



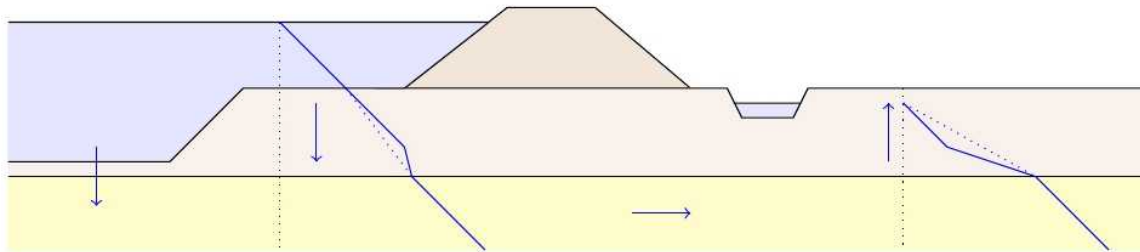
# Rekenhart D-Geo Flow

John van Esch

# Inhoud



- Grondwaterstromingsmodel
- Randvoorwaarden
- Materiaalparameters
- Piping module



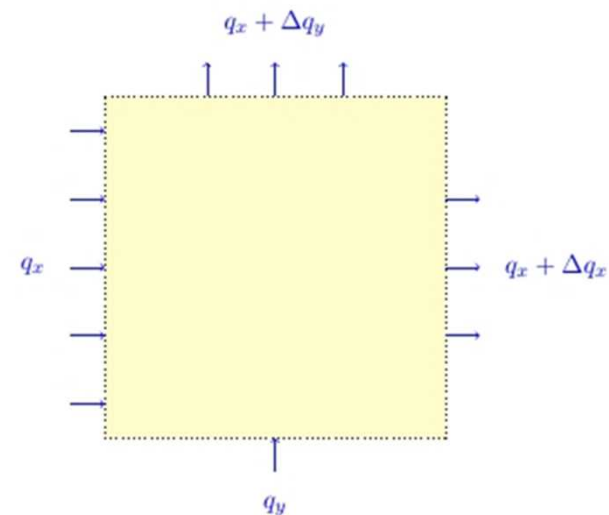
# Grondwaterstromingsmodel



Behoud van massa (volume)

$$\frac{dq_x}{dx} + \frac{dq_y}{dy} = 0$$

- plaats  $x, y$  (m)
- specifiek debiet  $q_x, q_y$  (m/s)



# Grondwaterstromingsmodel



Wet van Darcy, specifiek debiet  $q$  (m/s)

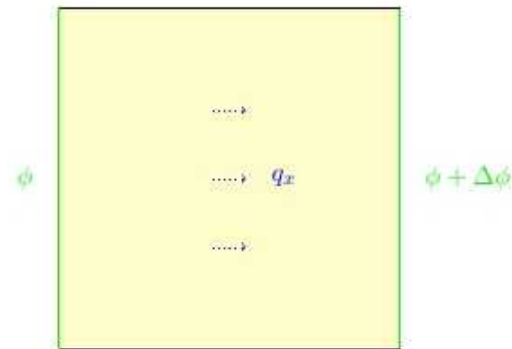
$$q_x = -K_{xx} \frac{d\phi}{dx}$$

$$q_y = -K_{yy} \frac{d\phi}{dy}$$

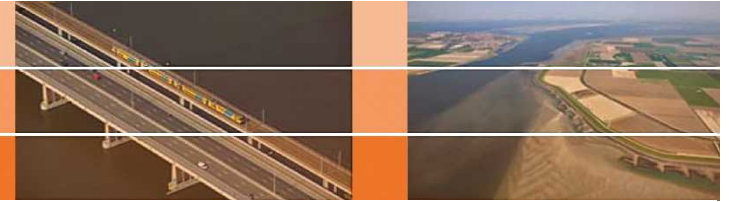
- doorlatendheid  $K$  (m/s)
- stijghoogte  $\phi$  (m)

$$\phi = \frac{p}{\rho g} + y$$

- druk  $p$  (N/m<sup>2</sup>)
- dichtheid  $\rho$  (kg/m<sup>3</sup>)
- gravitatie versnelling  $g$  (m/s<sup>2</sup>)
- plaatshoogte  $y$  (m)



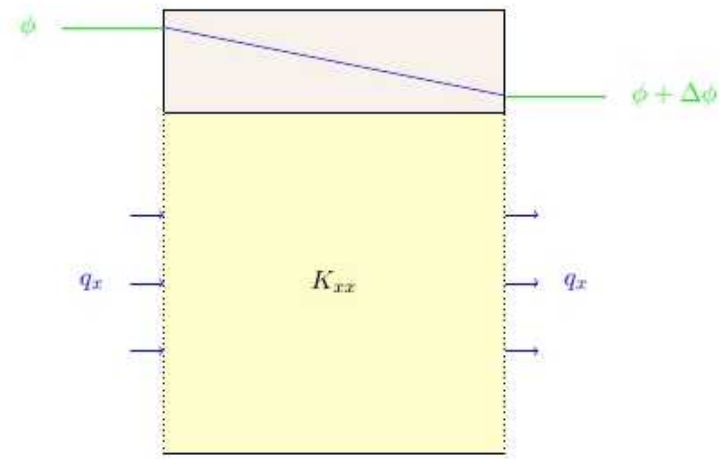
# Grondwaterstromingsmodel



Bergingsvergelijking stationair

$$\frac{d}{dx} \left( K_{xx} \frac{d\phi}{dx} \right) + \frac{d}{dy} \left( K_{yy} \frac{d\phi}{dy} \right) = 0$$

- doorlatendheid  $K$  (m/s)
- stijghoogte  $\phi$  (m)
- plaats  $x, y$  (m)



