Implementation of Tailings Rheology in a Predictive Open-Channel Beaching Model

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ABSTRACT

Tailings basins and dry-stacks constitute a micro-environment in comparison to natural beaches and natural river delta systems. High-tech three dimensional numerical models are available for simulating these natural systems. Extension of these models to slurries (rheology, channelization, etc.) offers the opportunity to answer questions regarding capacity (fines capture, segregation, and deposit composition), water management, and safety of tailings management.

This paper describes 1) the fundamental rheological models of slurries that are being included in an existing river and coastal sediment morphological model to capture beach and delta-formation (i.e. Delft3D numerical modeling suite, developed and maintained by Deltares), and 2) preliminary model results from one-dimensional test runs.

The water content with respect to the fines (carrier fluid) controls the rheology, which is augmented by adding coarse solids. Three different existing rheological formulations, from different fields (industrial concentrates, tailings and fluid mud flow in natural environments), are analyzed and compared, showing similarities and differences. One of these is issued here for the first time in its entirety. Results of test runs with the numerical model are summarized. Comparison with a velocity profile measured in discharged tailings shows good agreement.

INTRODUCTION

Sediments and tailings management represents a major factor that affects cost, risk and liability in mining, land reclamation and contaminated sediments clean-up operations. This is especially so when these activities move toward application of thicker and paste-like slurries comprising of higher fines (mud) and less sand. Some of the key questions related to tailings or slurry management refer to volume; beach slopes; deposition behavior; segregation of coarse and fine fractions; consolidation, strength development and stability of ultra-soft deposits.

Low-complexity predicting tools for beach slope and stack formation have been developed by for example Fitton and Slatter (2013) and Pirouz et al. (2013). These tools are largely based on experimental and field observation, and serves good to engineering purposes. Next to these low-complexity tools, Deltares develops and maintain the open source Delft3D modeling suite (https://www.deltares.nl/en/software/delft3d-4-suite/). This is a process based numerical modeling suite designed and utilized for prediction of flow, sediment transport and deposition (and water quality) in numerous research and engineering projects worldwide. Recently it was successfully implemented to analyze alluvial delta development (Figure 1) (van der Vegt et al. 2015) and slope (Sheets, et al. 2014). With deltas we refer to the larger depositional system at the mouth of a river, but also to smaller deltas developing from a pipe or specific discharge point in tailings basins or dredging disposals (Sheets). In all these cases the model was successfully used to predict delta growth, sediment sorting and average beach slope.

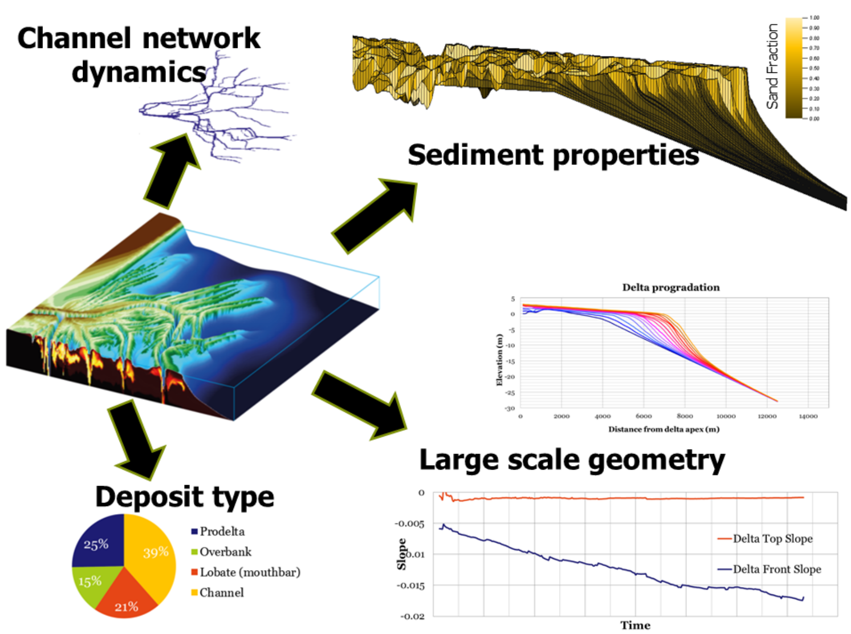

Figure 1 Morphodynamics and associated stratal response of a cross-section in the proximal delta (from van der Vegt et al. 2015)

As industrial sediment management applications move toward thicker paste like slurries and fines mixtures, rheology and non-Newtonian physics becomes relevant. With these needs in mind Deltares is developing a new module, Delft3D-Slurry, which is designed to predict the flow and deposition behavior of thick sand-mud slurries or tailings, and to predict beach slope and particle size distribution of the deposit (Sheets et.al, 2014).

