Singular value decomposition and correlation analysis to optimize in-situ observation networks.

Maria Fattorini, Carlo Brandini

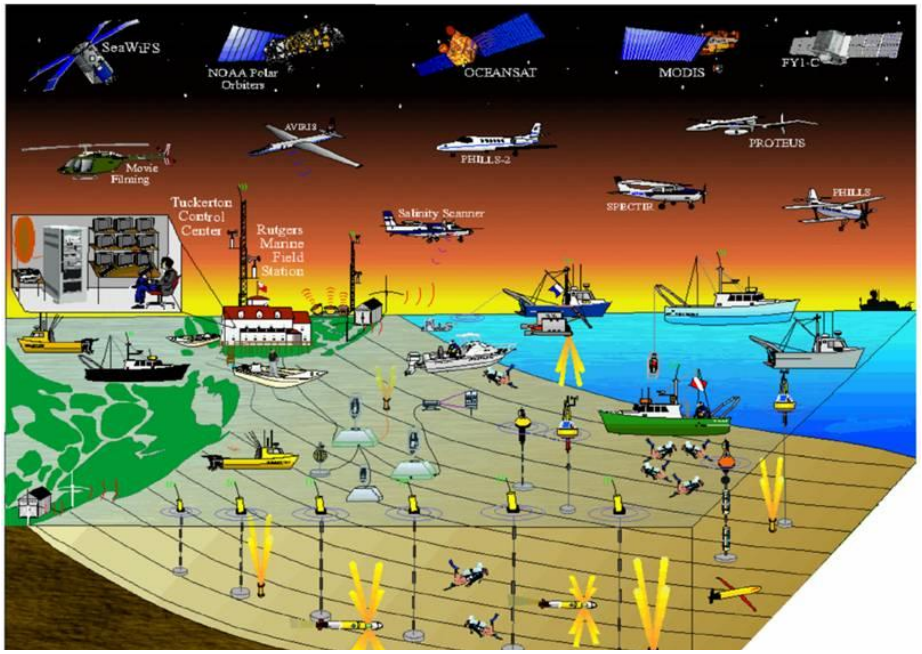
1) CNR-IBIMET Institute of biometeorology, Firenze, Italy
2) Consorzio LaMMA, Sesto Fiorentino, Firenze, Italy

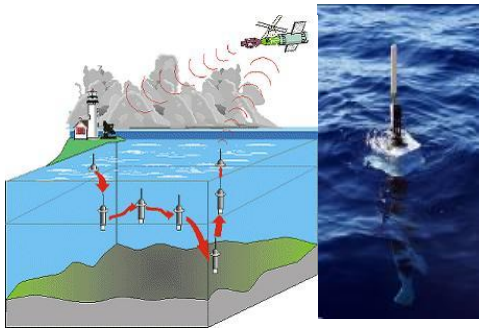
JONSMOD

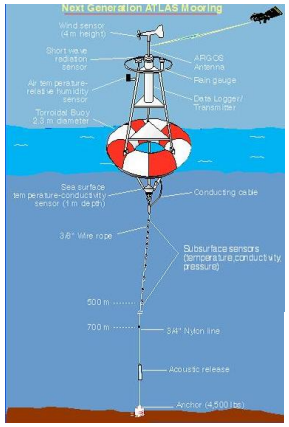
17-19 October 2018

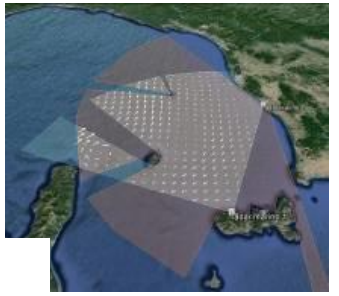
Accademia dei Georgofili - Firenze

Design of observation networks

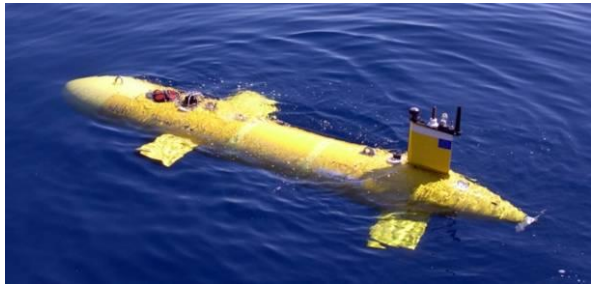




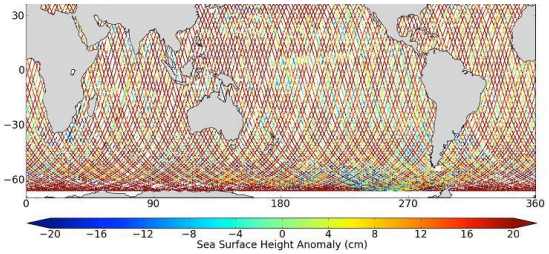


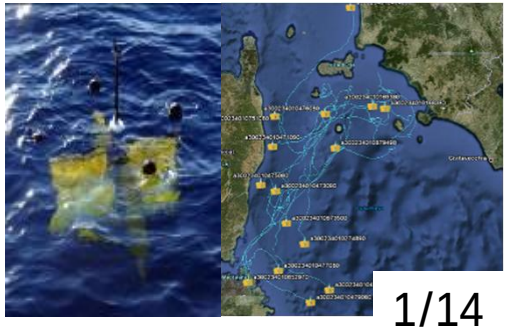


The core design of ocean observing networks:
Which tools? How many? Which positions?



It depends on our goal

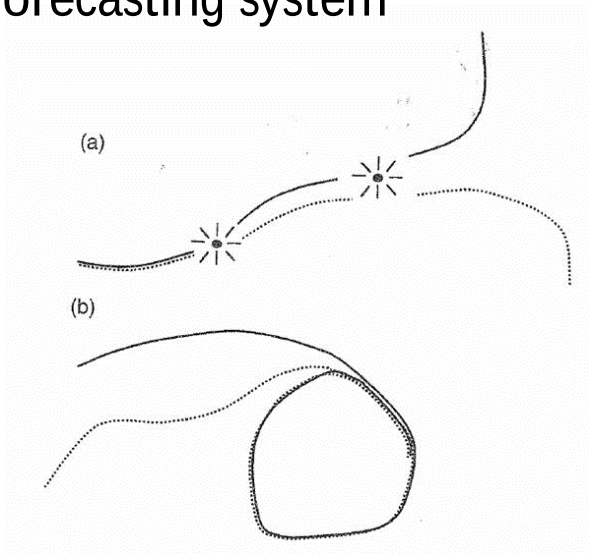




Error dynamics and SVD

GOAL: to increase the reliability of the ocean forecasting system through data assimilation

Errors in Initial Conditions evolve by the dynamics equations:
some of them grow
some decay



Singular Value Decomposition (SVD) of the tangent linear propagator:
A tool from the generalized stability theory (Farrell and Joannou, 1996).

DATA TO MODEL
MODEL TO DATA

The use of SVD in sampling is not new [FASTEX, NORPEX,... Langland et al 1999, Bishop and Toth 1999, Szunyogh et al. 1999,...]

Non linear model, tangent liner model and adjoint model

NLM: $x_{t+1} = M(x_t)$

TLM: $dx_{t+1} = TLM(x_t) = L\, x_t$

ADM: $dx^*_t = ADM(x^*_{t+1}) = L^T x^*_{t+1}$

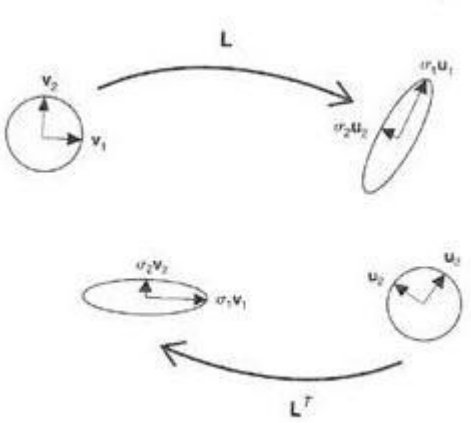
Singular Values Decomposition

SVD of the tangent propagator L: eigenproblem of LL^T or L^TL

•Initial singular vectors: perturbations which grow the most over a given time period (T_op)

•Final singular vectors: evolved initial singular vectors by tangent linear model

•Singular values: growth factor of perturbations



Use of TANGENT and ADJOINT models:

-SVD (implemented in ROMS, Moore et al 2009)

-4DVar (in ROMS, Arango et al 2003)

-Obs impact (in ROMS, Moore et al 2011)

-Obs sentitivity (in ROMS, Moore et al 2011)

How to use SVD in practice

Where to install instruments in order to reduce errors in a forecast systems?

