Stochastic modelling of the coastline position along the Holland Coast (The Netherlands)

C. Brière and H.F.P. Van den Boogaard
Deltares, PO Box 177, 2600 MH, Delft, Netherlands

ABSTRACT: In this paper stochastic regression models are used for a quantitative analysis of the coastline position. This modelling is based on a long term trend analysis of historic measurement data gathered along the Holland Coast (The Netherlands), and includes the effect of nourishment interventions. The description of the actual state of the system is performed now with more insight in the structural developments, than was done in the past. The application of the model leads to an indication for the (expected) coastal state in coming years T(=0) to T+5. Moreover, predictions by the stochastic model can be used to verify and compare the system response to alternative nourishment scenarios, and in this way supports the design of optimal nourishment strategies.

1 INTRODUCTION

The Dutch coastline along the South-East part of the North Sea is about 350 *km* long. Commonly the Dutch coast is divided into three regions, *viz.* (1) the Delta coast in the south, (2) the Holland coast in the centre and (3) the Wadden coast in the north. When focusing on the Holland coast between Hoek van Holland and Den Helder, the morphology is typically a storm-dominated sandy coast. Its primary function is to protect the low-lying hinterland from flooding. The sandy coast, however, represents important value to other functions as well: *e.g.* ecological value, drinking water supply, recreation, residential and industrial functions. Coastal erosion, dominant along half of the Dutch coast, is endangering these functions.

In order to stop any further structural recession of the coastline the Dutch government initiated the development of a new coastal policy, at the end of the 1980's, the so-called "Dynamic Preservation of the Coastline" (Min V&W, 1990). The strategic objective was to guarantee a sustainable safety level and sustainable preservation of values and functions in the dune area. To reach this strategic objective, a clear operational target has been specified: the coastline will be maintained at its position in the year 1990.

The bathymetry of the Holland coast is monitored on an annual basis through the program JARKUS which has been operational since 1963. The coastal depth profiles are measured from the first dunes up to 1 *km* in a seaward direction, at alongshore intervals of 250 *m*. The most compact and/or most convenient way to summarize the above mentioned type of information is in terms of sediment budgets and cross-shore positions. Among others, the Momentary CoastLine or MCL position is defined as a Coastal State Indicator that suitably describes the coastline position as a function of the volume of sand in the nearshore zone.

The description of the actual state of the system (i.e. the expected state for the coming year) is performed each year by means of linear extrapolation of the MCL positions observed during the 10 previous years. The actual state of the system can be therefore compared with the reference state, i.e. the position in the year 1990. This comparison provides an indication for the (expected) coastal state in the coming year: an (expected) coastline position that moves landward with respect to the reference state represents a signal to the responsible coastal authority to consider intervention. As the name "Dynamic Preservation" implies the goal to make optimal use of natural processes, the principal intervention procedure is sand nourishment.

However, we assume here that moderate rates of structural erosion or accretion should be described on a larger time scale than merely one decade. By visual inspection, the JARKUS data-set reveals typical properties of cyclic behaviour on a decadal scale. Amongst the analyses is that of Wijnberg and Terwindt (1995) pointing towards the existence of four distinct morphodynamic regimes, each characterised by their own particular sub-aqueous bar dynamics, that may affect the behaviour of the MCL position at a corresponding temporal scale. In some parts of the

Dutch coast, the return period of a certain bar topography is estimated to be about 15 *years*.

Besides, a nourishment intervention clearly represents the introduction of a disturbance to the natural configuration of a coastal system. The impact of the nourishment can be seen as a shift of the after-project MCL position-value due to the superposition to the reference state. According to the saw-tooth concept (e.g. Roelse, 2002), the erosion of the nourished sand over time implies moreover the tail off of the initial input spike in the MCL position time series. In such a case, a 10 *years* linear trend extrapolation of the MCL positions is not valid anymore, and coastal managers must predict the actual state of the system on the basis of a limited amount of observations and/or must rely on expert judgment.