A stepwise theoretical and numerical development approach is pursued to reach this objective, which includes: collection and uniformization of the available theory regarding the fundamental physical processes related to thick sand-mud slurry theories, with focus on non-Newtonian rheology and sand settling; embedding in Delft3D and thorough validation; 1DV (vertical dimension), followed by 2DV and finally 3D testing and application for theoretical and industrial cases.

In this paper, the results of the developments to date are reported, indicating possibilities and ranges for application and with focus on uniform (i.e. non-settling) thick sand-mud mixtures flowing in the laminar regime. An overview of the analytical description of the sand-mud mixture models considered is described, with indication of range of applicability and uncertainty. A summary of the embedding of the theory in Delft3D and validation in 1DV follows.

RHEOLOGICAL MODELS FOR SAND-MUD MIXTURES

The theoretical framework to describe sand-fines slurry flow and segregation behavior includes a dual rheology approach (Spelay 2007, Talmon et al. 2014). The rheology of the sand-mud mixture is quantified for flow momentum simulations. The rheology (inherent viscosity, Thomas 2010) of the carrier fluid (fines+water only) determines sand segregation (e.g. settling of coarse particles within the carrier fluid), which includes shear induced settling.

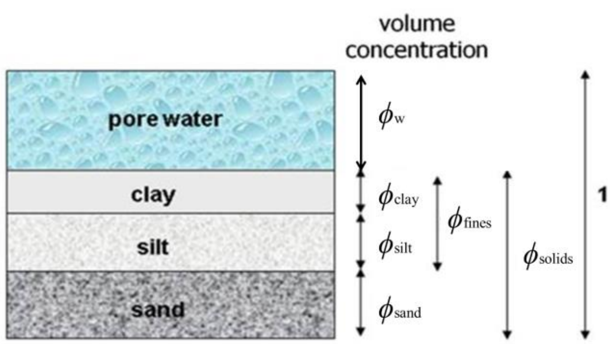

Figure 2 Definition of volume concentrations (adapted from Winterwerp and van Kesteren 2004)

This paper focuses on the derivation, implementation into Delft3D and testing of the rheology of sand-mud mixtures assuming near-zero sand settling velocity. Extension to sand settling is the subject of current developments and will be discussed in follow-up publications.

Three different existing rheological formulations which are traditionally utilized in different fields, i.e., industrial concentrates, tailings and fluid mud flow in natural environments, are compared and included in Delft3D. The implemented formulation utilizes the Bingham model concept (Eq 1). But in case a functional dependency of the plastic viscosity μ on shear rate $\dot{\gamma}$ is formulated, it becomes effectively a Hershel-Bulkley model (non-linear in shear rate). The fundamental rheological relation between shear stress τ, yield stress τ_y, plastic viscosity and shear rate is:

$$\tau = \tau_y + \mu\dot{\gamma} \tag{1}$$

Rheological model – Winterwerp and Kranenburg (Model 1: M1)

This rheological model is based on fractal-dimension theory developed by Kranenburg (1994). This approach is often used in siltation and fluid mud studies; effectively this is a Hershel-Bulkley type of model, Eq (2) and Eq (3). The various symbols are defined at the end of this paper.

$$\tau_y = A_y \left(\frac{\phi_{clay}}{\phi_{water} + \phi_{clay}} \right)^{2/(3-n_f)} \exp(\beta\lambda) \tag{2}$$

$$\mu = \left[\mu_w + A_\mu \left(\frac{\phi_{clay}}{\phi_{water} + \phi_{clay}} \right)^{\frac{2(a+1)}{3}} \left[\frac{1}{\dot{\gamma}} \right]^{\frac{(a+1)(3-n_f)}{3}} \right] \exp(\beta\lambda) \tag{3}$$

Kranenburg derived a shear-thinning model, assuming self-similarity of the cohesive structure (the mud flocs which constitute the carrier fluid). Self-similarity implies that the larger particle aggregates have similar structure as the smaller aggregates. This assumption generally yields a power law behavior. The structure of such self-similar mixtures can be expressed by the fractal dimension n_f. Depending on the clay composition the value of n_f (Eqns 3 and 4) varies between 2.5 and 2.8. Changes in the fractal dimension also influence the yield strength and shear rate in the formula (Eqs 3 and 4). An increase in clay content raises the rheological properties. The increase of internal friction induced by sand particles is captured by an exponential term containing the linear sand concentration λ (Eqs 2 and 3), Bagnold (1956).