SVD (errors growth areas)

+

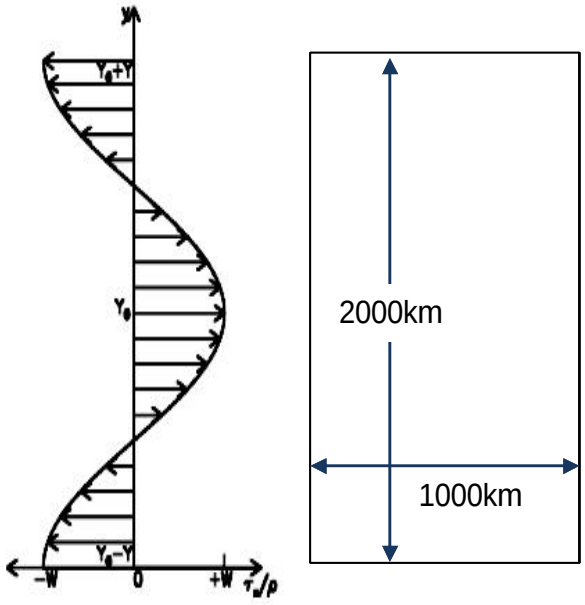
Minimum distance among instruments (for avoid redundancy)

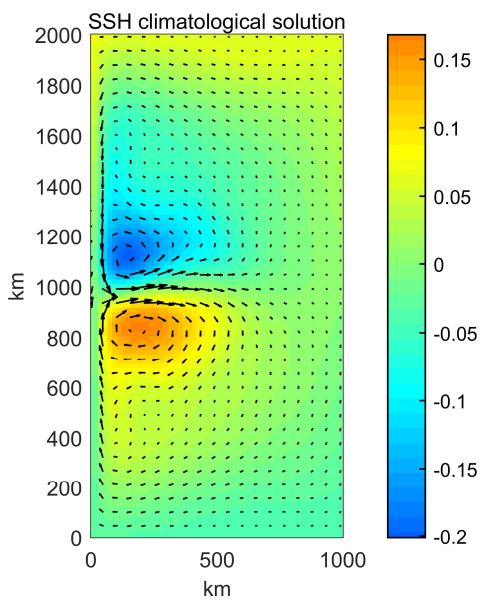
a fixed distance

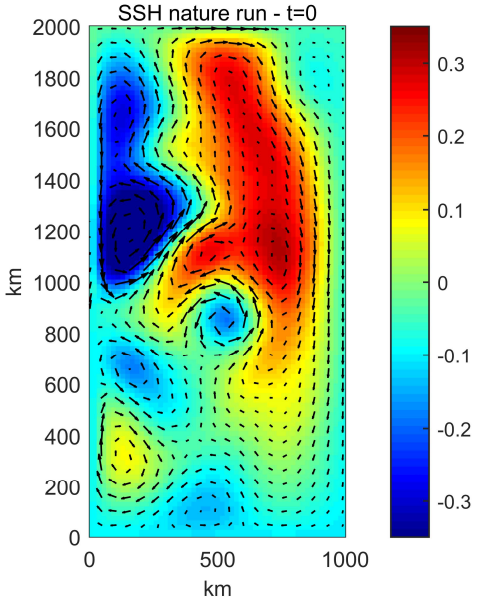
a distance from a threshold of max correlation (points with higher correlation are not considered)

SET-UP OF NUMERICAL EXPERIMENTS

- Double gyre model (flat-basin, all closed boundaries, wind-forced) implemented in ROMS
- 20 velocity profile observations at fixed points (ie ADCP) to be installed
- Climatological initial condition (average of 10-years-long solution)
- OSSE-like terminology: Observations are extracted from a “nature run” (taken as truth)
- Assimilation window 5-days-long (ROMS 4DVar)
- Ensuing 5-days-long forecast







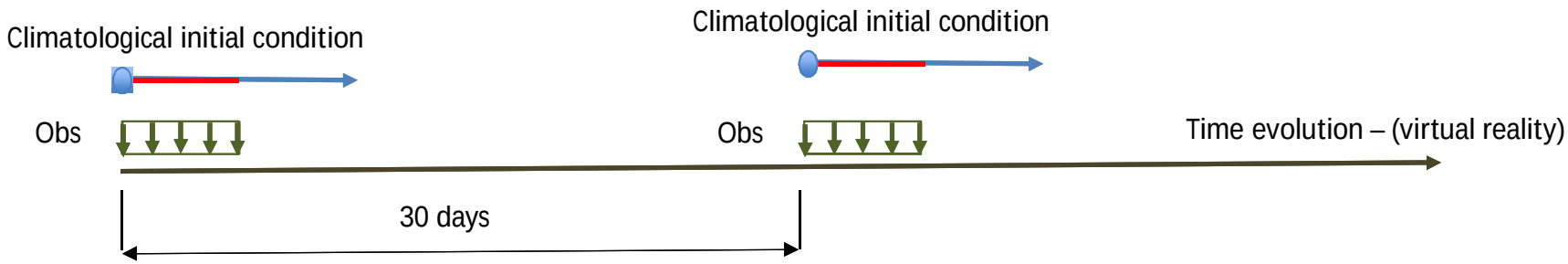
SET-UP OF NUMERICAL EXPERIMENTS

We compare two different criteria:

1.Random observations

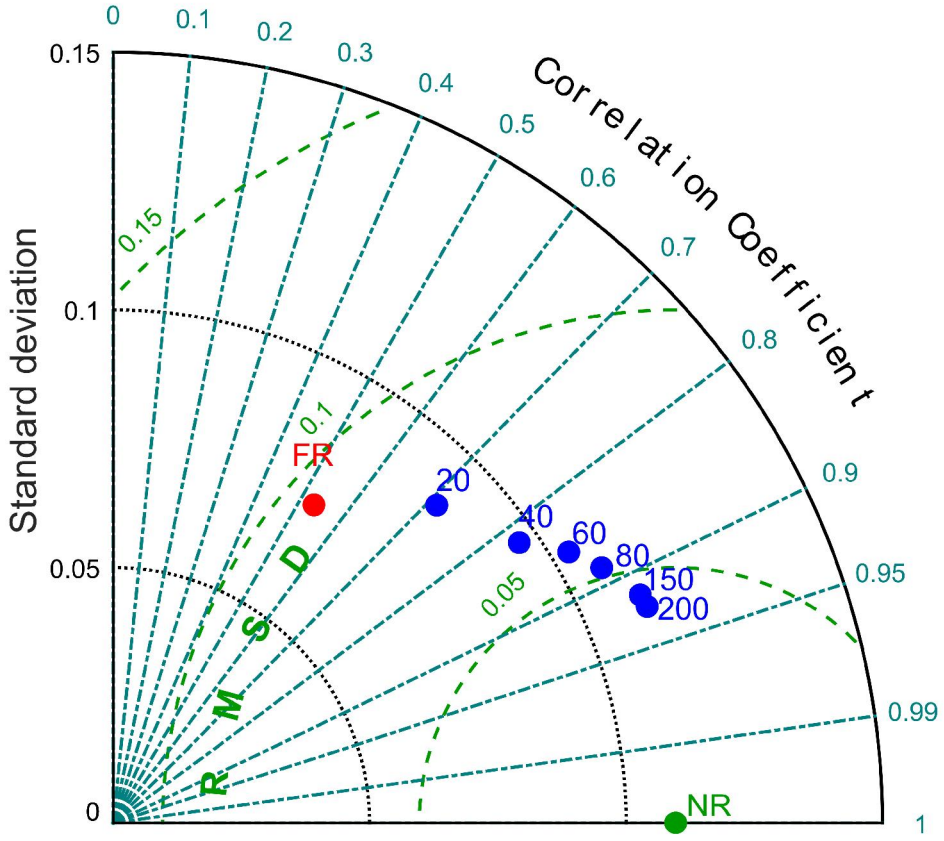
2. A SVD based criteria, which is based on both SVD and the correlation of the velocity field to avoid information redundancy in assimilated data.

We have repeated the experiment for time windows 30 days apart, to cover a year.
Every run starts from the climatological state.