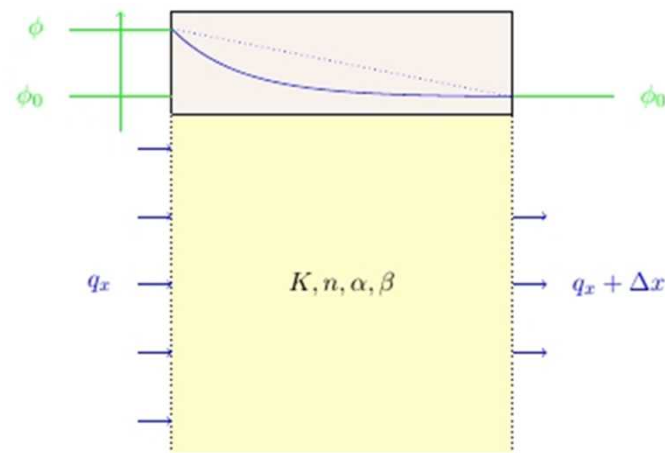
# Grondwaterstromingsmodel



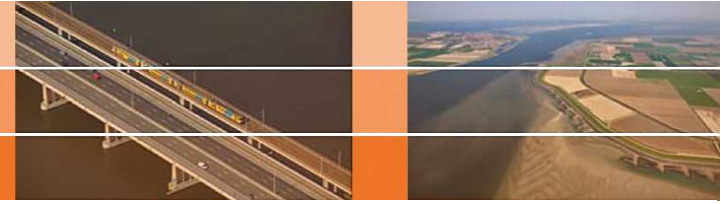
Bergingsvergelijking tijdsafhankelijk

$$\frac{\partial}{\partial x} \left( K_{xx} \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{yy} \frac{\partial \phi}{\partial y} \right) = \rho g (\alpha + n\beta) \frac{\partial \phi}{\partial t}$$

- doorlatendheid  $K$  (m/s)
- stijghoogte  $\phi$  (m)
- plaats  $x, y$  (m)
- dichtheid  $\rho$  (kg/m<sup>3</sup>)
- gravitatie versnelling  $g$  (m/s<sup>2</sup>)
- porositeit  $n$  (-)
- samendrukbaarheid skelet  $\alpha$  (m<sup>2</sup>/N)
- samendrukbaarheid water  $\beta$  (m<sup>2</sup>/N)



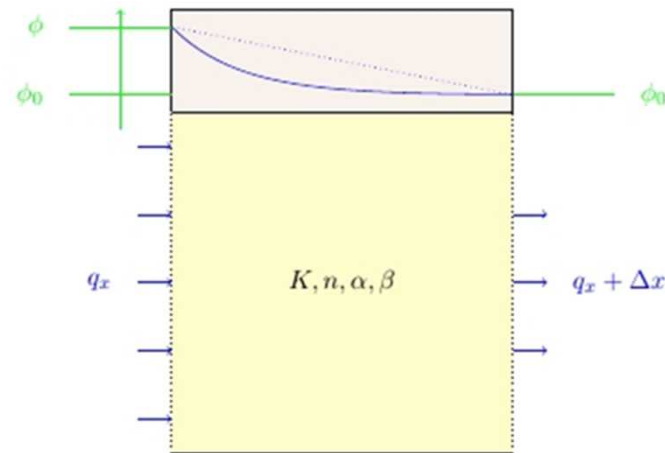
# Grondwaterstromingsmodel



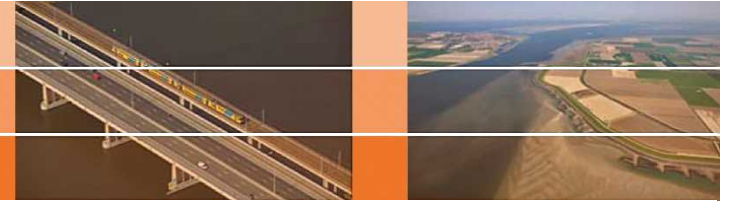
## Bergingsvergelijking

$$\frac{\partial}{\partial x} \left( \frac{\kappa_{xx}}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\kappa_{yy}}{\mu} \frac{\partial p}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{\kappa_{yy}}{\mu} \rho g \right) = (\alpha + n\beta) \frac{\partial p}{\partial t}$$

- plaats  $x, y$  (m)
- intrinsieke doorlatendheid  $\kappa$  (m<sup>2</sup>)
- dynamische viscositeit  $\mu$  (Pas)
- druk  $p$  (N/m<sup>2</sup>)
- dichtheid  $\rho$  (kg/m<sup>3</sup>)
- gravitatie versnelling  $g$  (m/s<sup>2</sup>)
- porositeit  $n$  (-)
- samendrukbaarheid skelet  $\alpha$  (m<sup>2</sup>/N)
- samendrukbaarheid water  $\beta$  (m<sup>2</sup>/N)



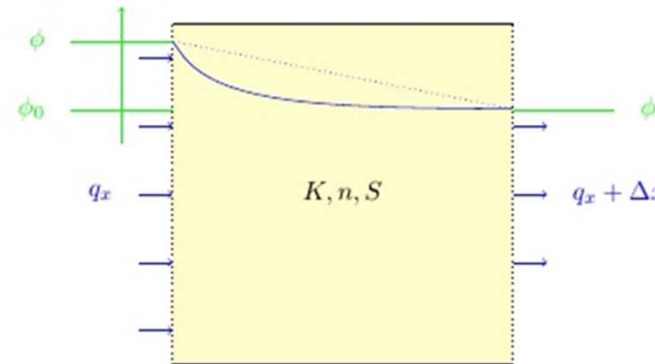
# Grondwaterstromingsmodel



## Bergingsvergelijking

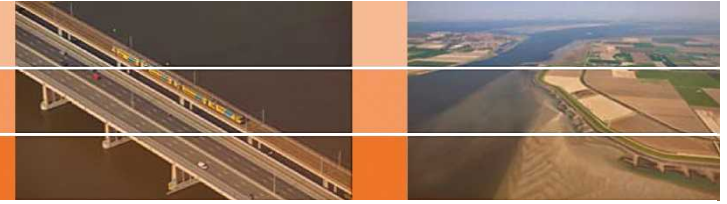
$$\frac{\partial}{\partial x} \left( \frac{k_r \kappa_{xx}}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{k_r \kappa_{yy}}{\mu} \frac{\partial p}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{k_r \kappa_{yy}}{\mu} \rho g \right) = (\alpha + n\beta) \frac{\partial p}{\partial t} + n \frac{dS}{dp} \frac{\partial p}{\partial t}$$

- relatieve doorlatendheid  $k_r$  (-)
- verzadigingsgraad  $S$  (-)
- plaats  $x, y$  (m)
- intrinsieke doorlatendheid  $\kappa$  ( $\text{m}^2$ )
- dynamische viscositeit  $\mu$  (Pas)
- druk  $p$  ( $\text{N}/\text{m}^2$ )
- dichtheid  $\rho$  ( $\text{kg}/\text{m}^3$ )
- gravitatie versnelling  $g$  ( $\text{m}/\text{s}^2$ )
- porositeit  $n$  (-)
- samendrukbaarheid skelet  $\alpha$  ( $\text{m}^2/\text{N}$ )
- samendrukbaarheid water  $\beta$  ( $\text{m}^2/\text{N}$ )