Rheological model – Jacobs and van Kesteren (Model 2: M2)

The second model is based on Jacobs et al. (2008) for yield stress, and a similar approach for associated Bingham plastic viscosity, van Kesteren (personal communication). This Bingham model is employed in non-segregating and thickened tailings characterization (in mining and oil sands):

$$\tau_y = K_y \left(\frac{W}{PI} \right)^{B_y} \exp(\alpha\lambda) \tag{4}$$

$$\mu = \left[\mu_w + K_\mu \left(\frac{W}{PI} \right)^{B_\mu} \right] \exp(\alpha\lambda) \tag{5}$$

This model is based on the ratio between the water content (*W*) and the plasticity index (*PI*) of the carrier fluid, which determines the rheology. An increase in water content decreases the rheological properties. Like in Model 1, the increase in friction due to sand is represented by an exponential function with the linear sand concentration λ (Eqs 4 and 5).

The mass based *W* and *PI* can be rewritten in terms of clay volumetric concentration, and clay activity (A_{clay}) characterizing clay type, assuming equal solids densities, Jacobs et al. 2008:

$$\frac{W}{PI} \approx \frac{\rho_w}{\rho_{solids}} \frac{\phi_{water}}{\phi_{solids}} \frac{1}{A_{clay}\xi_{clay}} = \frac{\rho_w}{\rho_{solids}} \frac{\phi_{water}}{\phi_{solids}} \frac{\phi_{solids}}{A_{clay}\phi_{clay}} = \frac{\rho_w}{A_{clay}\rho_{solids}} \frac{\phi_{water}}{\phi_{clay}} \quad (6)$$

Rheological model – Thomas (Model 3: M3)

The third model is a Bingham model presented by Thomas (1999), developed for sand-slime mixtures in mining operations (reformulated in our generalized format):

$$\tau_y = C_y \left(\frac{\phi_{fines}}{\phi_{water} + \phi_{fines}} \right)^p \left[1 - \frac{\phi_{sand}}{k_{yield}\phi_{sand\max}} \right]^{-2.5} \quad (7)$$

$$\mu = \exp\left(D \frac{\phi_{fines}}{\phi_{water}} \right) \left[1 - \frac{\phi_{sand}}{k_{visc}\phi_{sand\max}} \right]^{-2.5} \quad (8)$$

Thomas describes the influence of the fines in the carrier fluid (only clay) by a power function for the yield strength (Eq 7) and an exponential function for the viscosity (Eq 8). The formula for the yield strength (Eq 7) consists of two parts: the first describes the yield strength of the carrier fluid; the second the effect of sand particles. The addition of sand is accounted for with respect to the maximum sand concentration and a parameter *k* which may differ a bit between yield stress and viscosity formula.

For all models, the rheological parameters are established with dedicated rheological experiments. The presence of silt particles is neglected, but in all three models it is possible to include a silt fraction within the carrier fluid or model its presence as a separate solids fraction.

Effect of sand

The effect of sand particles on the behavior of a mixture is twofold: on one side it increases internal friction; on the other side it introduces non-cohesive particles. The resulting effect is that at equal volume concentration or density, yield stress and viscosity decreases with increasing sand content. Yet, adding sand to a specific sample increases its density, thus generally its yield stress and viscosity.

Models 1 and 2 make use of an exponential expression including the linear concentration λ (Eq 9). For the value of β an empirical value of 0.27 in combination with a maximum sand concentration of 0.6 has been utilized before. But in case of silt+sand conditions the maximum concentration is to be calculated by a procedure provided in Winterwerp and van Kesteren (2004).

$$\lambda = \frac{1}{\left(\phi_{sasi,\max}/\phi_{sasi}\right)^{1/3} - 1} \tag{9}$$

The parameter group $k_{visc}\phi_{sand\max}$ ranges between 0.6-0.9 in experiments. Thomas also found that the formulation is applicable for the effect of sand on the yield stress as well.