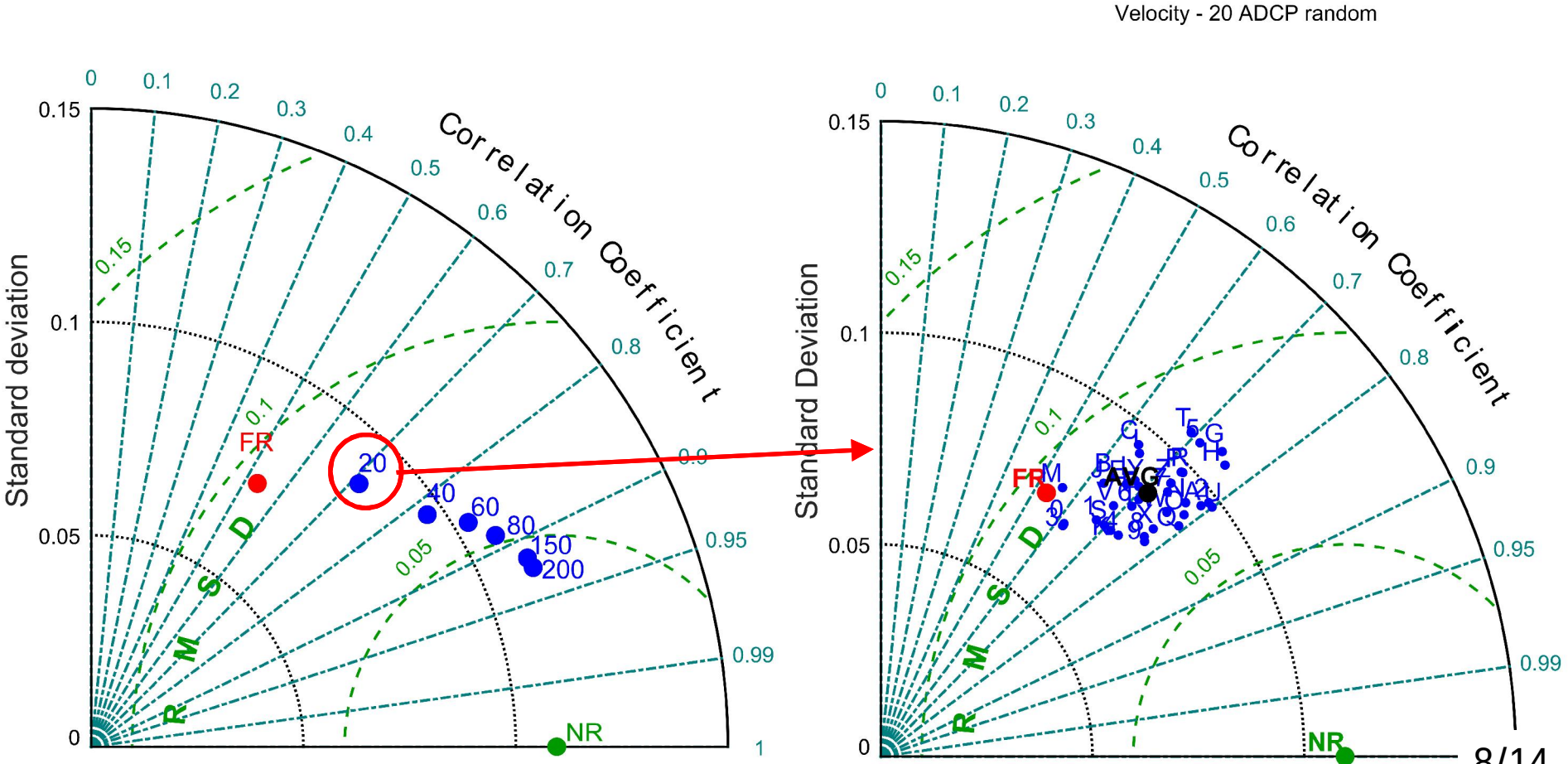
SAMPLING AT RANDOM POSITIONS

Averaged analysis statisticsfor an increasing number of observing tools



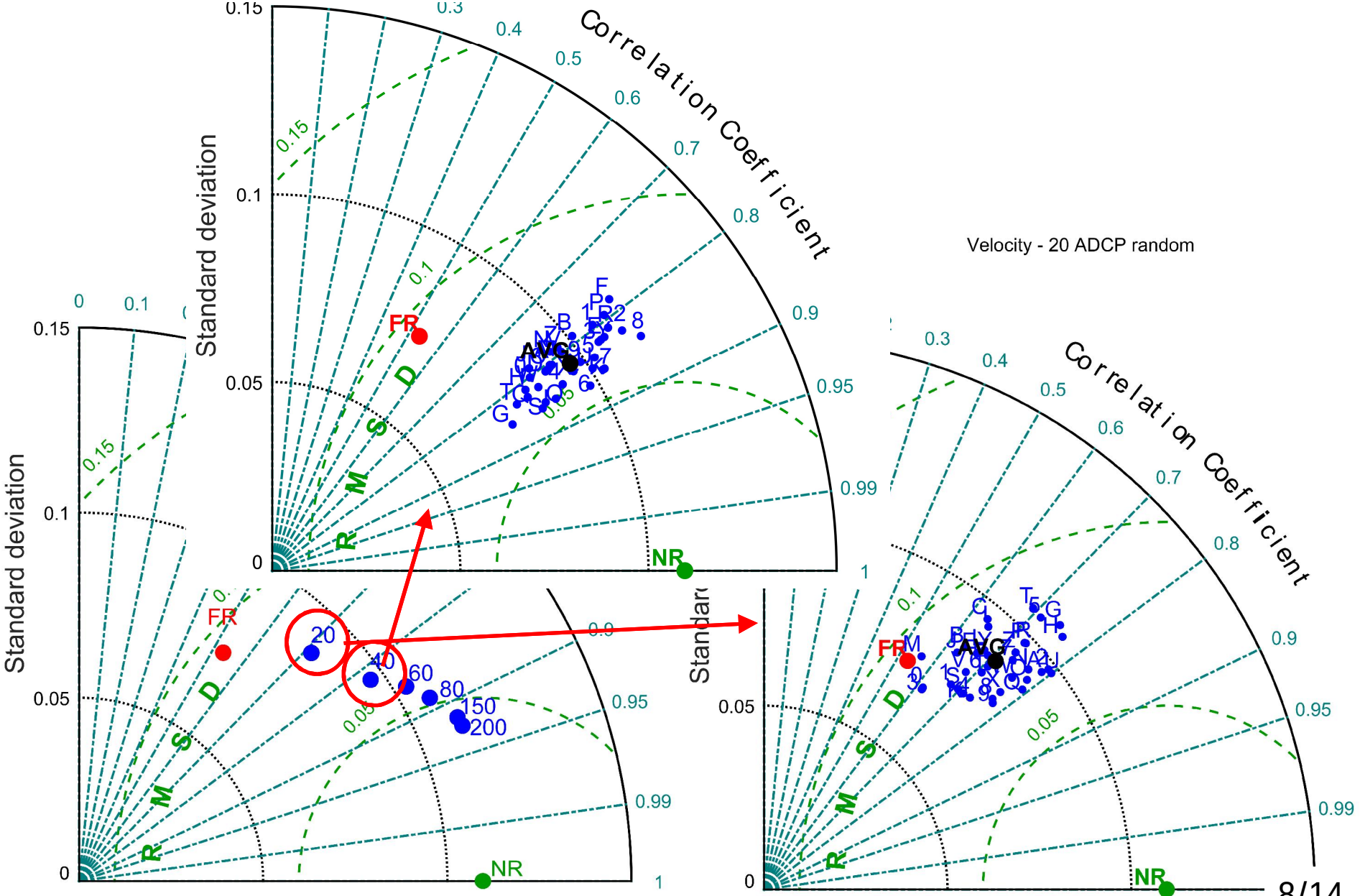
SAMPLING AT RANDOM POSITIONS

Averaged analysis statisticsfor an increasing number of observing tools



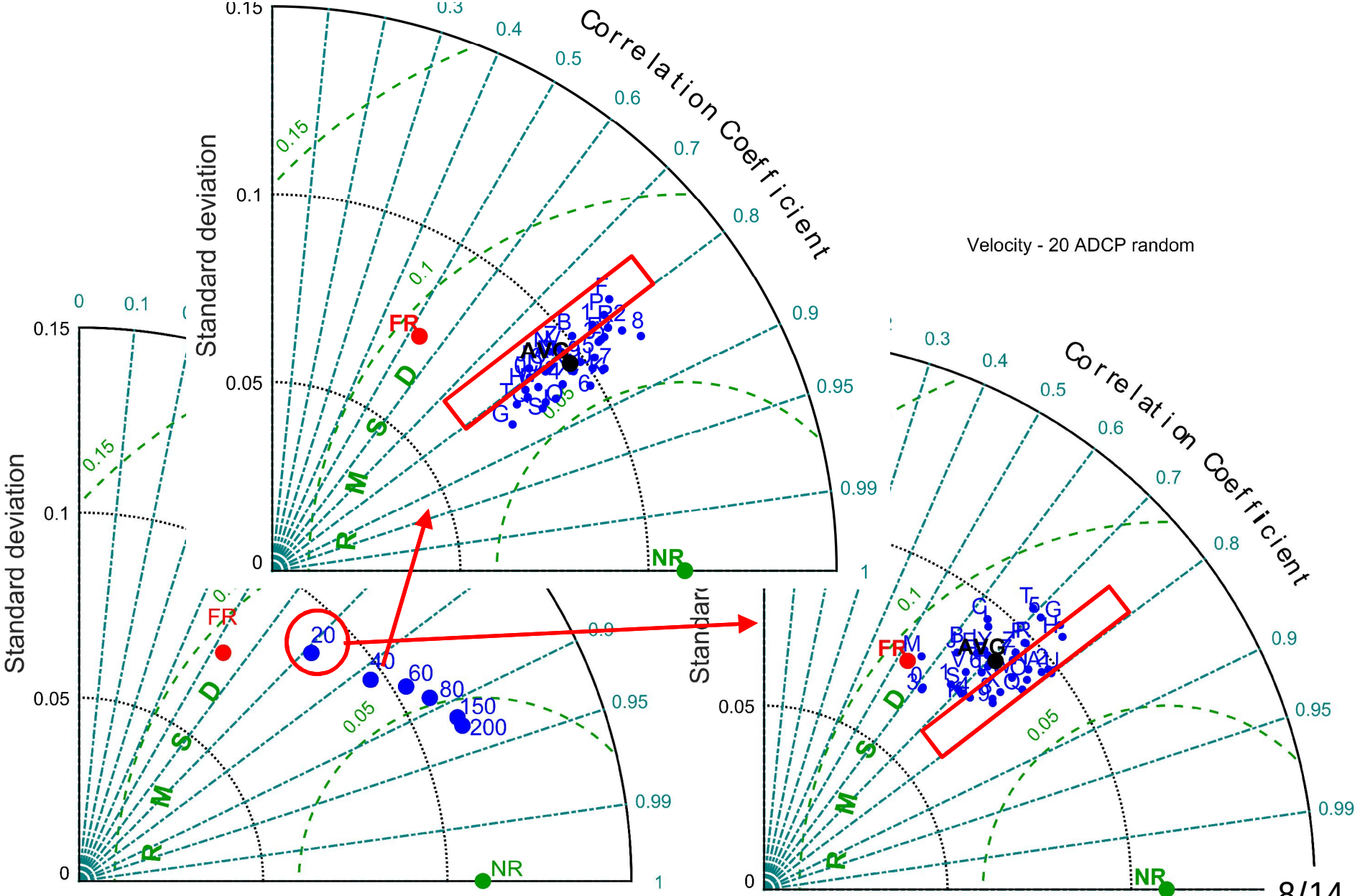
Velocity - 40 random obs points

SAMPLING AT RANDOM POSITIONS



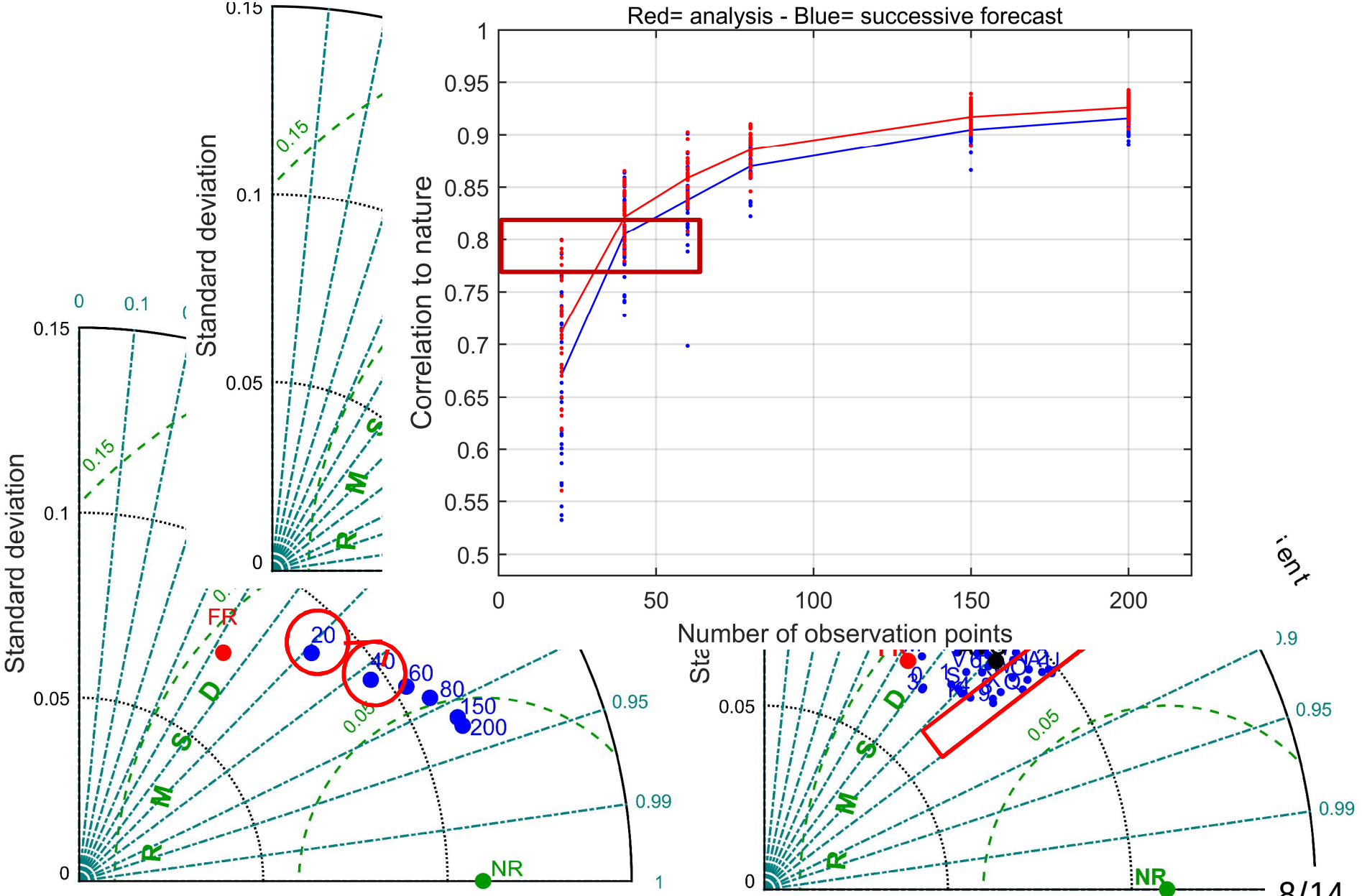
Velocity - 40 random obs points

SAMPLING AT RANDOM POSITIONS



Velocity - 40 random obs points

SAMPLING AT RANDOM POSITIONS

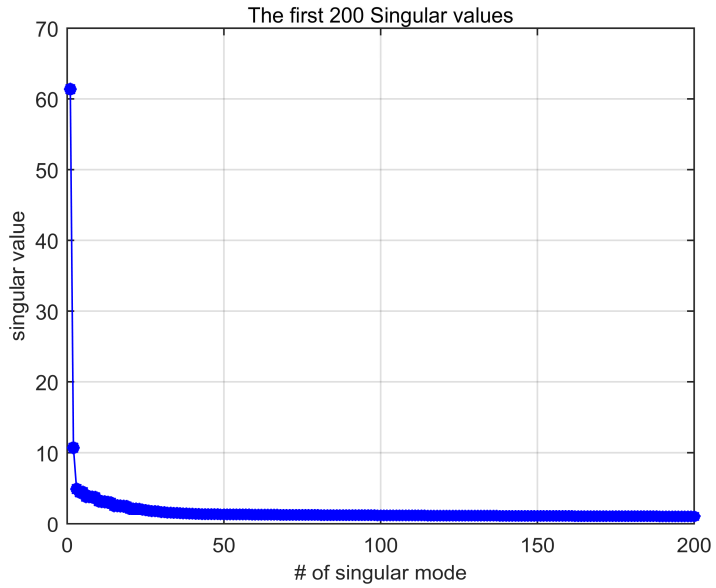