Although the choice for a (10 *years*) linear trend extrapolation of the MCL positions to describe the actual state of the system was inspired by the objective to counter structural rather than incidental erosion, such a linear description has limited accuracy because of not taking into account the influence of the nearshore bars dynamics and the effect of the nourishment interventions. It appears therefore necessary to model the coastal variability, as represented here by the MCL position, by means of parameterised functions of time, representing the deterministic long-term systematic variations in the temporal evolution of the MCL position. Still, the embedding of the model in a stochastic environment should be considered, as having the important advantage that apart from estimates for the parameters also uncertainties can be derived for these model parameters, as well as uncertainties in the model and uncertainties in its predictions.

In this paper, stochastic regression models are used for a quantitative analysis of the long-term behaviour of the MCL position. The model includes several components for the mathematical representation of the most relevant physical phenomena such as: long-term trends, effects induced by the nearshore bar dynamics, and the impact of nourishments. The modelling is based on the long term trend analysis of yearly-recorded JARKUS measurement data gathered along the Holland Coast (The Netherlands). The stochastic model has been identified and applied for 3 sites along the Holland Coast.

2 THE MODEL

2.1 *Model formulation*

The starting point in the approach is that the temporal evolution of the MCL position at a particular spatial position is described by a parameterised time series model of the form:

$$Z_t = \Phi\left(t \,|\, \vec{\Theta}\right) + V_t \qquad (1)$$

For the meaning of the symbols in this equation, the following must be mentioned:

The Z_t denotes the MCL position as function of a (continuous) time *t*.

The $\Phi\left(\cdot \,|\, \vec{\Theta}\right)$ is also a function of time and represents (a parameterised model for) the deterministic long(er) term temporal variations in the MCL position.

The $\vec{\Theta}$ are one or more uncertain parameters in the description of the systematic temporal variations. Estimates of these parameters must be obtained on the basis of measurements of the MCL position.

The V_t is a random noise that accounts for errors in the modelling of the MCL position, and/or the errors in the observations. The noise V_t is particularly introduced to deal with the shorter term, non-systematic, and non-deterministic fluctuations.

As a result, Equation 1 represents a stochastic time series model for the MCL position.

An important issue in the modelling is that an accurate formulation of the deterministic component $\Phi\left(\cdot \,|\, \vec{\Theta}\right)$ is derived. Preferably this formulation is based upon physical (system and process) knowledge, and/or is based on observed patterns in measured time series of the MCL position. Such temporal patterns may represent important sub-processes of different time scales. Separate models can be derived for the sub-processes and/or time scales, leading to a superposition according to:

$$\begin{aligned}\Phi\left(\cdot \,|\, \vec{\Theta}\right) = {} & \Phi_1\left(\cdot \,|\, \vec{\Theta}_1\right) + \Phi_2\left(\cdot \,|\, \vec{\Theta}_2\right) + \\ & \Phi_3\left(\cdot \,|\, \vec{\Theta}_3\right) + \cdots\cdots + \Phi_N\left(\cdot \,|\, \vec{\Theta}_N\right)\end{aligned} \qquad (2)$$

Both from a physical viewpoint, and from visual inspection of the presently available observed time series of the MCL position, and from knowledge about human interferences in the coastal zone, it was readily concluded that in the present case at least three sub-processes/sub-models must be taken into account: (1) long term trends in the MCL position with gradual variations that extend over at least one or more decades of years, (2) cyclic or quasi-periodic variations with periods in the range of 7 to 15 *years*, representing the influence of the migratory nearshore bar behaviour, and (3) the effects of (beach, shore face) nourishments.

In the present modelling, a superposition of these sub-processes is considered. In fact, if the nourishment source material does not differ too much from the native material, we may expect that the natural variability remains approximately constant. Consequently, a linear representation of the relevant phenomena is assumed to be acceptable.

