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Enabling Delta Life

# Nonlinear optimization of hydraulic systems

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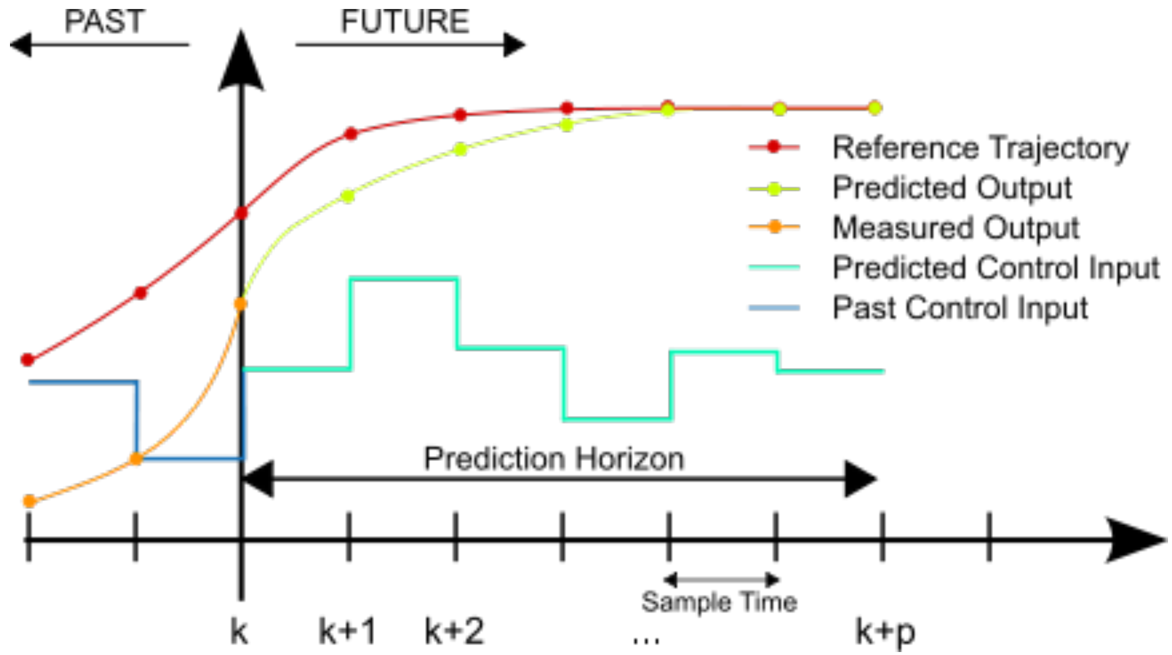
Liège, October 13, 2017

# Model predictive control

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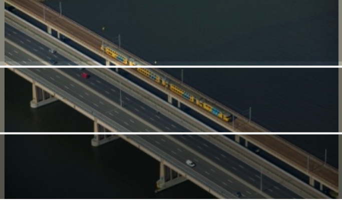
- Predict system state based on model
- Compute control inputs that maximize performance over prediction horizon
- Implement first computed control input
- Repeat procedure at next time step