SAMPLING based on SVD criteria

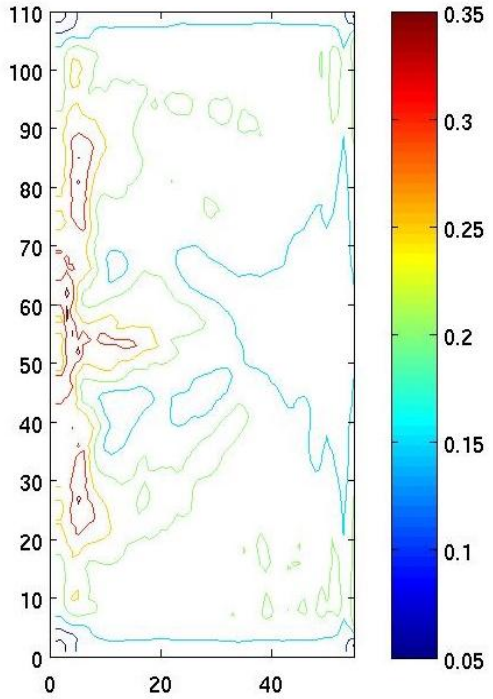
A criteria based on two elements:

1. SVD of the tangent linear propagator of the model

Singular Values



Initial SVs. weighted on the relative Singular value)

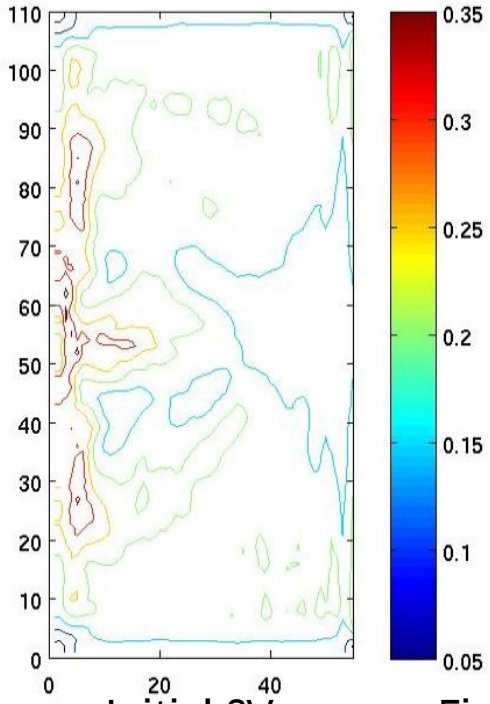


Optimization time = 5 days

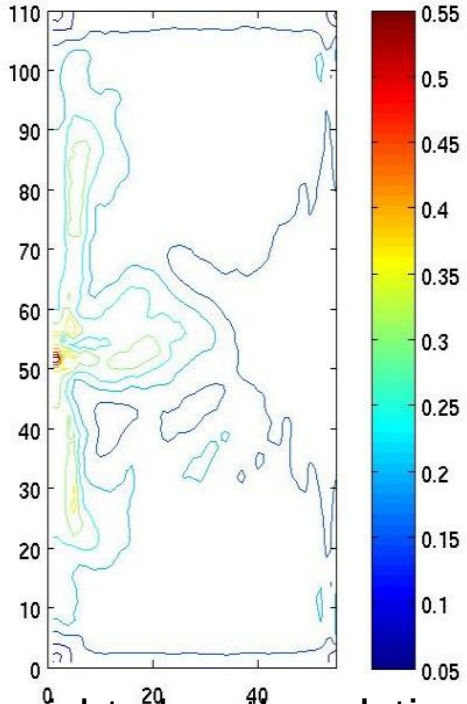
SAMPLING based on SVD criteria

A criteria based on two elements:

1. SVD of the tangent linear propagator of the model



Initial SVs. - Final SVs



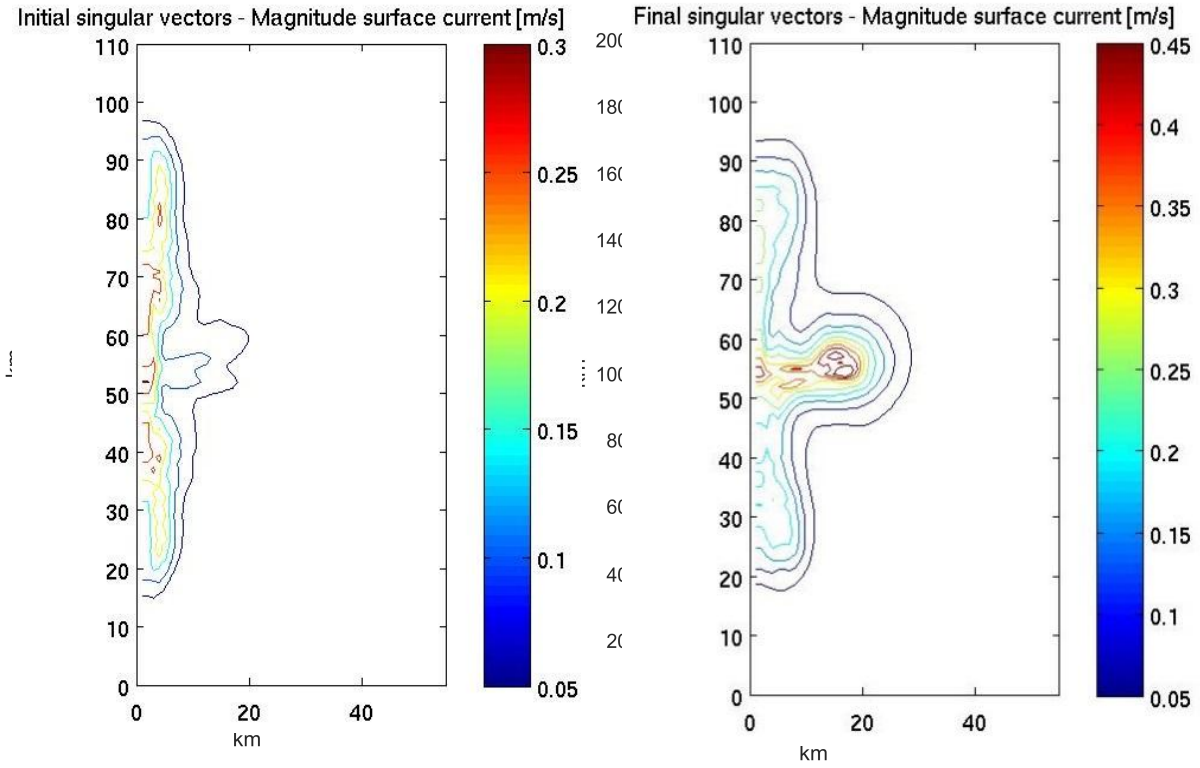
(weighted on the relative Singular value)

