

An adaptive Ensemble Optimal Interpolation for cost-effective assimilation in the Red Sea

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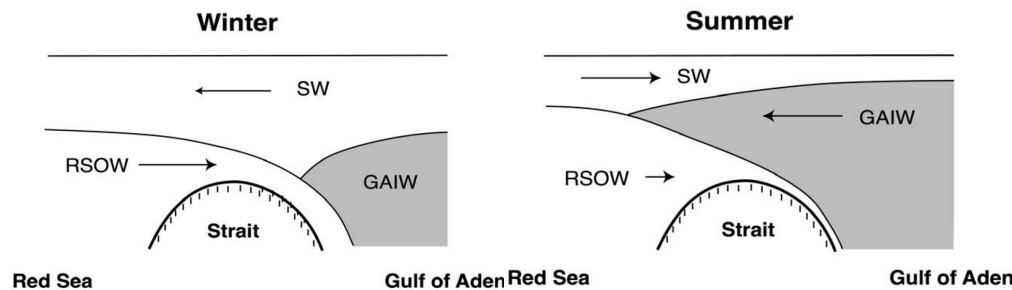
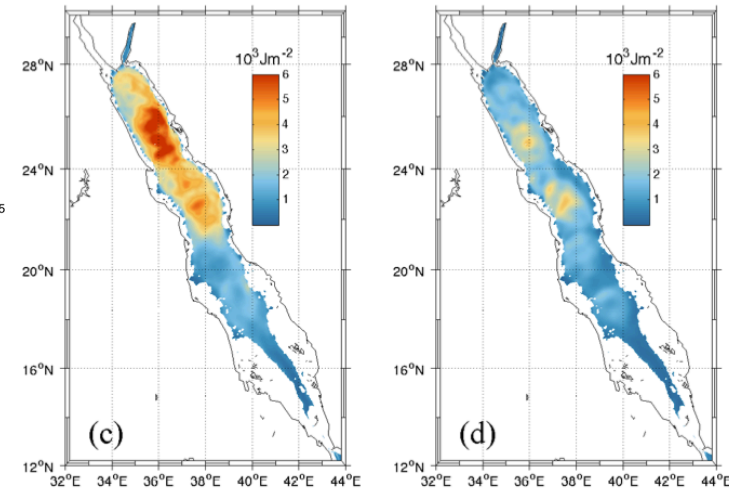
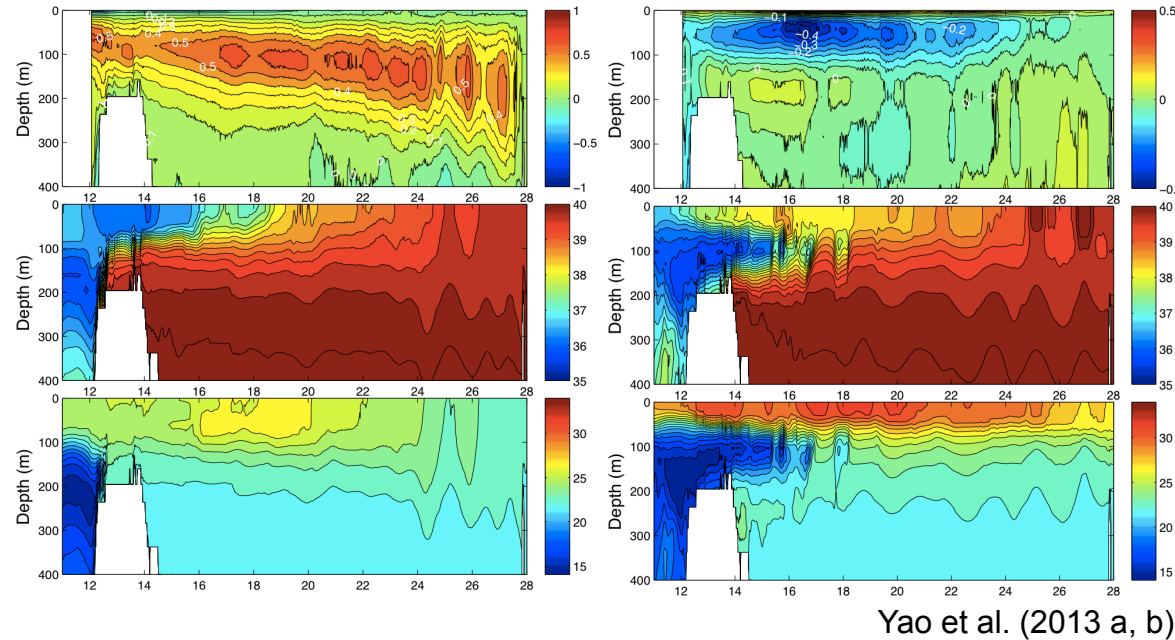








Seasonal Variability on the Dynamics



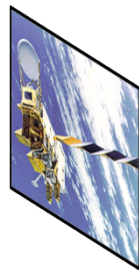
Smeed et al. (2004)

- **A stationary ensemble-covariance may not be appropriate for the Red Sea**
- ➔ to sample stationary-variant covariances adapted for each “season”

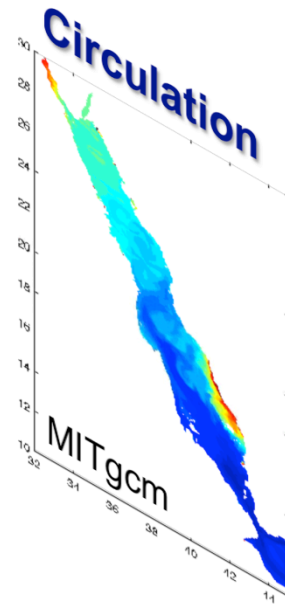
Data assimilation



Data assimilation

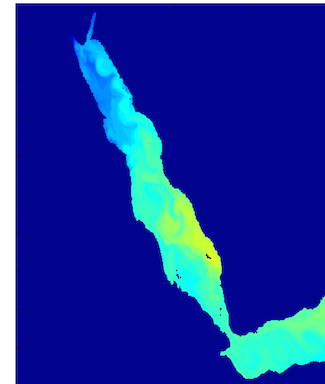
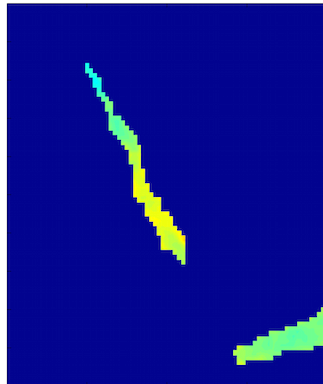
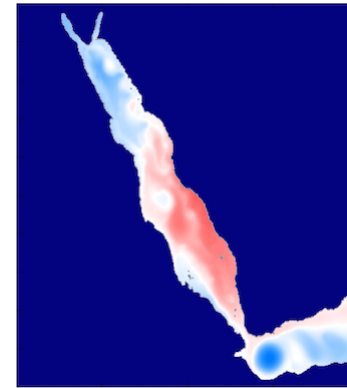
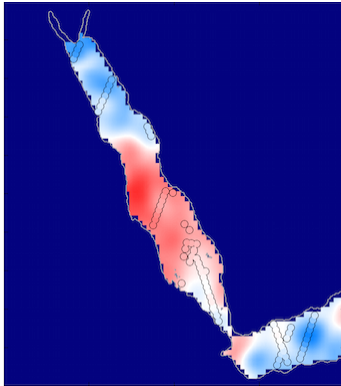