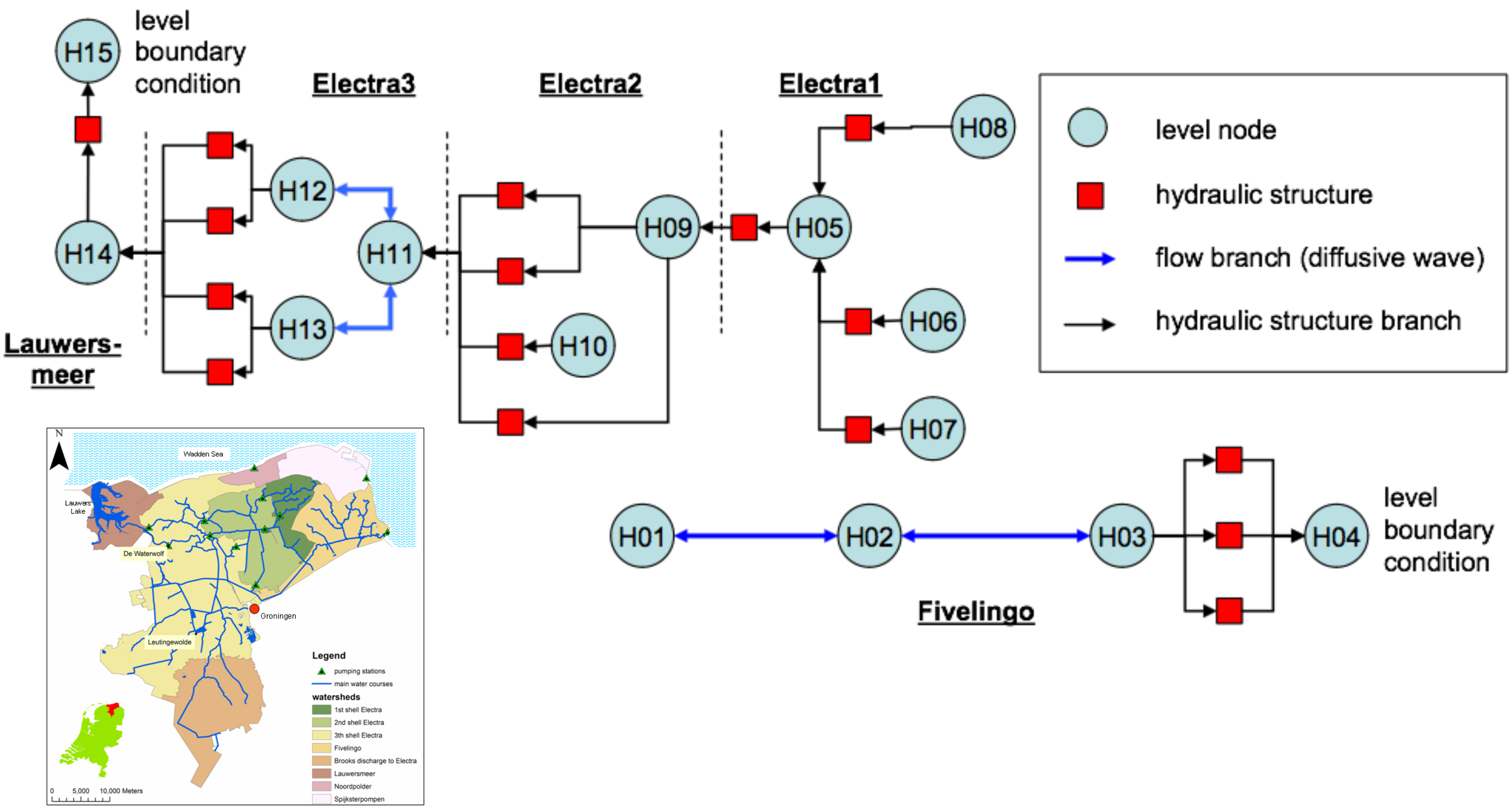


# Prediction model

A good prediction model satisfies several requirements:

- *Accurate*: It captures the relevant physical processes with sufficient accuracy.
- *Simple*: It focuses on the essential processes. Details are left out. Optimizing for details is a bad idea, considering the inaccuracies inherent in any inflow forecast. Less = more.
- *Quick*: As it will need to be evaluated many times during optimization, a single run needs to be computationally inexpensive.

# Prediction model

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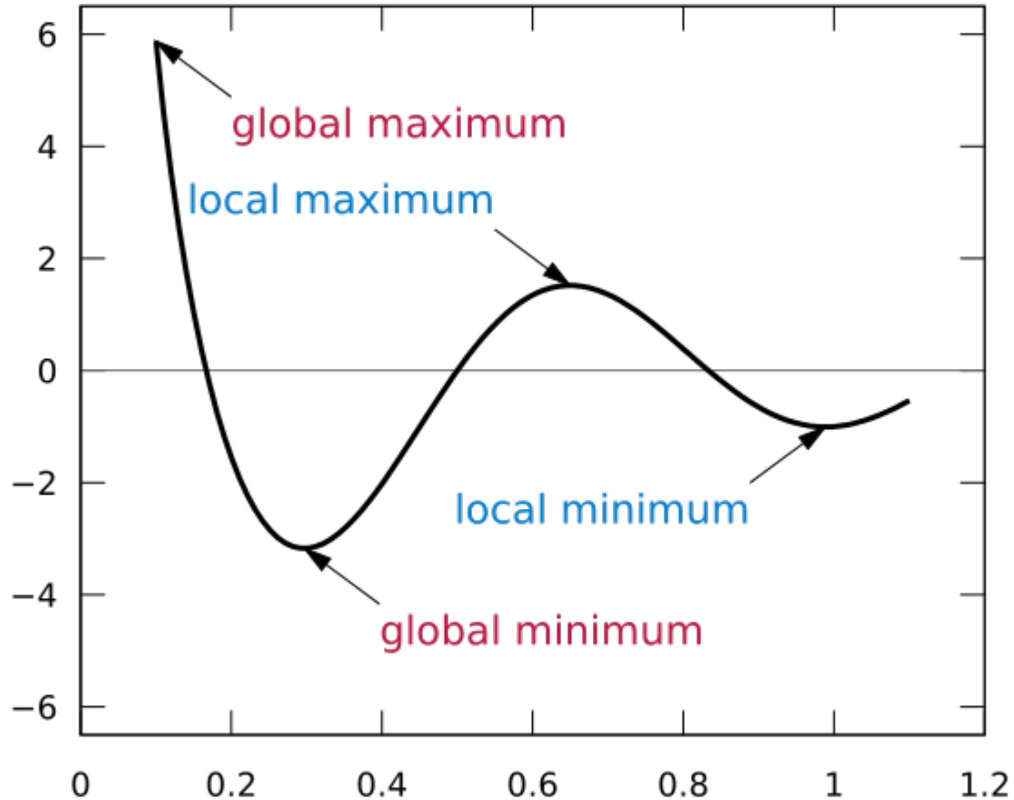
# Local and global optima




Source: Wikipedia. GFDL 1.2.