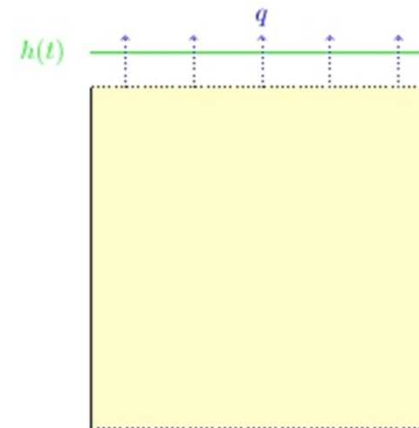
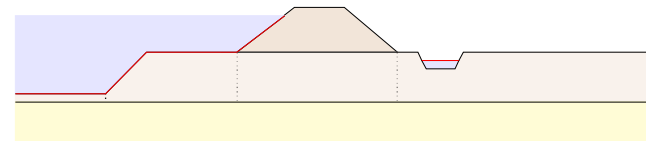
# Randvoorwaarden



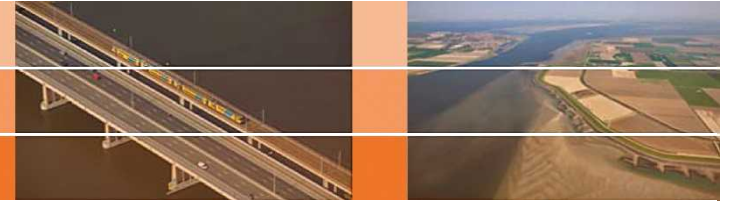
head boundary

$$\phi_i = h(t)$$

- stijghoogte  $\phi_i$  (m)
- buitenwaterstand  $h(t)$  (m)
- tijd  $t$  (s)



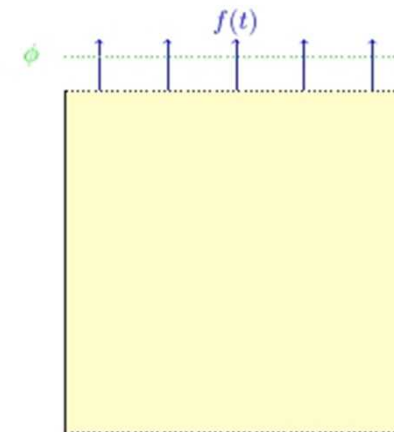
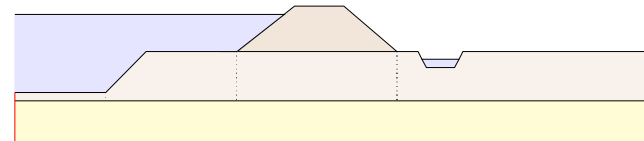
# Randvoorwaarden



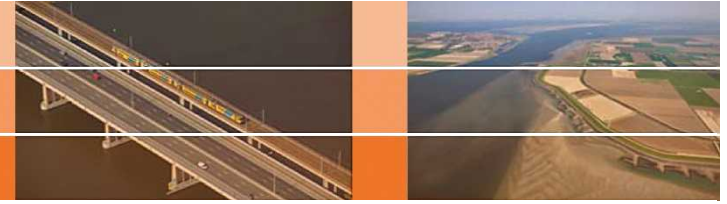
## Flux boundary

$$Q_i = f(t)$$

- debiet  $Q$  ( $\text{m}^2/\text{s}$ )
- rand lengte  $\Delta x$  (m)
- uit-flux  $f(t)$  (m/s)
- tijd  $t$  (s)



# Randvoorwaarden

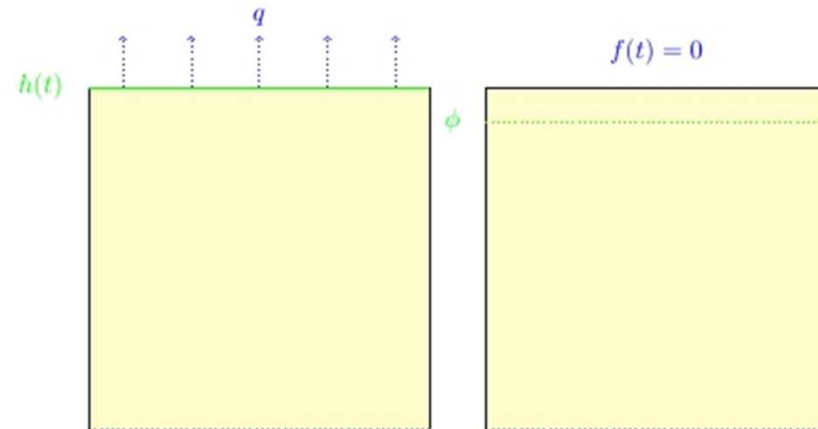
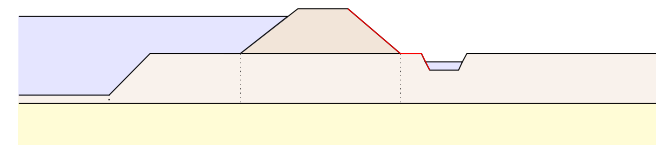


## Seepage boundary

$$Q_i > 0 \Rightarrow p_i = 0$$

$$p_i < 0 \Rightarrow Q_i = 0$$

- debiet  $Q$  ( $\text{m}^2/\text{s}$ )
- druk  $p$  ( $\text{N}/\text{m}^2$ )



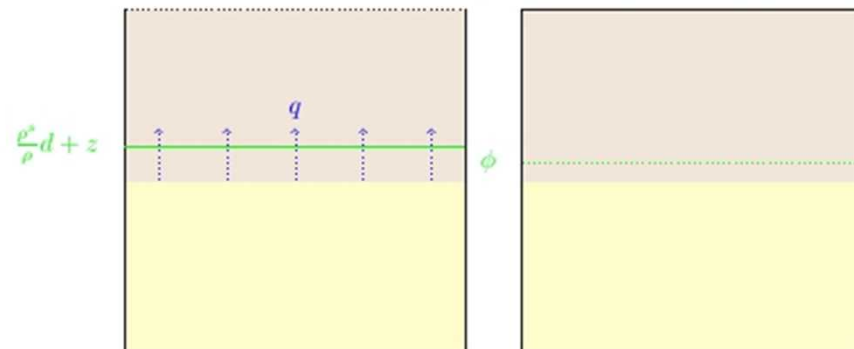
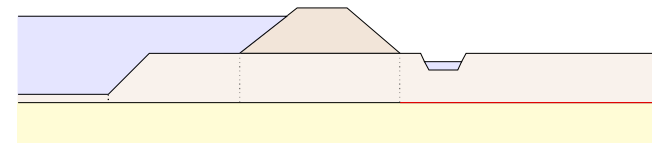
# Randvoorwaarden