data



model

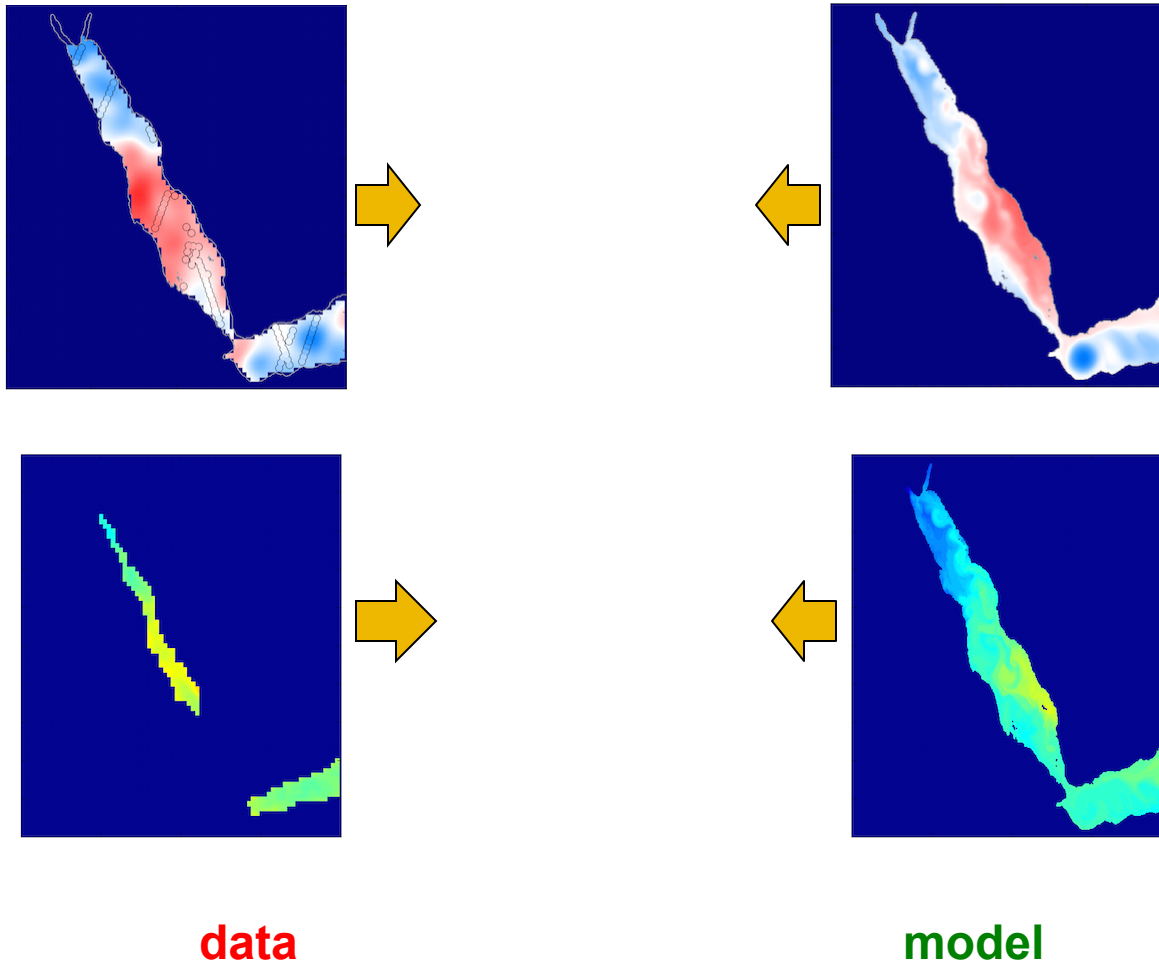
Data assimilation



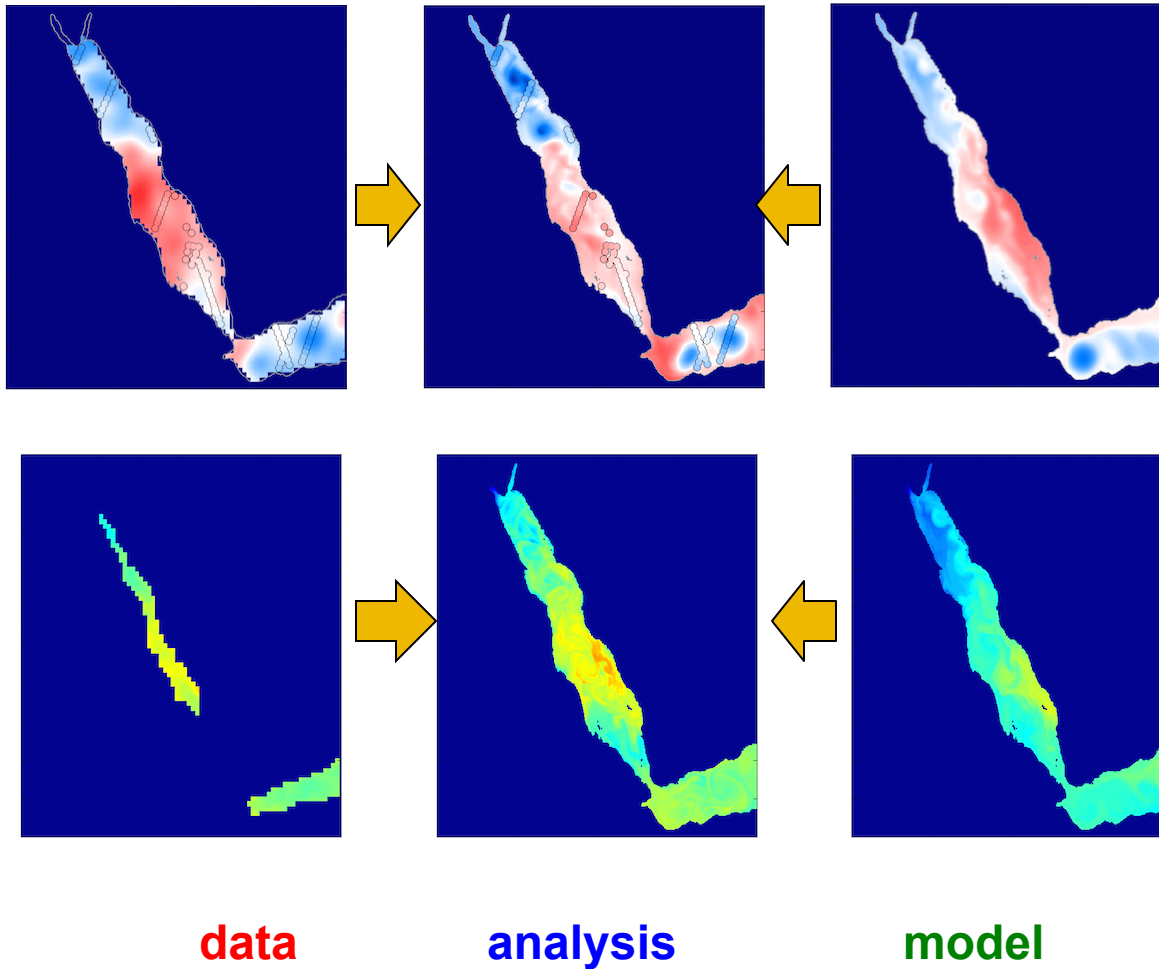
data

model

Data assimilation



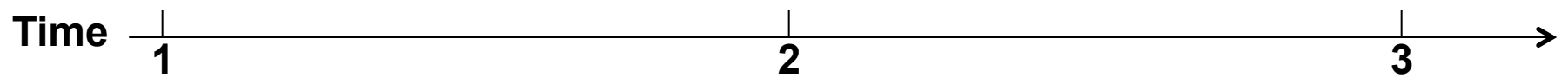
Data assimilation



Data assimilation



Data assimilation



Data assimilation



estimate ●

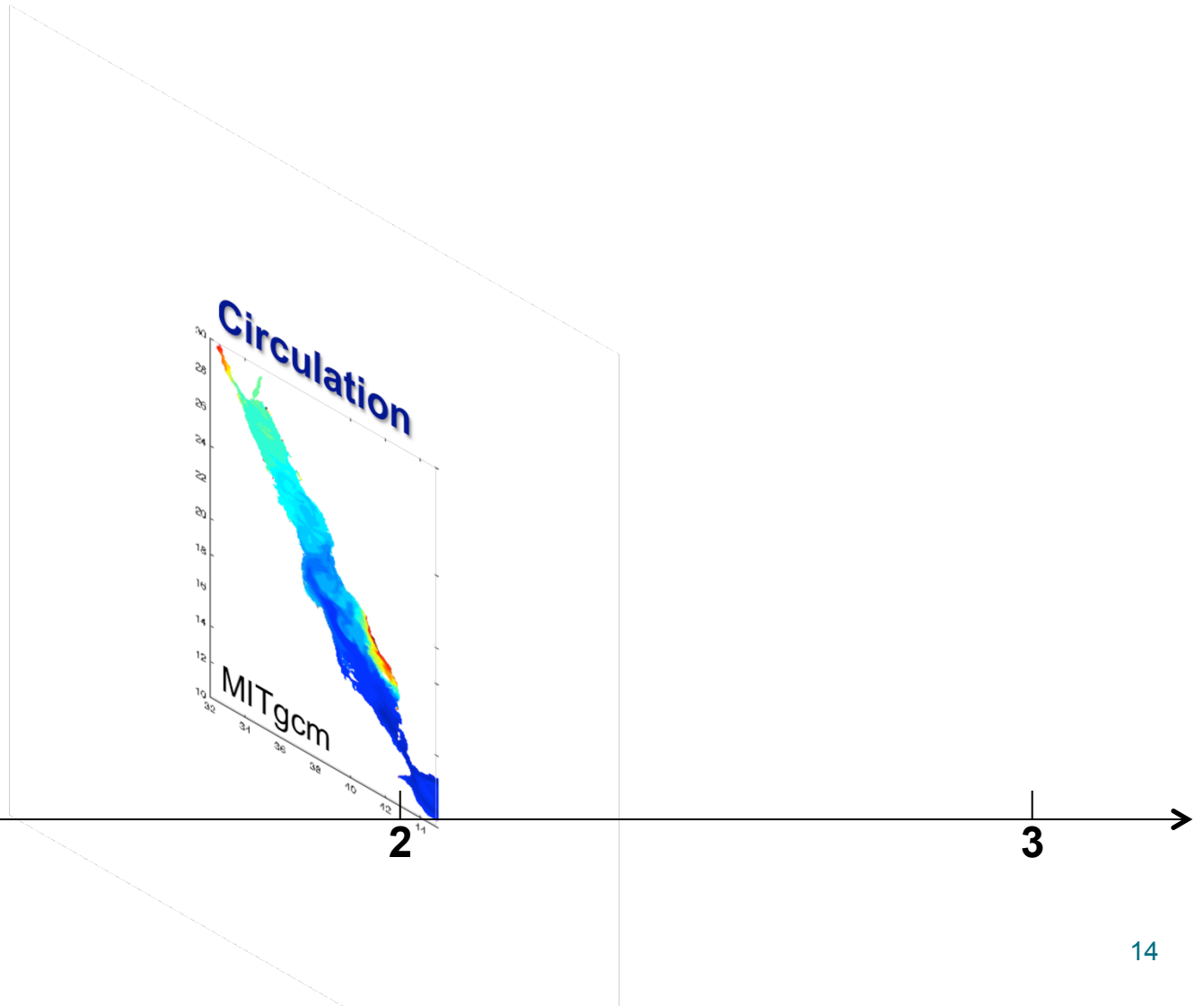
x^b

Time

1

2

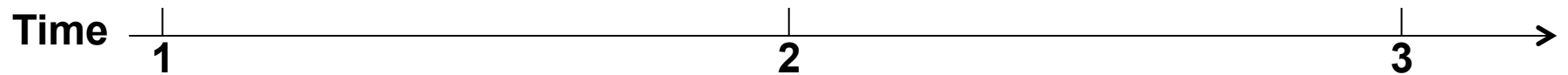
3



Data assimilation



estimate ●



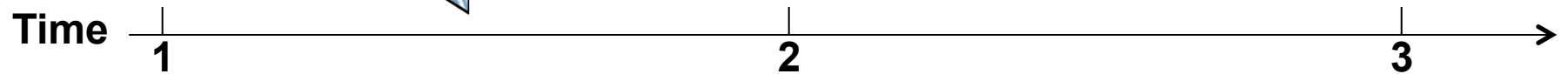
Data assimilation



estimate ●

data ●

y^o

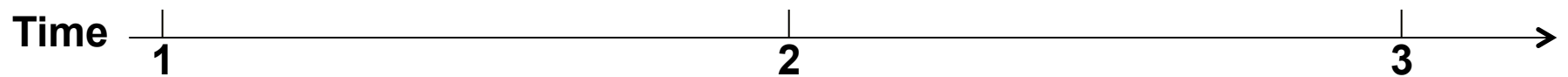


Data assimilation



estimate ●

data ●



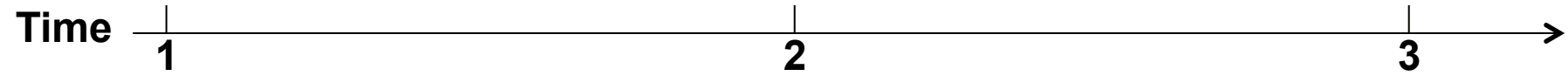
Data assimilation



update
↓

estimate ●

data ●



Data assimilation

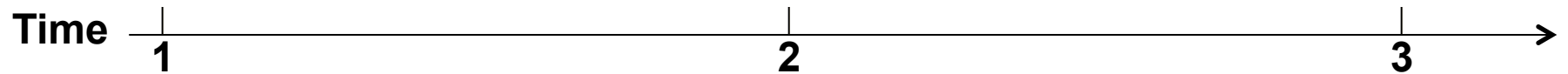


update
↓

estimate ●

analysis ●

data ●



Data assimilation

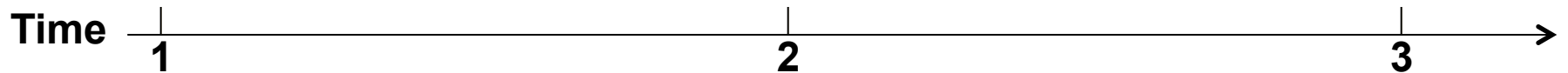


update
↓

estimate ●

analysis ● $x^a = x^b + K (y^o - h(x^b))$

data ●



Data assimilation

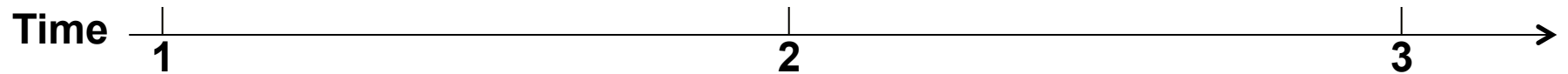