# Key to reliable operational optimization

Idea: Formulate optimization problems that only admit global minima**.**

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The mathematical term for such formulations is that they are convex.

Unfortunately, we will see that convexity is too restrictive when it comes to modelling hydraulic processes with large discharge variations.

# Open channel hydraulics

In open channel flow, a common approximation is to take the square of the discharge $Q$ proportional to the head difference $\Delta H$:

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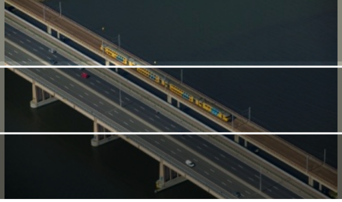

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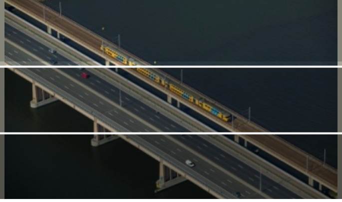

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➡ Nonlinear equality constraint.

➡ Nonconvex optimization problem.

➡ Finding even *any* feasible solution hard in general, and when one is found, no certificates on solution quality.

# Solution: Homotopy

Idea: **Interpolate between linear and nonlinear model.**

$$M = (1 - \theta)M_L + \theta M_{NL}$$

With continuation parameter $\theta \in [0, 1]$.

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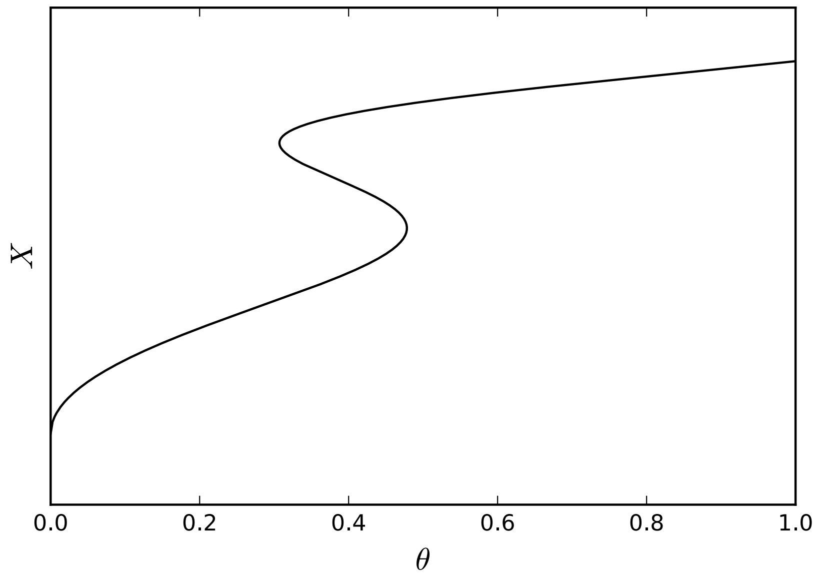
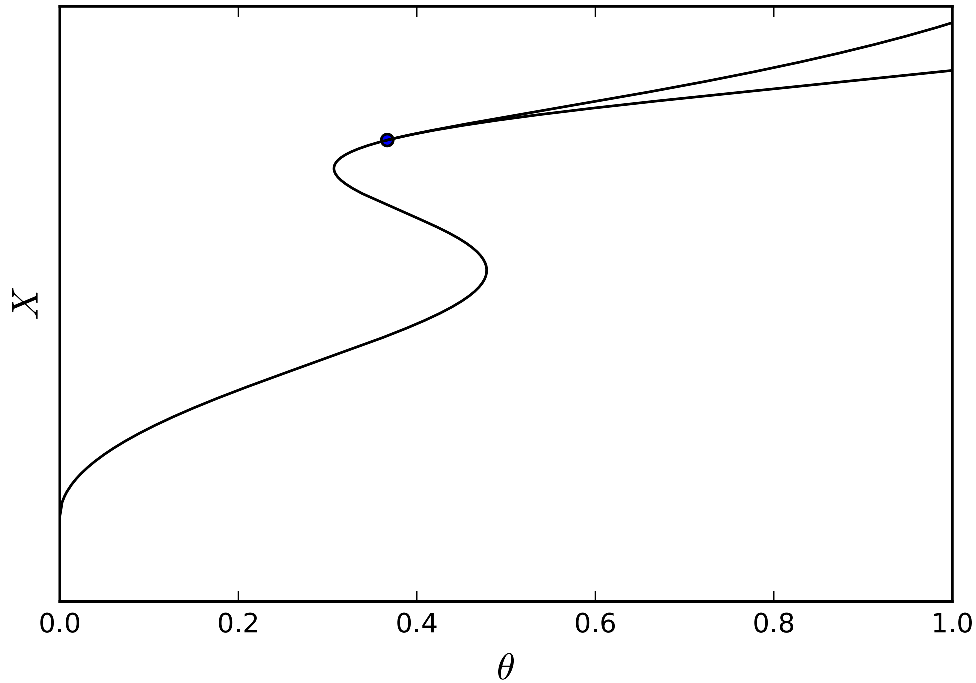
# Continuation method

Start solving at $\theta = 0$ continuing step–by-step until $\theta = 1$.