Optimization time = 5 days

SAMPLING based on SVD criteria

A criteria based on two elements:

1. SVD of the tangent linear propagator of the model

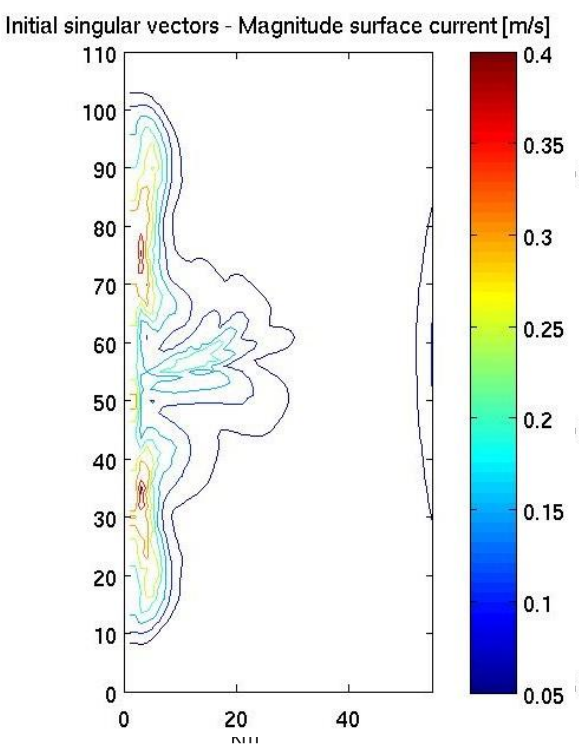


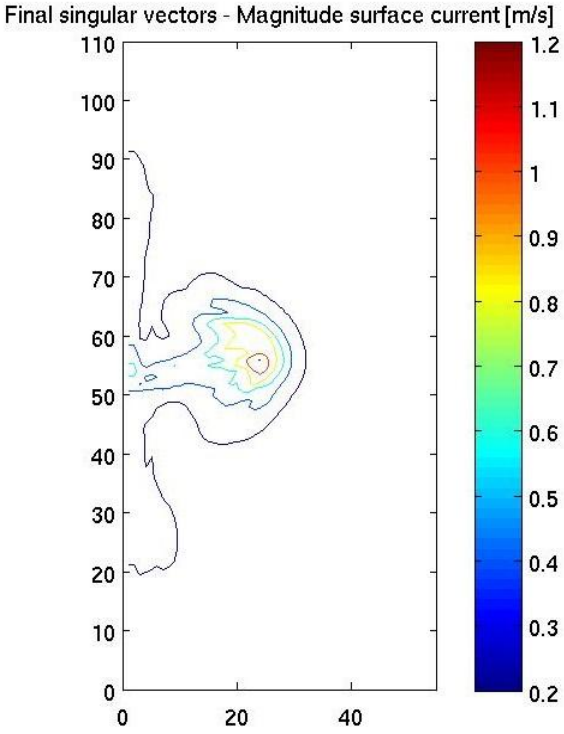
Optimization time = 10 days

SAMPLING based on SVD criteria

A criteria based on two elements:

1. SVD of the tangent linear propagator of the model



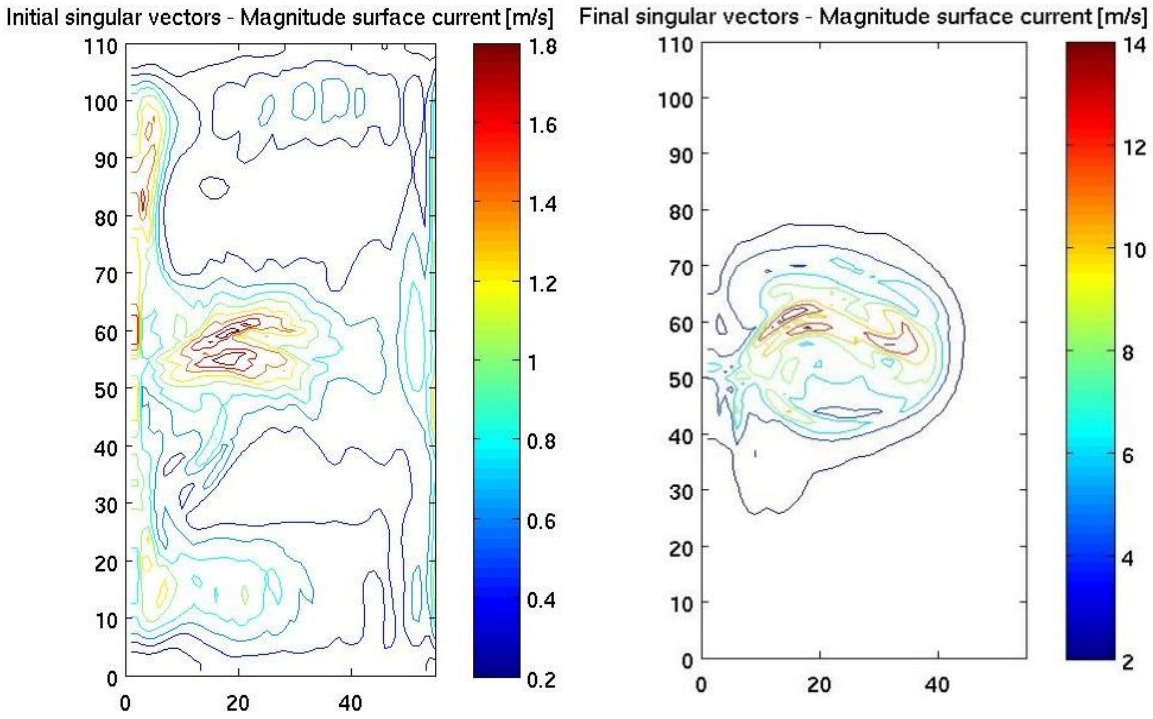


Optimization time = 20 days

SAMPLING based on SVD criteria

A criteria based on two elements:

1. SVD of the tangent linear propagator of the model



Optimization time = 60 days

SAMPLING based on SVD criteria

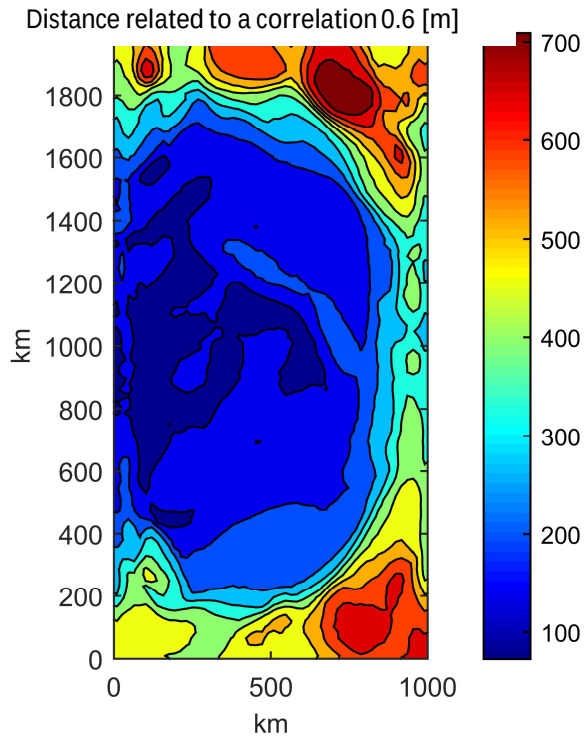
A criteria based on two elements:

1. the singular value decomposition (SVD) of the tangent propagator (weighted sum of the first initial singular vectors)

2. a. the minimum distance

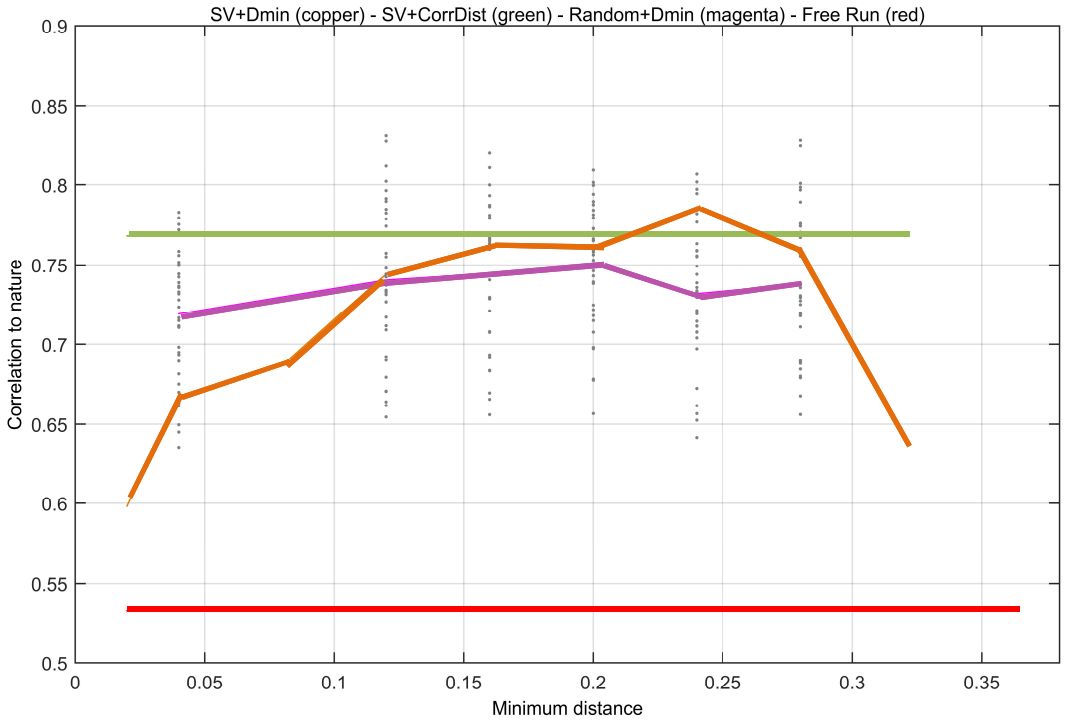
2.b. the correlation between the velocity fields (to avoid redundancy in neighboring measurements)

