Academy

Deltares

Enabling Delta Life

Rekenhart D-Geo Flow

John van Esch

Inhoud

- Verzadigde grondwaterstroming
- Onverzadigde grondwaterstroming
- Randvoorwaarden
- Piping module

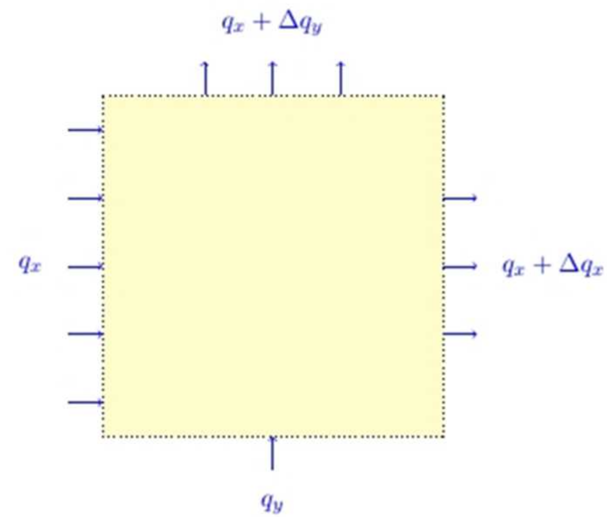
Deltares

Verzadigde grondwater stroming

Behoud van massa (volume)

$$\frac{dq_x}{dx} + \frac{dq_y}{dy} = 0$$

- plaats x, y (m)
- specifiek debiet q_x, q_y (m/s)



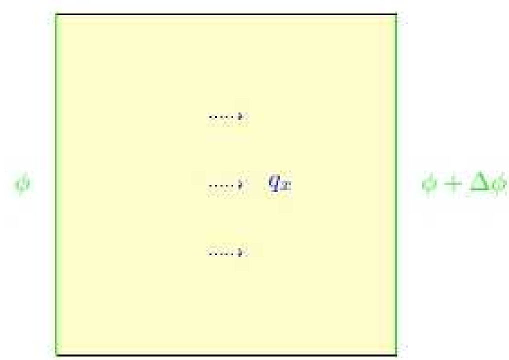
Verzadigde grondwater stroming

Wet van Darcy, specifiek debiet q (m/s)

$$q_x = -K_{xx} \frac{d\phi}{dx}$$

$$q_y = -K_{yy} \frac{d\phi}{dy}$$

- doorlatendheid K (m/s)
- stijghoogte ϕ (m)
- plaats x, y (m)



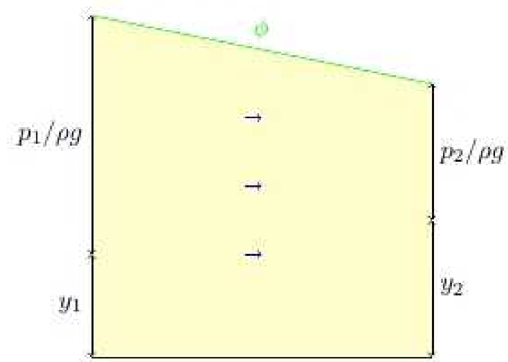
Deltares

Verzadigde grondwater stroming

Stijghoogte ϕ (m)

$$\phi = \frac{p}{\rho g} + y$$

- druk p (N/m^2)
- dichtheid ρ (kg/m^3)
- gravitatie versnelling g (m/s^2)
- plaatshoogte y (m)

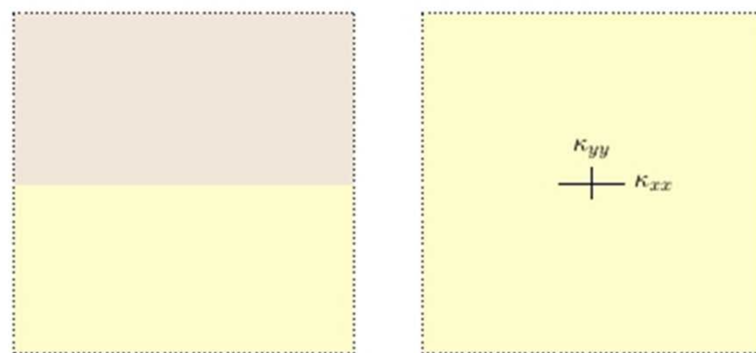


Verzadigde grondwater stroming

Intrinsieke doorlatendheid κ (m^2)

$$\kappa_{xx} = \frac{K_{xx}\rho g}{\mu}, \qquad \kappa_{yy} = \frac{K_{yy}\rho g}{\mu}$$

