Theme: A. Technologies for Water Management and Monitoring
Subtheme: A3. Real time control technologies and applications
Special session: Model predictive control for water management

Model predictive control of a river reach with weirs

Klaudia Horváth [1], Bart van Esch [1], Jorn Baayen [2], Ivo Pothof [2], Jan Talsma [2], Tjerk Vreeken[2]

[1] Eindhoven University of Technology, Department of Mechanical Engineering of the Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, the Netherlands
[2] Deltares, Boussinesqweg 1, 2629 HV, Delft

Corresponding author: Klaudia Horváth, hklau85@gmail.com

Abstract A decision support system, RTC-Tools 2, for water management is presented based on convex optimization. The advantage of convex optimization is the existence of a global optimum, which makes the decision support system robust and deterministic. In this work the convex modeling of open water channels and weirs is presented. The decision support system is illustrated using a river made of 12 river reaches divided by movable weirs. The goal of the decision support system is to calculate the necessary weir movements in order to keep the water levels in all reaches within the prescribed bounds. After this test the optimization can be applied to a realistic numerical model and model predictive control can be implemented.

Keywords: convex, hydraulic structures, model predictive control, optimization, weir

1 Introduction

Optimization methods are often used for managing water systems. Model predictive control is one of them and several studies have been carried out about its application [1, 2, 3]. In these studies linear models are used to preserve convexity even though the problem at hand is essentially nonlinear. However, when the nonlinearities are moved to the inequality constraints, it is possible to create a convex optimization problem and in some cases preserve non-linearity of the system. In this research such approach is demonstrated through the modelling of weirs by RTC-Tools 2 [4], a decision support system.

2 Material and methods

2.1 Convex optimization approach

The decision support system is using convex optimization which guarantees that the global optimum is reached. This property is crucial for a decision support system. If the problem was not convex, a local optimum can be reached instead, and a small change in the initial conditions might direct the solution into an entirely different local optimum. This fact would reduce the credibility of the decision support system by the user. Therefore, we aim at describing the water system as convex optimization problem in the form [5]:

$$\begin{array}{ll} \text{minimize} & f_0(x) \\ \text{subject to} & f_i(x) \le 0 \quad i=1,\dots,m \\ & a_i^T x = b_i \quad i=1,\dots,\mathrm{p} \end{array}, \tag{1}$$

where $f_0,\dots,f_m$ are convex functions. The objective and the inequality constraints are convex functions, but the equality constraints should be affine. The objective can be minimizing energy used by pumps, or minimizing water level error. Constraints can be for example lower and upper bunds within the water levels should be kept.

2.2 Modelling

The water systems are modelled consistent with the structure of Eq. 1. In this example the river reaches are modelled as reservoirs:

$$\frac{dH}{dt}A = Q_{in}(t) - Q_{out}(t) \tag{2}$$

where A is the surface of the river reach, H is the water level and Q_{in} and Q_{out} are the in- and outflow. As Eq. 2 discretized is affine, it can be used as equality constraint.

The modelling of the weirs needs extra steps as the weir equation is non-linear, hence it cannot be used as equality constraint. In this work weirs under free-flow are modelled by the general weir equation:

$$Q = C_d \frac{2}{3}\sqrt{2g}B\left(H - H_w\right)^{3/2} \tag{3}$$

where C_d is the weir discharge coefficient (approximated as 0.61), g is the acceleration of gravity, B is the width of the weir, H_w is the crest height, and H is the upstream water level. In the optimization problem with controllable weirs, the goal is to find the appropriate crest height so that the constraints (keep the water level in bounds) are satisfied. Therefore the following approach is used: in the optimization only the discharge is used and the crest height is calculated as post-processing. However, it should be ensured that all the computed discharges are feasible for the weir: for example the discharge cannot be larger than the discharge corresponding to the minimum crest level. Thus at each step, the discharge to be calculated is limited by the minimum and maximum discharges that the current water levels and the minimum and maximum crest heights allow. An example for such "working area" of the weir is shown in Figure 1. The possible discharge is bounded by horizontal lines of Q_{min} and Q_{max}: these values should be approximated based on the characteristics of the system. The left side of the area is bounded by the line corresponding to Eq. 3 with $H_w = H_{w,min}$, when the weir is in the lowest position. The area is bounded to the right by the maximum crest height, the line shows the plot of Eq. 3 when $H_w = H_{w,max}$.

3 Results and discussion

3.1 Results

The decision support system is applied to a river containing 12 short reaches divided by weirs. The goal of the operation of the system is to keep the water levels within certain bounds. This example is a feasibility problem: there is no objective function and a solution is valid if the constraints are satisfied, in this case, the water levels are kept within the bounds. The system has an upstream inflow with a step (Figure 2) and a constant outflow downstream (0.5m^3/s). The goal of the decision support system is to propose weir movements such that the water level stays within the prescribed bounds in all reaches.