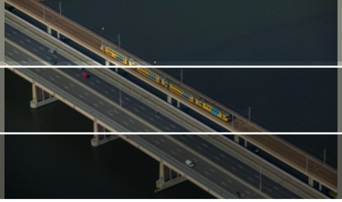
At $\theta = 0$, the linear problem only admits global optima. We find a global optimum, and trace this through to a nonlinear solution at $\theta = 1$: Solution to nonlinear problem can be *traced back* to globally optimal solution of linearized problem.



# Continuation method: Bifurcations




Bifurcations occur at so-called *singular points*.



# Nonlinear hydraulic model

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\frac{Q^2}{A} + gA\frac{\partial H}{\partial x} + g\frac{Q|Q|}{C^2 RA} = 0$$

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

Idea:

- Discretize 1D shallow water (Saint-Venant) equations on staggered grid;
- Drop advection term (valid when $\frac{\partial A}{\partial x}$ and $\frac{\partial Q}{\partial x}$ are small); of minor importance for typical NL water board applications.

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# Linear hydraulic model

Idea: Take nonlinear discretization, and fix terms that vary little compared to other terms.

($\neq$ linearization, which does not work well for large variations in flow variable $Q$.)

# Homotopy linear → nonlinear: Main result

**Theorem** A necessary condition for a singular point is for either of the following alternatives to hold:

- Wetting and drying ($H = H_b$)
- Flow reversal ($Q = 0$)

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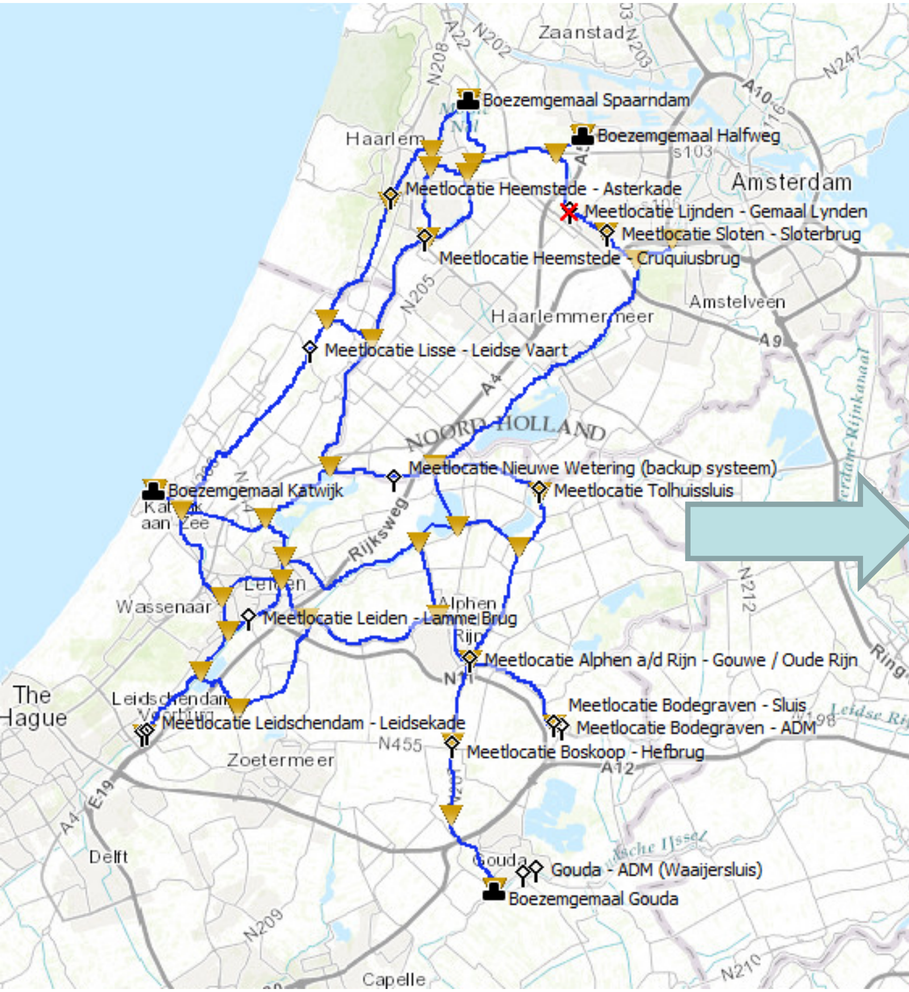
N.B. If these conditions are avoided, local, nonglobal minima may still arise (originating at $\infty$)!