- doorlatendheid K $(\mathrm{m/s})$
- dichtheid ρ $(\mathrm{kg/m^3})$
- gravitatie versnelling g $(\mathrm{m/s^2})$
- dynamische viscositeit μ (Pas)



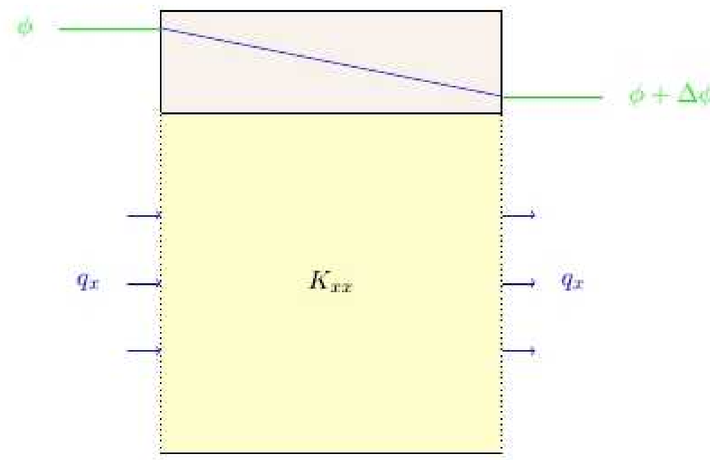
Deltares

Verzadigde grondwater stroming

Bergingsvergelijking stationair

$$\frac{d}{dx}\left(K_{xx}\frac{d\phi}{dx}\right)+\frac{d}{dy}\left(K_{yy}\frac{d\phi}{dy}\right)=0$$

- doorlatendheid K (m/s)
- stijghoogte ϕ (m)
- plaats x, y (m)

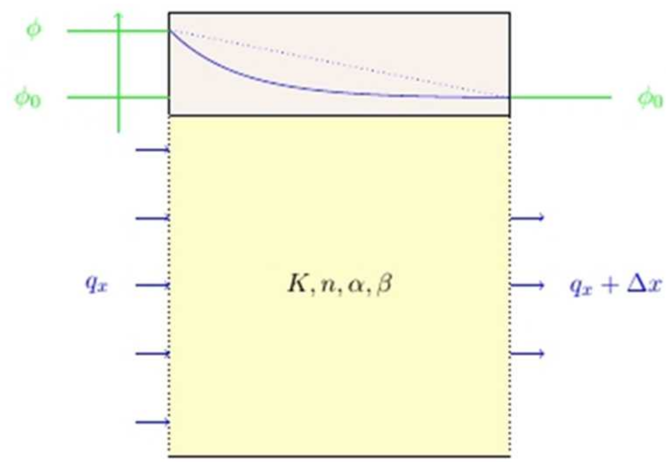


Deltares

Bergingsvergelijking tijdsafhankelijk

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial \phi}{\partial y}\right)=\rho g\left(\alpha+n\beta\right)\frac{\partial \phi}{\partial t}$$

- doorlatendheid K (m/s)
- stijghoogte ϕ (m)
- plaats x, y (m)
- dichtheid ρ (kg/m^3)
- gravitatie versnelling g (m/s^2)
- porositeit n (-)
- samendrukbaarheid skelet α (m^2/N)
- samendrukbaarheid water β (m^2/N)



