Surface Lagrangian Coherent Structures in the Gulf of Trieste

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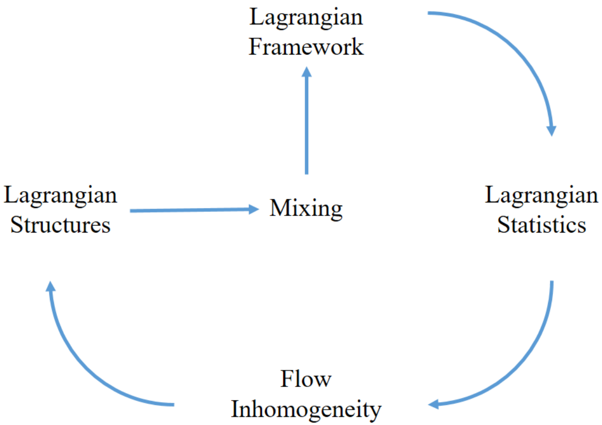
Conceptual Map

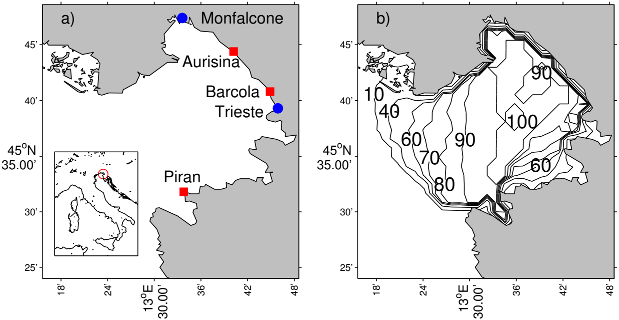


Research Topics

- Mixing in the Gulf of Trieste
  - Introduction to the dataset: velocity fields and drifter trajectories
  - Statistics from Lagrangian numerical observations
  - LCS as finite-time Lyapunov Exponents
  - LCS as tensorlines of the Cauchy-Green Tensor

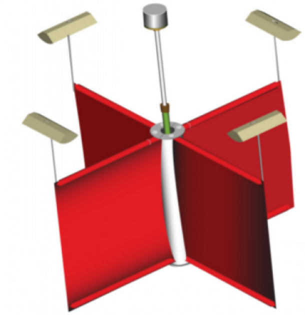
Gulf of Trieste dataset

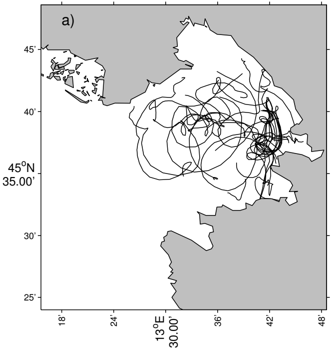
The dataset consists in surface velocity fields recorded by Coastal Radars (CODAR) in the Tracking Oil Spills & Coastal Awareness Network (TOSCA) Project.



Gulf of Trieste dataset

CODE drifters were deployed. 26 trajectories were adopted in order to validate the results.





Statistics from Numerical Observations

Absolute Dispersion

Absolute dispersion tensor is calculated above particle trajectories:

$$A_{ij}^2(t) = \frac{1}{M}\sum_{m=1}^{M}\left\{\left[x_i^m(t) - x_i^m(t_0)\right]\left[x_j^m(t) - x_j^m(t_0)\right]\right\} \tag{1}$$

The total absolute dispersion:

$$a^2(t) = \mathrm{Tr}\left[\boldsymbol{A}^2(t)\right] \tag{2}$$

The total absolute diffusivity is:

$$K^{(1)}(t) = \frac{1}{2}\frac{d}{dt}\left\{\mathrm{Tr}\left[\boldsymbol{A}^2(t)\right]\right\} \tag{3}$$