## Heave boundary

$$Q_i > 0 \Rightarrow p_i = \rho g d$$
$$p_i < \rho g h \Rightarrow Q_i = 0$$

- debiet  $Q$  ( $\text{m}^2/\text{s}$ )
- druk  $p$  ( $\text{N}/\text{m}^2$ )
- dichtheid grond  $\rho$  ( $\text{kg}/\text{m}^3$ )
- gravitatieversnelling  $g$  ( $\text{m}/\text{s}^2$ )
- laagdikte  $d$  (m)

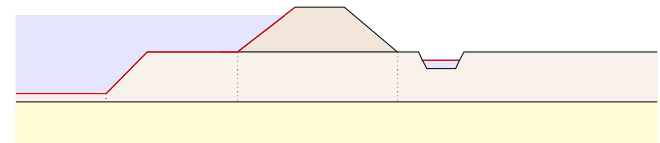


# Randvoorwaarden

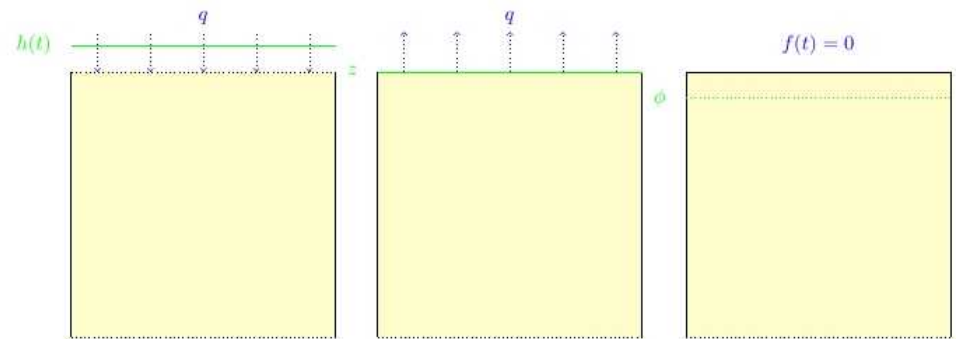


## Submerging boundary

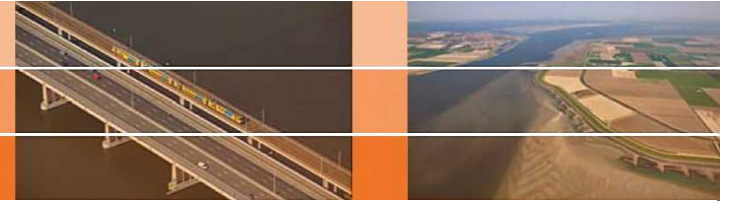
$$\begin{aligned}
 h(t) \geq z_i &\Rightarrow \phi_i = h(t) \\
 h(t) < z_i \cap F(t) > 0 \cap Q_i > F(t) &\Rightarrow p_i = g_1(t) \\
 h(t) < z_i \cap g_1 < p_i < g_2 &\Rightarrow Q_i = F(t) \\
 h(t) < z_i \cap F(t) < 0 \cap Q_i > F(t) &\Rightarrow p_i = g_2(t)
 \end{aligned}$$



- debiet  $Q$  ( $\text{m}^2/\text{s}$ )
- druk  $p$  ( $\text{N}/\text{m}^2$ )
- stijghoogte  $\phi$  (m)
- buitenwaterstand  $h(t)$  (m)
- verdampingsflux  $F(t)$  ( $\text{m}^2/\text{s}$ )
- ponding pressure  $g_1$  ( $\text{N}/\text{m}^2$ )
- osmotic pressure  $g_2$  ( $\text{N}/\text{m}^2$ )
- tijd  $t$  (s)



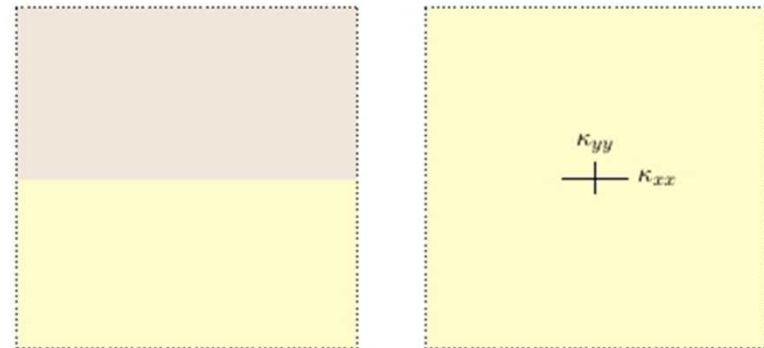
# Materiaalparameters



Intrinsieke doorlatendheid  $\kappa$  (m<sup>2</sup>)

$$\kappa_{xx} = \frac{K_{xx}\mu}{\rho g}, \quad \kappa_{yy} = \frac{K_{yy}\mu}{\rho g}$$

- doorlatendheid  $K$  (m/s)
- dichtheid  $\rho$  (kg/m<sup>3</sup>)
- gravitatie versnelling  $g$  (m/s<sup>2</sup>)
- dynamische viscositeit  $\mu$  (Pas)