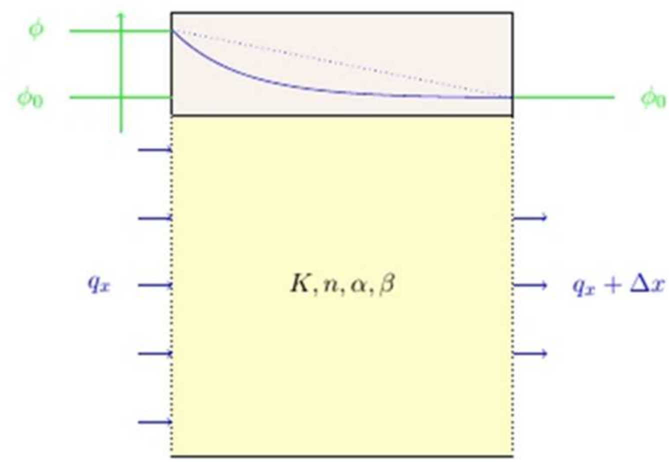
Deltares

Verzadigde grondwater stroming

Bergingsvergelijking

$$\frac{\partial}{\partial x}\left(\frac{\kappa_{xx}}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\kappa_{yy}}{\mu}\frac{\partial p}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\kappa_{yy}}{\mu}\rho g\right)=\left(\alpha+n\beta\right)\frac{\partial p}{\partial t}$$

- plaats x, y (m)
- intrinsieke doorlatendheid κ (m^2)
- dynamische viscositeit μ (Pas)
- druk p ($\mathrm{N/m}^2$)
- dichtheid ρ ($\mathrm{kg/m}^3$)
- gravitatie versnelling g ($\mathrm{m/s}^2$)
- porositeit n (-)
- samendrukbaarheid skelet α (m^2 /N)
- samendrukbaarheid water β (m^2 /N)



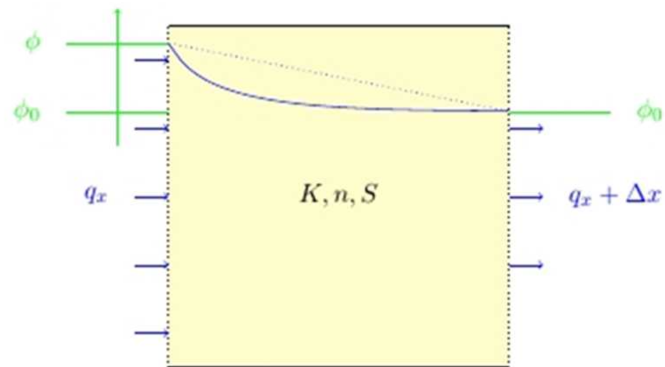
Deltares

Onverzadigde grondwater stroming

Bergingsvergelijking

$$\frac{\partial}{\partial x}\left(\frac{k_r \kappa_{xx}}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{k_r \kappa_{yy}}{\mu}\frac{\partial p}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{k_r \kappa_{yy}}{\mu}\rho g\right)=\left(\alpha+n\beta\right)\frac{\partial p}{\partial t}+n\frac{dS}{dp}\frac{\partial p}{\partial t}$$

- relatieve doorlatendheid k_r (-)
- verzadigingsgraad S (-)
- plaats x, y (m)
- intrinsieke doorlatendheid κ (m^2)
- dynamische viscositeit μ (Pas)
- druk p ($\mathrm{N/m}^2$)
- dichtheid ρ ($\mathrm{kg/m}^3$)
- gravitatie versnelling g ($\mathrm{m/s}^2$)
- porositeit n (-)
- samendrukbaarheid skelet α (m^2/N)
- samendrukbaarheid water β (m^2/N)



Deltares

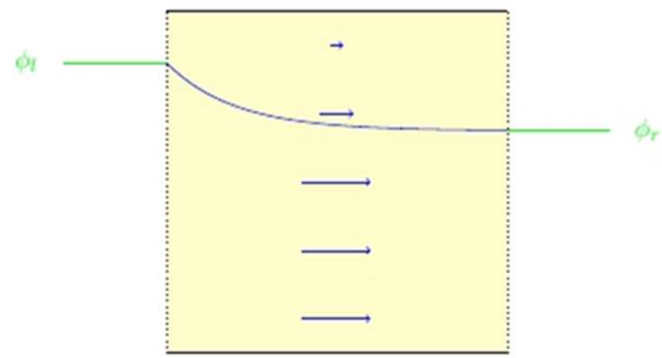
Onverzadigde grondwater stroming

Van Genuchten verzadigingsgraad S (-)

$$\psi < \psi_a \Rightarrow S = S_r + \left(S_s - S_r\right)\left[1 + \left|g_a \psi\right|^{g_n}\right]^{-g_m}$$

$$\psi \geq \psi_a \Rightarrow S = S_s$$

- drukhoogte ψ (m)
- air entry hoogte ψ_a (m)
- rest verzadiging S_r (-)
- maximale verzadiging S_s (-)
- materiaal parameters g_a, g_n, g_m (-)