Typical dispersion regimes are:

$$A_{ii}^2(t) = \rho_{L_{ii}}(0)t^2 \qquad t < T_{L_i} \tag{4}$$

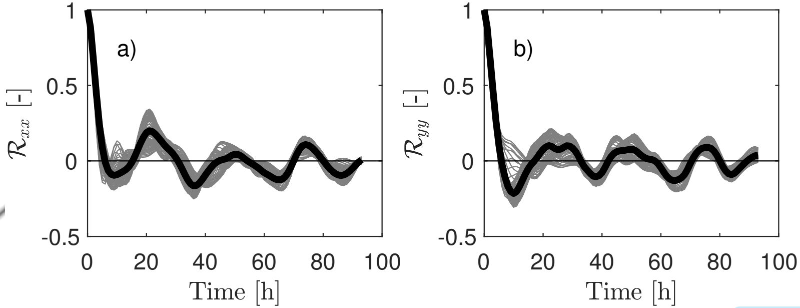
$$A_{ii}^2(t) = 2\rho_{L_{ii}}(0)T_{L_i}t + \text{const.} \qquad t > T_{L_i} \tag{5}$$

Velocity autocorrelation

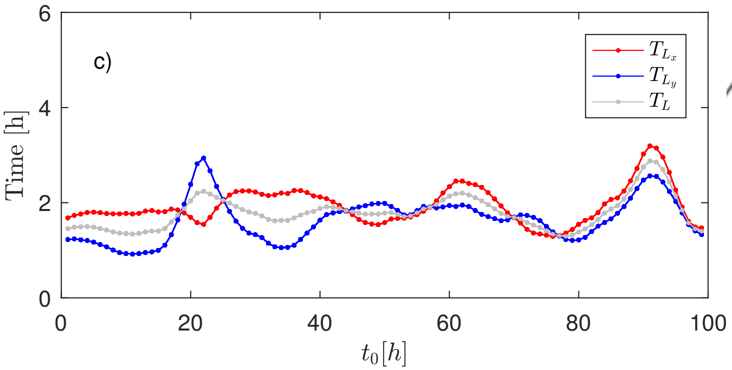
$$\mathscr{R}_{ii}(\tau) = \frac{\frac{1}{M}\sum_M \rho_{L_{ii}}(\tau)}{\sqrt{\rho_{L_{ii}}(0)\rho_{L_{ii}}(0)}} \tag{6}$$

where

$$\rho_{L_{ii}}(\tau) = \langle u'_{L_i}(t) u'_{L_i}(t+\tau)\rangle \tag{7}$$

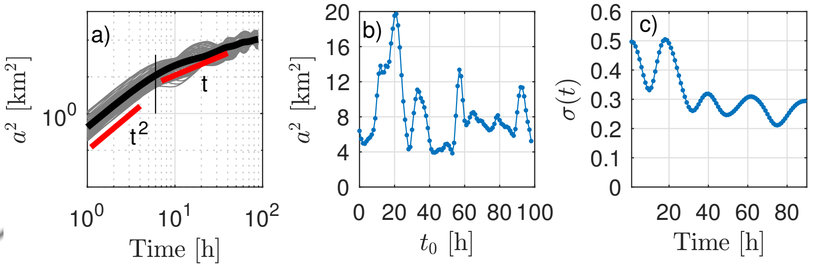


Lagrangian Integral Scale



Absolute Dispersion

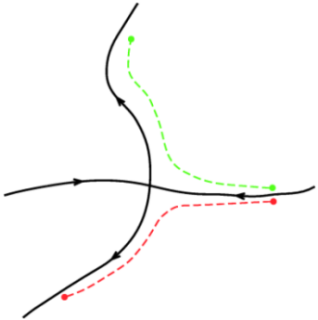
Panel a): absolute dispersion varying the initial conditions
Panel b): absolute dispersion at the same time-instance
Panel c): influence of the initial conditions $\sigma(t) = \text{std}[a^2(t)]/\mu(t)$



Lagrangian Coherent Structures

Lagrangian Coherent Structures

Lagrangian Coherent Structures (LCS) capture transient coherent transport dynamics in unsteady and aperiodic fluid flows that are known over finite time.