# Materiaalparameters

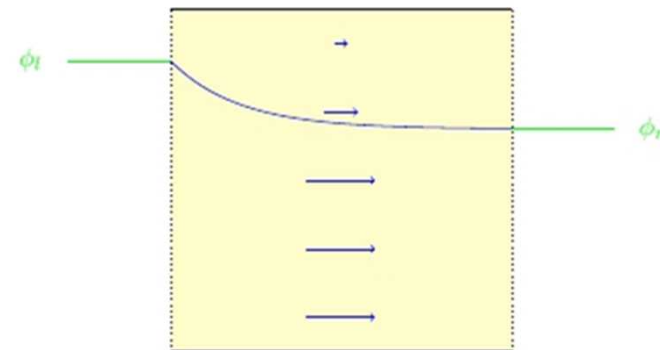
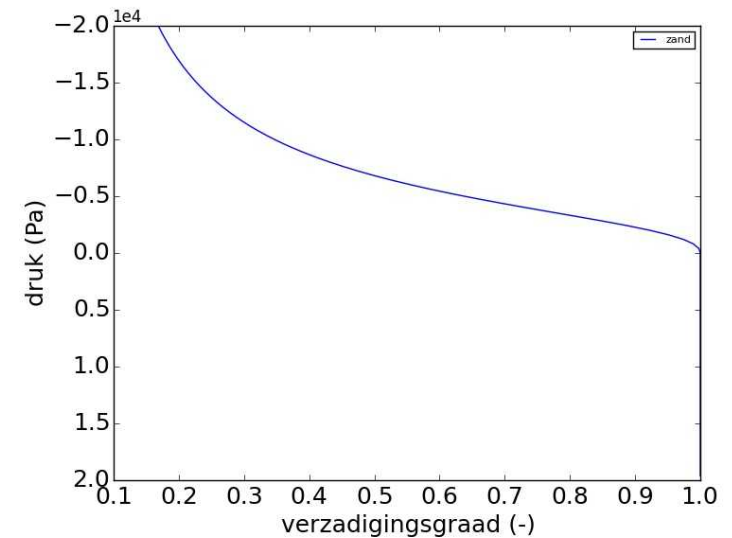


Van Genuchten verzadigingsgraad  $S$  (-)

$$\psi < \psi_a \Rightarrow S = S_r + (S_s - S_r) \left[ 1 + |g_a \psi|^{g_n} \right]^{-g_m}$$

$$\psi \geq \psi_a \Rightarrow S = S_s$$

- drukhoogte  $\psi$  (m)
- air entry hoogte  $\psi_a$  (m)
- rest verzadiging  $S_r$  (-)
- maximale verzadiging  $S_s$  (-)
- materiaal parameters  $g_a, g_n, g_m$  (-)



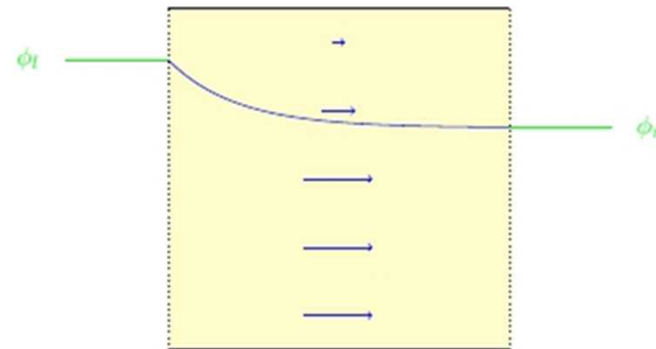
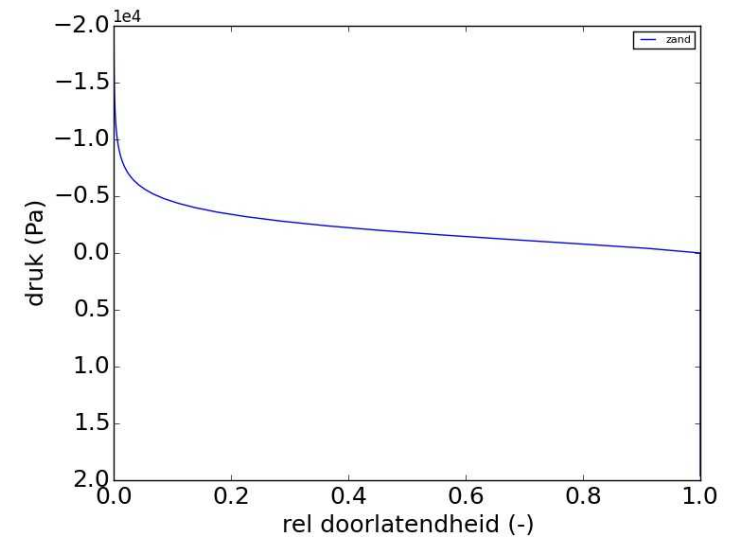
# Materiaalparameters



Van Genuchten relatieve doorlatendheid  $k_r$  (-)

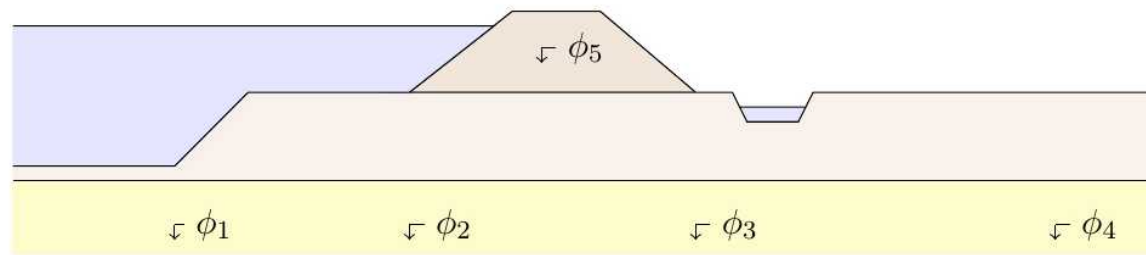
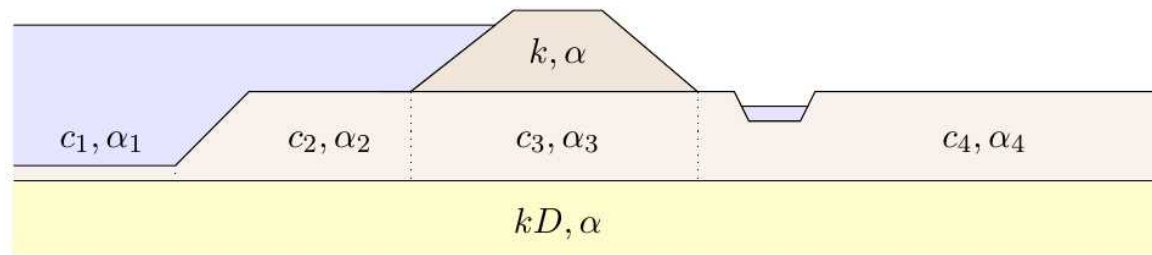
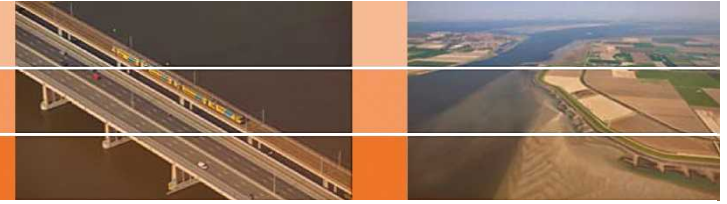
$$k_r = (S_e)^{g_l} \left[ 1 - \left( 1 - S_e^{1/g_m} \right)^{g_m} \right]^2 \quad S_e = \frac{S - S_r}{S_s - S_r}$$