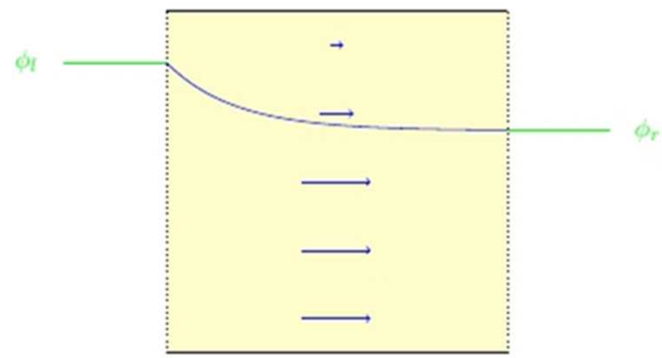
Deltares

Onverzadigde grondwaterstroming

Van Genuchten relatieve doorlatendheid k_r (-)

$$k_r = \left(S_e\right)^{g_l}\left[1-\left(1-S_e^{\,1/g_m}\right)^{g_m}\right]^2 \qquad S_e = \frac{S-S_r}{S_s-S_r}$$

- verzadigingsgraad S (-)
- effective verzadigingsgraad S_e (-)
- rest verzadiging S_r (-)
- maximale verzadiging S_s (-)
- materiaal parameters g_l, g_m (-)



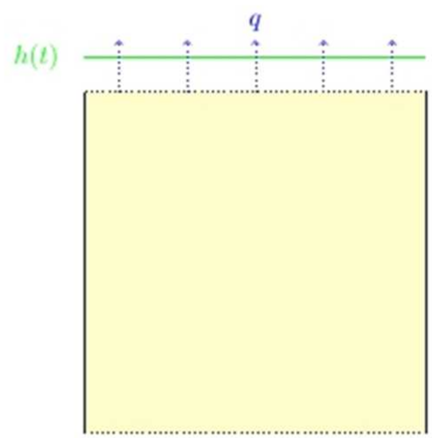
Deltares

Randvoorwaarden

head boundary

$$\phi_i = h(t)$$

- stijghoogte ϕ_i (m)
- buitenwaterstand $h(t)$ (m)
- tijd t (s)



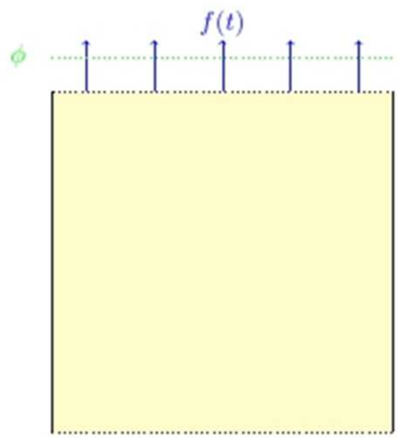
Deltares

Randvoorwaarden

Flux boundary

$$Q_i = f(t)$$

- debiet Q $(\mathrm{m}^2/\mathrm{s})$
- rand lengte Δx (m)
- uit-flux $f(t)$ (m/s)
- tijd t (s)

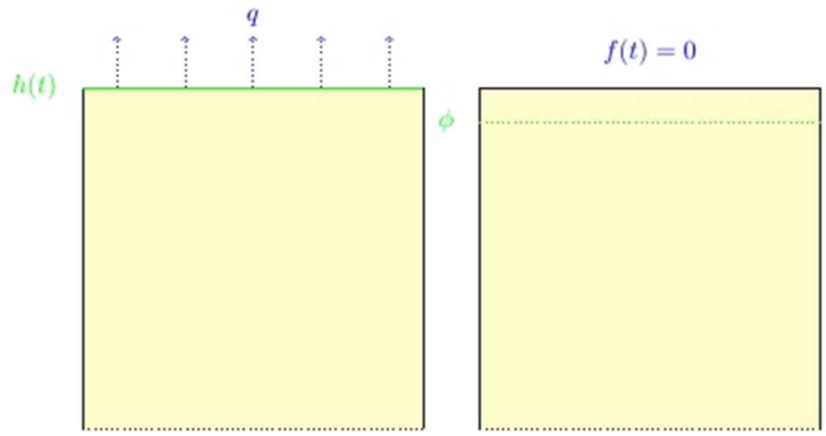


Randvoorwaarden

Seepage boundary

$$Q_i > 0 \quad \Rightarrow \quad p_i = 0$$
$$p_i < 0 \quad \Rightarrow \quad Q_i = 0$$