update
↓

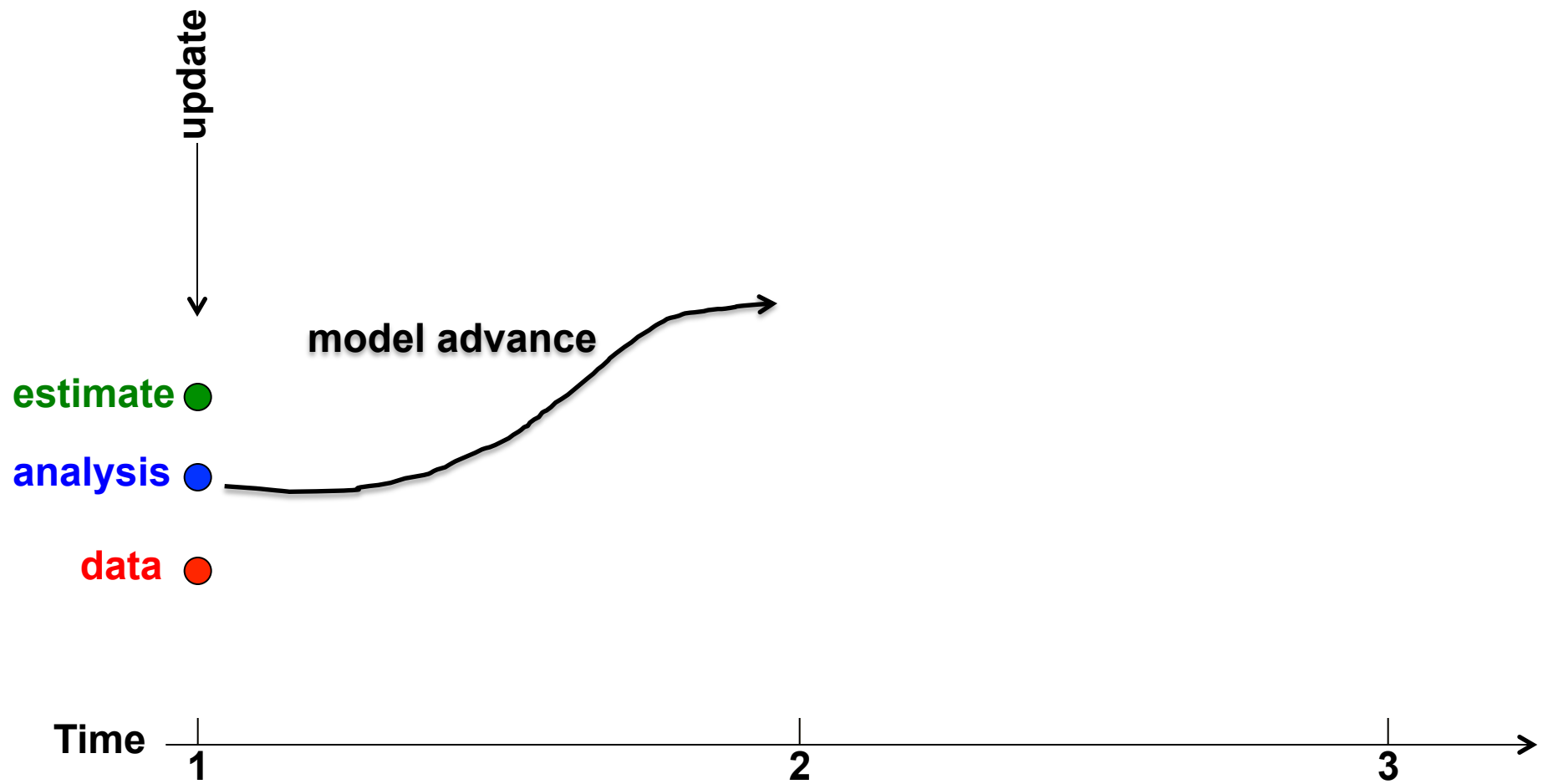
estimate ●

analysis ●

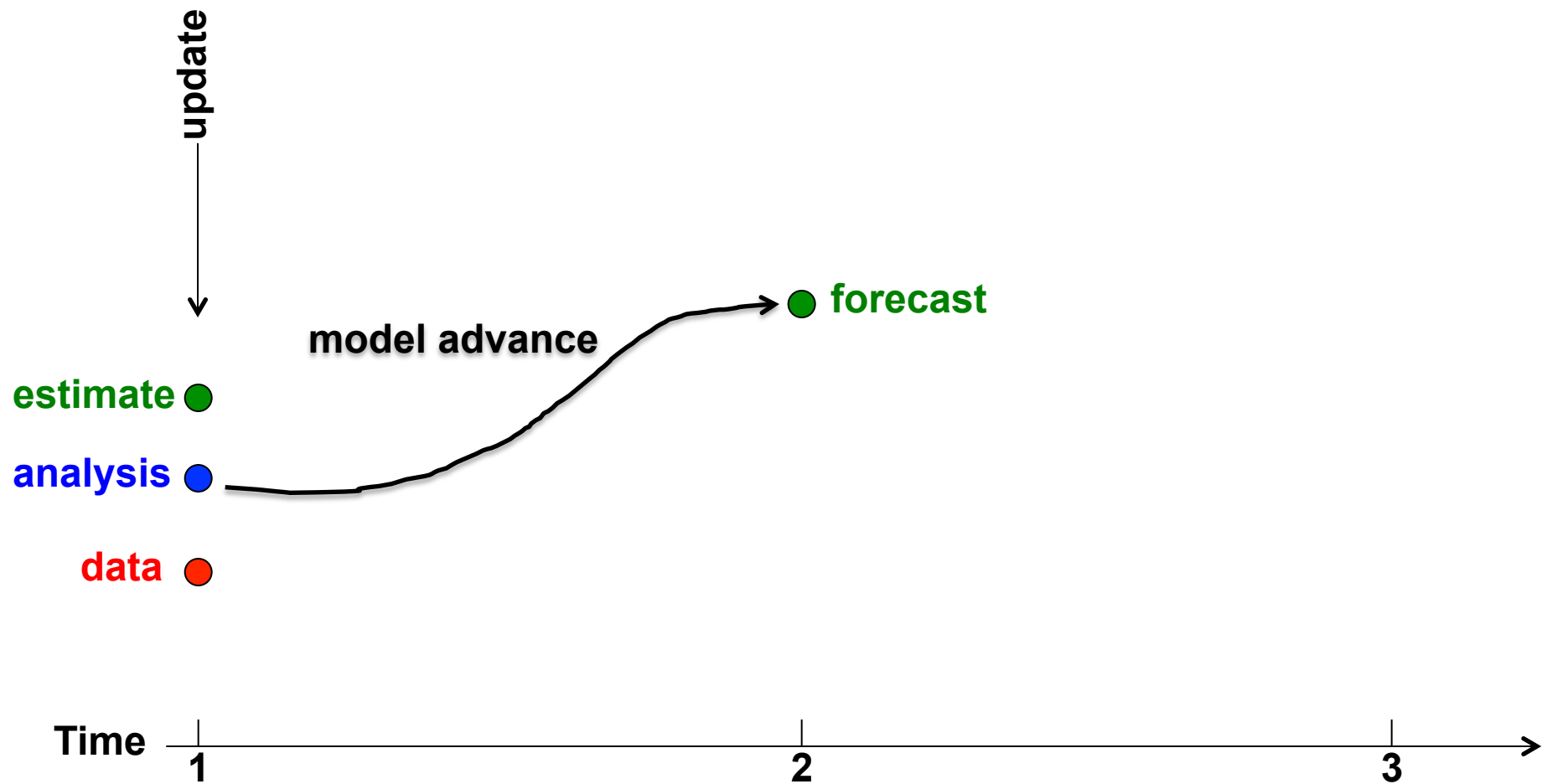
data ●



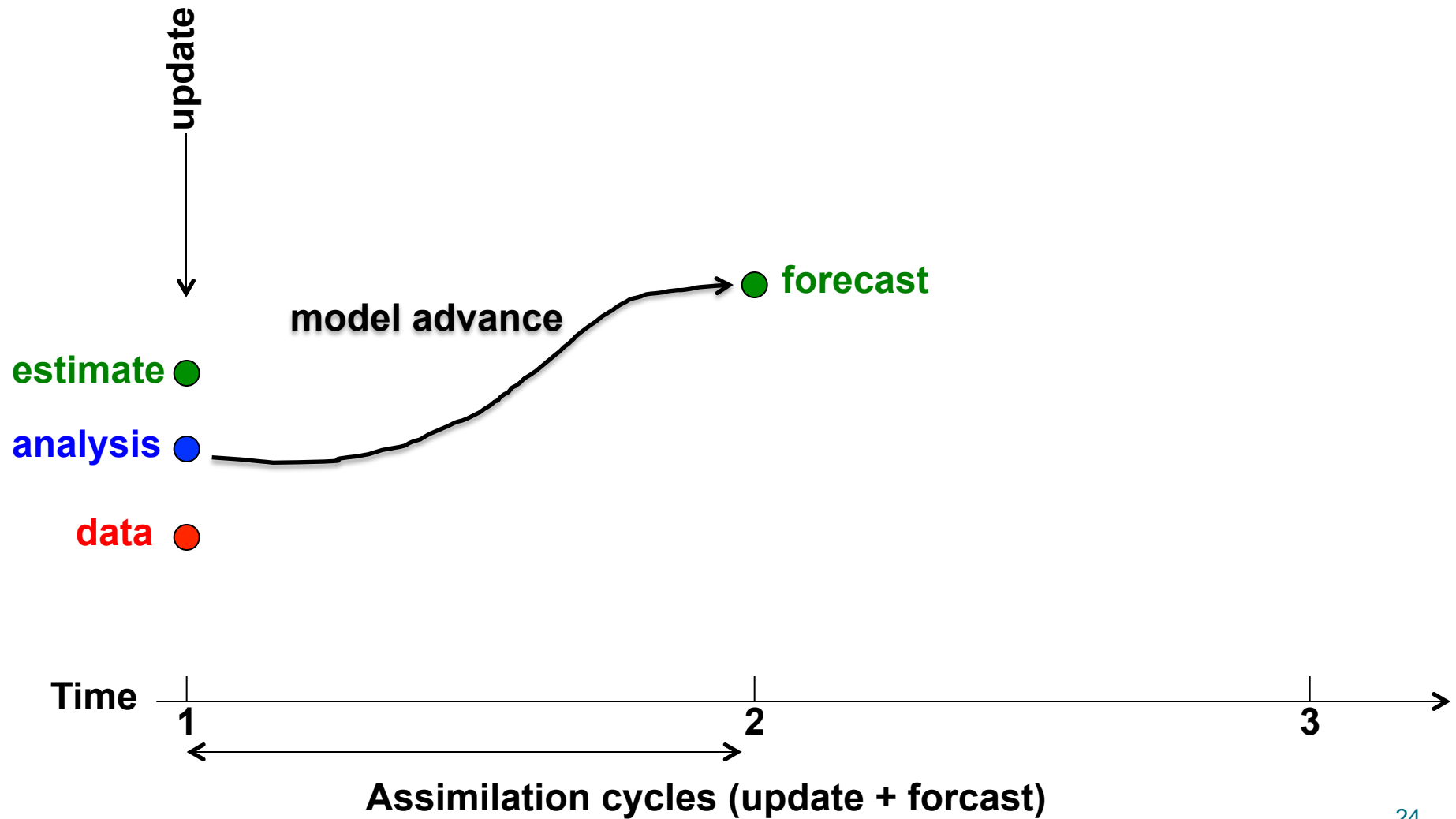
Data assimilation



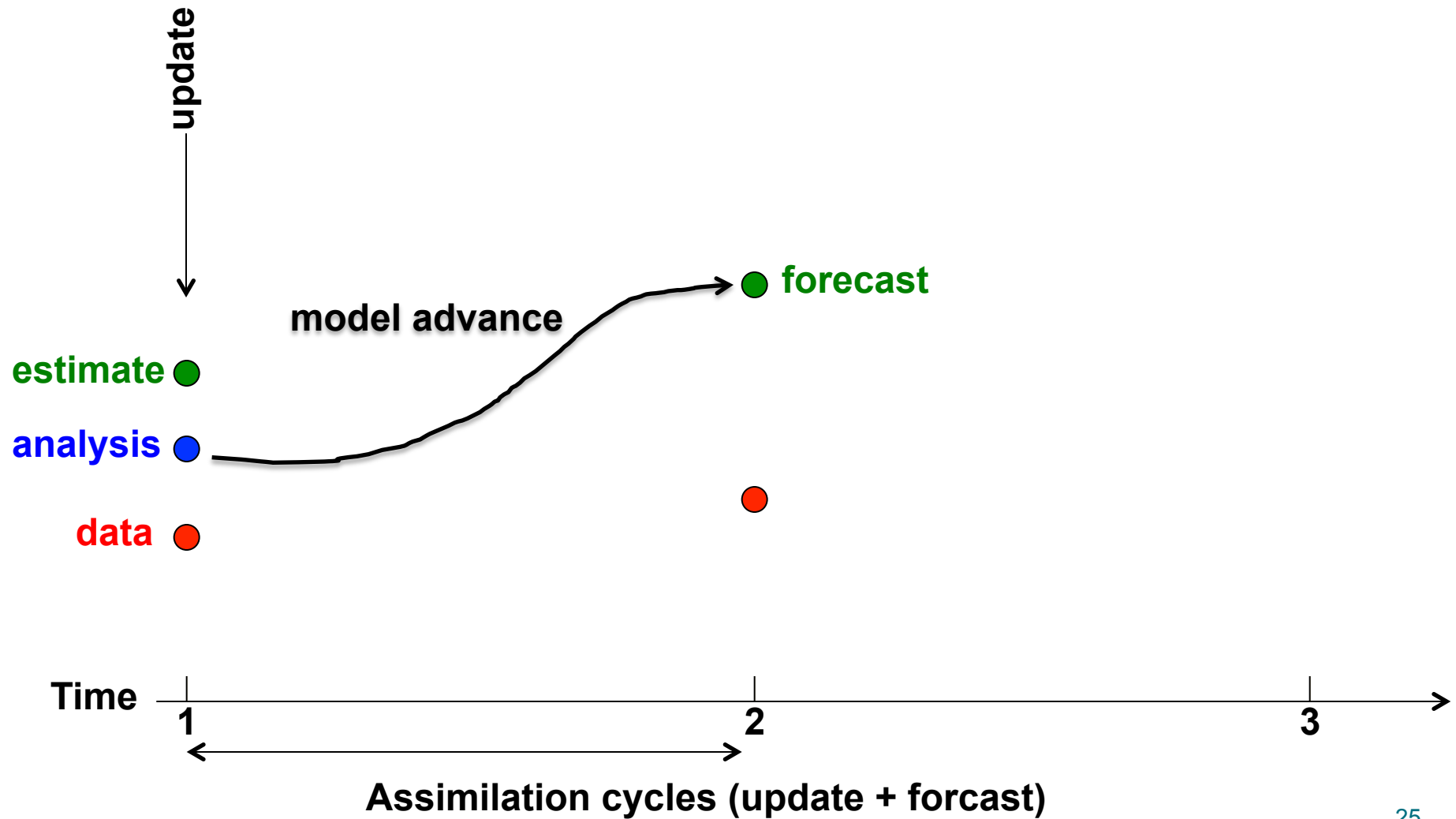
Data assimilation



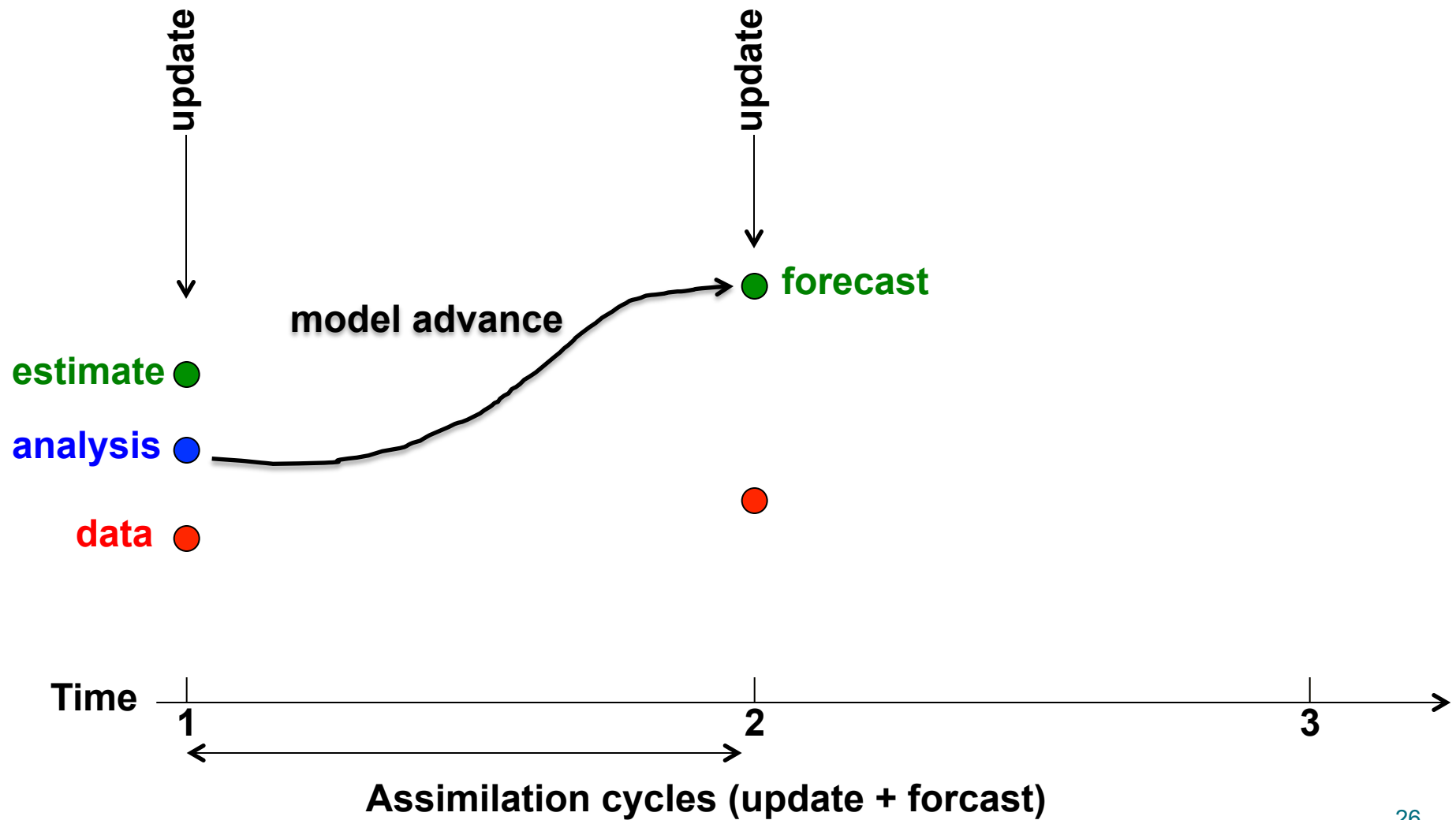
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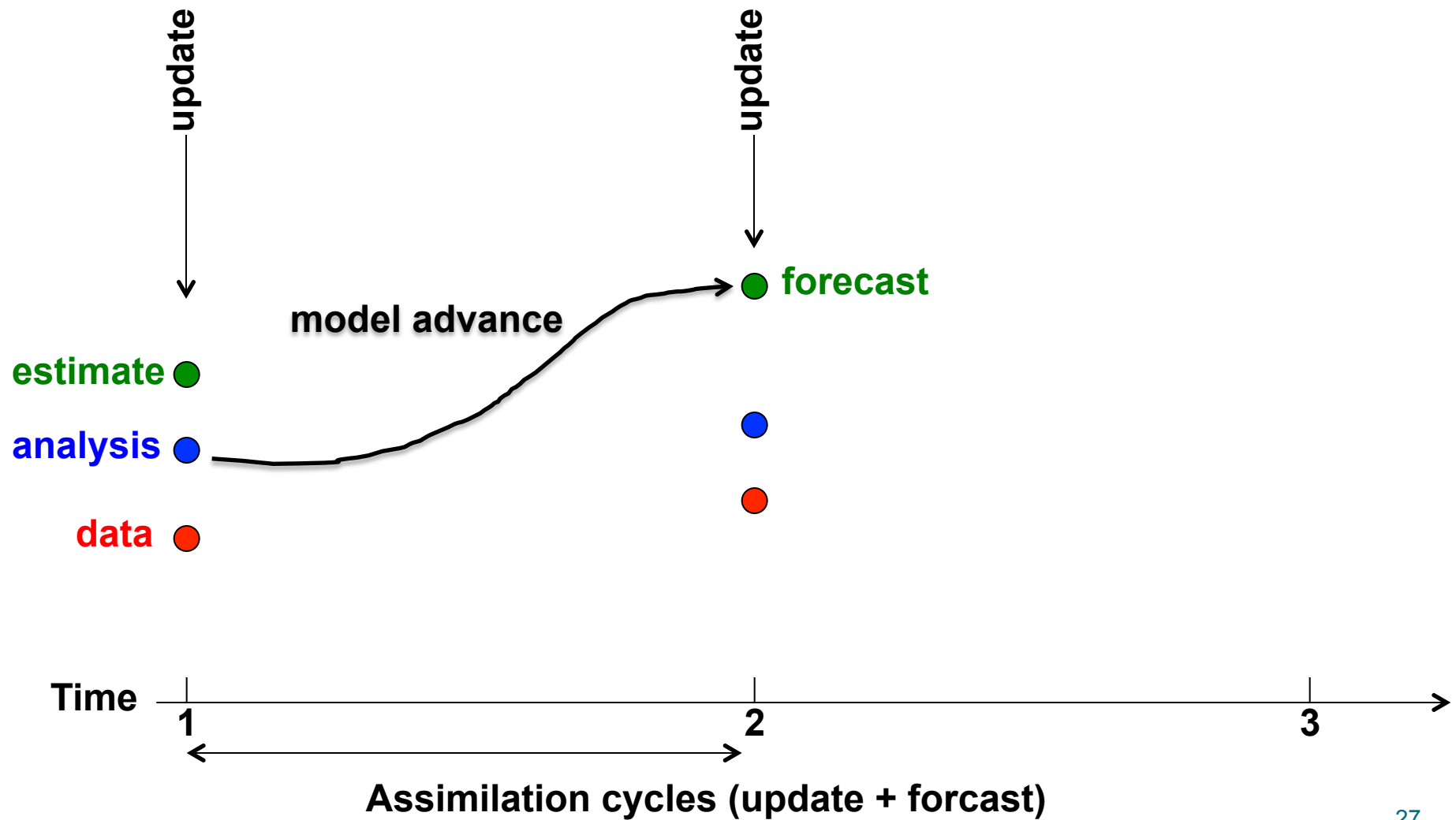
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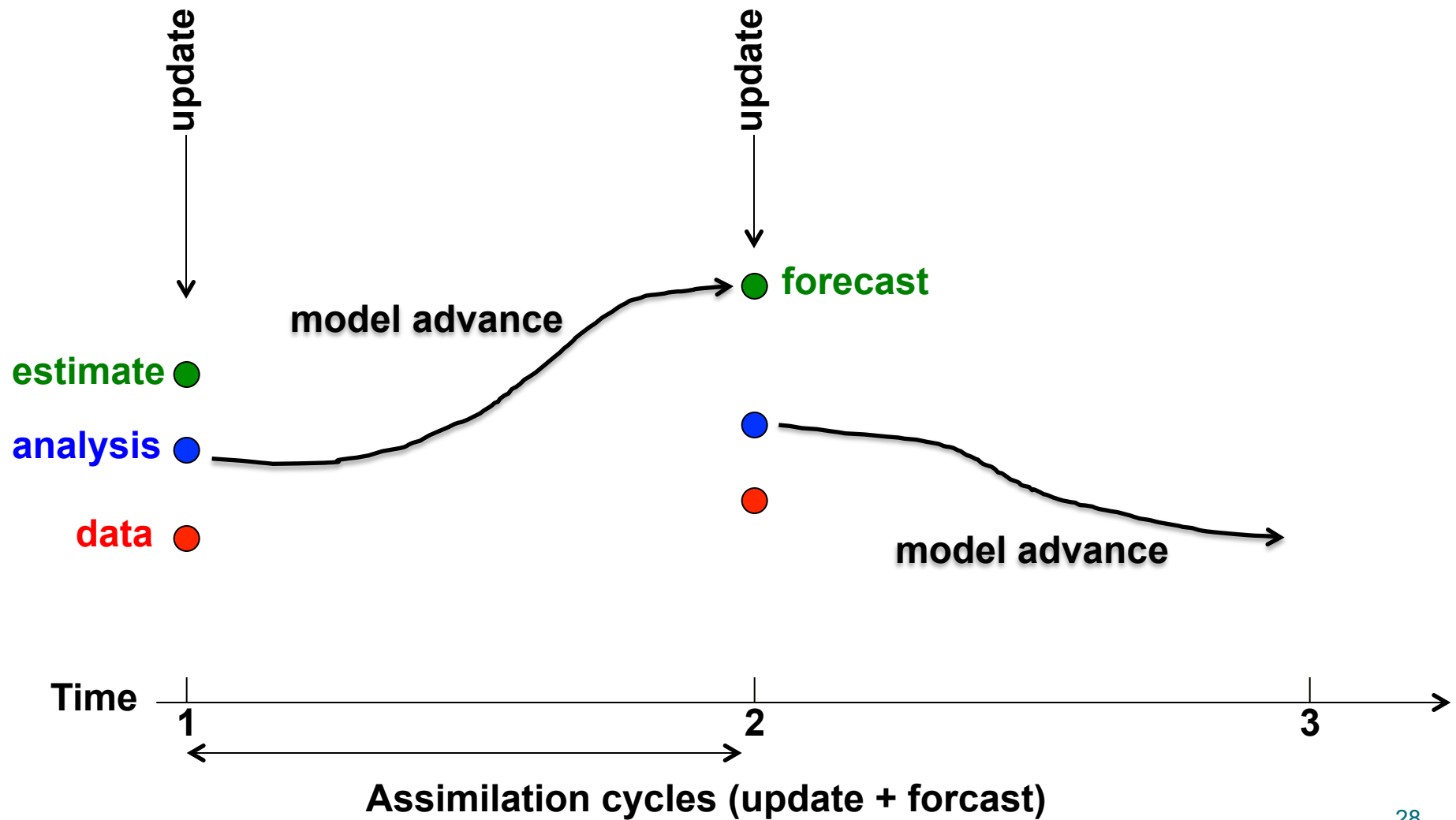
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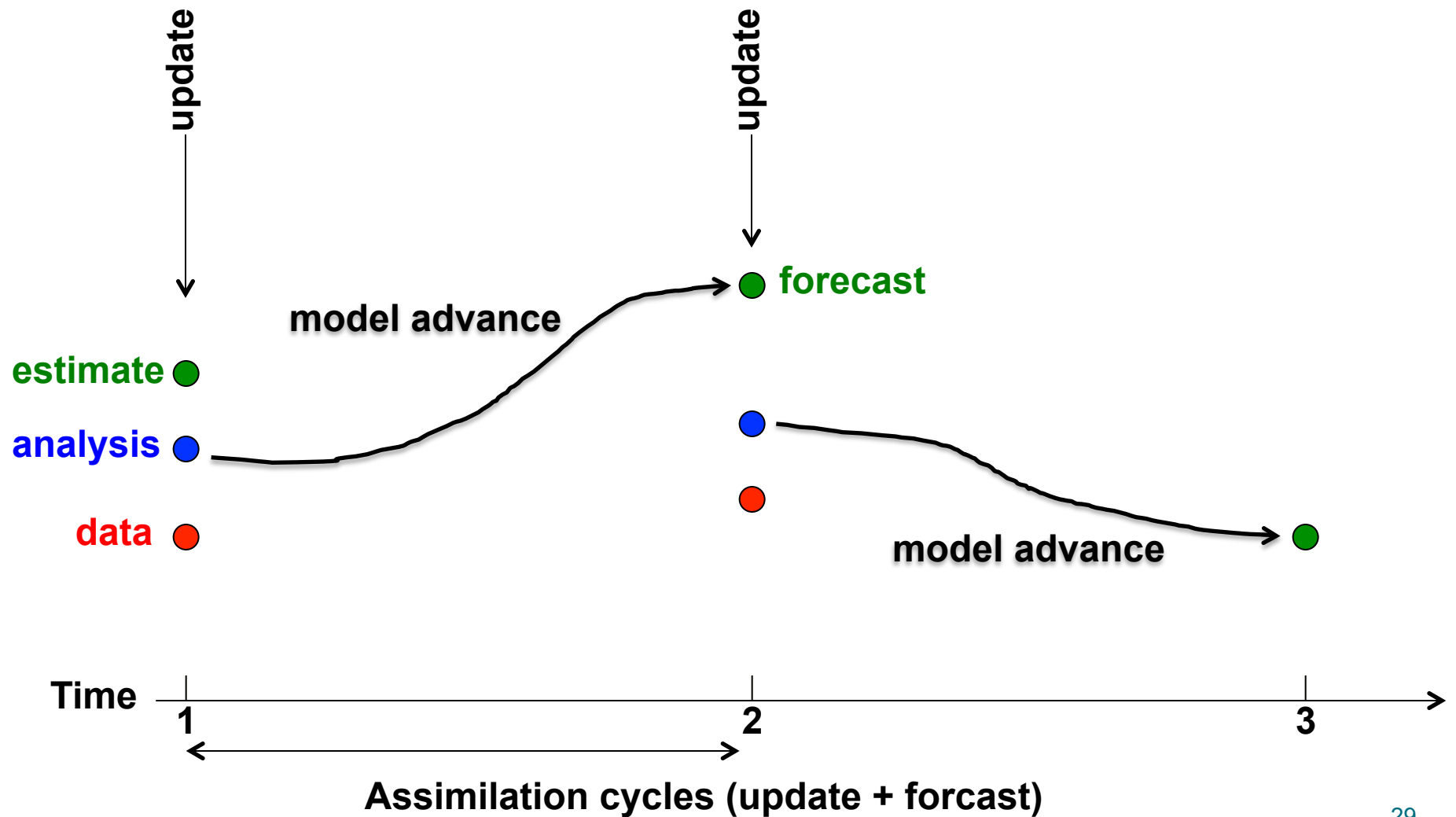
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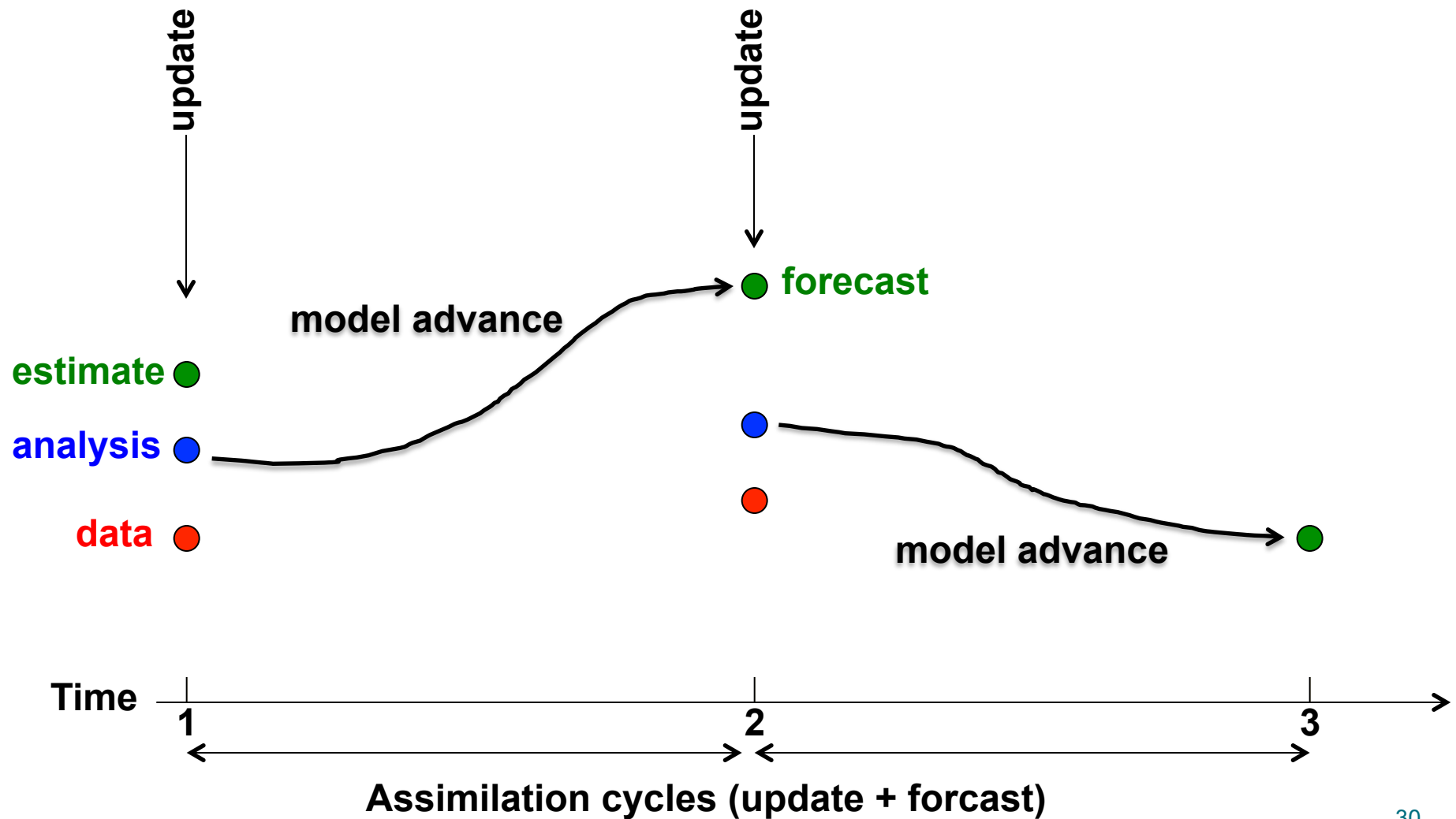
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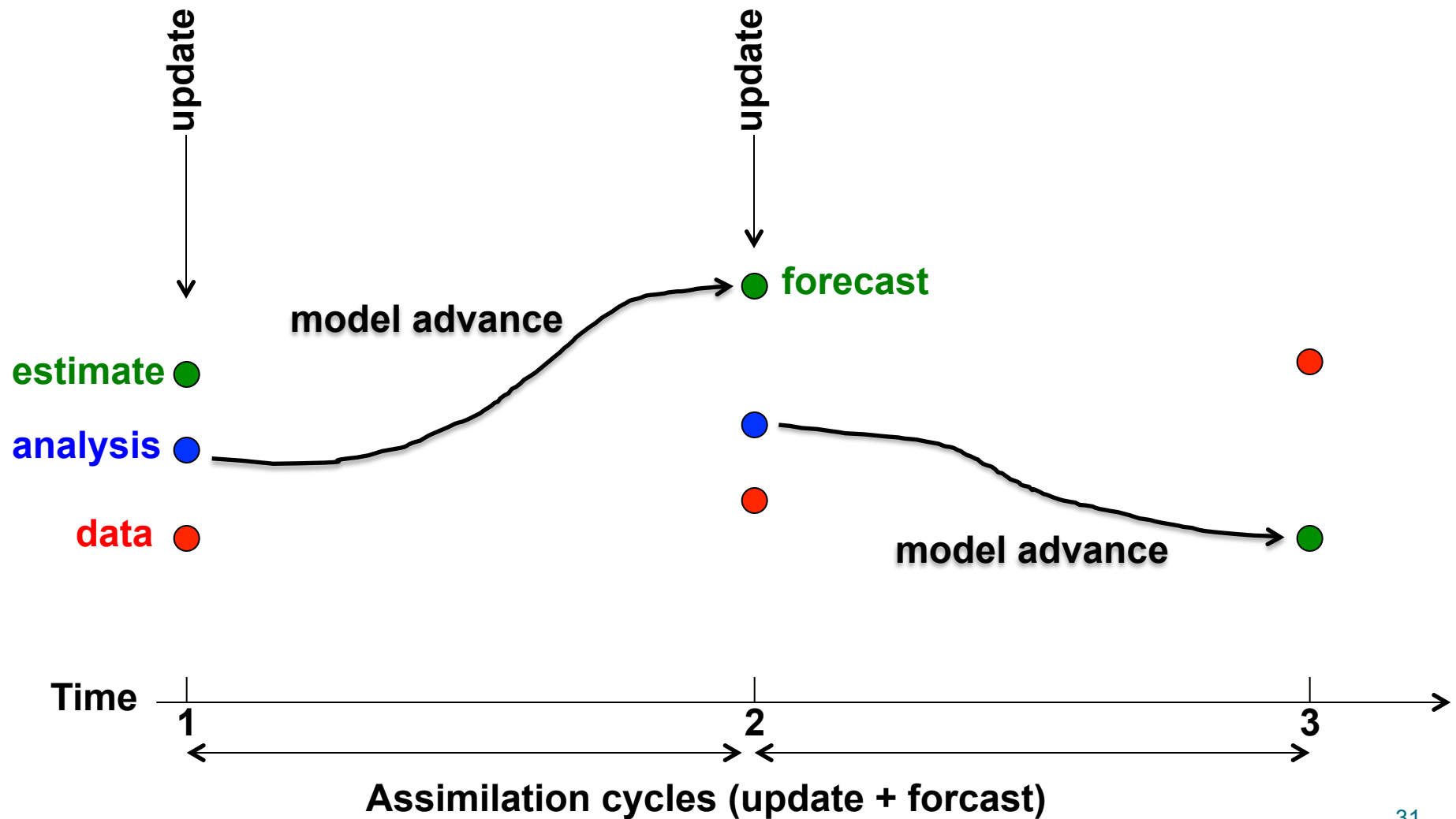
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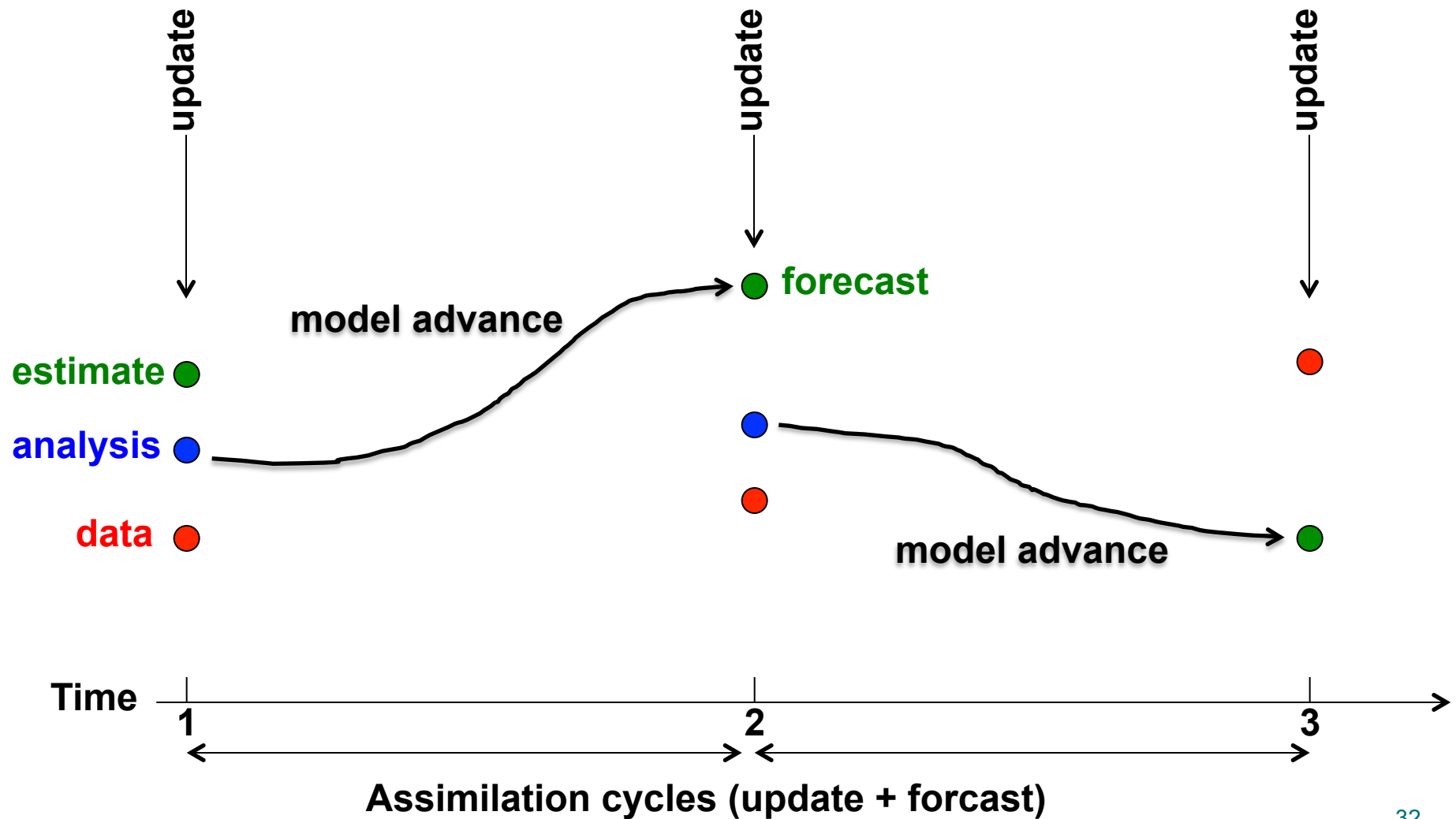
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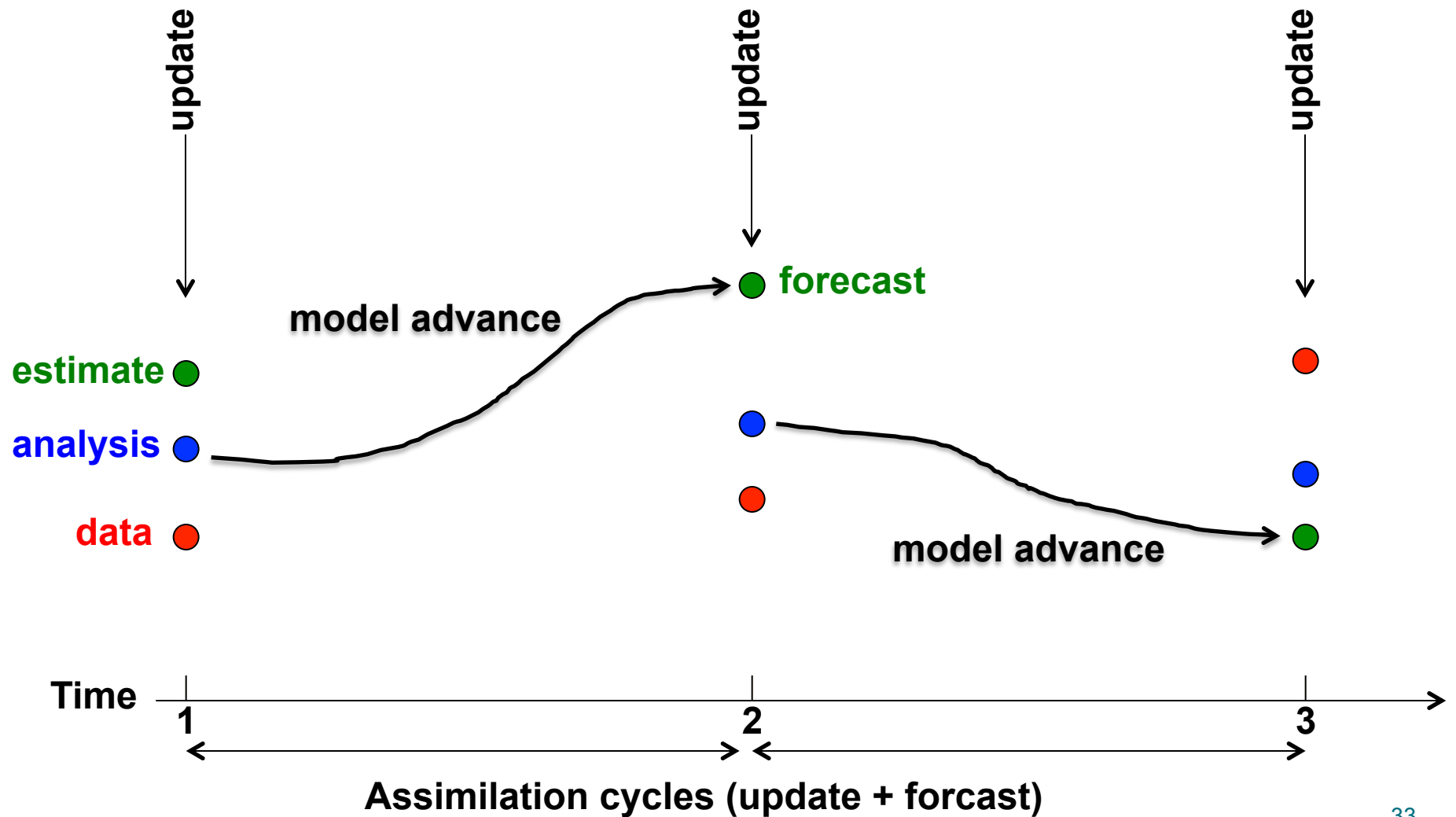
Data assimilation