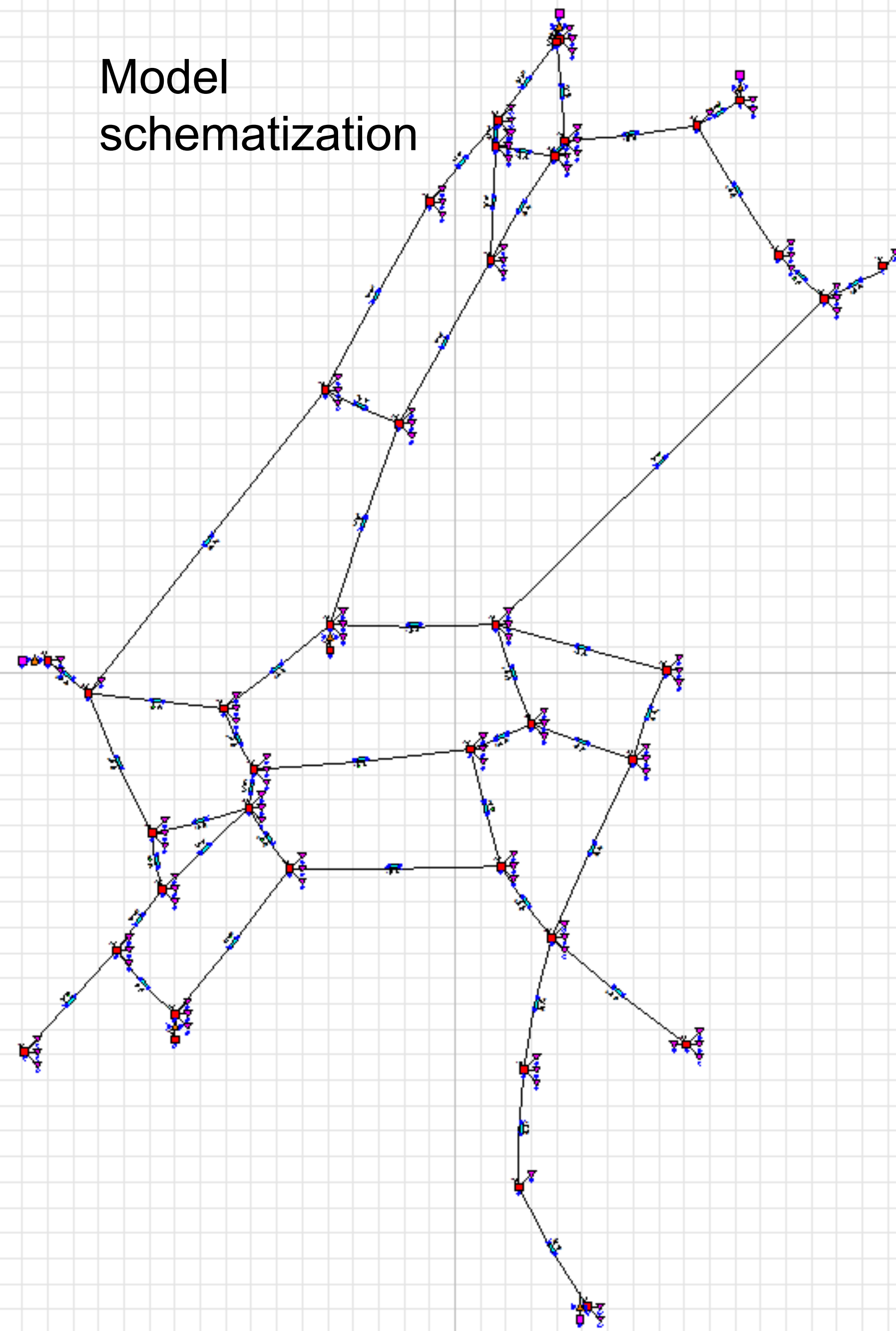
As long as the alternatives are avoided, a continuation method can be applied.