- debiet Q $(\mathrm{m^2/s})$
- druk p $(\mathrm{N/m^2})$



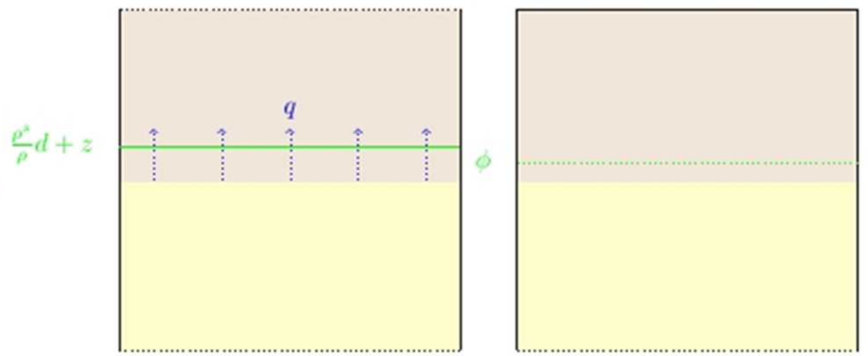
Deltares

Randvoorwaarden

Heave boundary

$$Q_i > 0 \quad \Rightarrow \quad p_i = \rho g d$$
$$p_i < \rho g h \quad \Rightarrow \quad Q_i = 0$$

- debiet Q $(\mathrm{m}^2/\mathrm{s})$
- druk p $(\mathrm{N/m}^2)$
- dichtheid grond ρ $(\mathrm{kg/m}^3)$
- gravitatieversnelling g $(\mathrm{m/s}^2$
- laagdikte d (m)



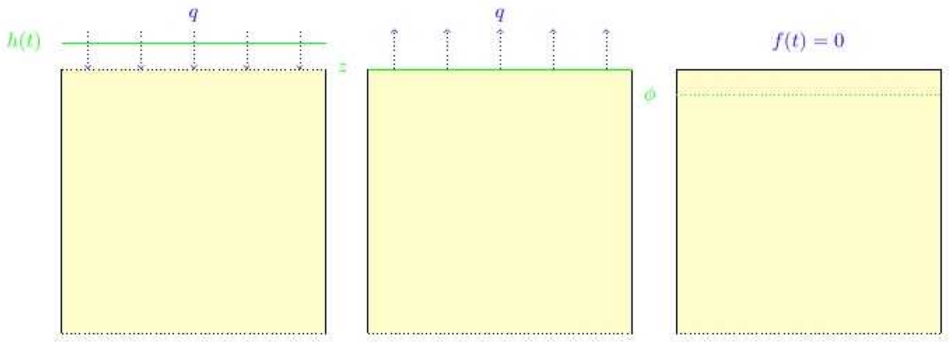
Deltares

Randvoorwaarden

Submerging boundary

$$
\begin{aligned}
h(t) \geq z_i \quad &\Rightarrow \quad \phi_i = h(t) \\
h(t) < z_i \cap F(t) > 0 \cap Q_i > F(t) \quad &\Rightarrow \quad p_i = g_1(t) \\
h(t) < z_i \cap g_1 < p_i < g_2 \quad &\Rightarrow \quad Q_i = F(t) \\
h(t) < z_i \cap F(t) < 0 \cap Q_i > F(t) \quad &\Rightarrow \quad p_i = g_2(t)
\end{aligned}
$$

- debiet Q $(\mathrm{m}^2/\mathrm{s})$
- druk p $(\mathrm{N/m}^2)$
- stijghoogte ϕ (m)
- buitenwaterstand $h(t)$ (m)
- verdampingsflux $F(t)$ $(\mathrm{m}^2/\mathrm{s})$
- ponding pressure g_1 $(\mathrm{N/m}^2)$
- osmotic pressure g_2 $(\mathrm{N/m}^2)$
- tijd t (s)