Data assimilation



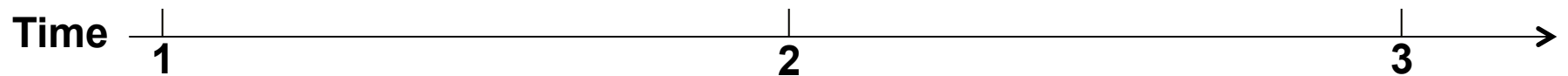
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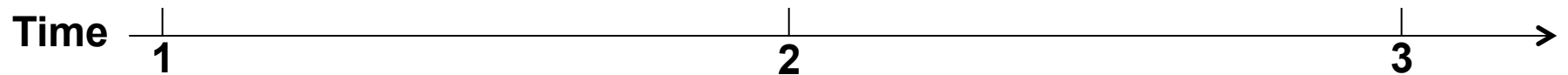
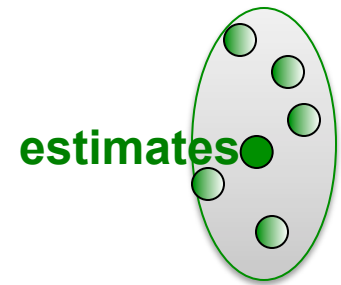
Ensemble data assimilation



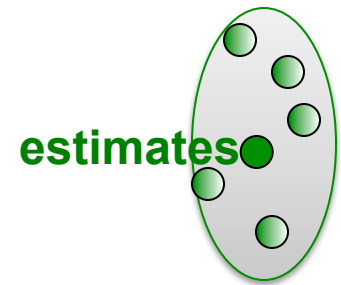
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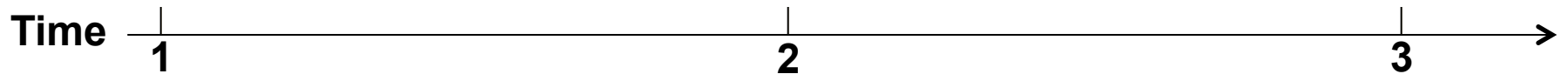
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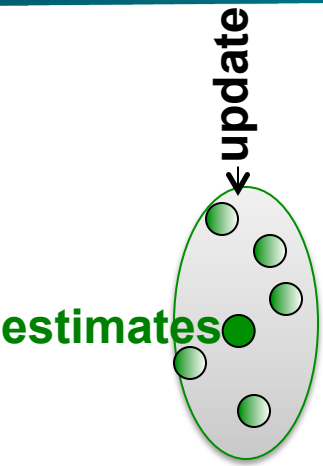
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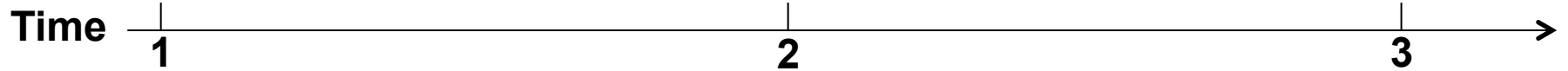
data ●



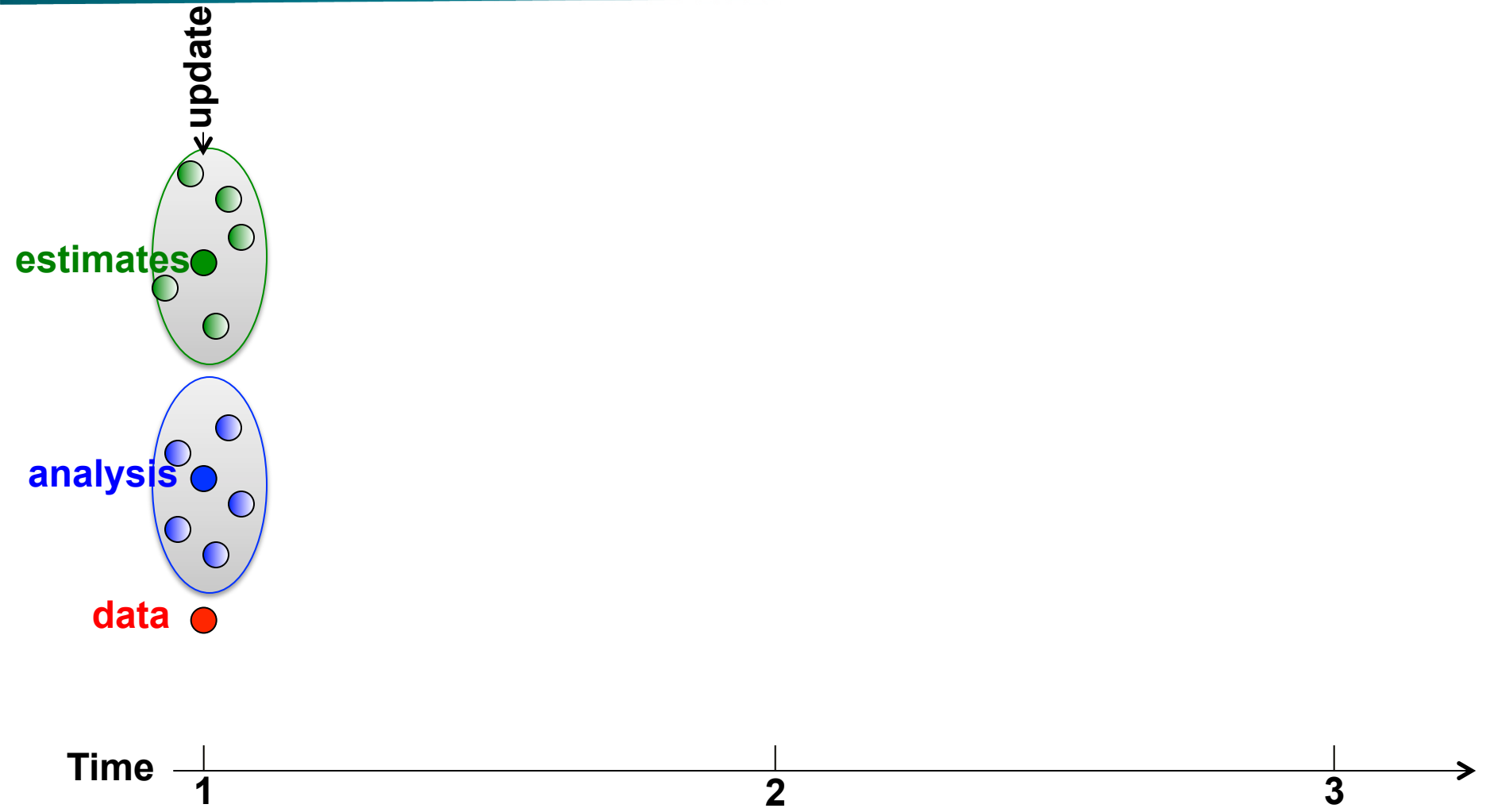
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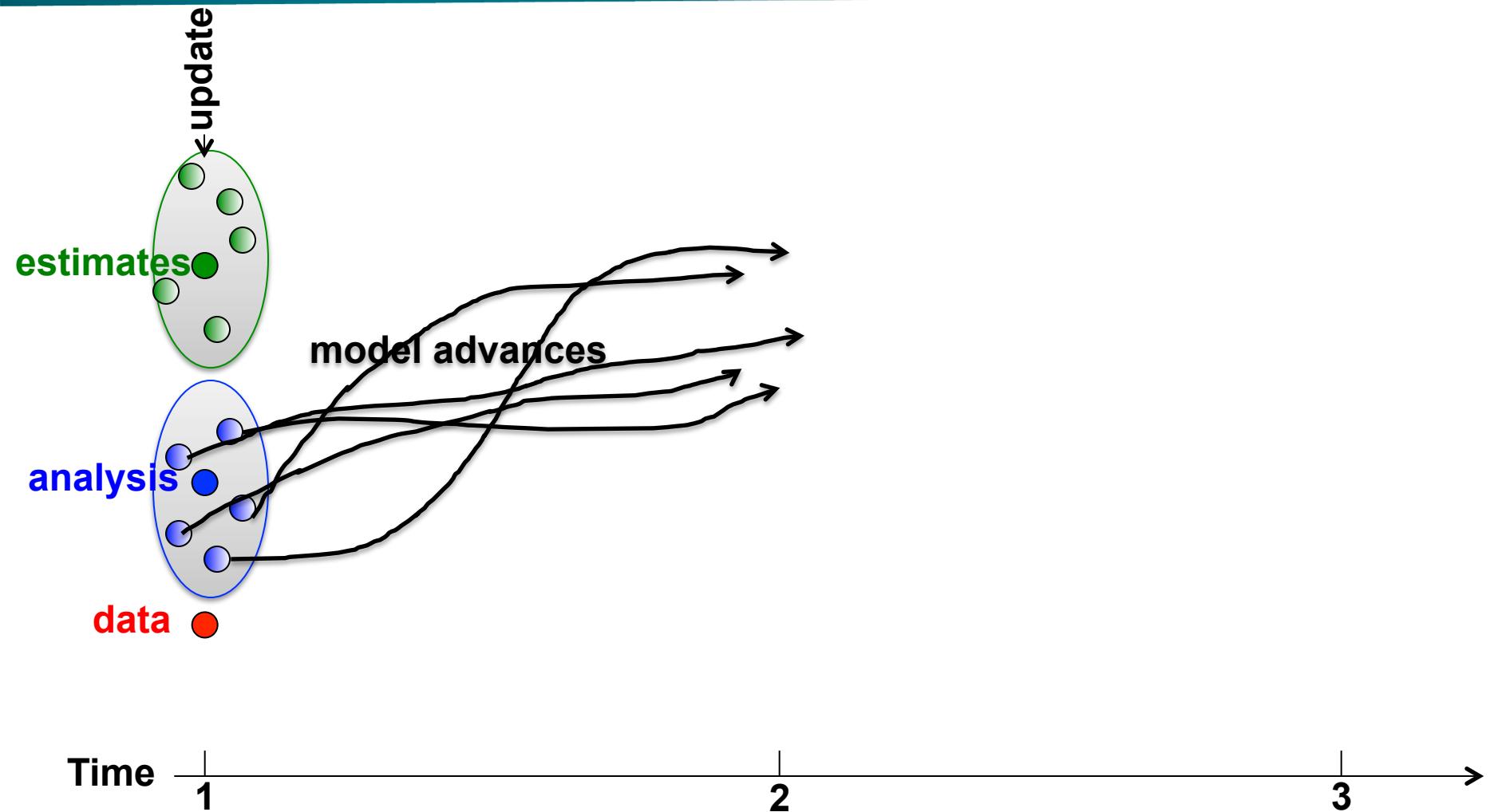
data ●



