An adaptive vertical coordinate system for a coastal flows discontinuous Galerkin finite element model

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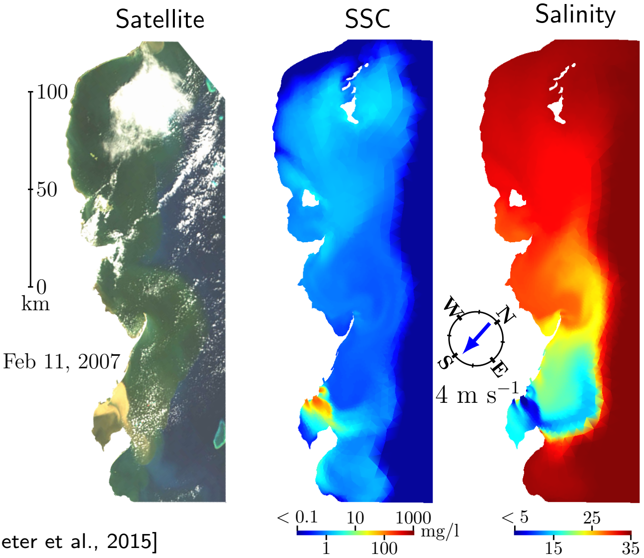
SLIM 3D

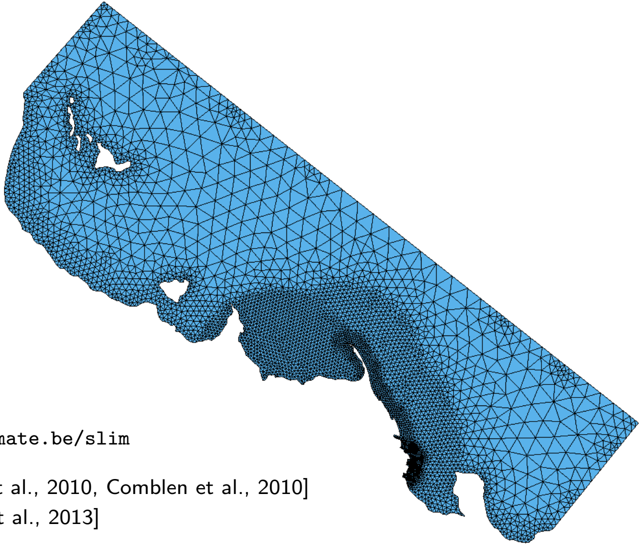
- Burdekin river flood (Australia)



[Delandmeter et al., 2015]

SLIM 3D: domain discretization

- unstructured in the horizontal direction

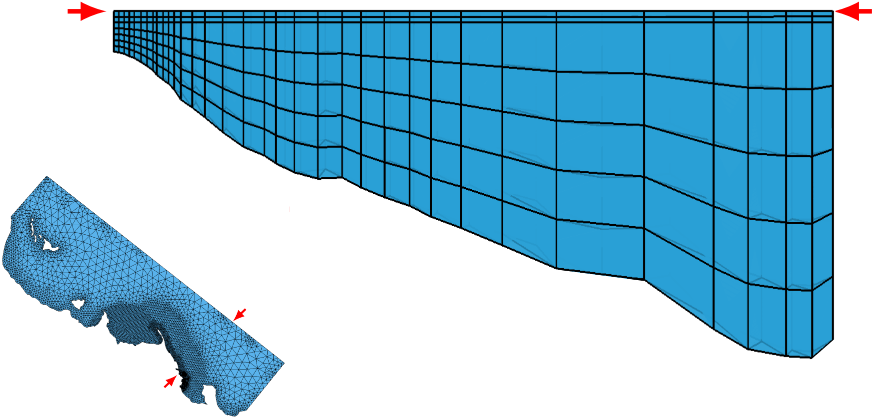


www.climate.be/slim

[Blaise et al., 2010, Comblen et al., 2010]
[Kärnä et al., 2013]