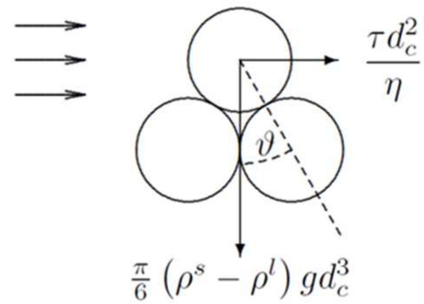


Deltares

Piping module

Kritische schuifspanning τ_c ($\mathrm{N/m^2}$)

$$\tau_c = \frac{\pi}{6}\left(\rho^s - \rho\right) g \eta D_{70} \tan \vartheta$$

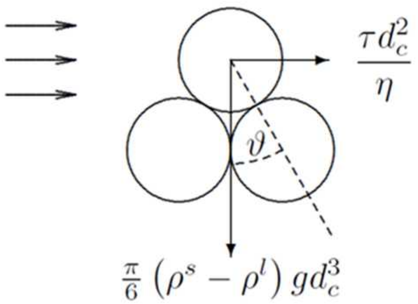


- dichtheid korrel ρ^s ($\mathrm{kg/m^3}$)
- dichtheid water ρ ($\mathrm{kg/m^3}$)
- gravitatie versnelling g ($\mathrm{m/s^2}$)
- constante van White η (-)
- korreldiameter D_{70} (m)
- bedding hoek ϑ (deg)

Piping module

Uitgeoefende schuifspanning τ (N/m^2)

$$\tau = \frac{a}{2}\frac{dp}{dx}$$



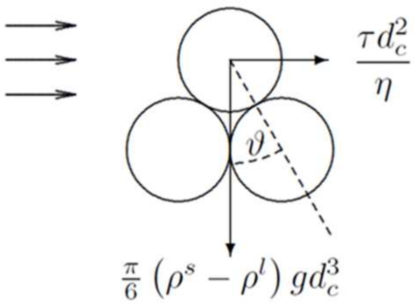
- hoogte pipe a (m)
- druk p (N/m^2)
- plaats x (m)

Deltares

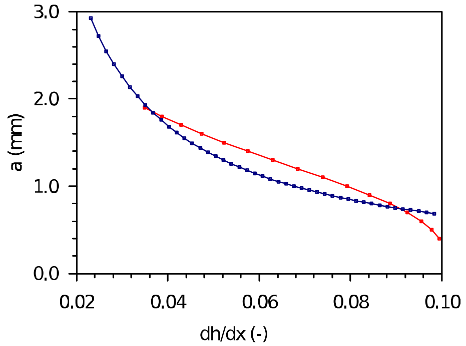
Piping module

Evenwichtsvergelijking

$$a\frac{dp}{dx}=\frac{\pi}{3}\left(\rho^{s}-\rho^{l}\right)gD_{70}\eta\tan\vartheta$$



- dichtheid korrel ρ^{s} $(\mathrm{kg/m^3})$
- dichtheid water ρ $(\mathrm{kg/m^3})$
- gravitatie versnelling g $(\mathrm{m/s^2})$
- constante van White η (-)
- korreldiameter D_{70} (m)
- bedding hoek ϑ (deg)
- hoogte pipe a (m)
- druk p $(\mathrm{N/m^2})$
- plaats x (m)

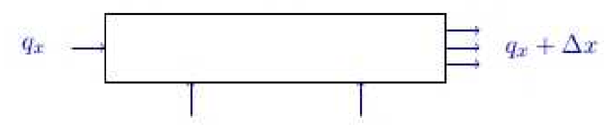


Deltares

Piping module

Behoud van massa (volume)

$$\frac{dq}{dx} + s = 0$$

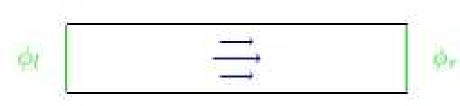


- flux q (m/s)
- plaats x (m)
- sink term s (1/s)

Piping module

Poisseuille stroming q (m/s)

$$q = -\frac{a^3}{12\mu}\frac{dp}{dx}$$



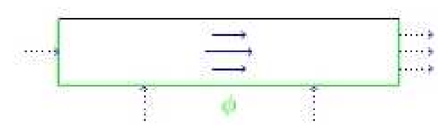
- hoogte pipe a (m)
- dynamische viscositeit μ (Pas)
- druk p (N/m^2)
- plaats x (m)