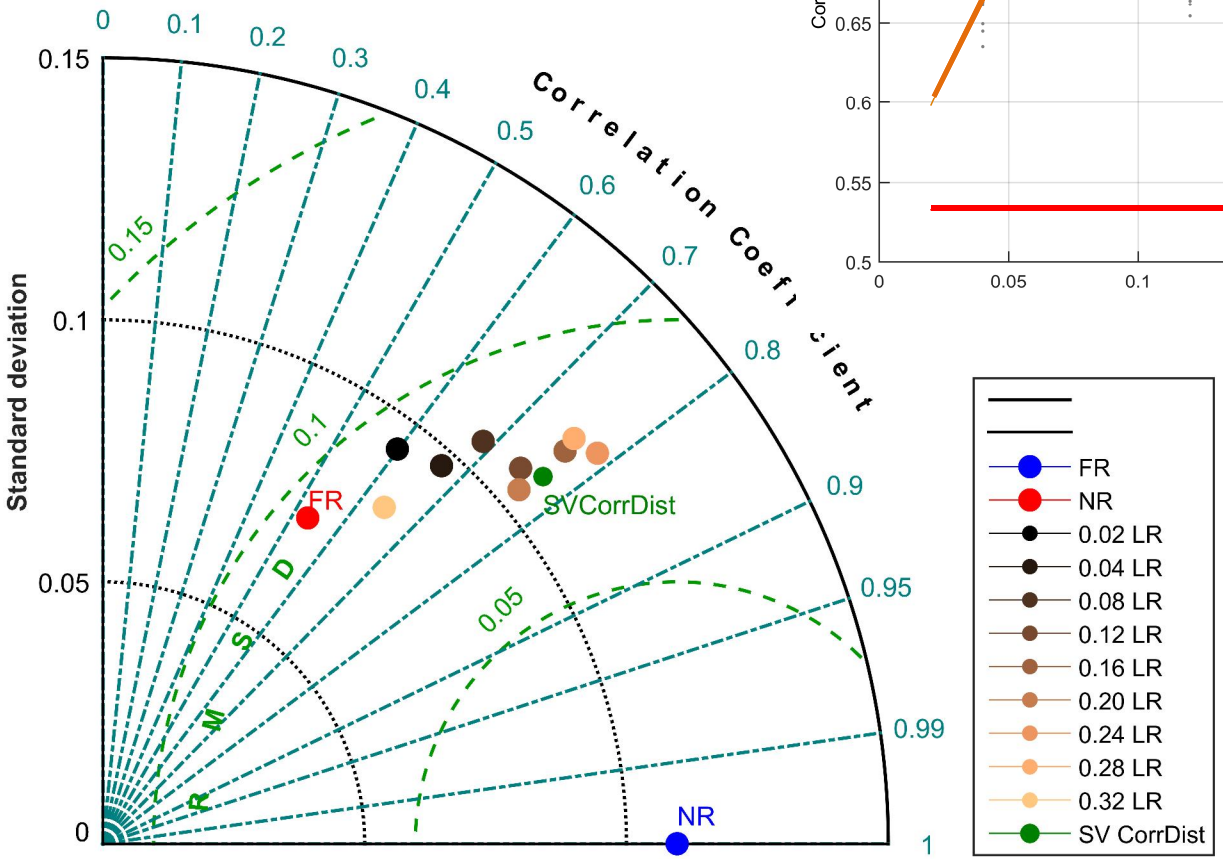
$$CORR_{ij} = \begin{vmatrix} < u_{ij}u_{hl} > & < v_{ij}u_{hl} > \\ < u_{ij}v_{hl} > & < v_{ij}v_{hl} > \end{vmatrix}$$