Lagrangian Coherent Structures

- Release a grid of tracers and see where they go: flow map

$$\boldsymbol{x}(t) = \boldsymbol{x_0} + \int_{t_0}^{t_0+T} \boldsymbol{u}(\boldsymbol{x}, t)dt \tag{8}$$

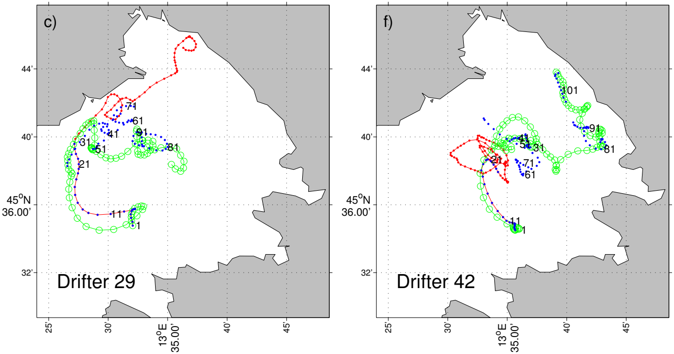
- Differentiate the flow map: compute Cauchy-Green Tensor

$$\boldsymbol{C}_{t_0}^{t_0+T} = \left[\frac{dF_{t_0}^{t_0+T}}{d\boldsymbol{x_0}}\right]^* \left[\frac{dF_{t_0}^{t_0+T}}{d\boldsymbol{x_0}}\right] \tag{9}$$

- Compute the maximum finite-time Lyapunov exponent (FTLE) field

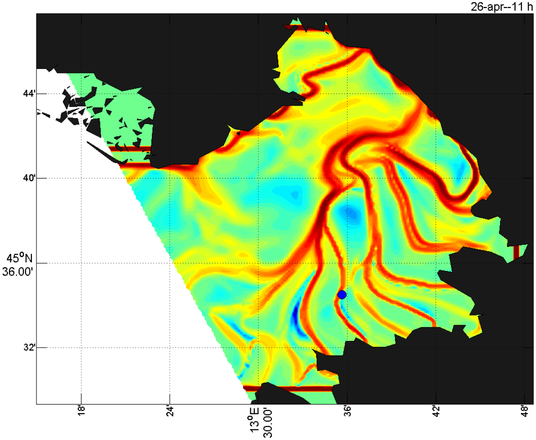
$$\sigma_{t_0}^{t_0+T} = \frac{1}{T}\log\sqrt{\lambda_{max}} \tag{10}$$

Drifter simulated as single particles

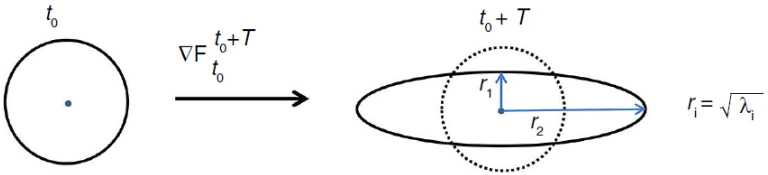


FTLE in The Gulf of Trieste

Green point: real drifter. Red point: simulated drifter. Blue point: simulated drifter with reseeding.



Tensorlines of the Cauchy-Green Tensor

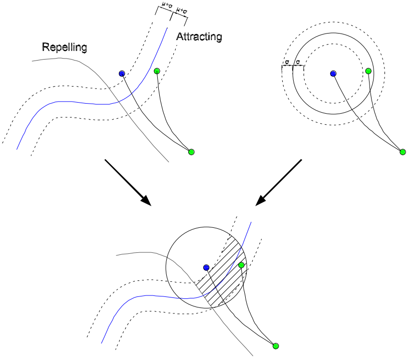


Tensorlines: $x' = \xi_1$, $x' = \xi_2$

Most repelling and attracting material lines

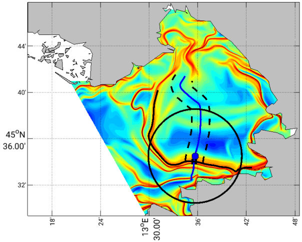
Search Approach

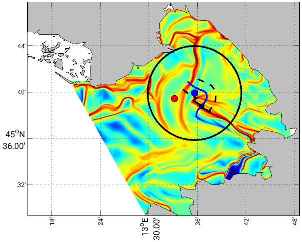
Joint application of LCS and single-particle tracking.



Search Approach

Example of two reseeding time windows:





Conclusions

- Mixing in a semi-enclosed basin:
  - Lagrangian statistics in a tidal dominated basin are influenced by initial conditions
  - Single-particle tracking simulations are affected by some uncertainties
  - Lagrangian Coherent Structures detected directions of transport
  - Joint approach: LCS/single-particle tracking

Thank you for your attention!