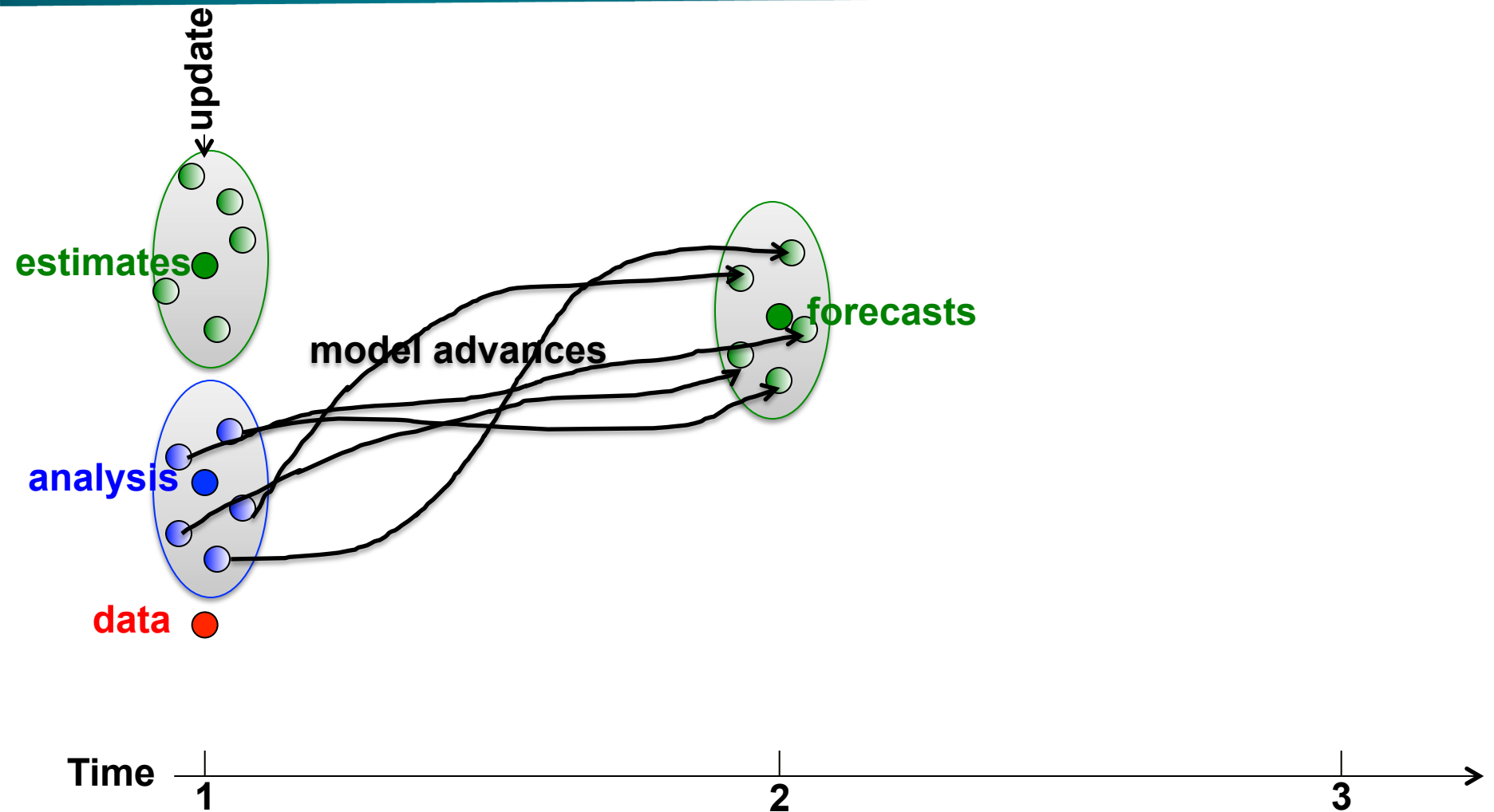
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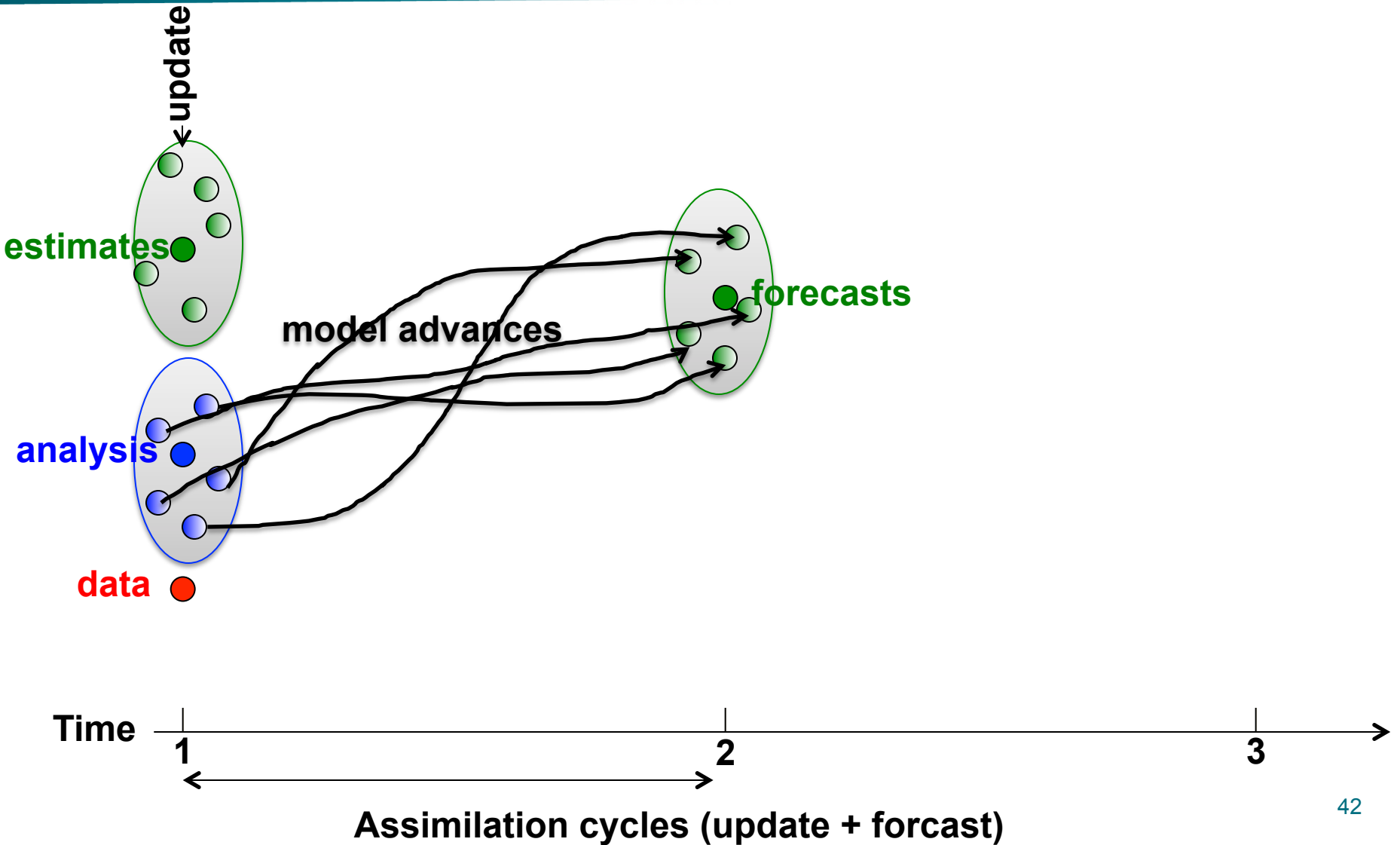
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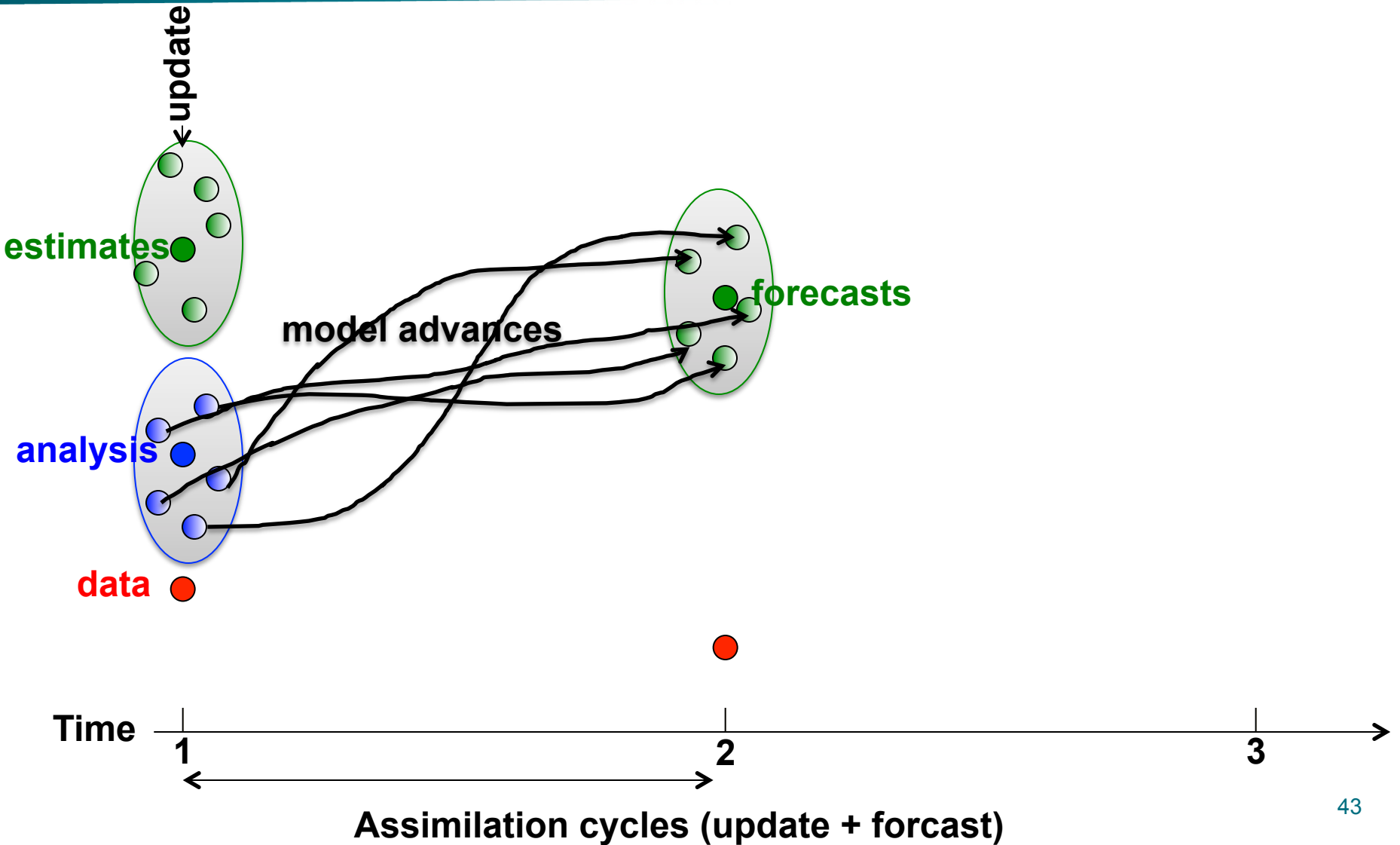
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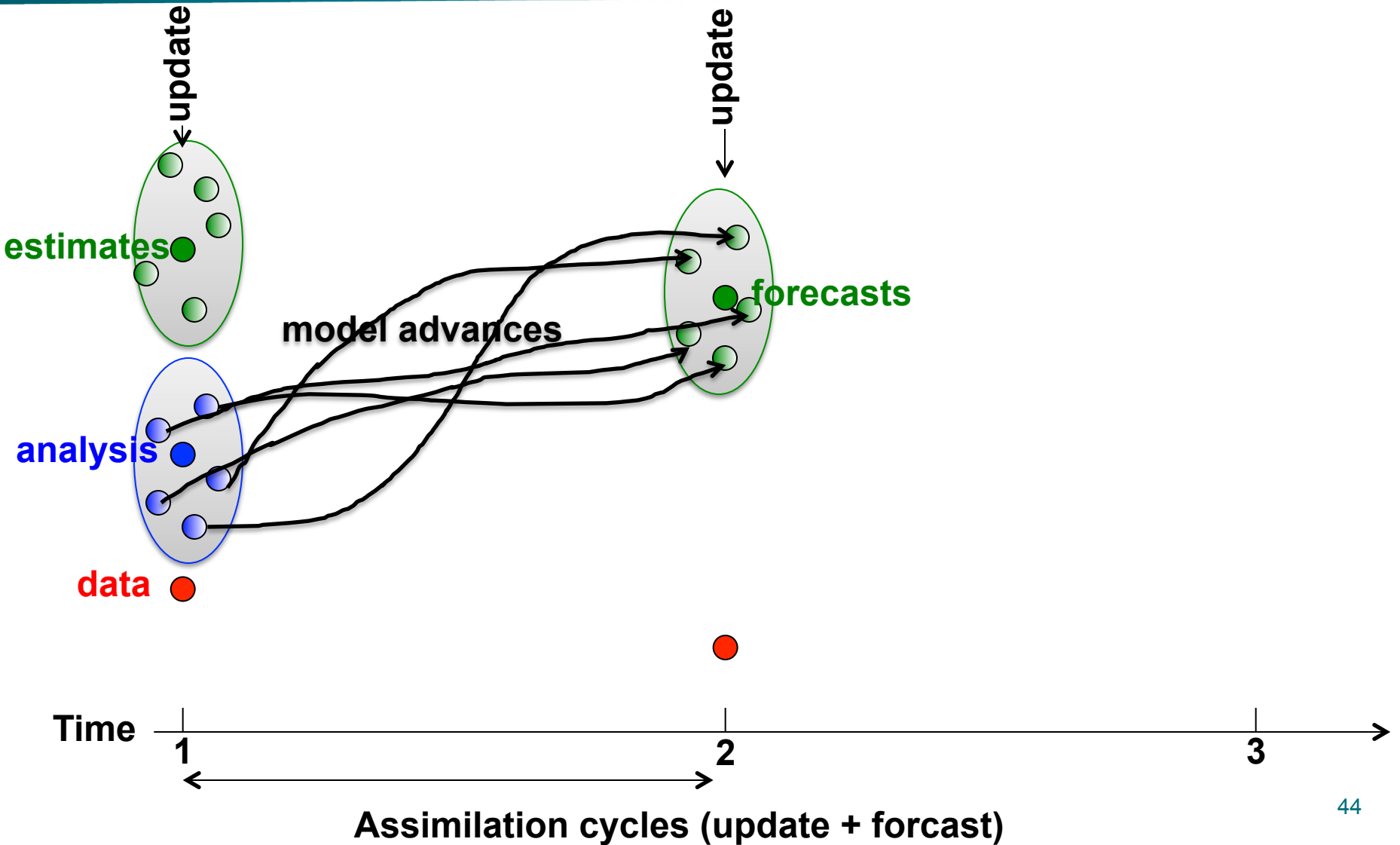
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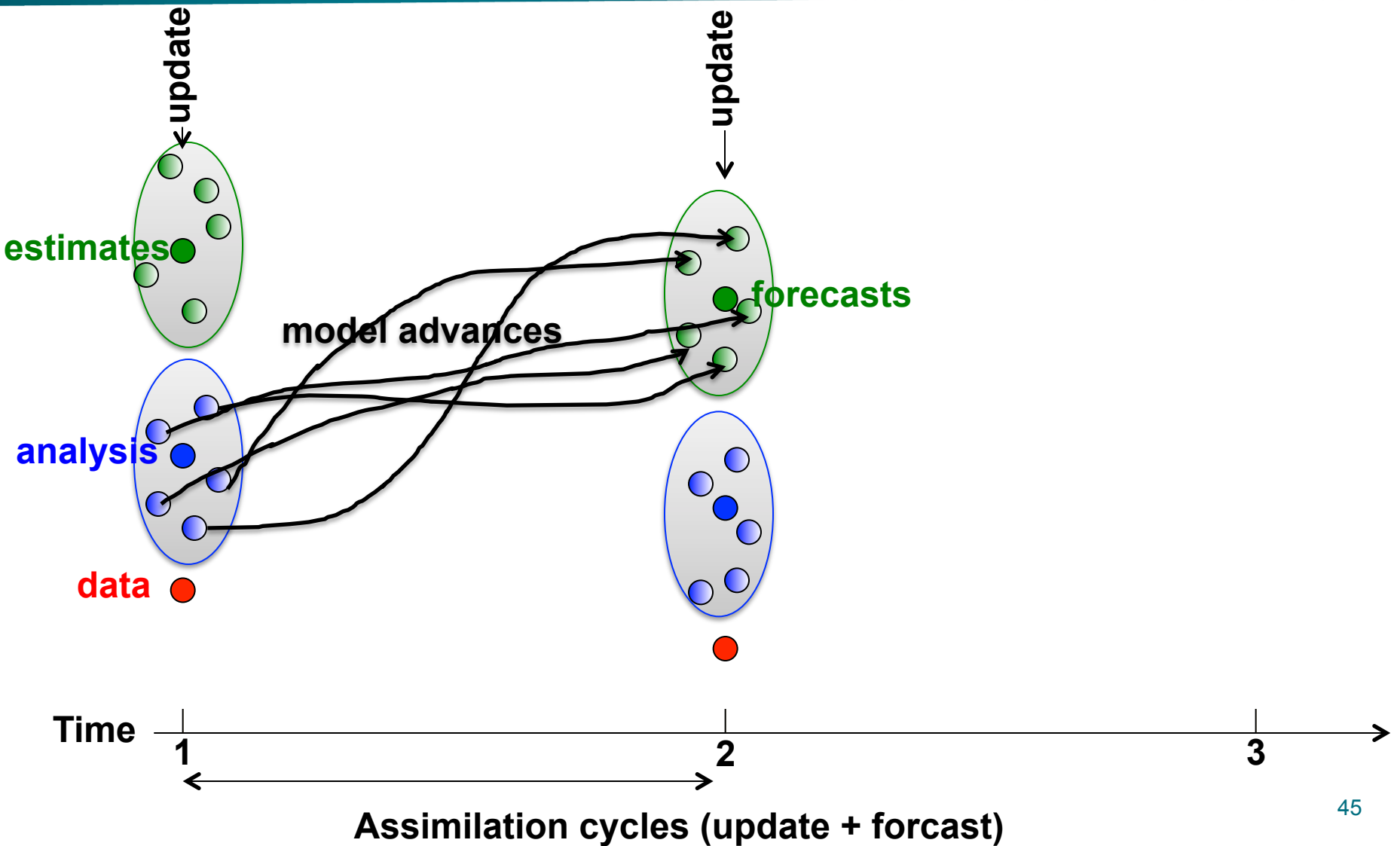
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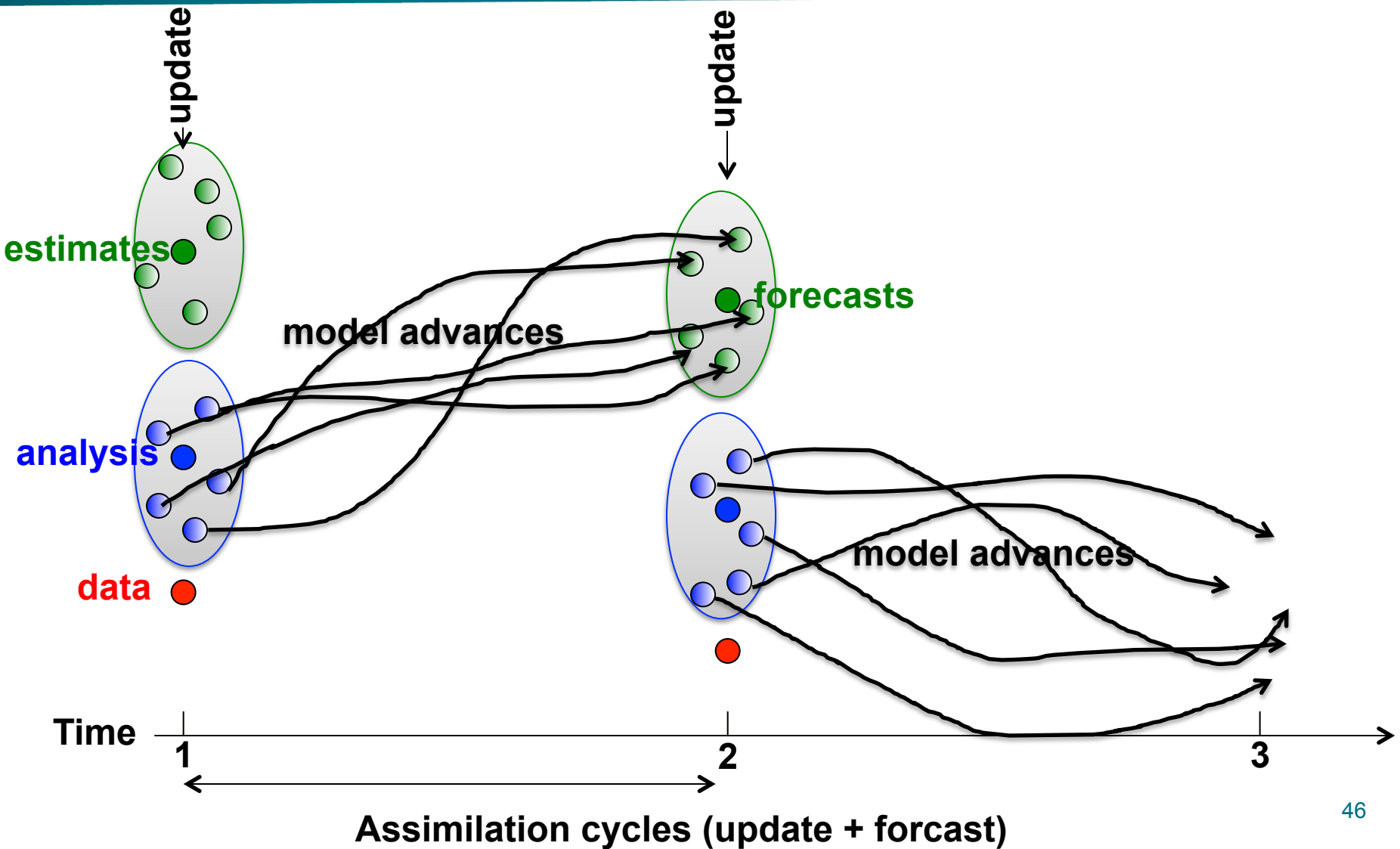