- verzadigingsgraad  $S$  (-)
- effective verzadigingsgraad  $S_e$  (-)
- rest verzadiging  $S_r$  (-)
- maximale verzadiging  $S_s$  (-)
- materiaal parameters  $g_l, g_m$  (-)

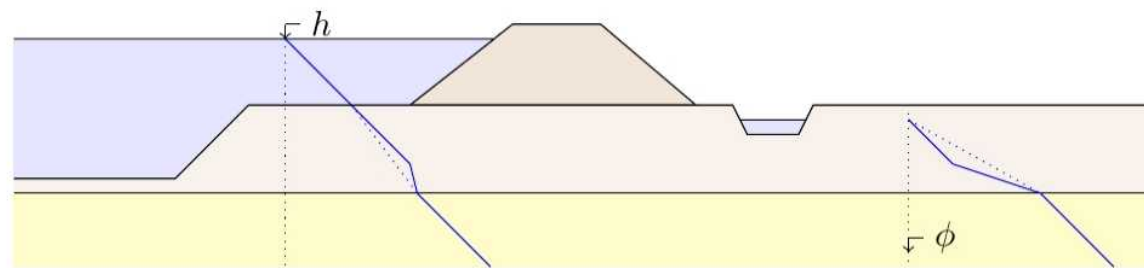
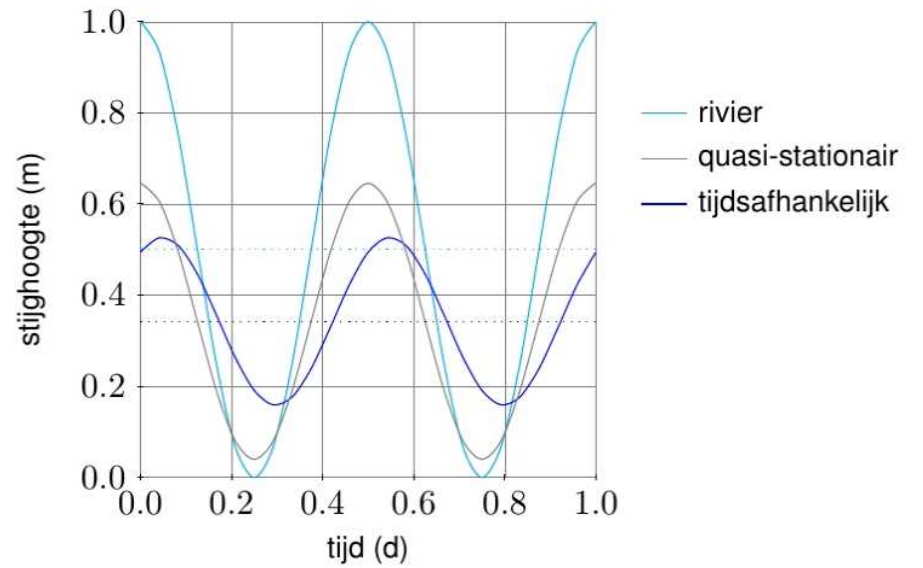




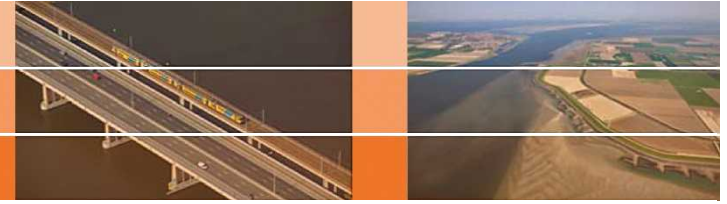
# Materiaalparameters



# Materiaalparameters



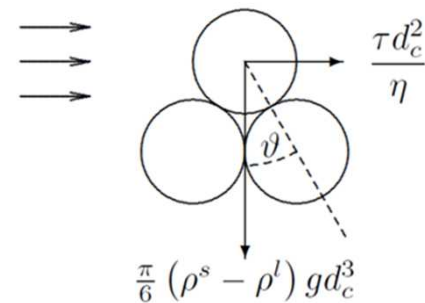
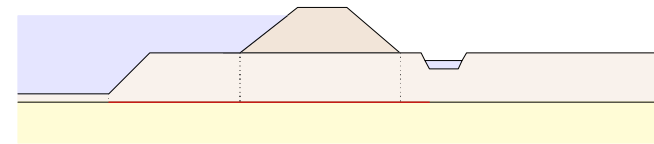
# Piping module



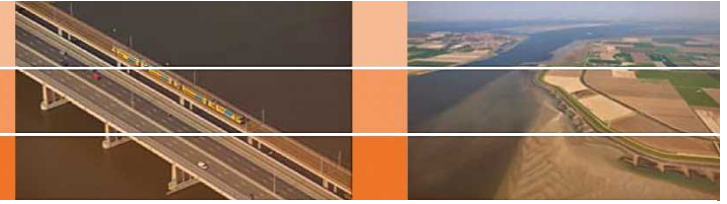
Kritische schuifspanning  $\tau_c$  (N/m<sup>2</sup>)

$$\tau_c = \frac{\pi}{6} (\rho^s - \rho) g \eta D_{70} \tan \vartheta$$

- dichtheid korrel  $\rho^s$  (kg/m<sup>3</sup>)
- dichtheid water  $\rho$  (kg/m<sup>3</sup>)
- gravitatie versnelling  $g$  (m/s<sup>2</sup>)
- constante van White  $\eta$  (-)
- korreldiameter  $D_{70}$  (m)
- bedding hoek  $\vartheta$  (deg)