Taking all these sub-processes into account the following extension of the model is obtained:

$$\Phi\left(t\mid\vec{\Theta}\right)=\alpha_0+\alpha_1\cdot t+A\cdot\cos\left(\frac{2\pi}{P}\cdot t\right)+B\cdot\sin\left(\frac{2\pi}{P}\cdot t\right) + \Phi^{(S)}\left(t\mid\vec{\Theta}^{(S)}\right) + \Phi^{(B)}\left(t\mid\vec{\Theta}^{(B)}\right) \tag{3}$$

Parameters α account for a long-term (linear) trend in the temporal evolution of the coastal state indicator. Parameter P denotes the period (here in years) of the harmonic variation, while A and B denote the amplitudes of the associated cosine and sine components. The effect of a shore face and/or beach nourishment is represented by the sub-models $\Phi^{(S)}(\cdot)$ and $\Phi^{(B)}(\cdot)$ respectively, where:

$$\Phi^{(S)}\left(t\mid\vec{\Theta}^{(S)}\right)=\sum\nolimits_i N_i^{(S)}\left(t-\tau_i^{(S)}\right) \tag{4}$$

and similarly

$$\Phi^{(B)}\left(t\mid\vec{\Theta}^{(B)}\right)=\sum\nolimits_j N_j^{(B)}\left(t-\tau_j^{(B)}\right) \tag{5}$$

The $\tau_i^{(S)}$ in Equation 4 denote the times of (individual) *shore face* nourishments. The response or transfer function $N_i^{(S)}(\cdot)$ represents the (after)effect of a nourishment at time $\tau_i^{(S)}$ on the coastal state indicator. This (impulse) response function is modelled as follows:

$$N_i^{(S)}(t)=K_i^{(S)}\cdot c^{(S)}\cdot\frac{\exp\left(\frac{t-\mu_1^{(S)}}{\lambda_1^{(S)}}\right)}{1+\exp\left(\frac{t-\mu_1^{(S)}}{\lambda_1^{(S)}}\right)}\cdot\frac{\exp\left(\frac{t-\mu_2^{(S)}}{\lambda_2^{(S)}}\right)}{1+\exp\left(\frac{t-\mu_2^{(S)}}{\lambda_2^{(S)}}\right)} \tag{6}$$

In this way, the effect of a shore face nourishment is represented by a product of two sigmoid functions, each including two parameters consisting of a shift parameter μ and a shape parameter λ. Parameter c in Equation 6 is a scaling factor that represents the effect of a nourishment of *unit* magnitude. At the same time, a parameter K is included to account for the *actual* amount (volume or density) of the nourishment at time $\tau_i^{(S)}$. This amount K_i will be known for each nourishment time $\tau_i^{(S)}$, but for the other five parameters $\vec{\Theta}^{(S)}:=\left(\mu_1^{(S)},\lambda_1^{(S)};\mu_2^{(S)},\lambda_2^{(S)};c^{(S)}\right)$, estimates must be obtained through model calibration.

For the modelling of the response to *beach* nourishments, here denoted by $N_i^{(B)}(\cdot)$, a fully similar approach is followed as described above for $N_i^{(S)}(\cdot)$, leading to the following formulation:

$$N_j^{(B)}(t)=K_j^{(B)}\cdot c^{(B)}\cdot\frac{\exp\left(\frac{t-\mu_1^{(B)}}{\lambda_1^{(B)}}\right)}{1+\exp\left(\frac{t-\mu_1^{(B)}}{\lambda_1^{(B)}}\right)}\cdot\frac{\exp\left(\frac{t-\mu_2^{(B)}}{\lambda_2^{(B)}}\right)}{1+\exp\left(\frac{t-\mu_2^{(B)}}{\lambda_2^{(B)}}\right)} \tag{7}$$

This model for the effect of beach nourishments induces another set of five parameters $\vec{\Theta}^{(B)}:=\left(\mu_1^{(B)},\lambda_1^{(B)};\mu_2^{(B)},\lambda_2^{(B)};c^{(B)}\right)$ that must be estimated through model calibration.