# Use Case - Rijnland



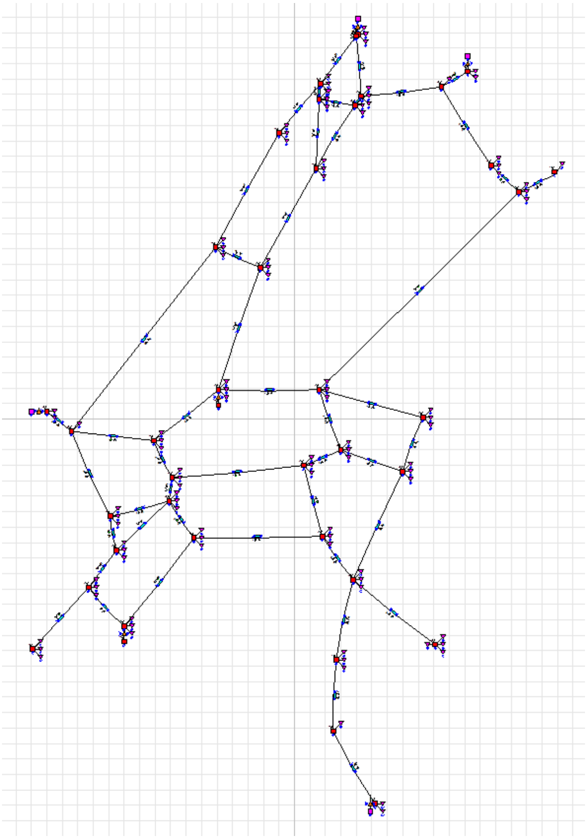

Primary channel system of Rijnland

Model schematization



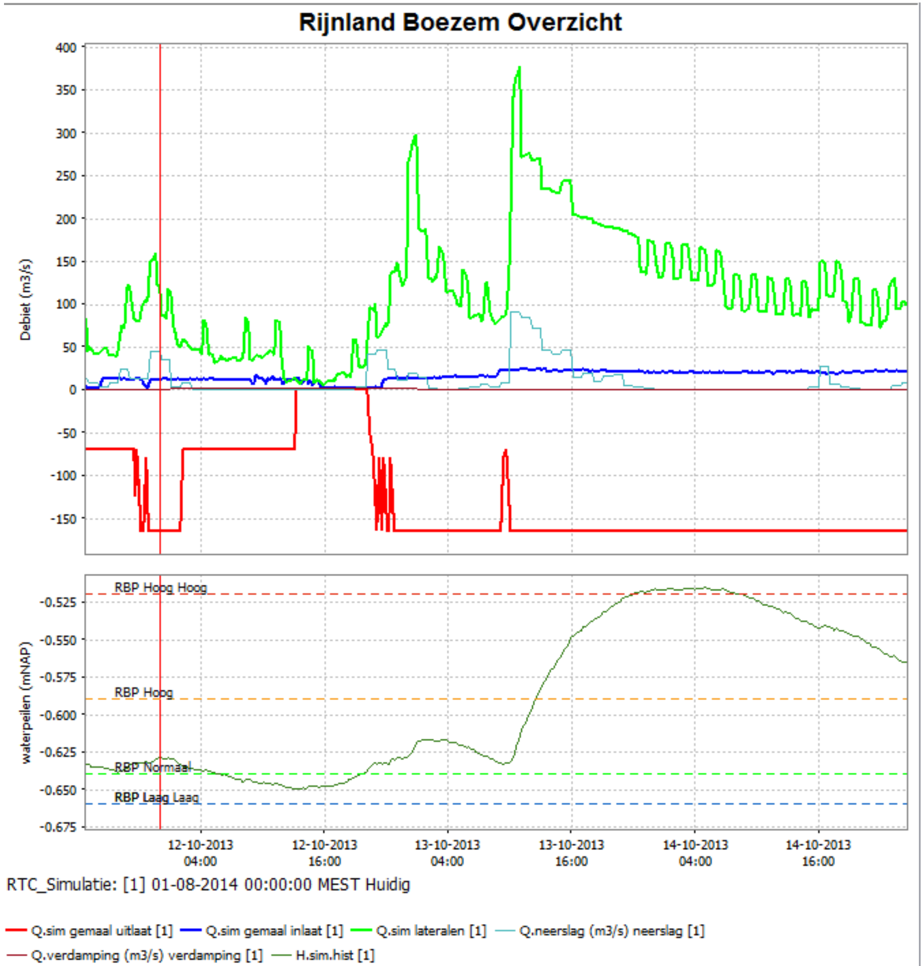
# Use Case – Primary channel system Rijnland

- Multi-objective optimization for safety, salt flushing, and energy use.
- Primary goal: Optimization of water level in primary channel system (the *boezem*)
  - Water level range as well as target level
- Salt and wind dynamics are relevant
  - Wind has an impact on relative water level differences and hence on circulation pattern
  - Multiple sources of salinity, flushing strategy required