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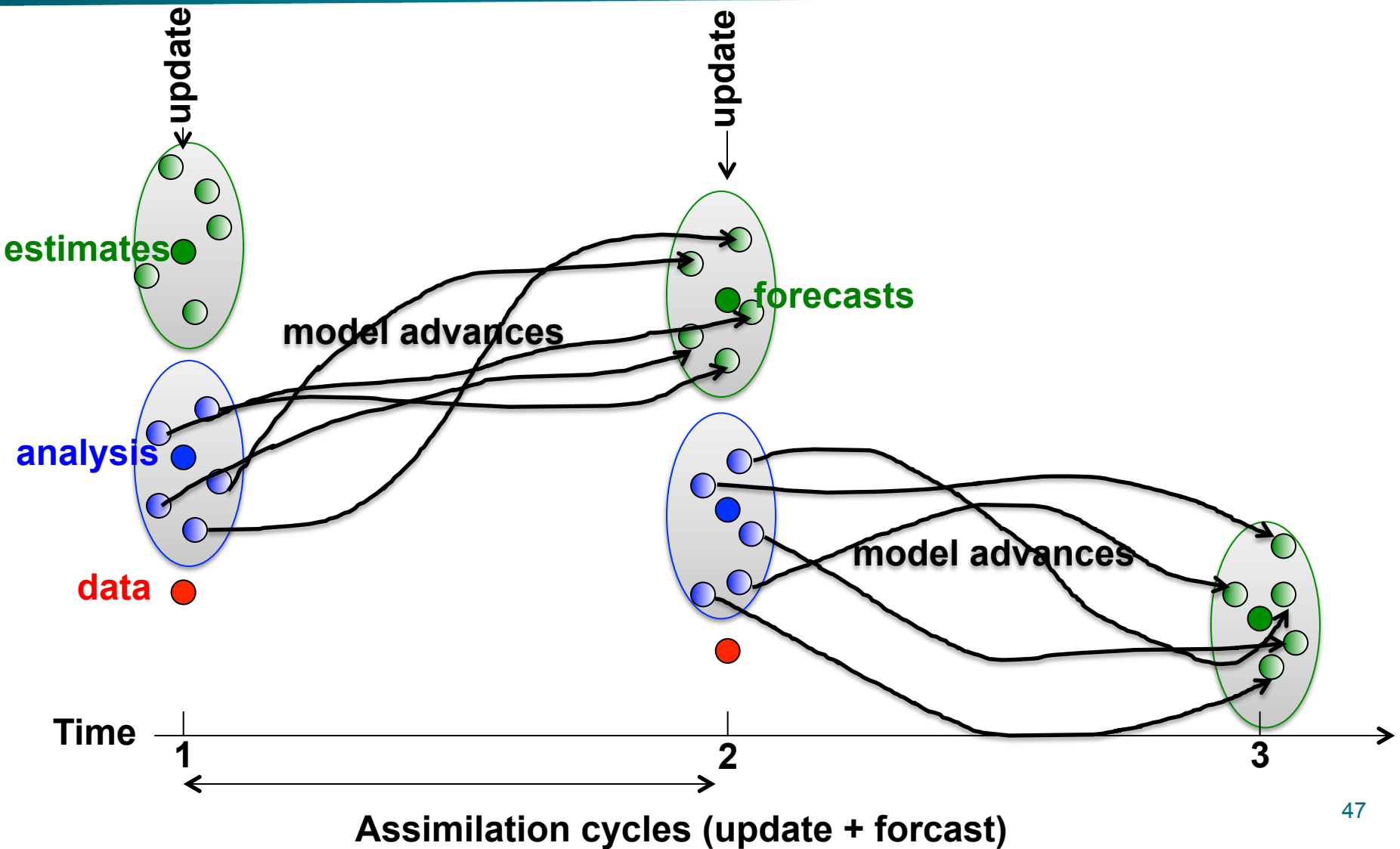
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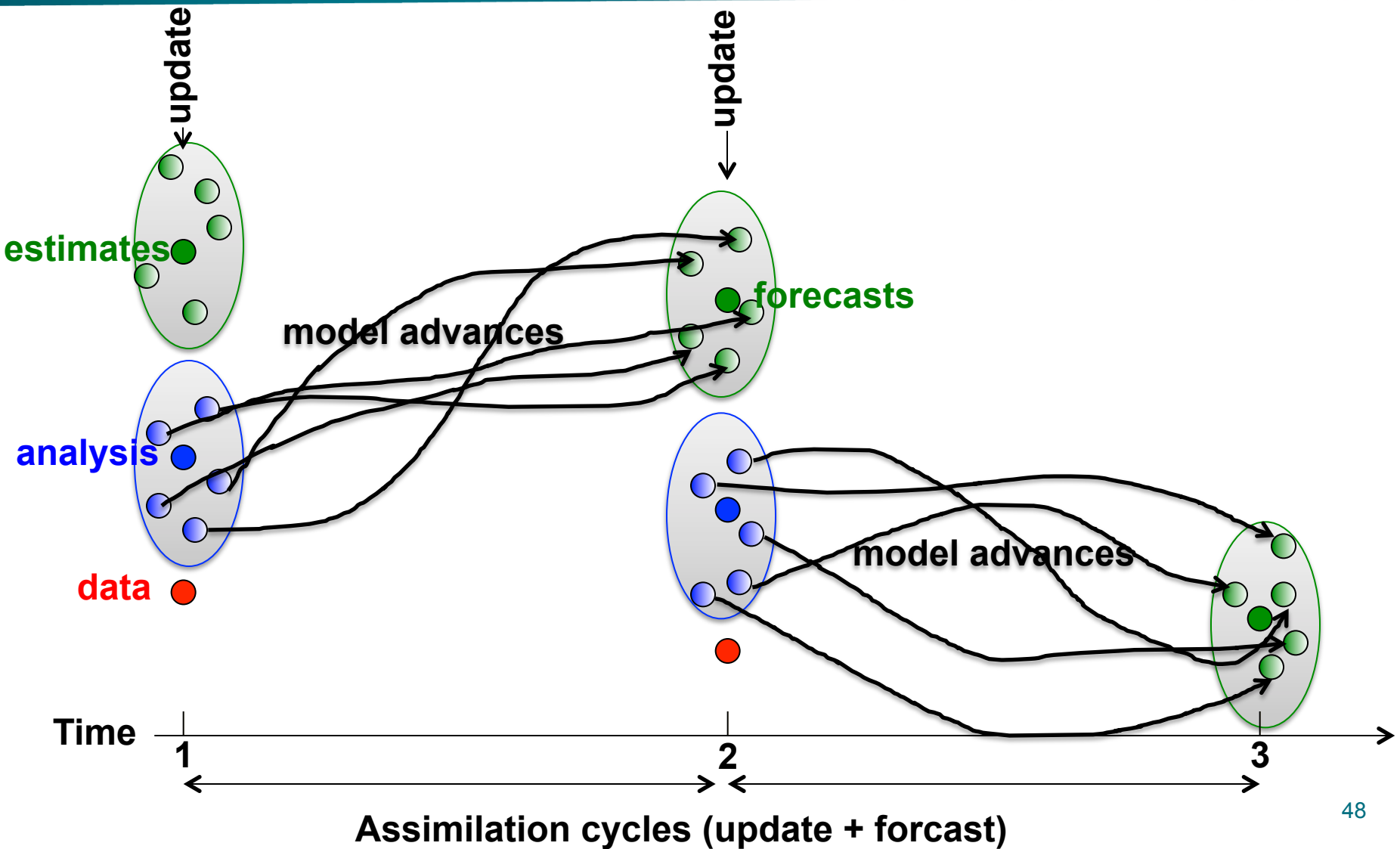
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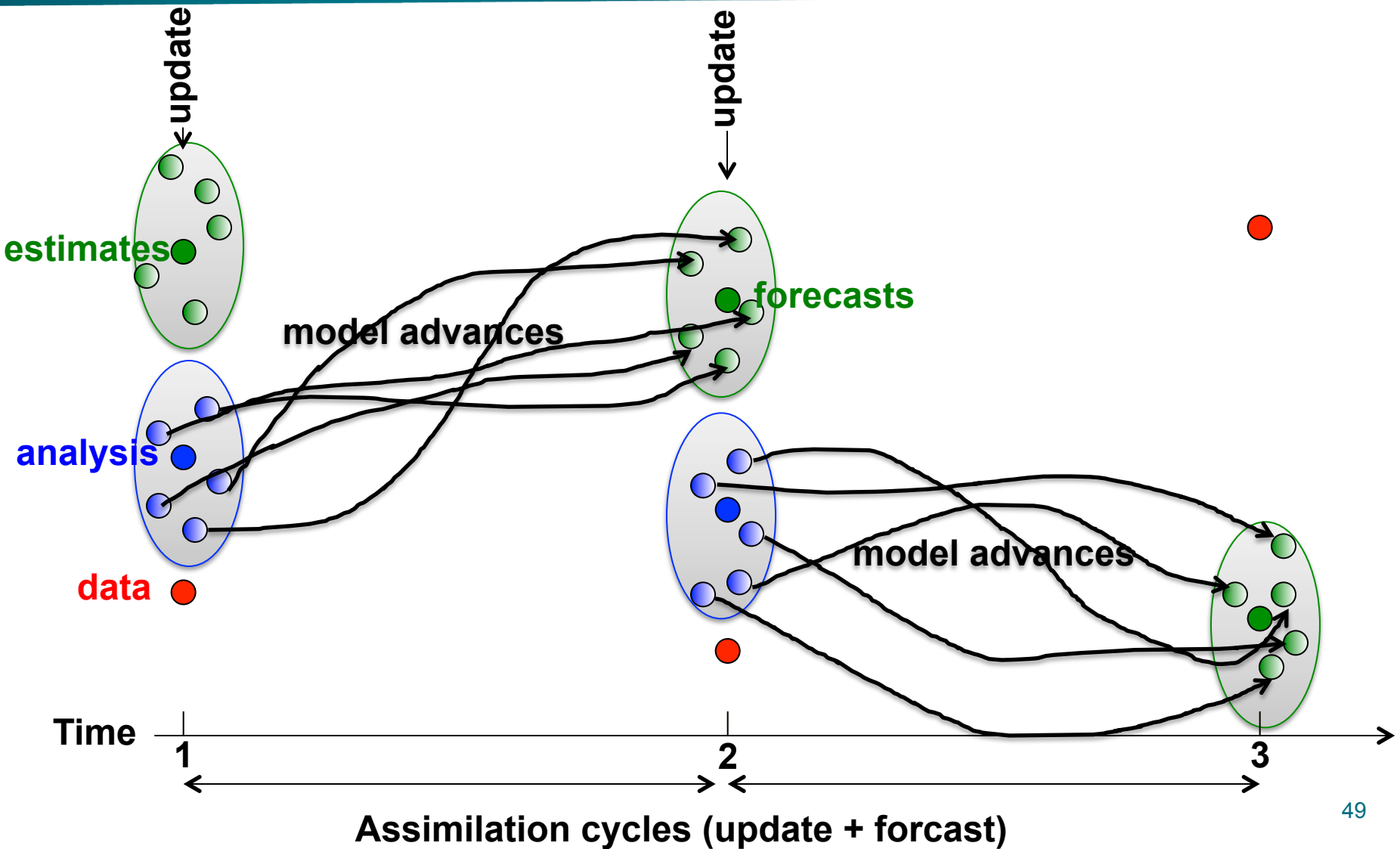
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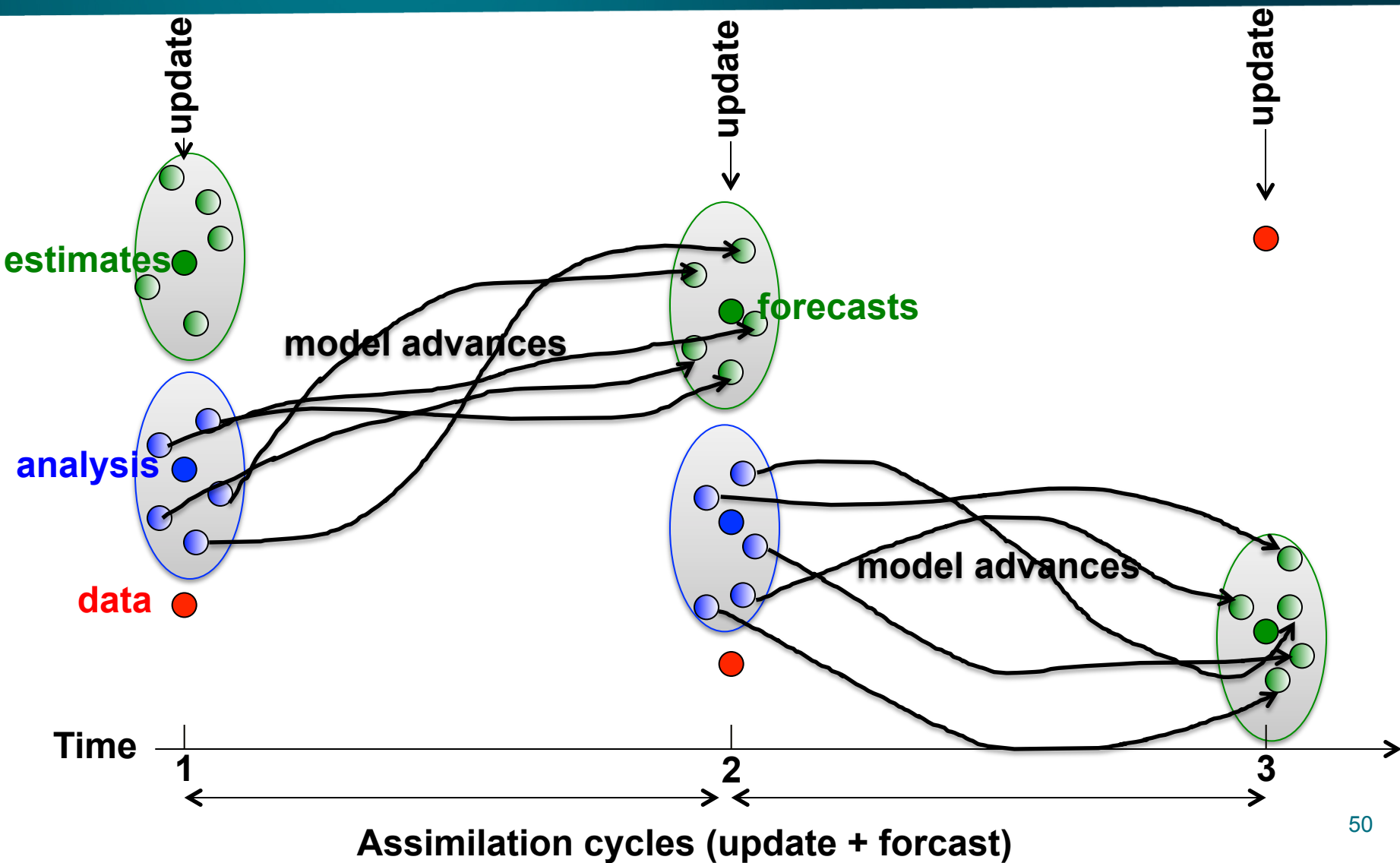
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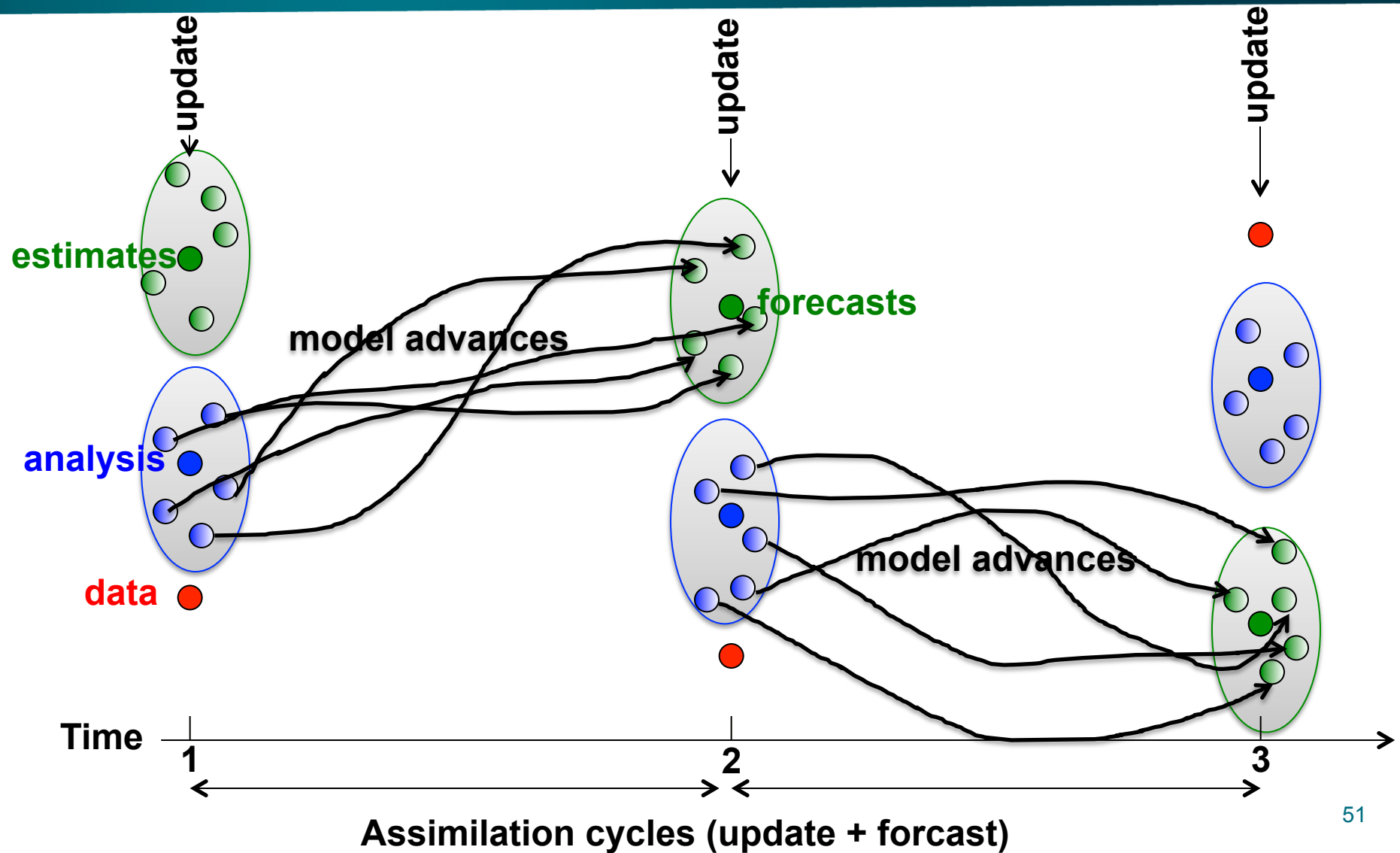
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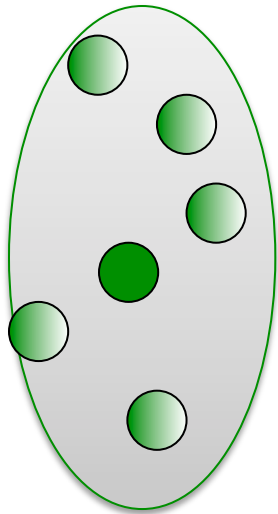
Ensemble data assimilation