SAMPLING based on SVD criteria

Results for the first time window

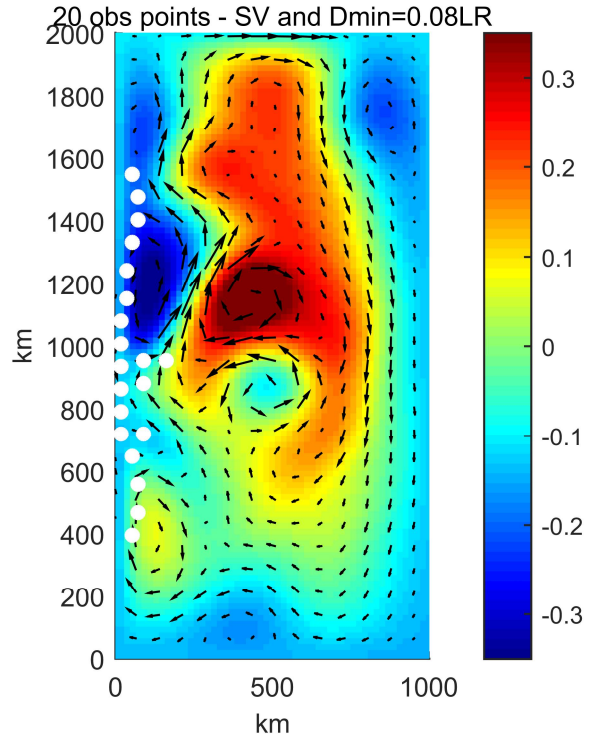


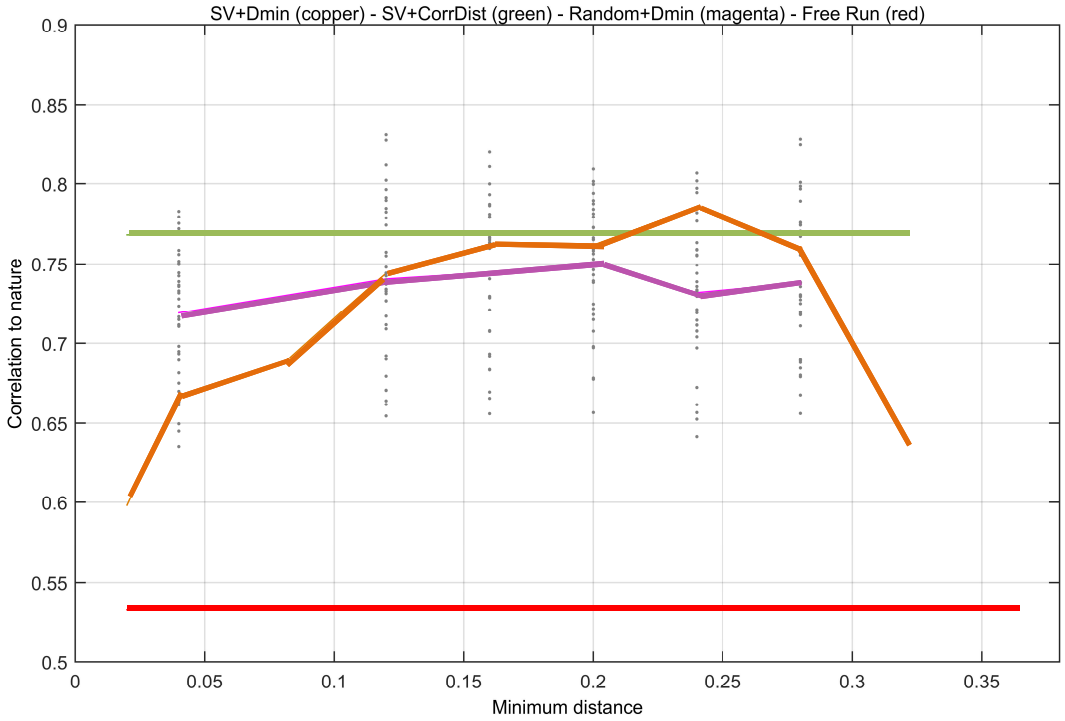


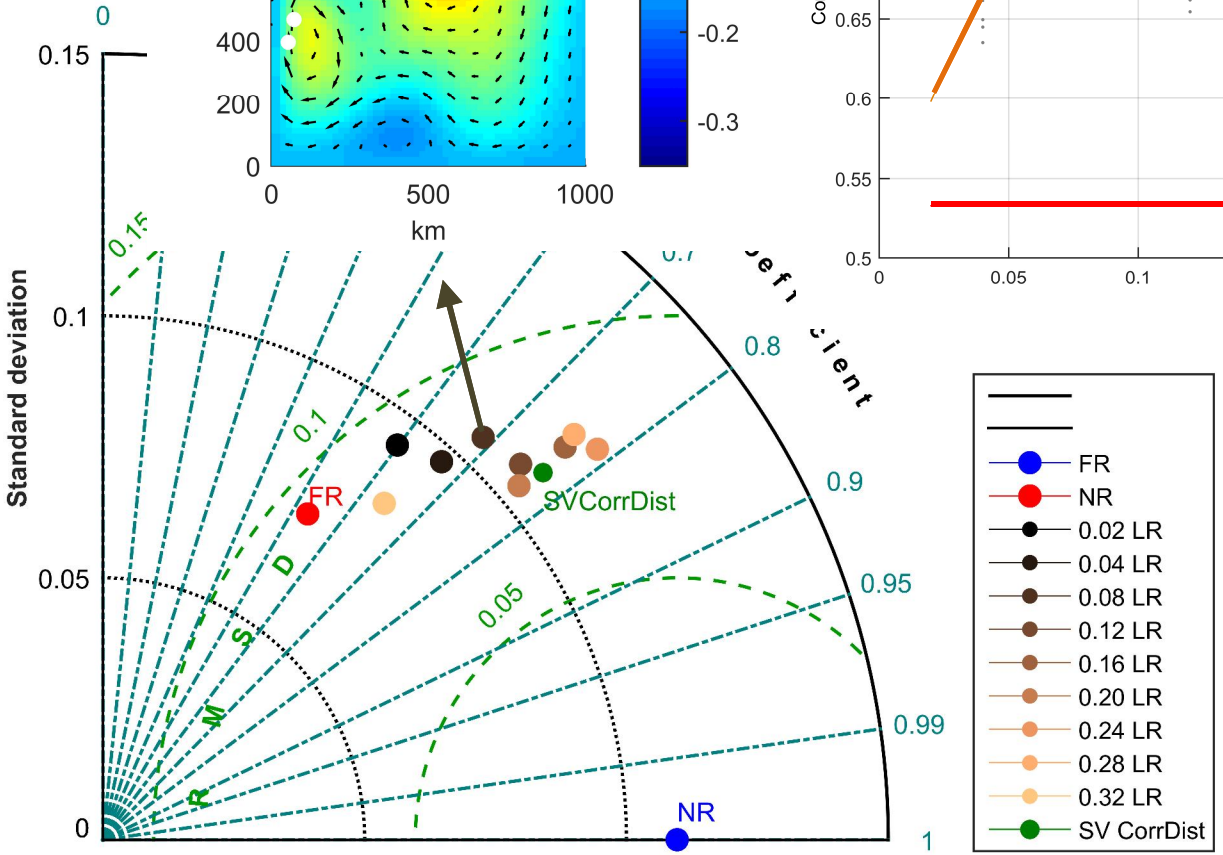
barotropic Rossby deformation radius

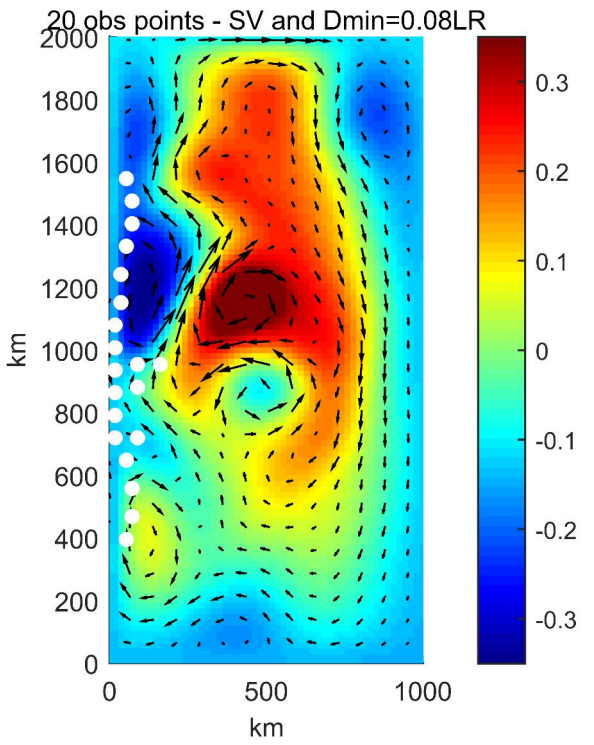
$L_R = (gh)^{1/2}/f \approx 900$ km.