SLIM 3D: domain discretization

- unstructured in the horizontal direction
- structured in the vertical direction: prismatic elements

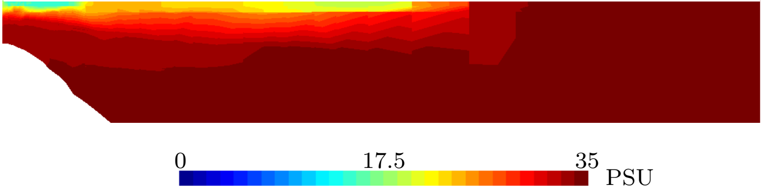


Layers distribution

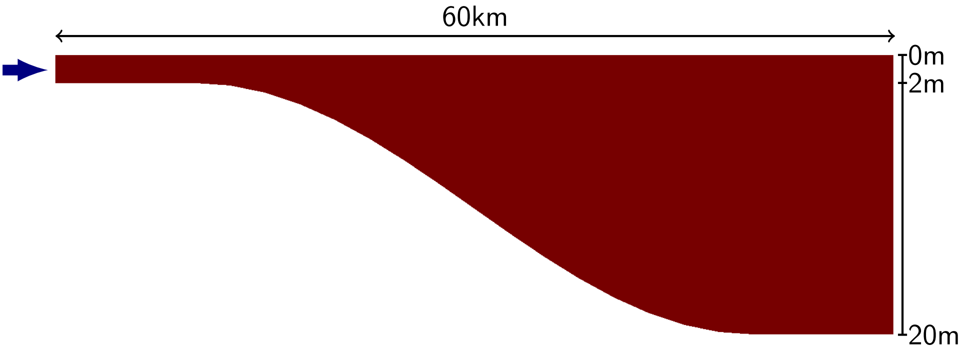
Salinity Profile

- 5 σ-layers

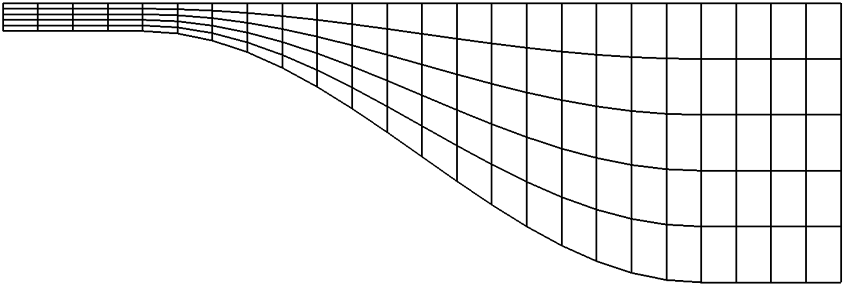
- 2 z-layers, followed by 5 σ-layers



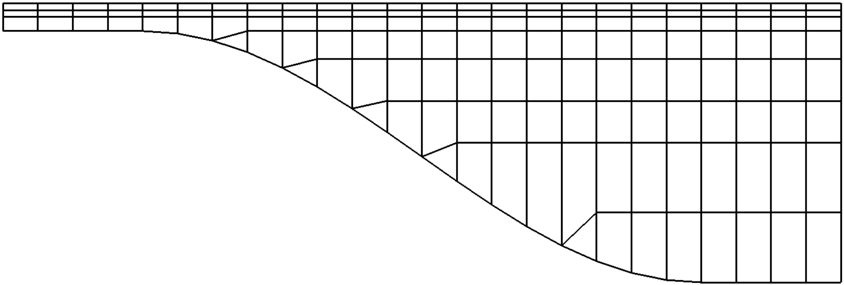
Test case: Idealized river plume



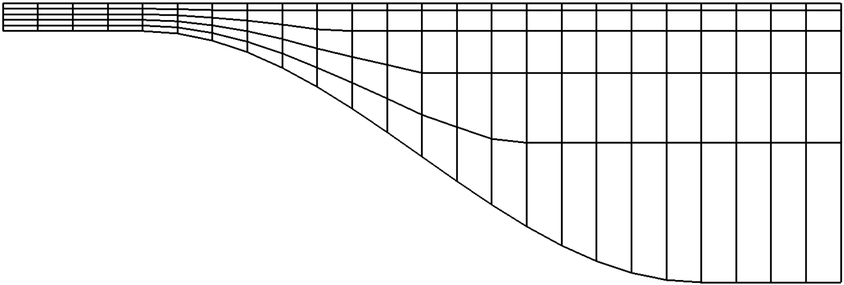
Idealized river plume: σ-layers



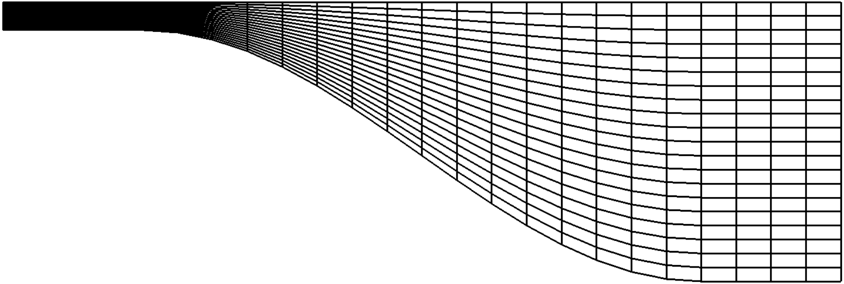
Idealized river plume: z-layers with shaved cells