The formulations describing the influence of sand on the rheology are compared in Figure 3. The Figure shows that for low volumetric sand concentrations (smaller than 40%) all three models are close.

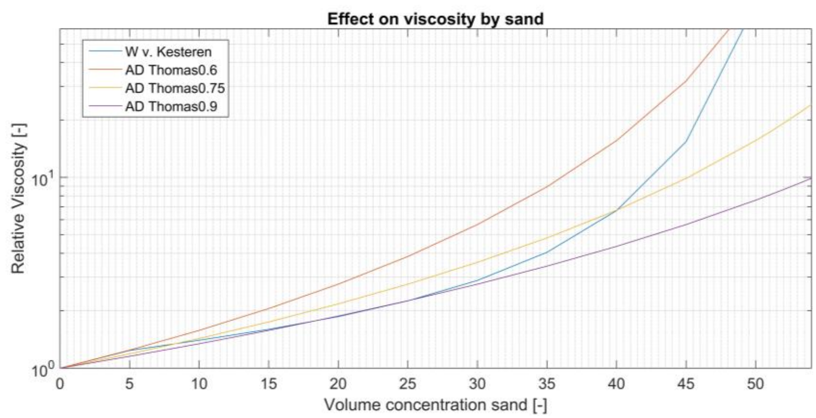

Figure 3 Influence of internal friction sand on viscosity

Comparison of Rheological Models for Sand-Mud Mixtures

To validate the performance and compare the three theoretical rheological models the experimental data of Thomas (1999) is applied. The data concerns Bingham yield stress and plastic viscosity of sand-mud-water mixtures of samples of increasingly high Sand To Solids ratio (STS: is equal to $\phi_{sand}/\phi_{solids}$, see Figure 2). The results of a calibration study are shown in Figure 4. The calculated yield stress and viscosity are plotted with the measured data. In general the performance of the models is comparable. All three models replicate the yield stress and viscosity more accurate at low total volume concentrations. Model 3 has the best fitting overall (note that this model was developed on this data). The applied values for the model parameters are: Model 1: n_f=2.64, a=3.65, A_y=7.3E5, A_μ=9.3, β=0.27, $\phi_{sasi,max}$=0.6, Model 2: ρ_{water}=1000 kg/m^3, ρ_{solids}=2750 kg/m^3, A_{clay} =1.0, K_y=548, B_y=-4.75, K_μ=0.17, B_μ=-2.64, α=0.27, $\phi_{sasi,max}$=0.6, Model 3: C_y=4.75E5, p=5.61, D=17.7, k_{yield} $\phi_{sasi,max}$=0.9, k_{visc} $\phi_{sasi,max}$=0.75. Common in models M1 and M2 are: $\phi_{sasi,max}$=0.6, μ_w=0.001 [Pa s].

The reason for the large error at STS 78.9% is that the mixture approaches the granular regime. Another reason for the differences between measured values and the predicted values is the error that might occur during the measurements. At high concentrations – and especially high sand concentrations – it is more difficult to measure the yield stress and viscosity.

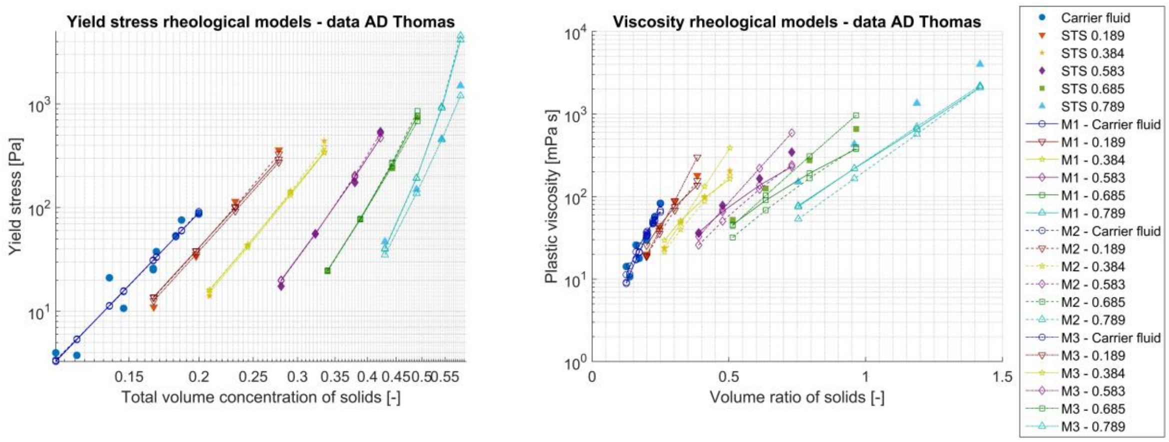

Figure 4 Yield stress and plastic viscosity of rheological models (M1, M2, M3) fitted to measurements (closed symbols). Volume ratio of solids = $\phi_{solids}/\phi_{water}$. M1: plastic viscosity at $\dot{\gamma}$ = 1 [1/s]