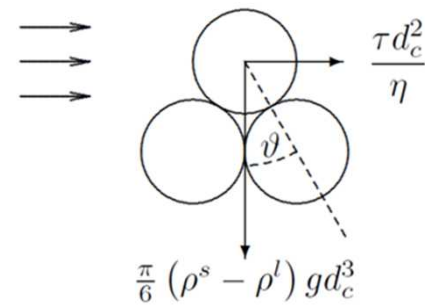
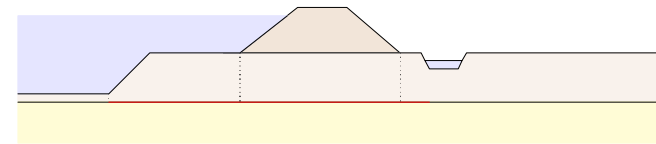
# Piping module



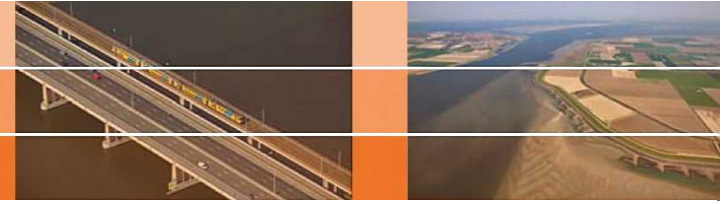
Uitgeoefende schuifspanning  $\tau$  (N/m<sup>2</sup>)

$$\tau = \frac{a}{2} \frac{dp}{dx}$$

- hoogte pipe  $a$  (m)
- druk  $p$  (N/m<sup>2</sup>)
- plaats  $x$  (m)



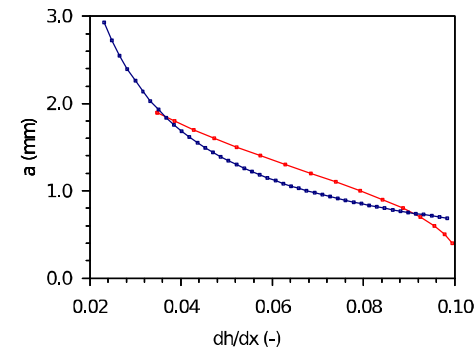
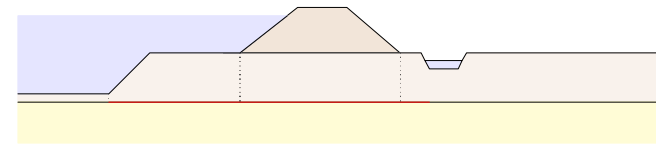
# Piping module



## Evenwichtsvergelijking

$$a \frac{dp}{dx} = \frac{\pi}{3} (\rho^s - \rho^l) g D_{70} \eta \tan \vartheta$$

- dichtheid korrel  $\rho^s$  ( $\text{kg/m}^3$ )
- dichtheid water  $\rho$  ( $\text{kg/m}^3$ )
- gravitatie versnelling  $g$  ( $\text{m/s}^2$ )
- constante van White  $\eta$  (-)
- korreldiameter  $D_{70}$  (m)
- bedding hoek  $\vartheta$  (deg)
- hoogte pipe  $a$  (m)
- druk  $p$  ( $\text{N/m}^2$ )
- plaats  $x$  (m)



# Piping module



Behoud van massa (volume)

$$\frac{dq}{dx} + s = 0$$

- flux  $q$  (m/s)
- plaats  $x$  (m)
- sink term  $s$  (1/s)

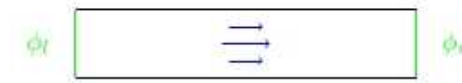


# Piping module



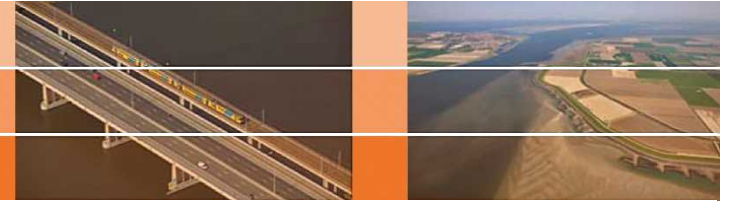
Poiseuille stroming  $q$  (m/s)

$$q = -\frac{a^3}{12\mu} \frac{dp}{dx}$$



- hoogte pipe  $a$  (m)
- dynamische viscositeit  $\mu$  (Pas)
- druk  $p$  (N/m<sup>2</sup>)
- plaats  $x$  (m)

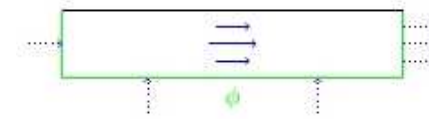
# Piping module



## Beringsvergelijking

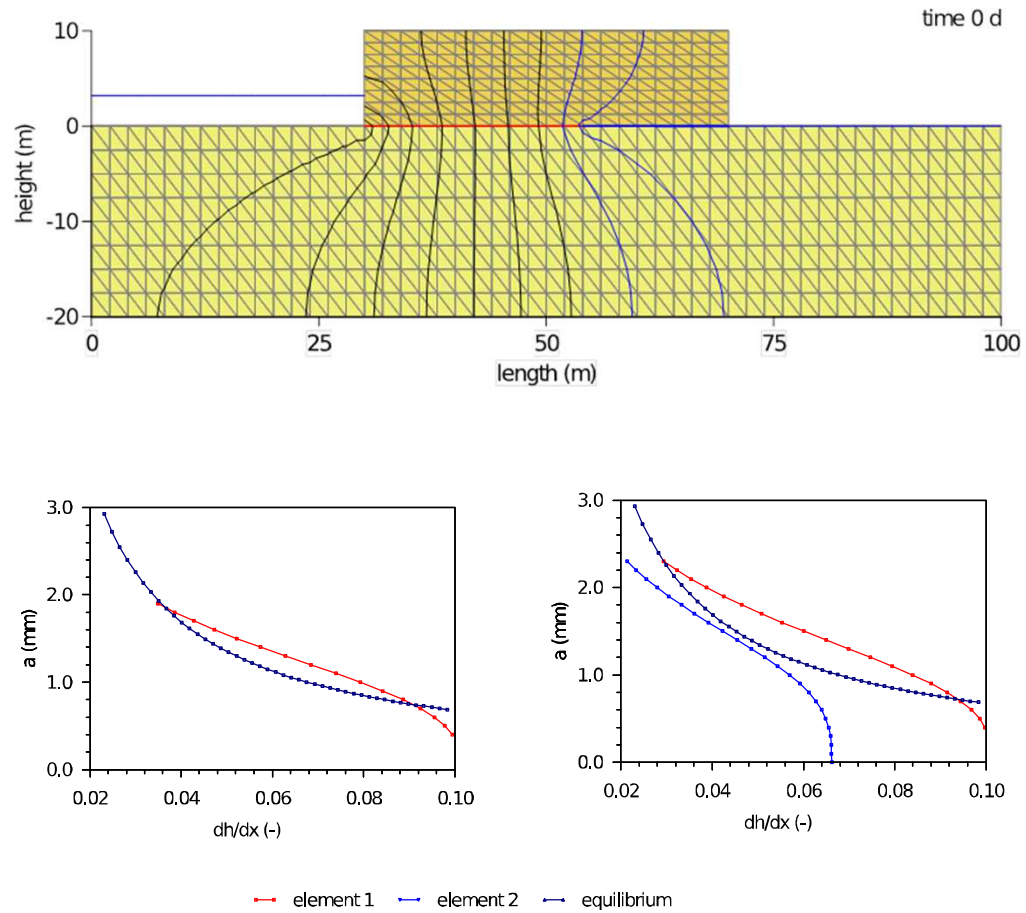
$$\frac{d}{dx} \left( \frac{a^3}{12\mu} \frac{dp}{dx} \right) - s = 0$$