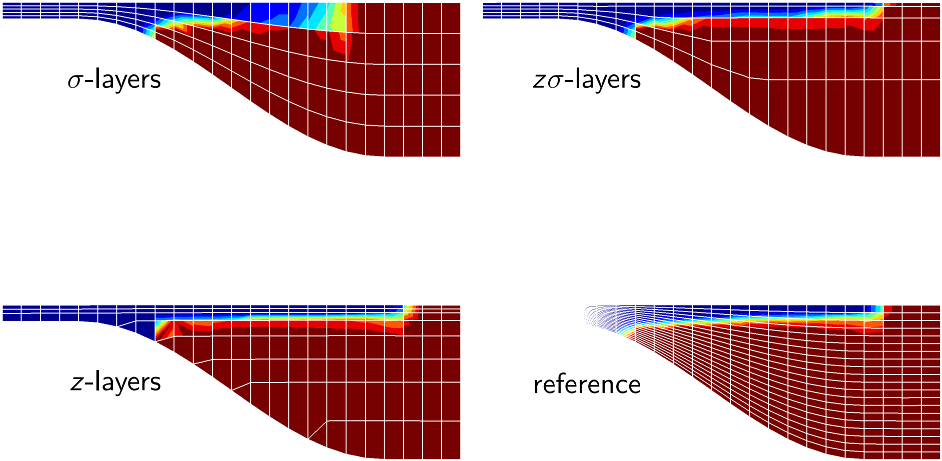
Idealized river plume: $z\sigma$-layers



Idealized river plume: reference σ-layers

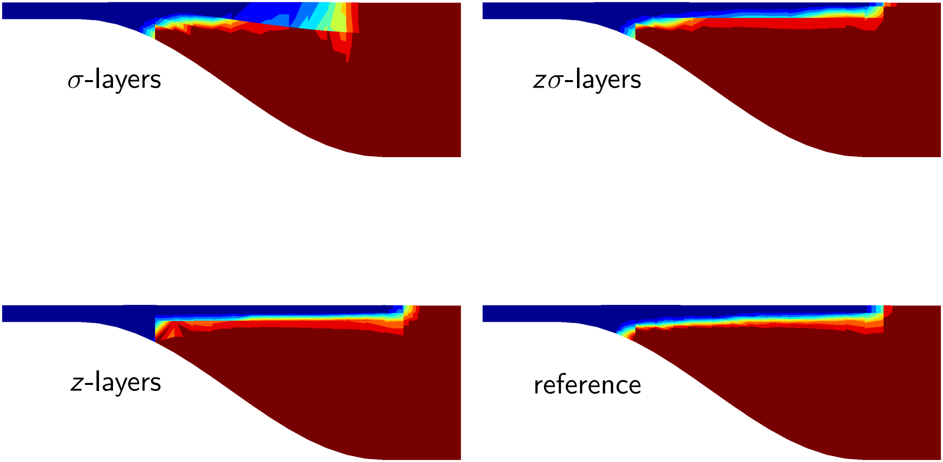


Idealized river plume



After 15 hours

Idealized river plume (mesh hidden)



After 15 hours

Non-uniform adaptive vertical grid

The grid should be adapted considering the:

- Stratification
- Shear
- Distance to the surface
- Distance to the bottom

Two options:

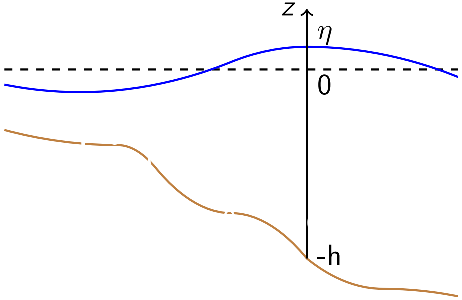
- Define the mesh resolution following an error function
- Diffuse the mesh following an error function

[Burchard and Beckers, 2004, Hofmeister et al., 2010]

Non-uniform adaptive vertical grid:

Moving mesh algorithm:

$$z_{\text{new}} = z + \frac{h+z}{h+\eta_{\text{old}}}\,(\eta - \eta_{\text{old}}) + dt\ v_z$$

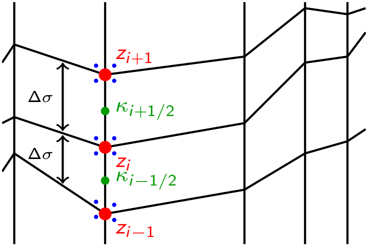


Non-uniform adaptive vertical grid:

Arbitrary moving mesh function:

$$v_z = \begin{cases} \dfrac{\partial}{\partial \sigma}\left(\kappa_z \, \dfrac{\partial z}{\partial \sigma}\right) \\ v_z(-h) = 0, \quad v_z(\eta_{\text{old}}) = 0 \end{cases}$$

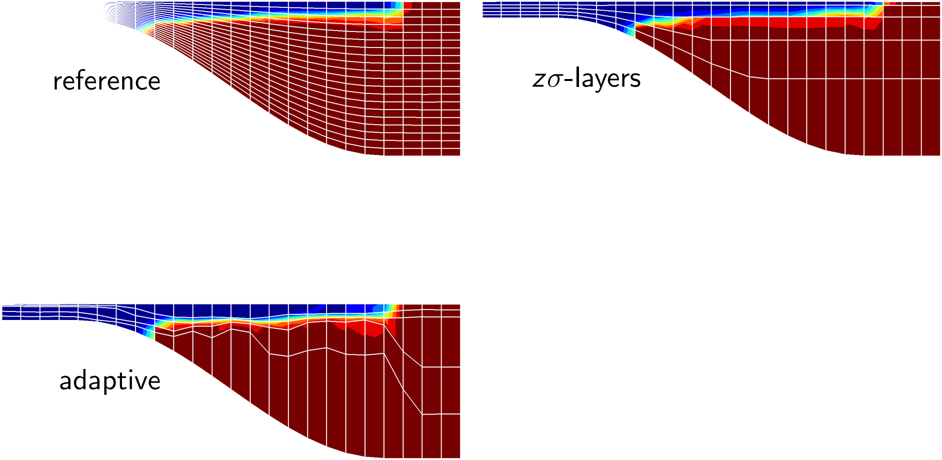
$$\kappa_z = \frac{1}{\tau}\big(c_{N^2}\kappa_{N^2} + c_{S^2}\kappa_{S^2} + c_d\kappa_d + c_b\kappa_b\big)$$



$$\kappa_{N^2} = \frac{[\rho]_v}{\rho_0}, \qquad \kappa_{S^2} = \frac{[u]_v}{u_0},$$