Ensemble data assimilation



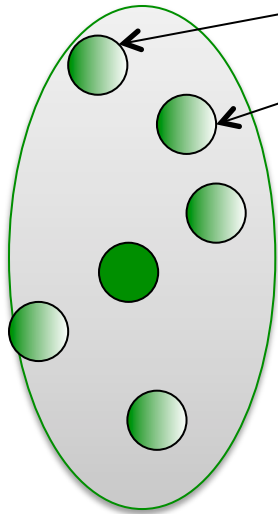
Ensemble data assimilation



Ensemble data assimilation



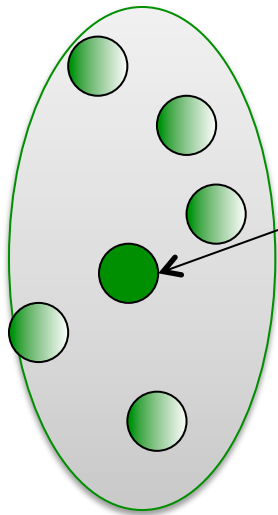
$$X = [x_1^f, x_2^f, \dots, x_N^f]$$



Ensemble data assimilation



$$X = [x_1^f, x_2^f, \dots, x_N^f]$$



$$\bar{x}^f = \frac{1}{N} \sum_{i=1}^N x_i^f$$



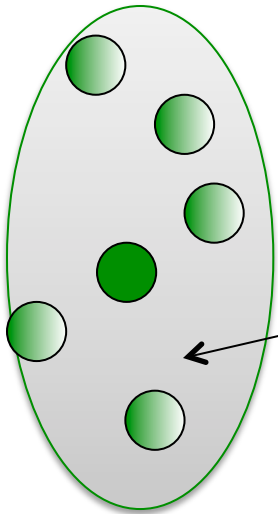
Ensemble data assimilation



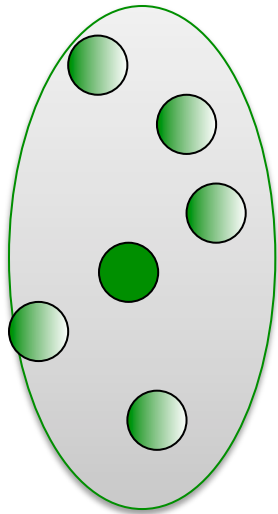
$$X = [x_1^f, x_2^f, \dots, x_N^f]$$

$$\bar{x}^f = \frac{1}{N} \sum_{i=1}^N x_i^f$$

$$X' = X - \bar{x}^f$$



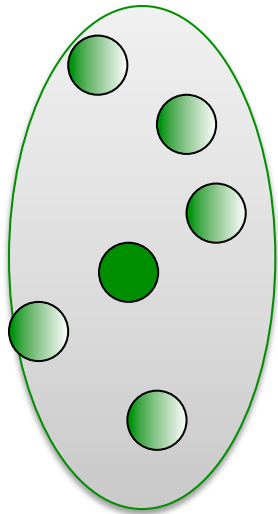
Ensemble data assimilation



$$X' = X - \bar{x}^f$$

$$P^f = \frac{1}{N-1} (X' X'^T)$$

Ensemble data assimilation



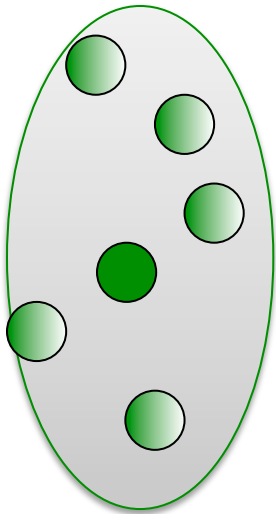
forecast error covariance

$$\downarrow P^f = \frac{1}{N-1} (X' X'^T)$$

$$K = P^f H^T (H P^f H^T + \underset{\uparrow}{R})$$

observational error covariance

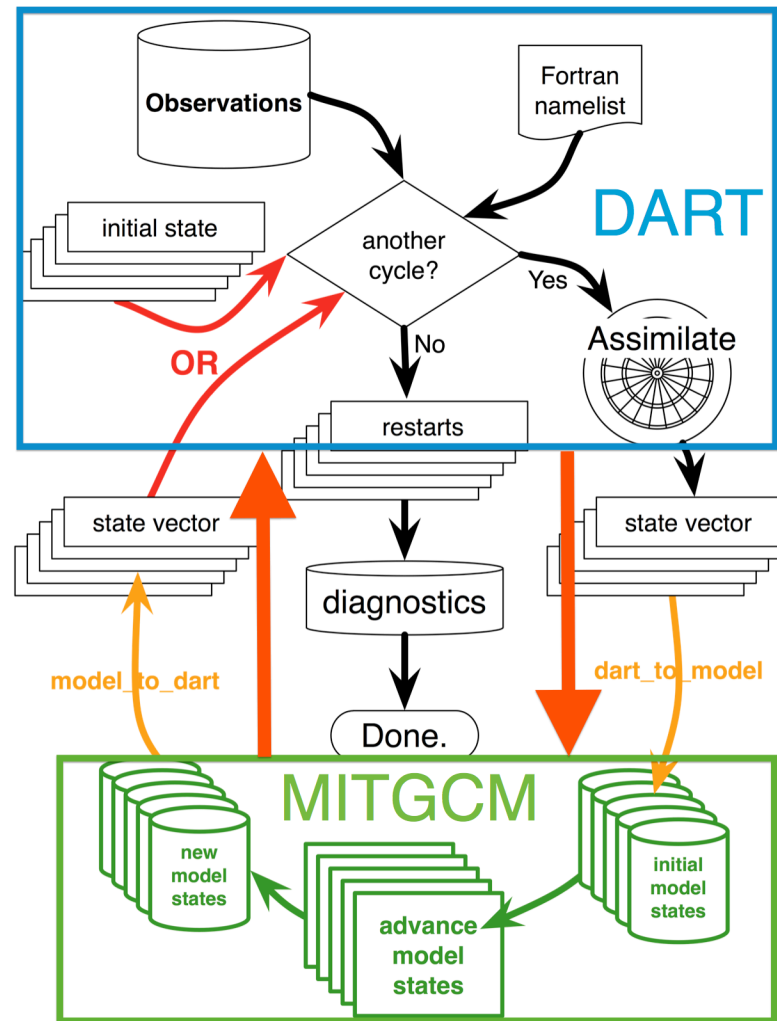
Ensemble data assimilation



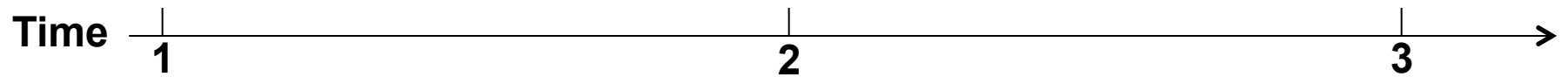
$$K = P^f H^T (H P^f H^T + R)$$

$$x_i^a = x_i^b + K (y^o - h(x_i^b))$$

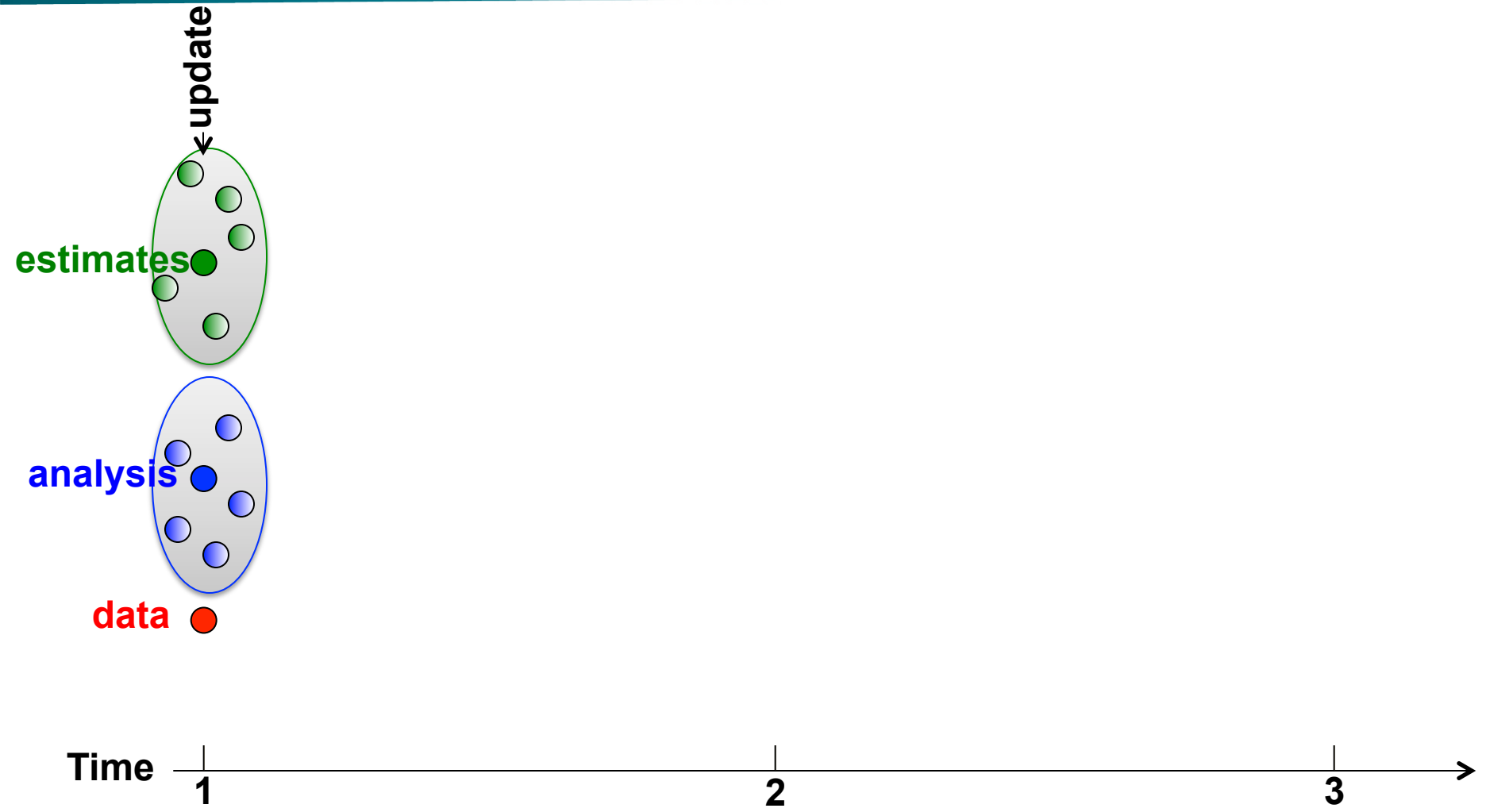
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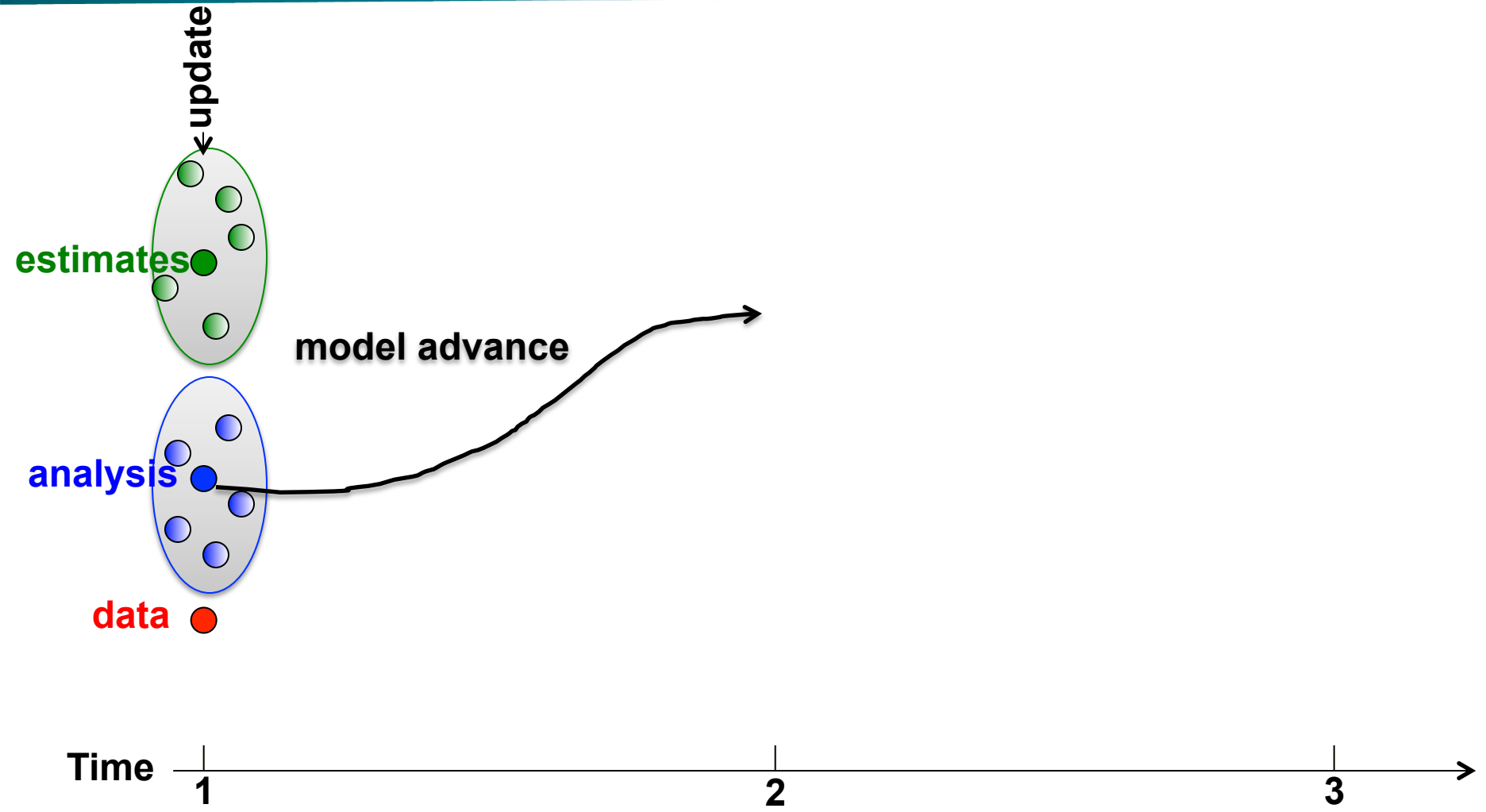
EnOI principle