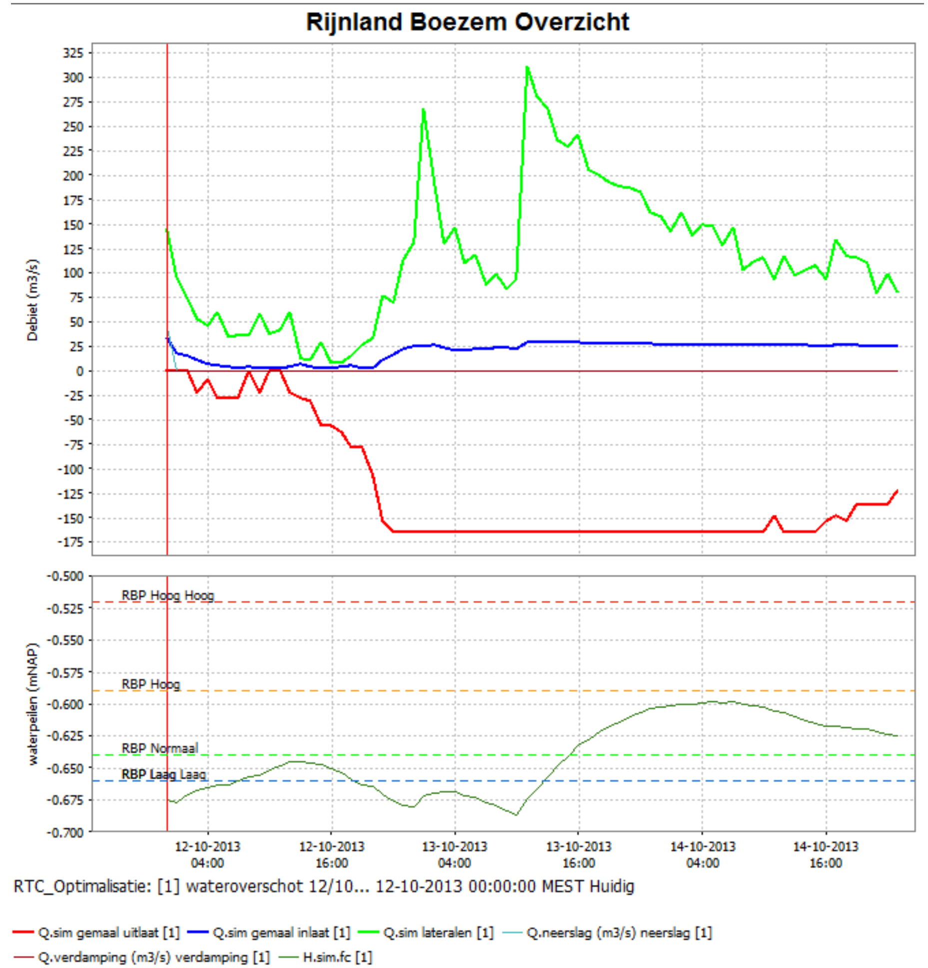


# Feedback control vs. Optimization – wet event

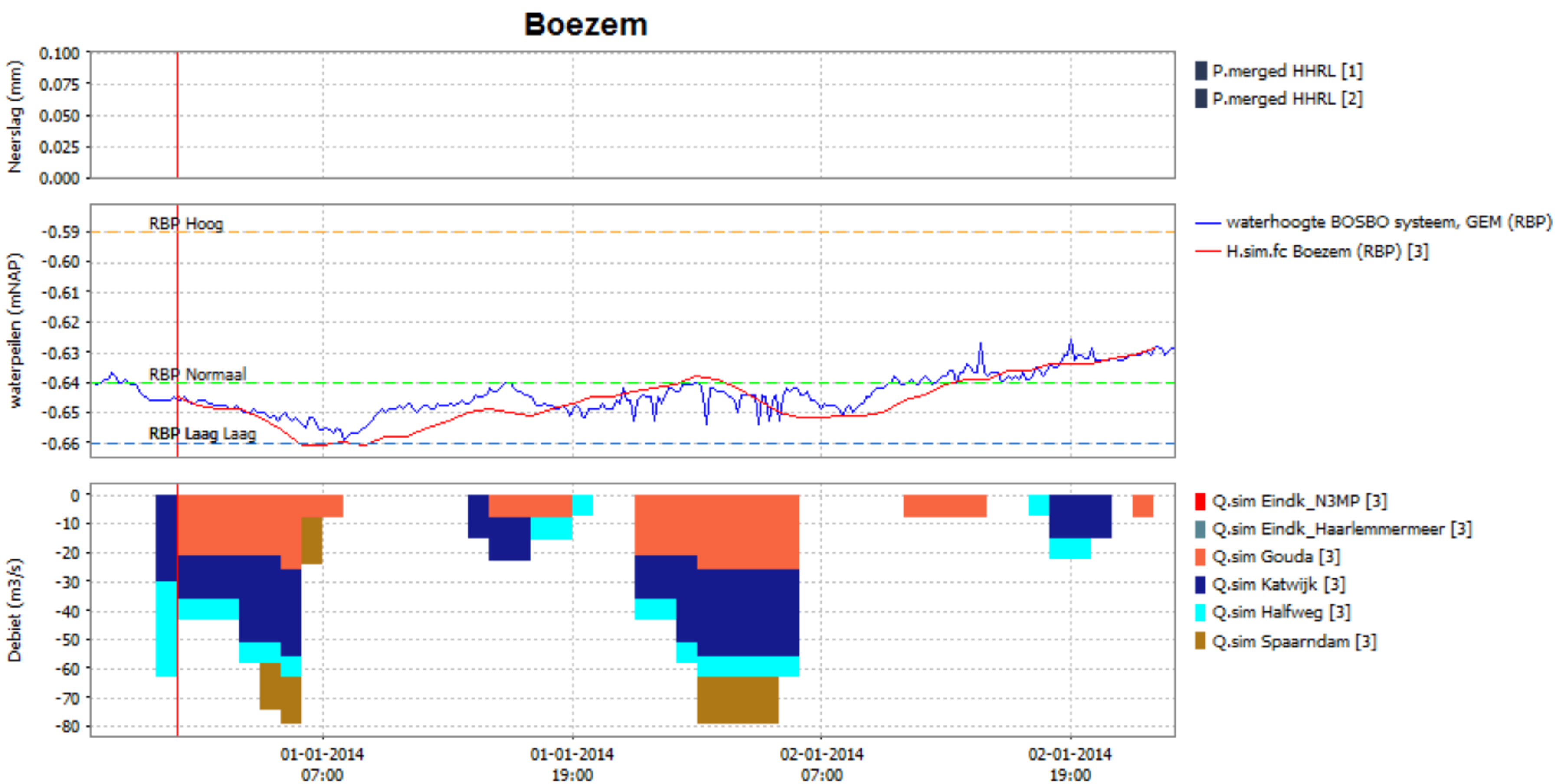
## Feedback control results




## Optimization results



# Measured vs. Optimized – similar pump action




Optimization prefers nighttime pumping (lower price)