The resulting crest heights are shown in Figure 3 and the weir discharges in Figure 4. It can be seen that the resulting discharge at the first weir (Figure 4) is higher than the base flow (0.5m^3/s) from the beginning of the period - the system is preparing for the discharge peak. The weir crest is increasing and then gradually lowered to avoid sending the discharge peak directly to the next reach. The position of the first weir crest can also be seen in Figure 3, as crosses in the working area. The flow over weir 1 is more than the double of the base flow (Figure 4). The water levels in all river reaches can be seen in Figure 5: the dashed line shows the minimum and maximum allowable water levels and the solid line is the calculated water level. The water level in reach 1 is kept almost constant in the first half of the simulation. With the weir movements the water levels change in several reaches, the discharge wave is dispatched. Note that there is only a 2 cm margin in the allowable water levels in reach 5, still the water level is kept between the bounds.

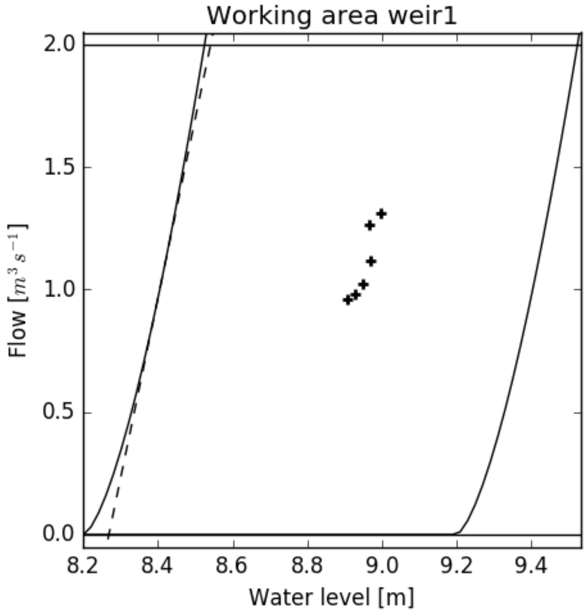

Figure 1 Working area of the weir

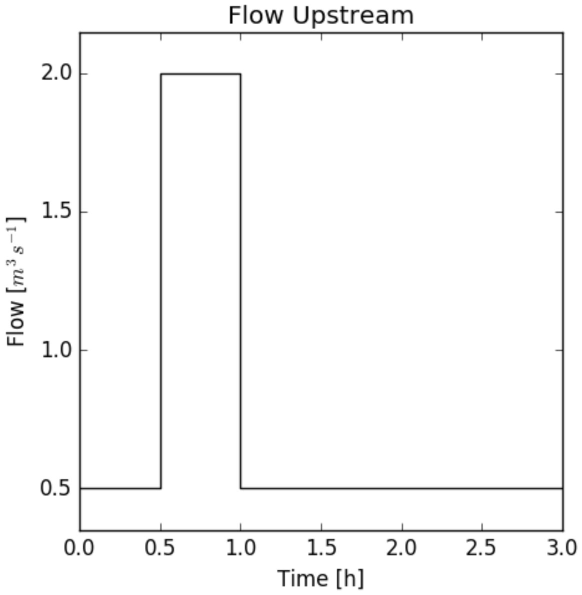

Figure 2 Upstream flow

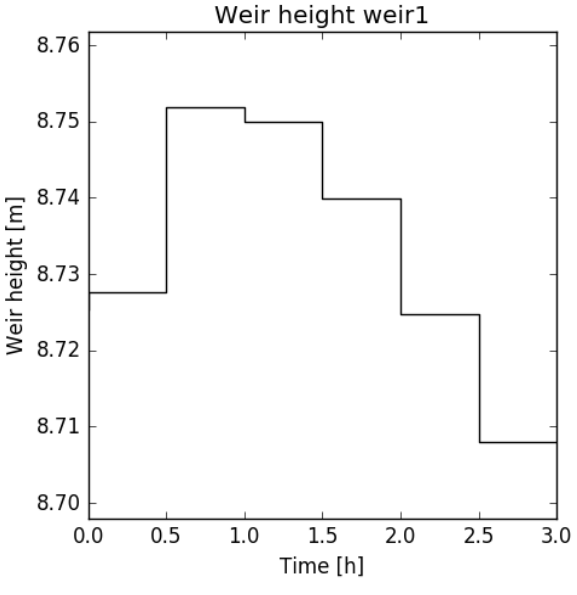

Figure 3 Crest height of the weir

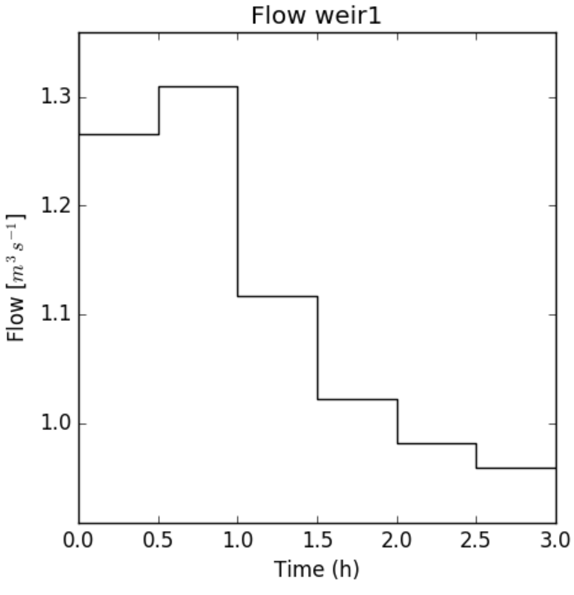

Figure 4 Discharge over the weir

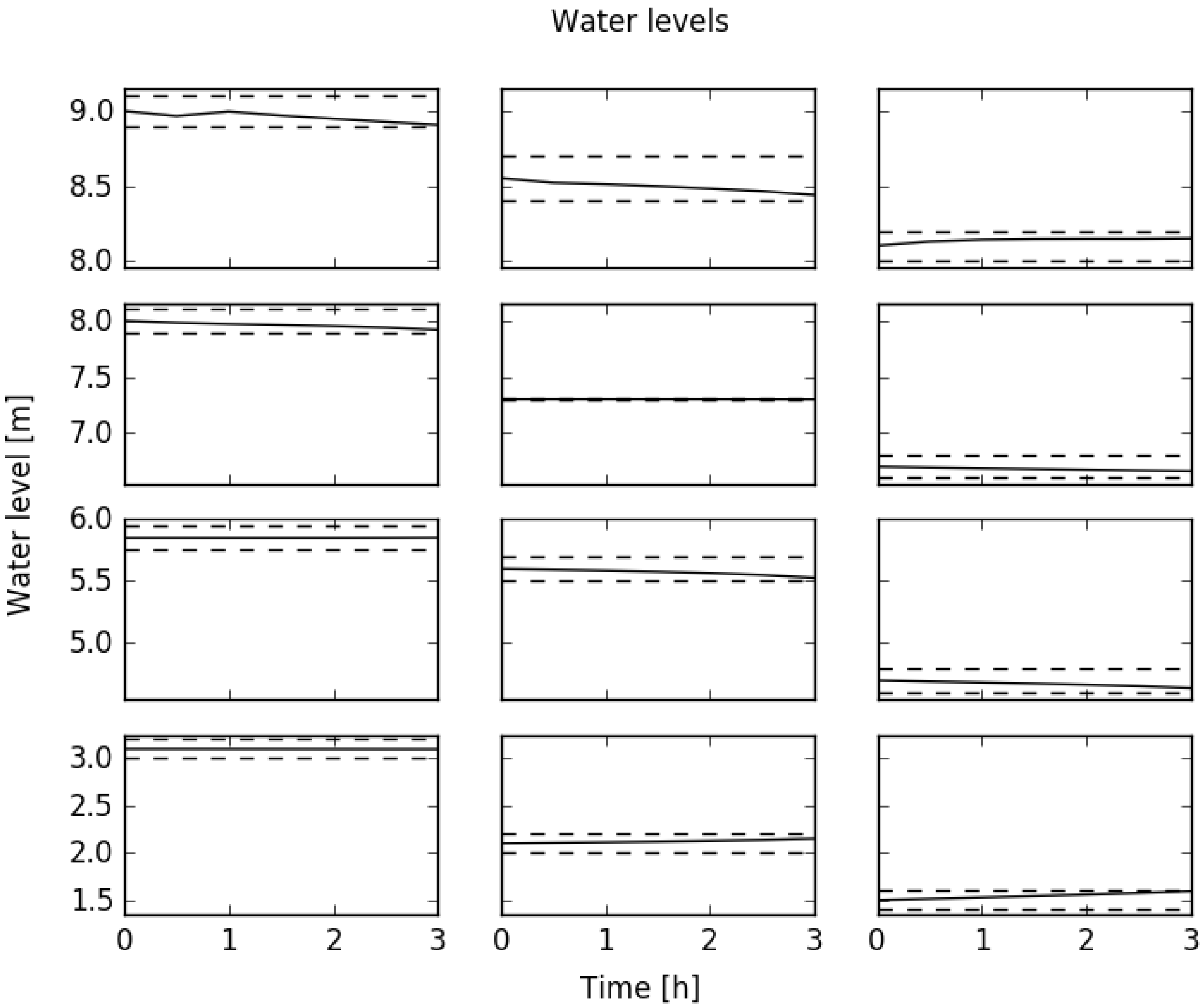

Figure 5 Water levels in the branches, the downstream direction is from left to right, top to bottom

4 Conclusions

A decision support system, RTC-Tools 2, based on convex optimization is presented. The convex modelling of open water channels and weirs is described. The system is illustrated through a base study with a river containing 12 reaches divided by weirs. It was shown that by applying the weir movements calculated by the decision support system, the water levels can be kept within the prescribed bounds. After this test the optimization can be applied to a realistic numerical model and model predictive control can be implemented.

Reference

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