The model parameters $\vec{\Theta}$ in Equation (3) then finally consist of:

$$\vec{\Theta}:=\left(\alpha_0,\,\alpha_1,\,A,\,B,\,P,\,c^{(S)},\,\mu_1^{(S)},\lambda_1^{(S)},\mu_2^{(S)},\lambda_2^{(S)},\,\ldots\,,\; c^{(B)},\mu_1^{(B)},\lambda_1^{(B)},\mu_2^{(B)},\lambda_2^{(B)}\right) \tag{8}$$

For more details on the mathematical formulations of the sub-models $\Phi_n\left(\,\cdot\mid\vec{\Theta}_n\right)$ (and the involved uncertain model parameters $\vec{\Theta}_n$), one is referred to Brière and Van den Boogaard (2008).

Finally, a model must be formulated for the random fluctuations V_t as well. In the present case this V_t is assumed to be a zero-mean Gaussian noise. The spread σ (or variance σ^2) of the noise V_t is usually not known. Together and simultaneously with the deterministic model parameters $\vec{\Theta}$, the spread σ must then be estimated from the observed MCL positions.

2.2 *Calibration of the model*

Dealing with a stochastic model implies that a statistically consistent and well defined calibration procedure must be applied. Here a so-called Maximum LikeliHood (MLH) approach is followed. This leads to a procedure where a function of the parameters, the so-called minus log-likelihood function, must be minimised. This log-likelihood function is closely related to a least squares criterion. Due to non-linearities in the modelling, numerical techniques had to be used for the actual minimisation of the log-likelihood function. The result of the calibration are optimal estimates $\hat{\Theta}$ and $\hat{\sigma}$ for the uncertain model parameters.

2.3 *Uncertainties in the parameters and model-predictions*

The stochastic formulation of the models, and the Maximum Likelihood based calibration procedure, has an important advantage that also uncertainties in the (estimates $\hat{\Theta}$ and $\hat{\sigma}$ for the) parameters can be derived in a fully quantitative form. In standard MLH procedures it is assumed that the estimates $\hat{\Theta}$ and $\hat{\sigma}$ satisfy a Gaussian distribution, and from the minimised log-likelihood function the spread in the $\hat{\Theta}$ and $\hat{\sigma}$ can be computed. From these spreads, and still assuming a Gaussian distribution, confidence intervals (e.g. 95%) can be constructed. These (symmetric) confidence intervals can be used for assessing the accuracy and (statistical) significance of the estimates for the parameters.

3 MODEL APPLICATION

3.1 *Heemskerk*

Heemskerk is located in the central part of the North-Holland coast, and belongs to the LSCB-region III following the classification of Wijnberg and Terwindt (1995). In this area, the return period of a certain bar topography is estimated to be about 15 *years*. No nourishments have been directly applied at Heemskerk and we assume that the lee-influence of adjacent nourishments is weak. Consequently, the sub-processes that have been taken into account are the long-term trend in the MCL position and the cyclic variation representing the influence of the migratory nearshore bar behaviour. The model of Equation 3 is therefore reduced to:

$$\Phi\left(t \mid \vec{\Theta}\right) = \alpha_0 + \alpha_1 \cdot t + A \cdot \cos\left(\frac{2\pi}{P} \cdot t\right) + B \cdot \sin\left(\frac{2\pi}{P} \cdot t\right) \quad (9)$$

The long-term behaviour of the MCL position is displayed in Figure 1. The MCL position exhibits a regressive evolution over the last decades, at a rate of –0.2 *m/year*. A cyclic component, representing the influence of the migratory bar behaviour, has been added, characterised by a calculated period of the oscillation of 12.8 *years*, that is consistent with the analysis of Wijnberg and Terwindt (1995).