Implementation in Delft3D

The three rheological models described in the section above were implemented in a new module dedicated to thick sand-mud mixtures, Delft3D-Slurry. A simplified 1DV version is utilized to verify matching with analytical solutions, and test application to tailings mixtures with different rheology.

Since solid state mechanics is not described by a computational flow model, the yield stress terms are modified with an exponential function (eqn 10) in order to create a finite viscosity at low shear rates. The constant *m* is a user input. Numerical solutions approach the analytical solution at high values of *m*.

$$\tau = \tau_y \left(1 - \exp\{-m\dot{\gamma}\}\right) + \mu\dot{\gamma} \tag{10}$$

RESULTS

A suite of test runs for velocity profile, viscosity profile, shear stress profile, boundary conditions and correspondence with analytical solutions was conducted. Figure 5 is an example of such a test run, showing a situation with a thin sheared wall layer, and differences in outcomes between the three rheological models, having different character (Bingham v/s Hershel Bulkley). Also the *m*-value has some influence and needs to be well chosen. At the same time it also appeared that when sand settling routines were activated (not described here), a challenging arena was entered, not uncommon in morphodynamics.

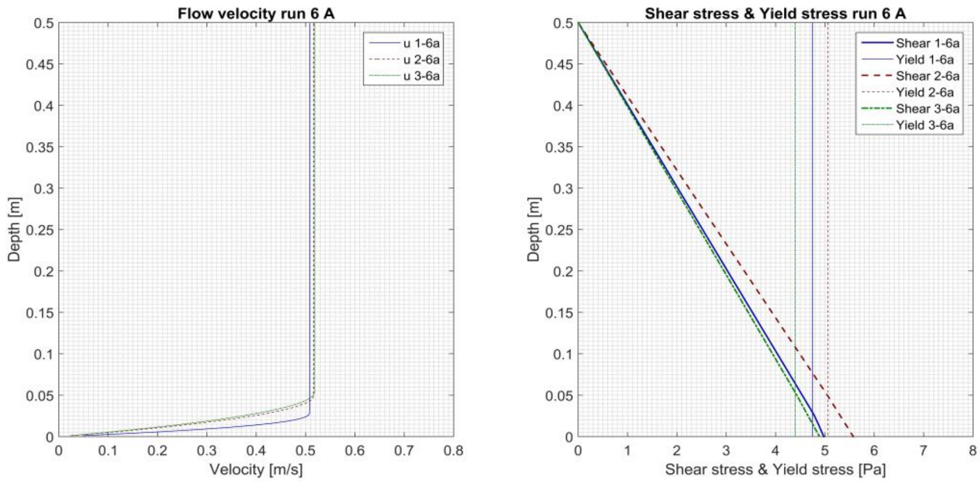

Figure 5 Example of a test run with the three models showing correspondences and differences due to rheological characterization, inputs: flow depth, mean velocity and rheology

At finishing of testing for homogenous mixture conditions, a comparison was made with one of the few tailings velocity profiles available in the open literature. A velocity profile measured with a Delft E30 EMS probe in a field flume running prototype tailings, Pirouz et al. (2013) is simulated. Figure 6 shows the measured and calculated velocity profile. Good agreement is found. Hershel-Bulkley mixture rheology as listed in Pirouz et al. is input. Flow depth and mean velocity are taken as evident from the reported measurement.