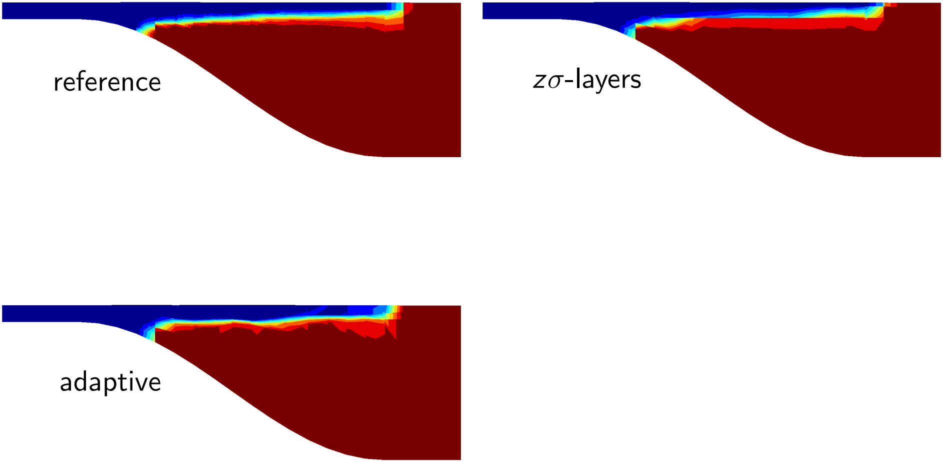
$$\kappa_d = \frac{h}{d + d_0}, \quad \kappa_b = 1.$$

Idealized river plume



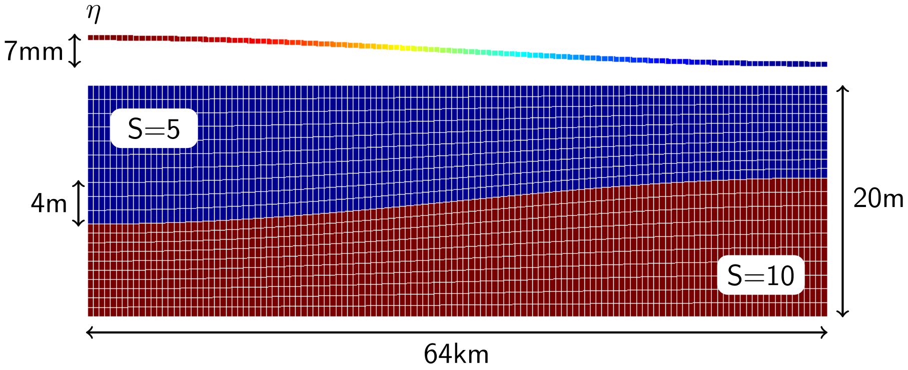
After 15 hours

Idealized river plume (mesh hidden)