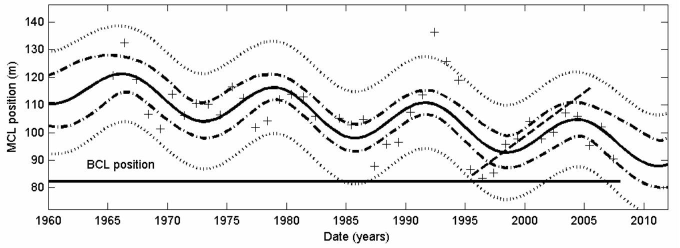

Figure 1. Long-term behaviour of MCL position (*m*), where the symbols '+' represent the observations. The solid line, the dashed-dotted lines, and the dotted lines represent the simulated mean trend, the boundaries of the 95% skew confidence interval, and the boundaries of the 95% skew prediction interval, respectively. The dashed line represents the linear trend for the period 1995-2005.

By taking into account the cyclic variation representing the influence of the migratory nearshore bar behaviour, instead of extrapolating the MCL positions by means of a 10 *years* linear trend, the stochastic model describes the long-term evolution of the MCL position with more insight into the existing physical phenomena. For example, Figure 1 shows that the linear trend estimate (dashed line) for the period 1995-2004 leads to a significant overestimation of the MCL position value in 2005 when compared to the model prediction.

3.2 *Zijpe*

Zijpe is located in the northern part of the North-Holland coast, and belongs to the LSCB-region II following the classification of Wijnberg and Terwindt (1995). In this area, the return period of a certain bar topography is estimated to be about 15 *years*. Along the considered coastal stretch, beach and shore face nourishments (Figure 4, bottom panel) have been applied in the past and the representation of their impacts has been considered when investigating the long-term behaviour of the MCL position.

Consequently, the sub-processes that have been taken into account in the modelling are the long-term trends in the MCL position, the cyclic variation representing the influence of the migratory near-shore bar behaviour, and the effects of both beach and shore face nourishments (see Equation 3).

The long- term behaviour of the MCL position is displayed in Figure 2 (top panel). The MCL position exhibits a slightly regressive evolution until 1987, at a rate of –0.1 *m/year*. A cyclic component, representing the influence of the migratory bar behaviour, has been added, and, within the model calibration, a period of 14.9 *years* was identified for this oscillation. This estimate agrees well with the literature, see Wijnberg and Terwindt (1995).

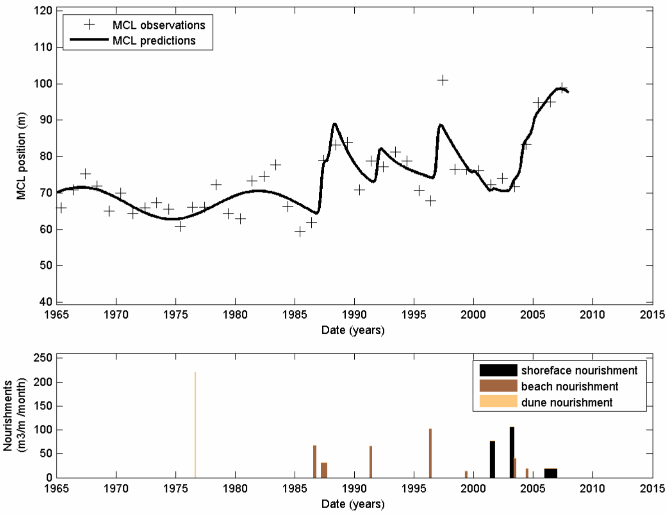

Figure 2. Long-term behaviour of MCL position (*m*), where dots and the solid line represent the observations and the identified mean trend, respectively.

Accretion (in terms of seaward migration of the MCL position) is noticed after the application of the nourishments (Figure 2), and the positive impacts are gradually disappearing after a couple of years. The model properly reproduces these phenomena. In fact, the transfer functions representing the impacts of a beach and a shore face nourishments on the

MCL position (Figure 3) exhibit the following behaviours: (1) in case of a beach nourishment, the maximum impact is found about 0.5 to 1 *year* after the application of the nourishment, and the lifetime is estimated to be about 7 to 8 *years*; (2) in case of a shore face nourishment, the maximum impact is delayed, occurring about 5 *years* after the application of the nourishment, and the lifetime is estimated to be about 8 to 9 *years*.

