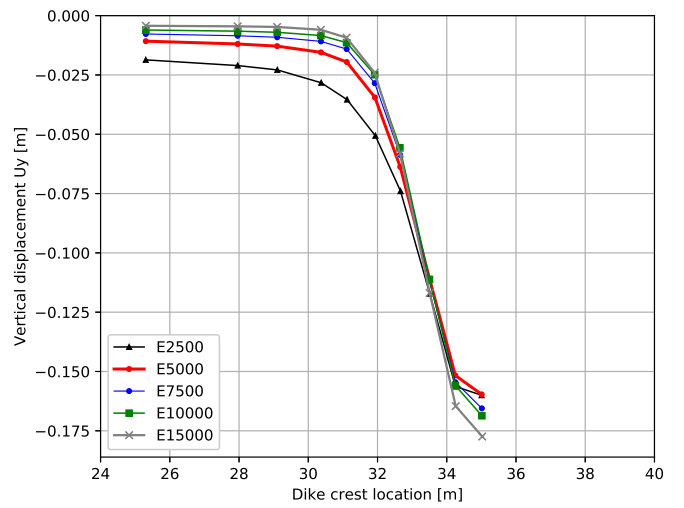


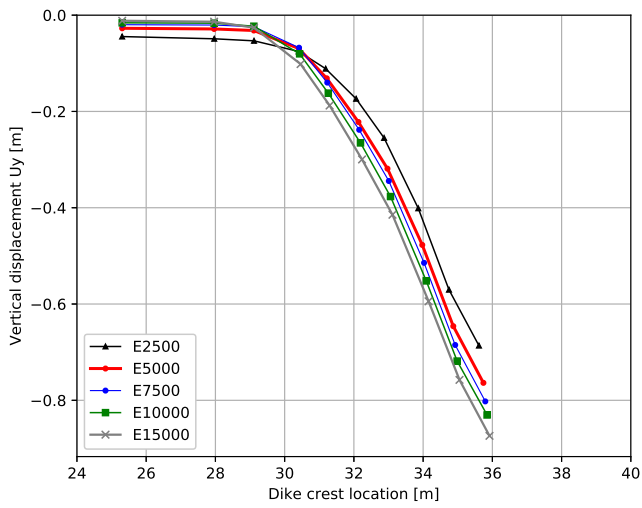
Figure 8: Displacement and deviatoric strain fields for different values of gravity: (a) and (b) 1.2g, (c) and (d) 1.5g, (e) and (f) 2g, (g) and (h) 3g, (i) and (j) 4g.



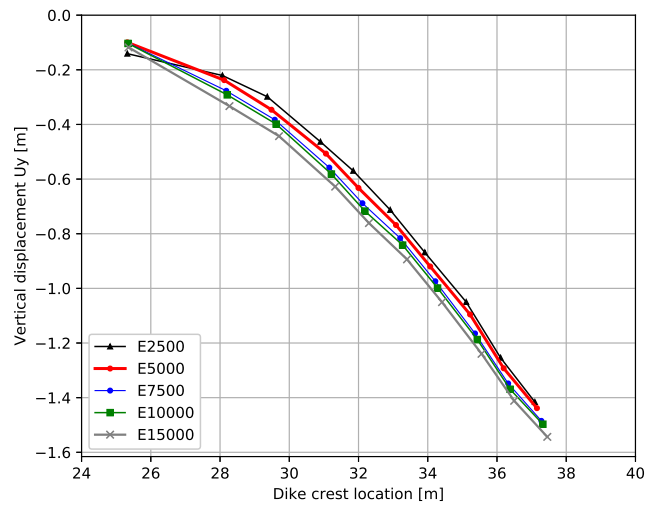
(a)



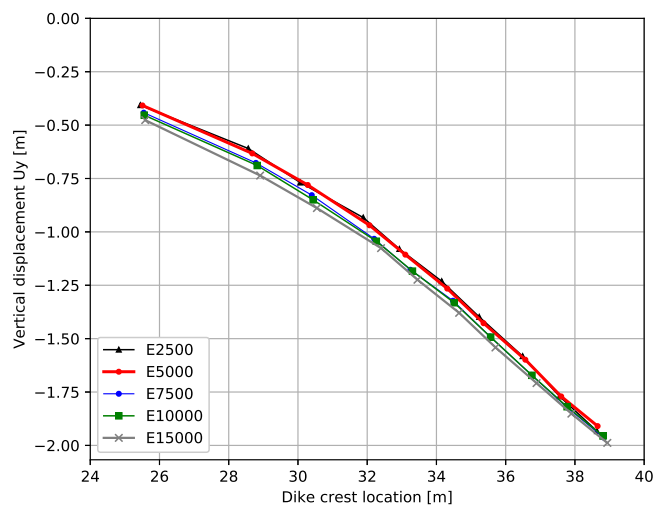
(b)



(c)



(d)



(e)

Figure 9: Effect of the Young modulus on the crest profile for different values of gravity: (a) 1.2g, (b) 1.5g, (c) 2g, (d) 3g and (e) 4g.

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