Deltares

Piping module

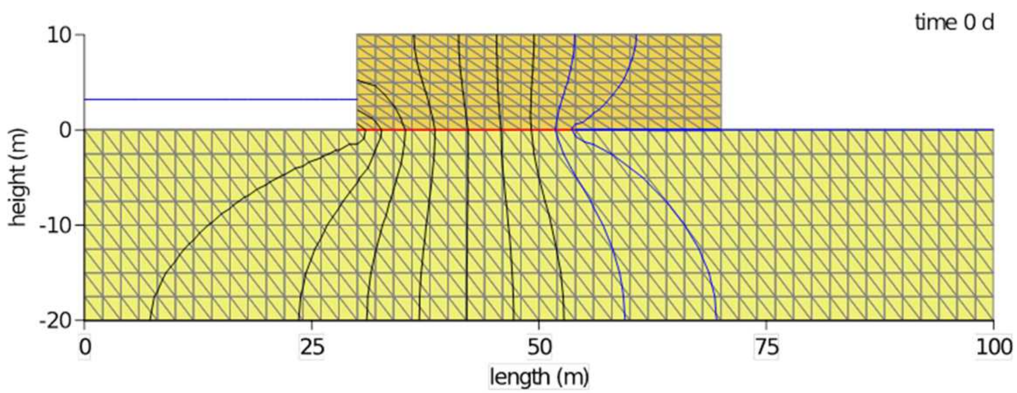
Beringsvergelijking

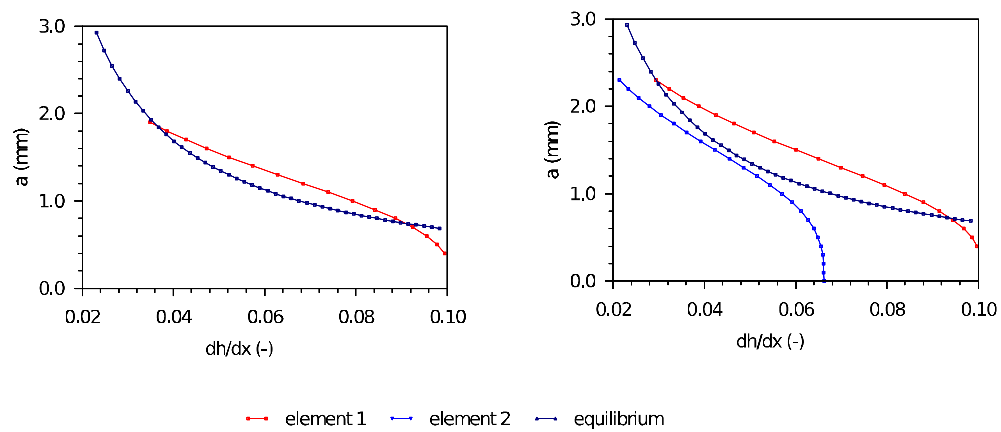
$$\frac{d}{dx}\left(\frac{a^3}{12\mu}\frac{dp}{dx}\right) - s = 0$$



- hoogte pipe a (m)
- dynamische viscositeit μ (Pas)
- druk p (N/m^2)
- plaats x (m)
- sink term s (1/s)

Piping module





Deltares

Rekenhart D-Geo Flow

Vragen ?

Randvoorwaarden

Climate boundary

$$
\begin{array}{ccc}
F(t) > 0 \cap Q_i > F(t) & \Rightarrow & p_i = g_1(t) \\
g_1 < p_i < g_2 & \Rightarrow & Q_i = F \\
F(t) < 0 \cap Q_i > F(t) & \Rightarrow & p_i = g_2(t)
\end{array}
$$

- debiet Q $(\mathrm{m}^2/\mathrm{s})$
- druk p $(\mathrm{N/m}^2)$
- verdampingsflux $F(t)$ $(\mathrm{m}^2/\mathrm{s})$
- ponding pressure g_1 $(\mathrm{N/m}^2)$
- osmotic pressure g_2 $(\mathrm{N/m}^2)$
- tijd t (s)

Deltares