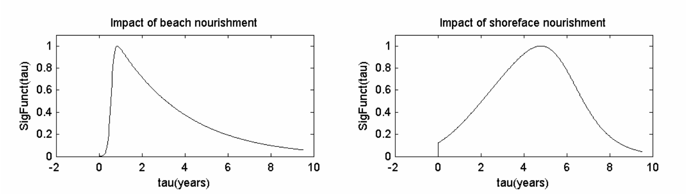

Figure 3. Normalised identified transfer functions representing (a) the impact of a beach nourishment on the MCL position, and (b) the impact of a shore face nourishment on the MCL position.

These findings agree well with the saw-tooth concept (see e.g. Roelse, 2002), which is used globally as a nourishment design basis.

As discussed by Lescinski *et al.* (2008), there are two key components in the saw-tooth design: first, a steeply positive-sloped trend, illustrating the initial increase in the beach volume due to the placement of the nourishment, and secondly, the tail off of the initial input spike depicting the erosion of the nourished sand over time; the slope of this tail being equal to the decay rate of the nourishment.

The model is able to provide a quantitative estimate of these parameters (i.e. initial increase, decay rate).

3.3 *Bergen-aan-Zee*

Bergen-aan-Zee is located in the central part of the North-Holland coast, and belongs to the LSCB-region III following the classification of Wijnberg and Terwindt (1995). In this area, the return period of a certain bar topography is estimated to be about 15 *years* as well, and shore face nourishments have been applied since 1990 (Figure 4, bottom panel).

Consequently, the sub-processes that have been taken into account in the present modelling are the long-term trends in the MCL position, the cyclic variation representing the influence of the migratory nearshore bar behaviour, and the effects of both beach and shore face nourishments (see Equation 3).

The long-term behaviour of the MCL position is displayed in Figure 4 (top panel). The MCL position exhibits a regressive evolution until 1990, at a rate of –0.4 *m*/*year*. A cyclic component, representing the influence of the migratory bar behaviour, has been added, and is characterised by a calculated period of the oscillation of 9.5 *years*, that is much lower than the *prior* expected 15 *years*-value.

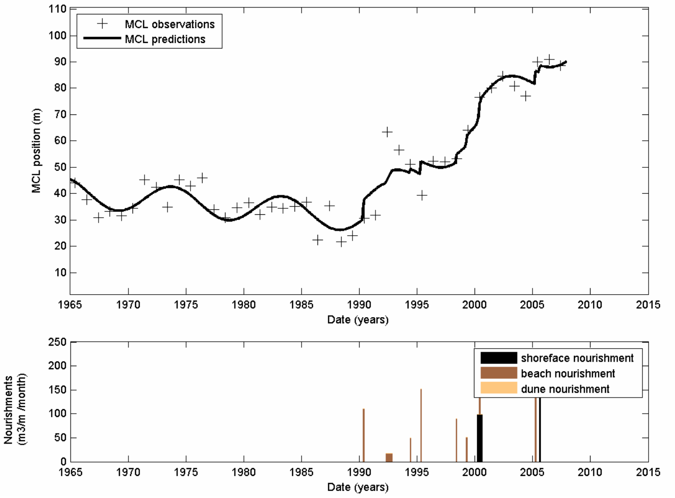

Figure 4. Long-term behaviour of MCL position (*m*), where dots and the solid line represent the observations and the simulated mean trend, respectively.

In case of a beach nourishment (Figure 5a), the first impact is found immediately after the application of the nourishment, and a gradual seaward migration of the MCL position is then observed. In case of shore face nourishment, the maximum impact is obtained immediately after the application of the nourishment, and the transfer function exhibits then an asymptotic behaviour. In such a case, the saw-tooth concept is not valid anymore. Although the initial spike in the saw-tooth is observed, the decreasing tail is not (re)produced through the modelling. Such shapes of the transfer functions are associated to a blocking mechanism, in combination with the net longshore transport occurring on the beach system, as also shown by Lescinski *et al.* (2008); sand is then transferred to the dune system. The accretion appearing after the nourishment and post initial blocking phase is consistent with van Duin *et al.* (2004) and with Grunnet and Ruessink (2005).