- hoogte pipe  $a$  (m)
- dynamische viscositeit  $\mu$  (Pas)
- druk  $p$  (N/m<sup>2</sup>)
- plaats  $x$  (m)
- sink term  $s$  (1/s)

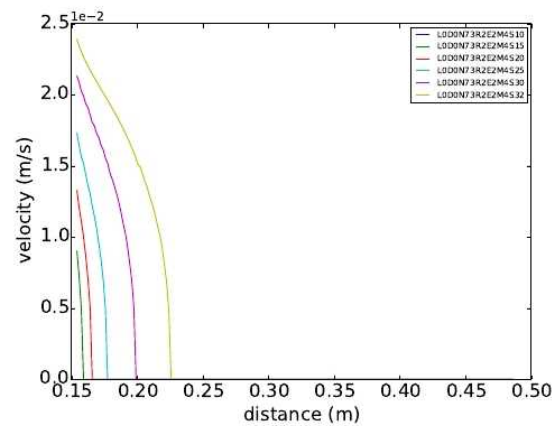
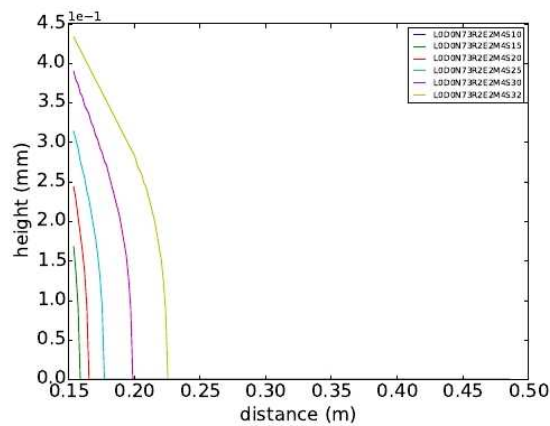
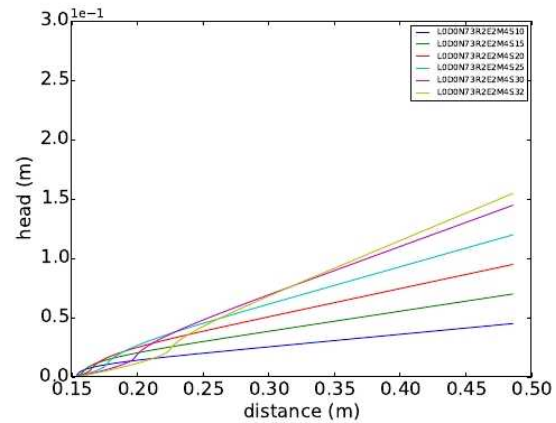
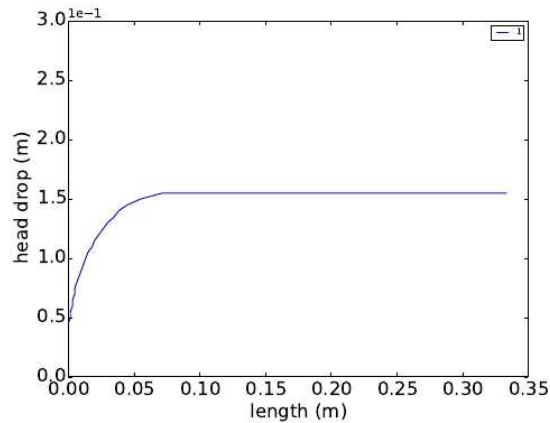
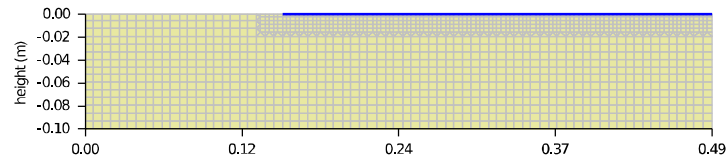




# Piping module



# Piping module

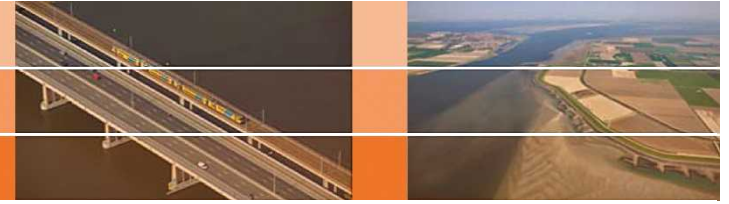


# Rekenhart D-Geo Flow



Vragen ?

# Randvoorwaarden



Climate boundary

$$F(t) > 0 \cap Q_i > F(t) \Rightarrow p_i = g_1(t)$$

$$g_1 < p_i < g_2 \Rightarrow Q_i = F$$

$$F(t) < 0 \cap Q_i > F(t) \Rightarrow p_i = g_2(t)$$

- debiet  $Q$  ( $\text{m}^2/\text{s}$ )
- druk  $p$  ( $\text{N}/\text{m}^2$ )
- verdampingsflux  $F(t)$  ( $\text{m}^2/\text{s}$ )
- ponding pressure  $g_1$  ( $\text{N}/\text{m}^2$ )
- osmotic pressure  $g_2$  ( $\text{N}/\text{m}^2$ )
- tijd  $t$  (s)