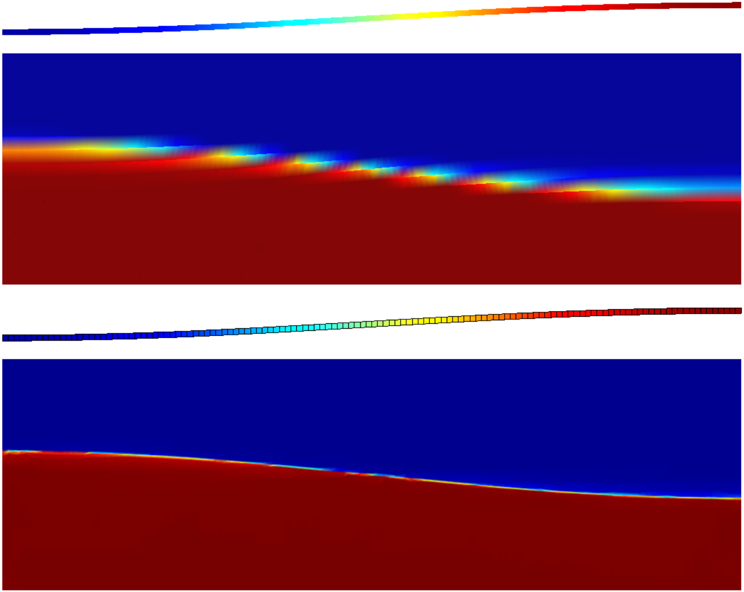
After 15 hours

Internal seiche



Oscillation period: T = 3.4 days

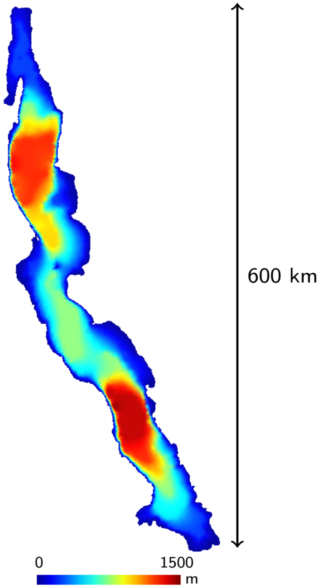
Internal seiche



After T/2

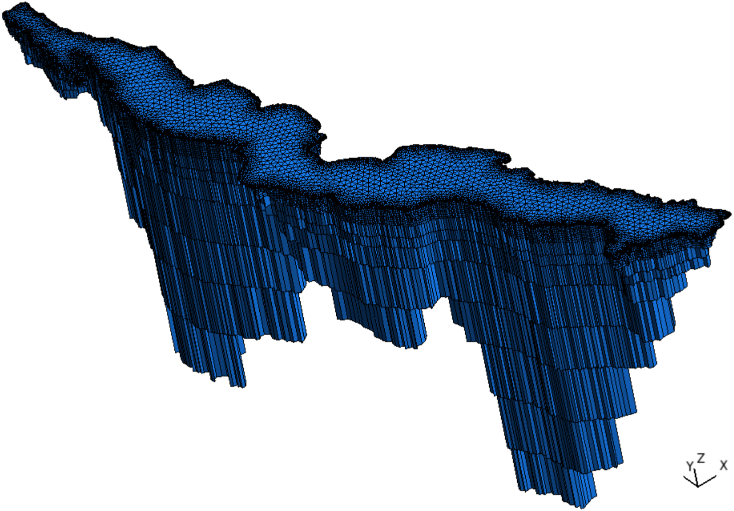
Lake Tanganyika

- Goal
  - Simulate the oscillations of the thermocline
  - Adaptive coordinates are necessary
- Difficulties
  - Deep lake
  - Strong bathymetry gradient



Lake Tanganyika

For this run, z-layers will be used.



Conclusions

- σ-layers
  - – Plume dynamics depends on bathymetry
- $z\sigma$-layers
  - + Simple and not noisy
  - + Good results on river plume test case (if enough z-layers)
  - – Constant grid
  - – Bad results on internal seiche test case
- Adaptive vertical layers
  - + Very good result for internal seiche
  - + Cheap implementation
  - – Difficult to get best diffusivity parameters
  - – Does not enhance the simulation for river plume
- Perspectives: test adaptive vertical coordinates on Lake Tanganyika!

Thank you for your attention !

References

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