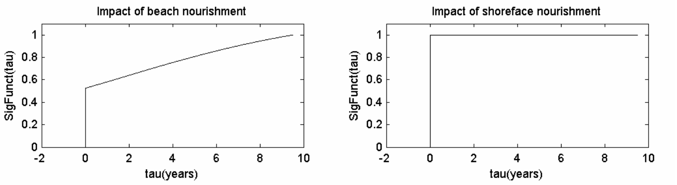

Figure 5. Normalised calculated transfer functions representing (a) the impact of a beach nourishment on the MCL position, and (b) the impact of a shore face nourishment on the MCL position.

4 CONCLUSIONS

Considering 43 *years* long records of measured JARKUS data, stochastic regression models have been identified for a quantitative description of the long-term behaviour of the Momentary Coastline or MCL position. The modelling includes mathematical representations of the following physical processes and/or interventions: (1) the long-term trend in the

MCL temporal evolution, (2) the influence of nearshore bars, and (3) the impact of sand nourishments.

The stochastic model has been identified and applied to 3 different sites (with one site not influenced by nourishments, and two others with significant nourishments), showing its ability to deal with structural evolutions of the MCL position (e.g. effects induced by the nearshore bar dynamics).

Besides, the stochastic model appears to be a useful tool to describe mathematically the impact of nourishments on the long-term evolution of the MCL position. The modelling displays either a regressive (in time) behaviour of the transfer function representing their impact, that is consistent with the saw-tooth concept, or a retentional or progressive (in time) behaviour of the transfer function, that is associated to a blocking mechanism in combination with the net longshore transport. Still, these inconsistencies in the system response to nourishment interventions carry interesting implications for current coastal management practices, and they especially support the use of the stochastic model for the quantitative description of the actual state of the coastal system.

Moreover, the description of the actual state of the system is performed with more insight into the structural developments (in particular related to the nearshore bars dynamics) when using the stochastic model rather than simply extrapolating the MCL positions by means of a 10 *years* linear trend. Furthermore, the model application leads to an indication for the (expected) coastal state in the years T to T+5.

More generally, with a quantitative measure for the model parameters and the uncertainties, the MCL positions can be predicted for a quite extended period out of the range of the observations, and the model can therefore be used in decision support issues and/or for the design (length, volume, implementation time) of optimal nourishment strategies.

REFERENCES

Brière, C., Van den Boogaard, H.F.P. 2008. Statistical uncertainty assessment of Coastal State Indicators. *International Conference on Coastal Engineering*. Hamburg. Germany. August 2008.

Grunnet, N.M., Ruessink, B.G. 2005. Morphodynamic response of nearshore bars to a shoreface nourishment. *Journal of Coastal Engineering* 52: 119-137.

Lescinski, J., De Sonneville, B., Van Koningsveld, M. 2008. Nourishment effectiveness along the Holland Coast. Research note. Deltares. Delft. The Netherlands.

Min V&W. 1990. Coastal Defence after 1990, a Policy Choice for Coastal Protection. 1st Coastal Policy Document, Ministry of Transport, Public Works and Watermanagement. The Hague. The Netherlands.

Roelse, P. 2002. Water en Zand in Balans. Evaluatie zandsuppleties na 1990; een morfologische beschouwing. Report RIKZ-2002.003. ISBN 90-36-369-3426-5. (In Dutch)

Van Duin, M.J.P., Wiersma, N.R., Walstra, D.J.R., van Rijn, L.C., Stive, M.J.F. 2004. Nourishing the shoreface: observations and hindcasting of the Egmond case, The Netherlands. *Journal of Coastal Engineering*. 51: 813–837.

Wijnberg, K.M, Terwindt, J.H.J. 1995. Extracting decadal morphological behaviour from high-resolution, long-term bathymetric surveys along the Holland coast using eigenfunction analysis. *Marine Geology* 126: 301-330