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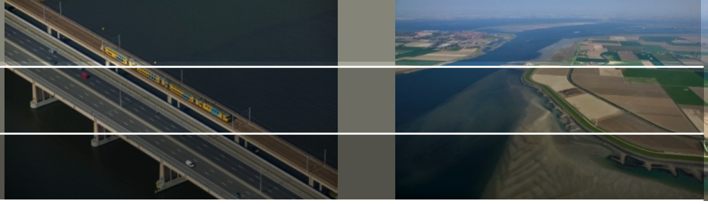
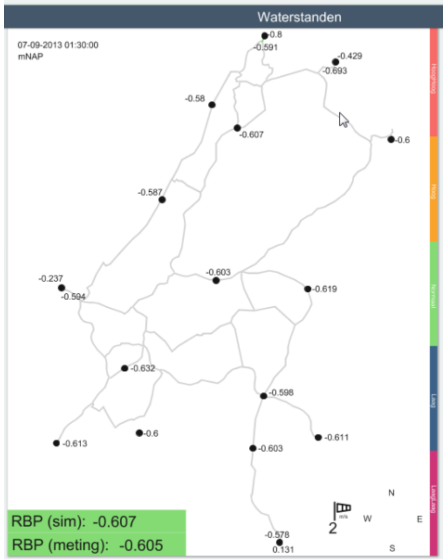
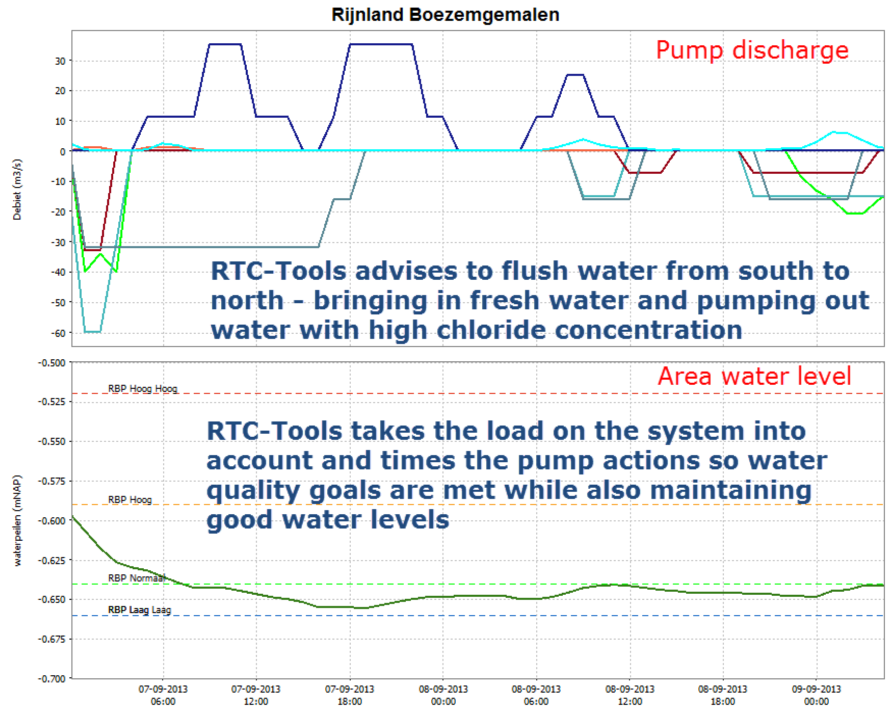
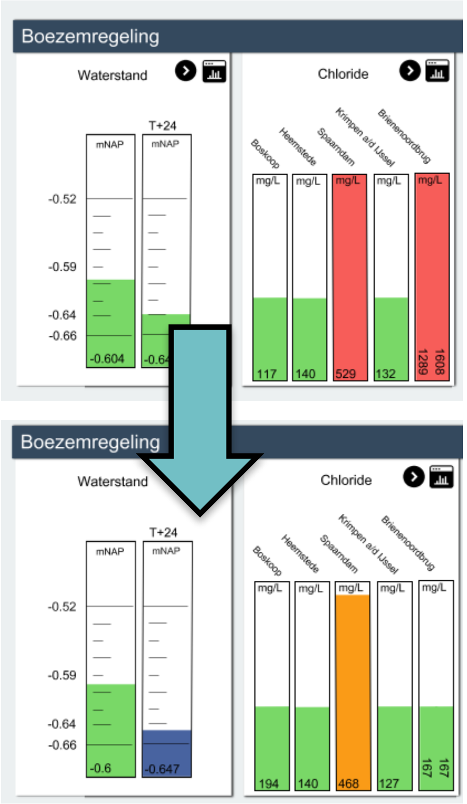


# Rijnland

A decision support and control system for the water board of Rijnland was brought online earlier this year. The system provides advice on the dispatch of pumping stations, taking into account the operational objectives of flood control, <u>water quality</u>, and cost savings.

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Thank you for your attention!

Feel free to contact me any time at

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