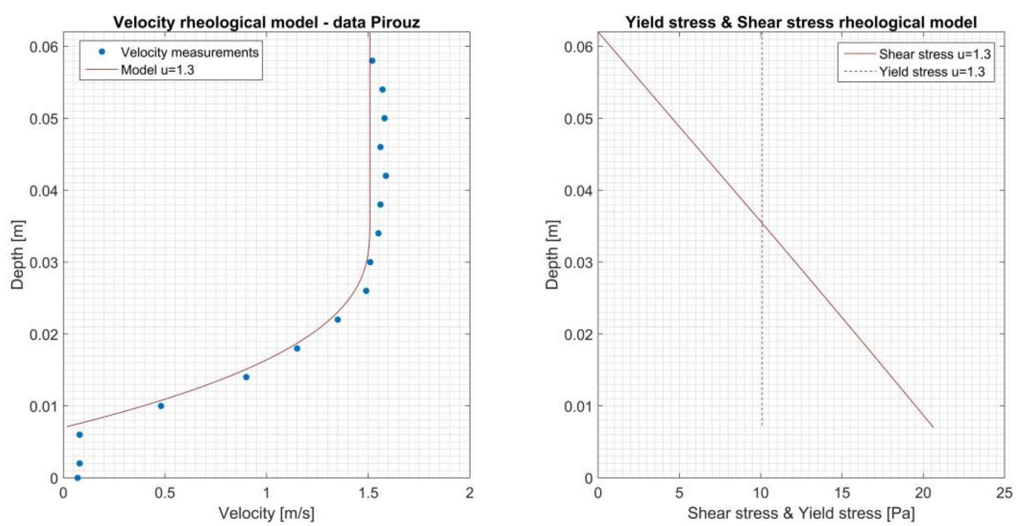

Figure 6 Calculated velocity and shear stress profile for half pipe open channel test of Pirouz et al. (2013)

The corresponding model parameters (Model 1) are: ϕ_{clay}=0.16, ϕ_{sand}=0.24, n_f =2.5, A_y=2403, β=2.7, μ_w=0.001 Pa s, A_μ=3.975, *a*=1.56 and *m*=5000. The corresponding measured solids concentration profile did not show any non-uniformity in the flowing slurry. The measured stagnant part, with higher solids concentration, is to be addressed in upcoming analyses when settling processes are activated in the model.

CONCLUSION

The larger objective of this on-going research program is to improve our physical understanding and predictive capabilities of the transport and fate of sand-mud-water slurry of various composition and rheology. Delft3D has demonstrated to be a robust numerical tool to predict delta development and sediment composition. The specific objective of this study is extending model capabilities to solve flow and sediment distribution, hence delta development, of thick non-Newtonian sand settling mixtures.

Three different rheological models for clay-sand-water mixtures have been compared with experimental data. Testing in 1DV-mode in laminar open channel non-segregating flow conditions was carried out for different types of slurries and rheology, and compared with analytical solutions. All three rheological models performed similarly. A comparison with a measured velocity profile in discharged tailings shows good agreement. Based on these encouraging results, similar testing on sand settling mixtures is on-going.

While this model can provide indication for flow (and soon) sand settling characteristics of different types of tailings, embedding in standard Delft3D will allow expanding applications to 2DV cross section profiles and 3D entire delta beach deposits.

ACKNOWLEDGEMENTS

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NOMENCLATURE

a	anisometric parameter M1	α	constant linear concentration M2
A_{clay}	activity of clay M2	β	constant linear concentration M1
A_y	yield stress constant M1	ϕ_{clay}	volume concentration, see Figure 2
A_μ	viscosity constant M1	ϕ_{fines}	volume concentration, see Figure 2
B_y	power function yield M2	ϕ_{sand}	volume concentration, see Figure 2
B_μ	power function viscosity M2	ϕ_{sasi}	volume concentration sand+silt
C_y	yield stress constant M3	$\phi_{sasi,\ max}$	maximum concentration sand+silt
D	viscosity constant M3	ϕ_{silt}	volume concentration, see Figure 2

K_y	yield stress constant M2	ϕ_{solids}	volume concentration, see Figure 2
K_μ	viscosity constant M2	ϕ_{water}	volume concentration, see Figure 2
k_{yield}	yield constant w.r.t. max. conc. M3	$\dot{\gamma}$	shear rate
k_{visc}	visc. constant w.r.t max. conc. M3	λ	linear solids concentration
m	constant modifying yield stress	μ	(plastic) viscosity
n_f	fractal dimension M1	μ_w	dynamic viscosity water
PI	plasticity index M2	ρ_w	density water
p	power function yield M3	ρ_{solids}	density solids
W	water content	τ	shear stress
	(mass water/mass solids)	τ_y	yield stress
		ξ_{clay}	mass clay/total mass solids

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