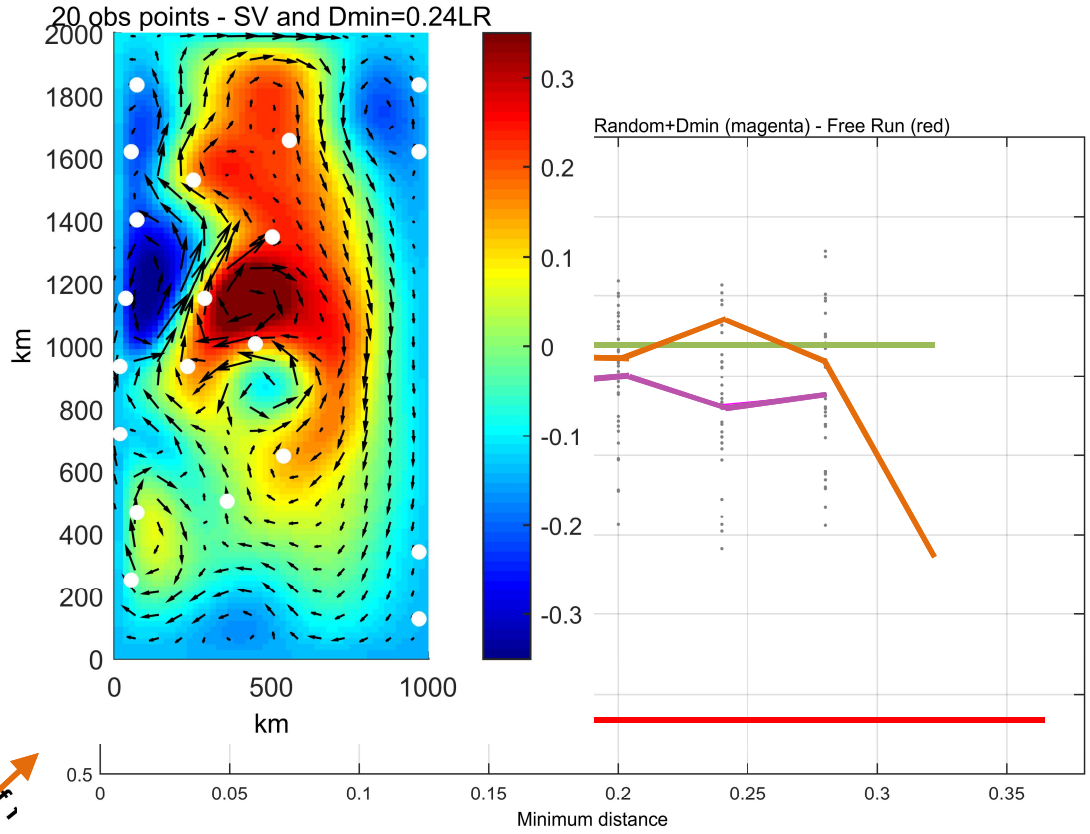
Tes ed on SVD criteria



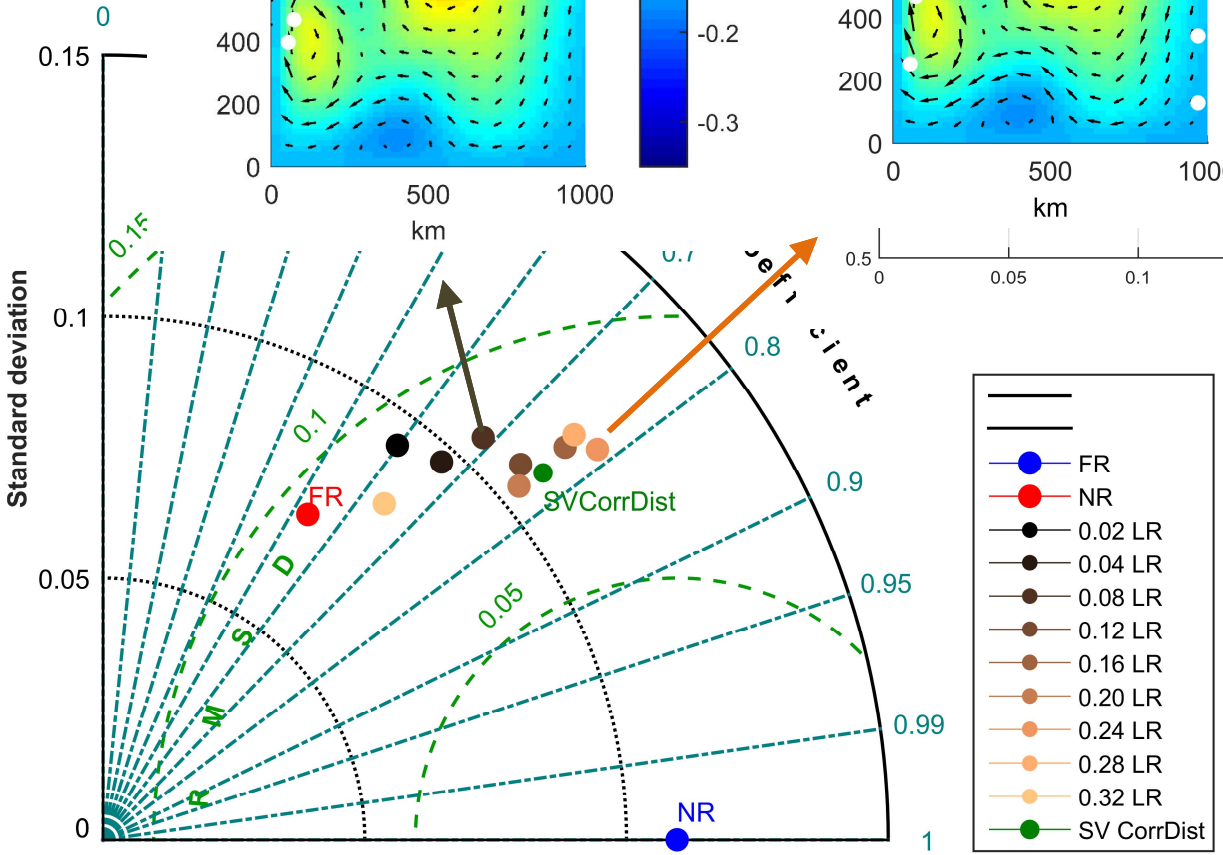




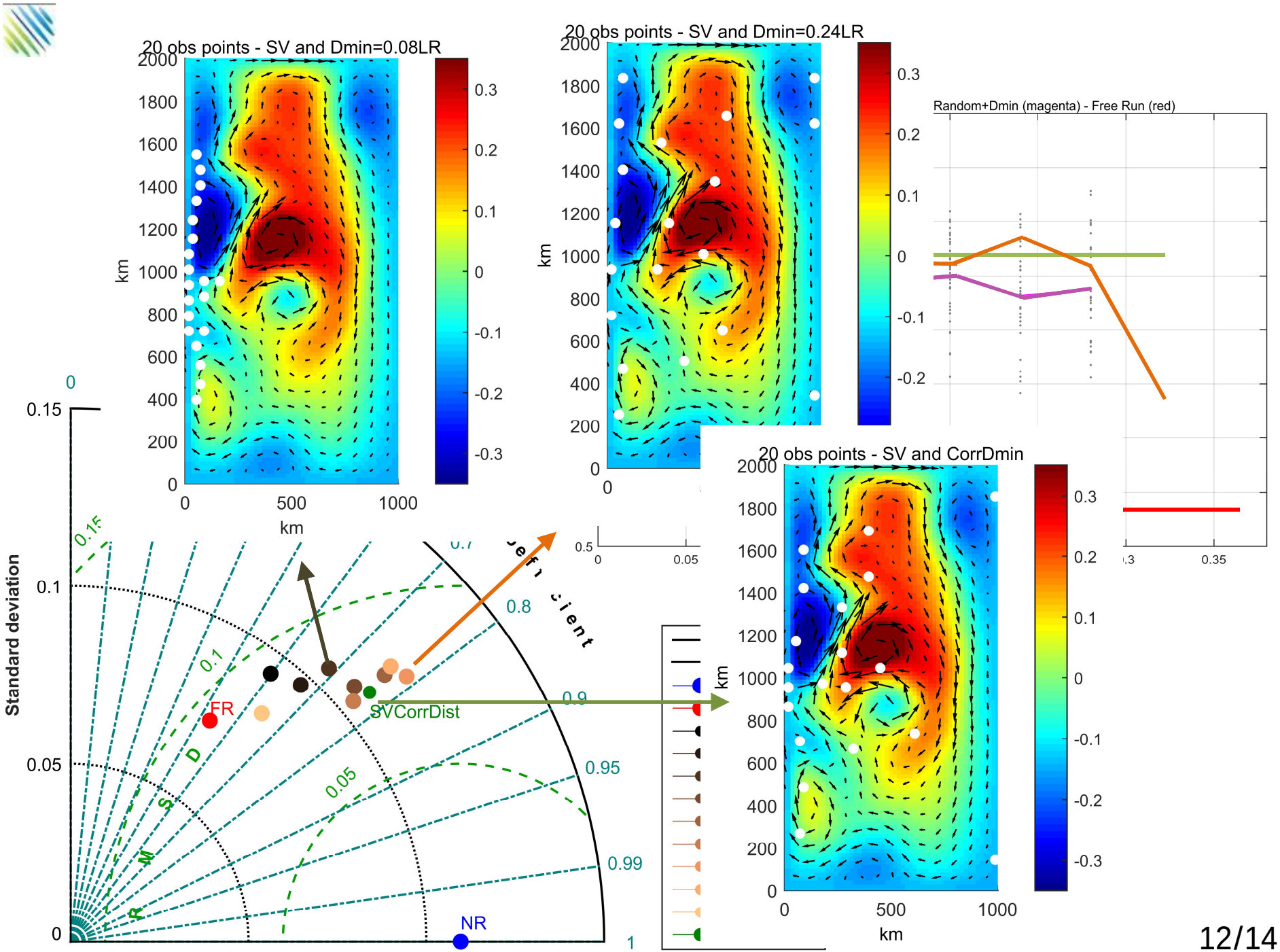

20 obs points - SV and Dmin=0.08LR
km
km


20 obs points - SV and Dmin=0.24LR
km
km

Random+Dmin (magenta) - Free Run (red)
Minimum distance

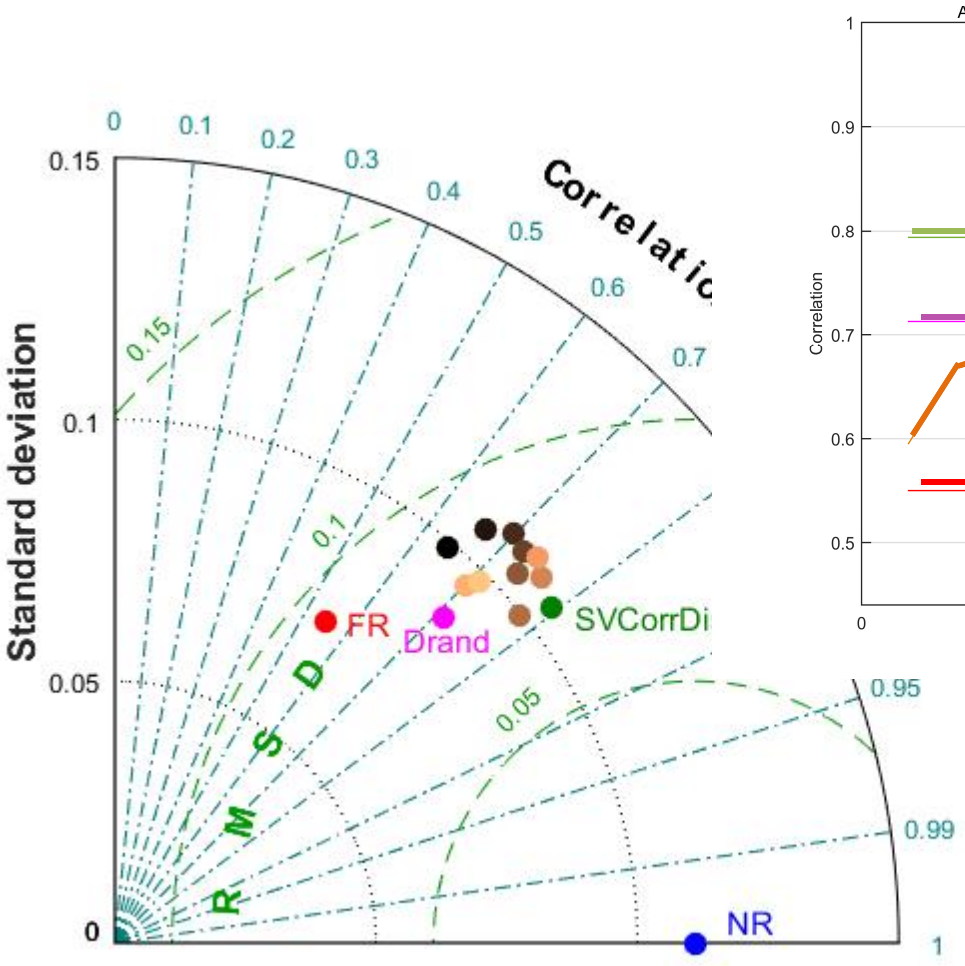

Standard deviation
FR
SVCorrDist
NR

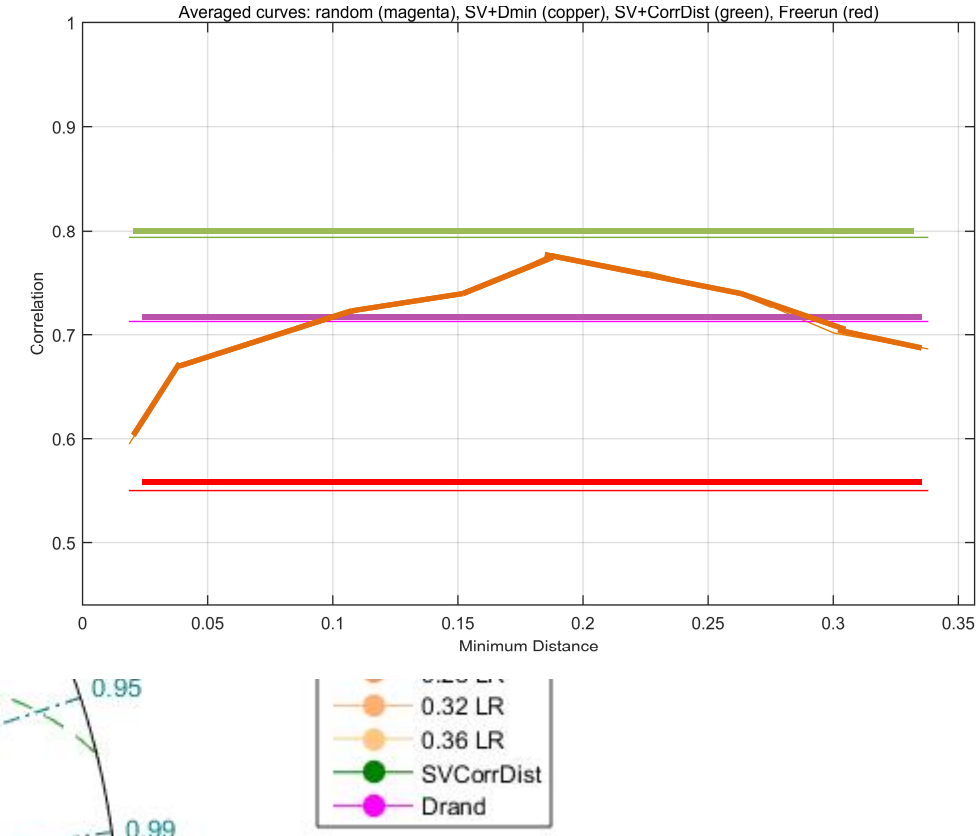
FR
NR
0.02 LR
0.04 LR
0.08 LR
0.12 LR
0.16 LR
0.20 LR
0.24 LR
0.28 LR
0.32 LR
SV CorrDist


20 obs points - SV and Dmin=0.08LR
20 obs points - SV and Dmin=0.24LR
Random+Dmin (magenta) - Free Run (red)
20 obs points - SV and CorrDmin
km
Standard deviation
FR
NR
SVCorrDist

SAMPLING based on SVD criteria

Averaged results for all time windows





Future works

The criteria adopted is easily replicable in practical applications and requires rather standard studies to obtain prior information (i.e. climatological and correlation studies), to be carried out in order to properly design observation networks.

The sampling strategy we present is a preliminary tool to understand the observation needs for real analysis and forecasting systems.

Next steps will be:

1.to compute the Hessian Singular Vectors, which are the EOFs of the analysis error covariance matrix [Smith et al. 2015];

2.Application of the observation sensitivity and assessment of assimilation process by using observation impact [Moore et al 2011];

3.Application to more realistic models, in a baroclinic circulation.