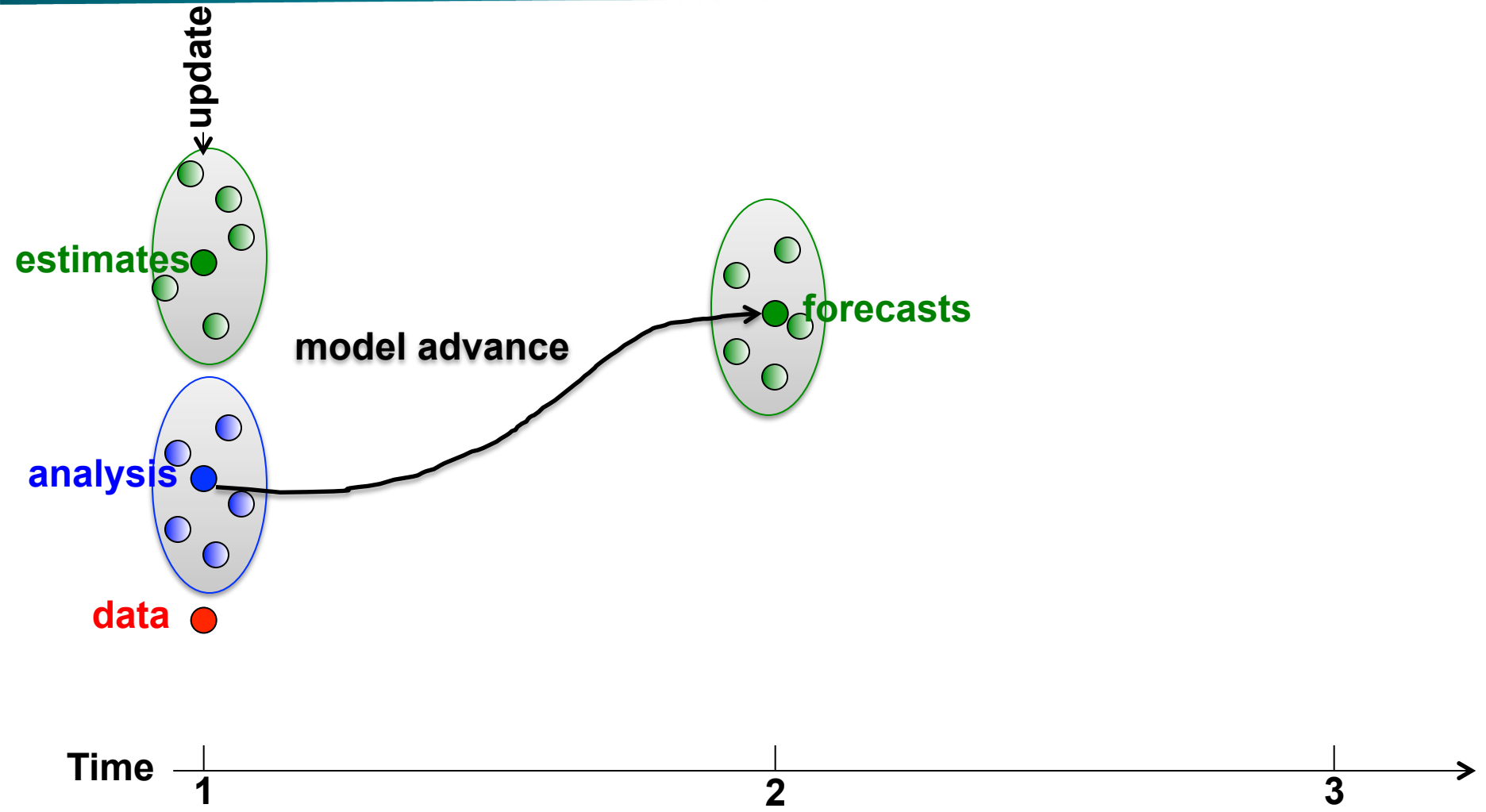
EnOI principle



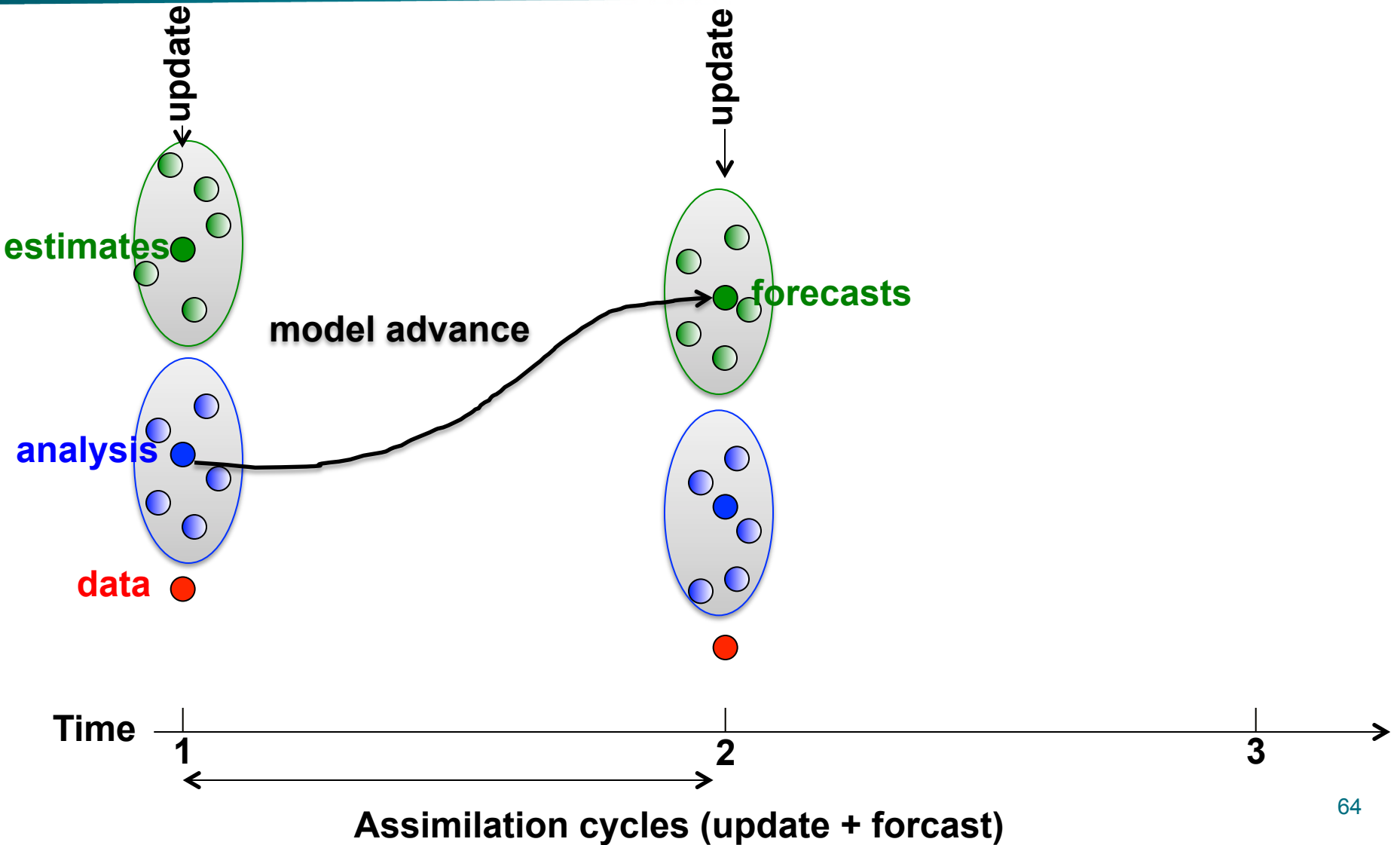
EnOI principle



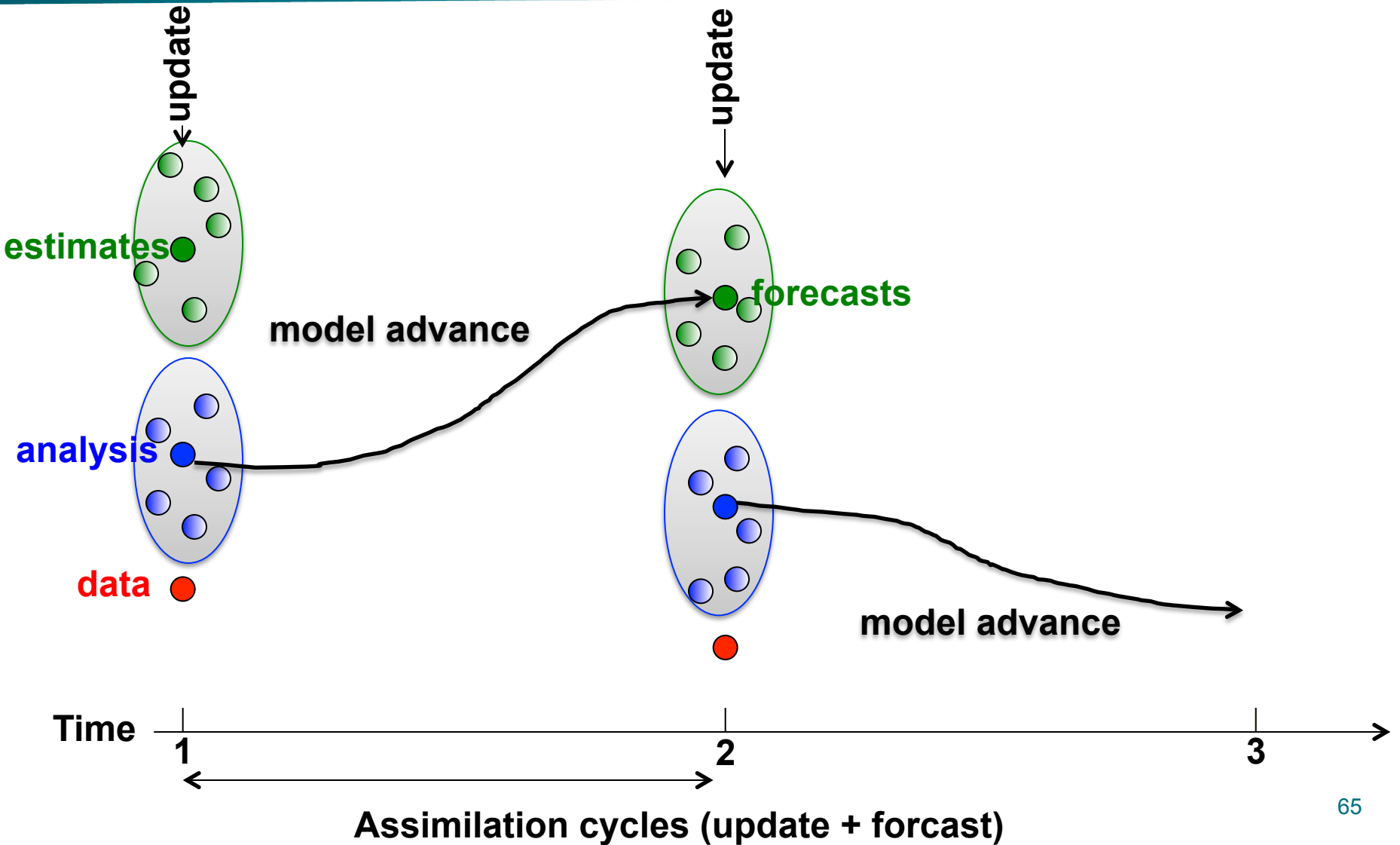
EnOI principle



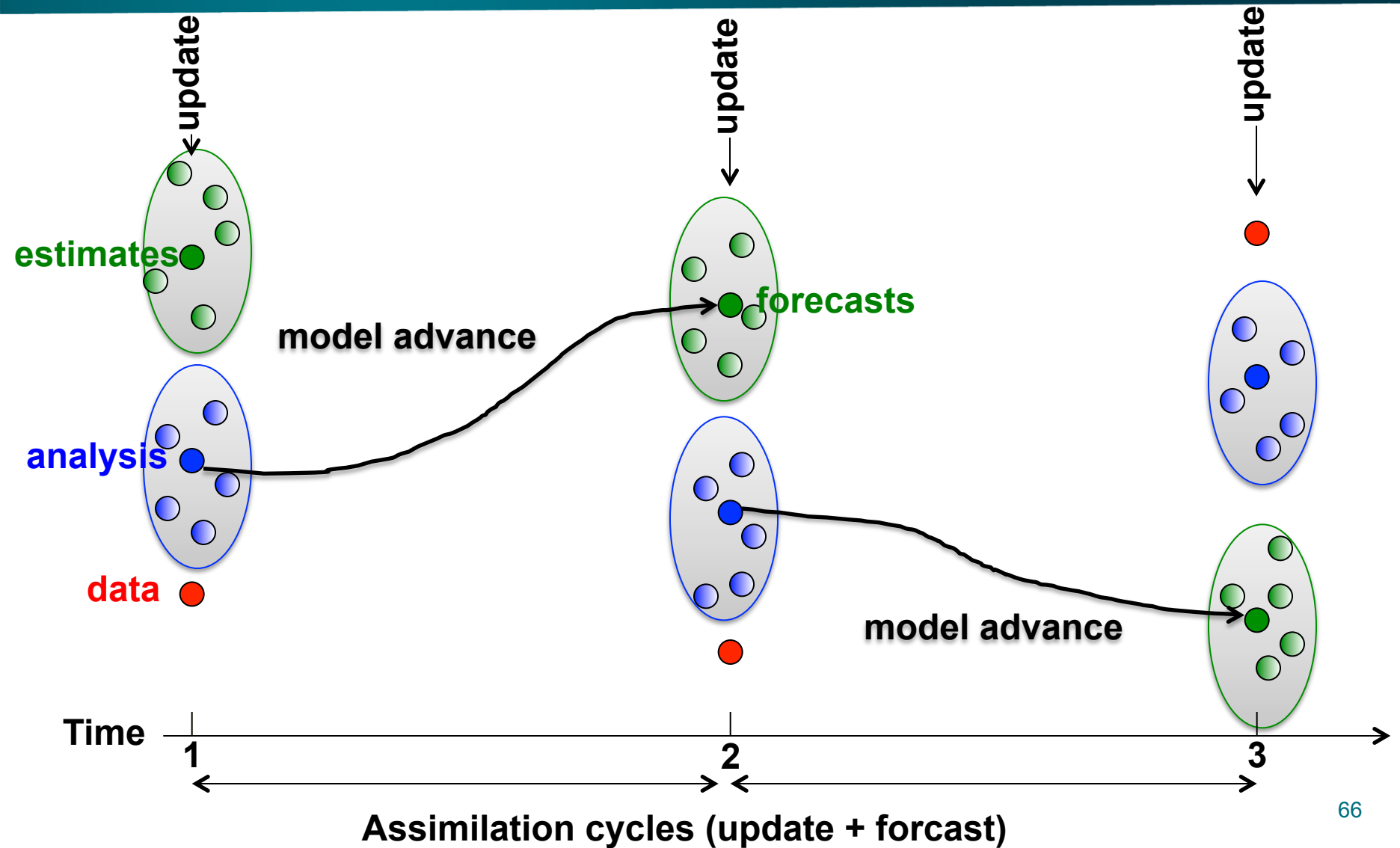
EnOI principle



EnOI principle

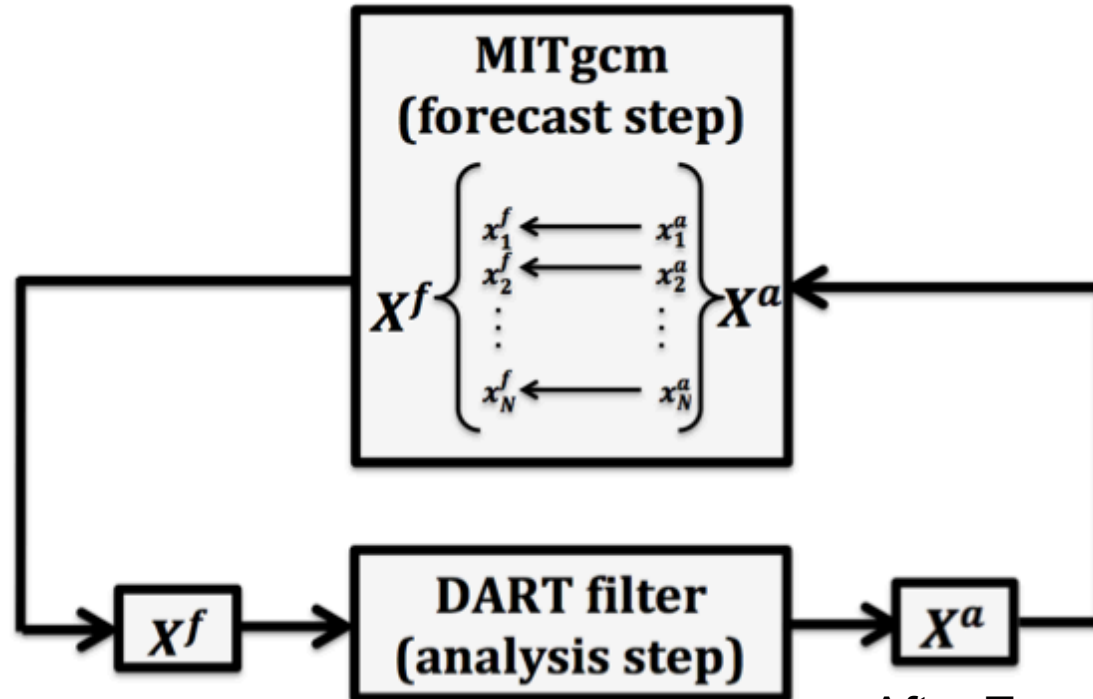


EnOI principle



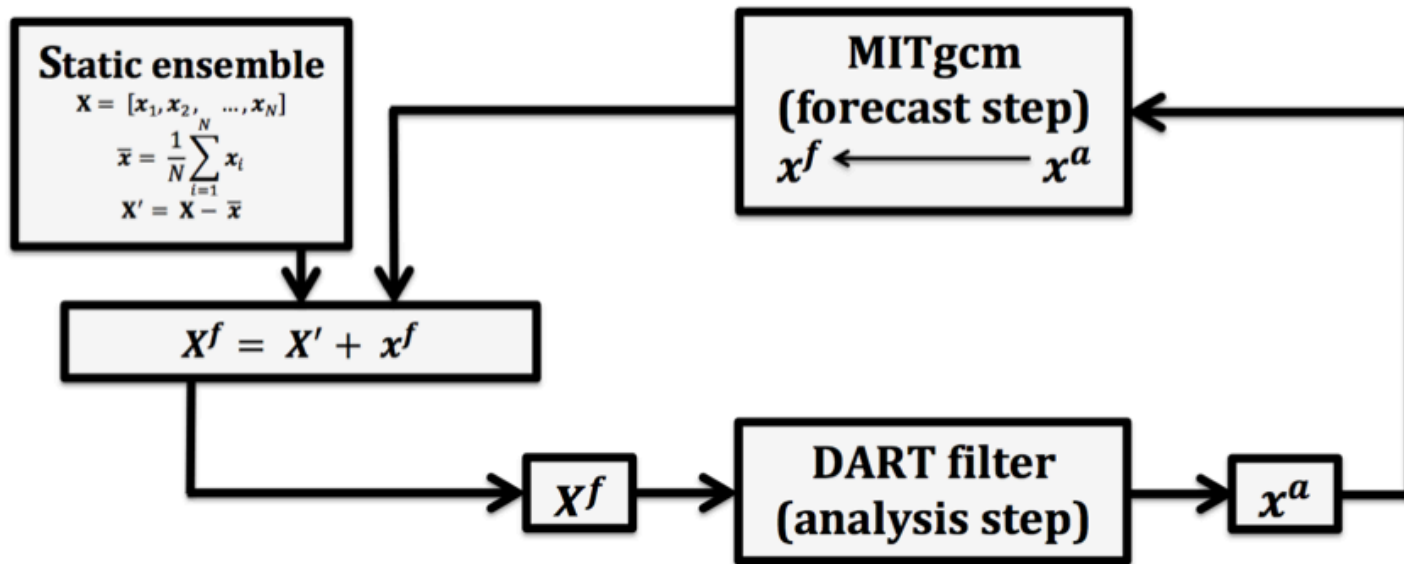
EnOI





After Toye et al. 2017

In the EnKF configuration, all the members are advanced by the model.



After Toye et al. 2017

- With the EnOI, only the analysis is forecasted
- Reduce the cost of the EnKF forecast step by a factor N
- The forecast ensemble needed in the analysis step is built by adding preselected static anomalies to the forecast

Seasonal EnOI



Seasonal EnOI



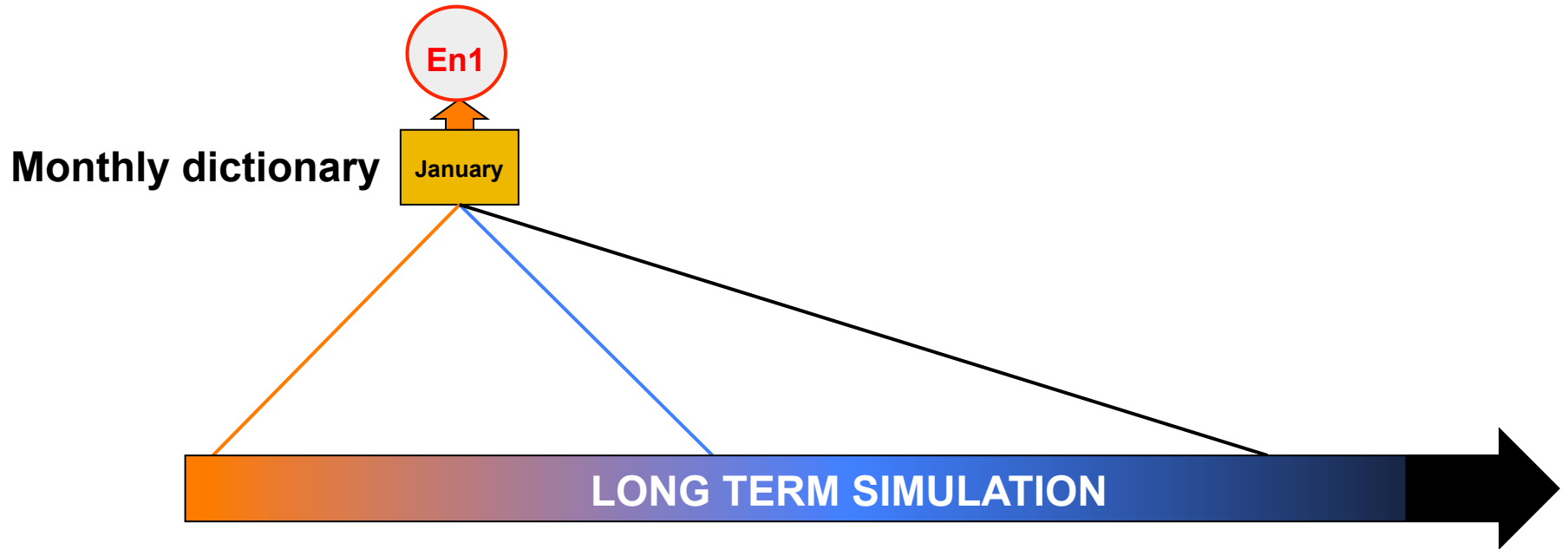
Seasonal EnOI



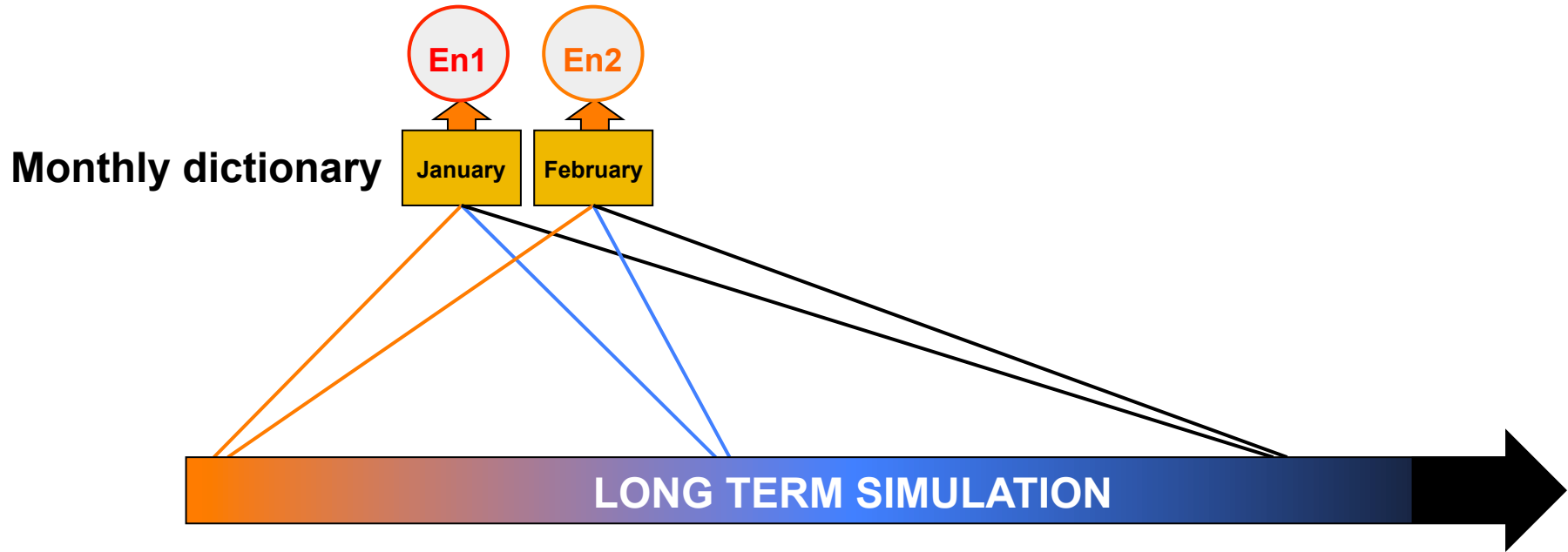
Monthly dictionary



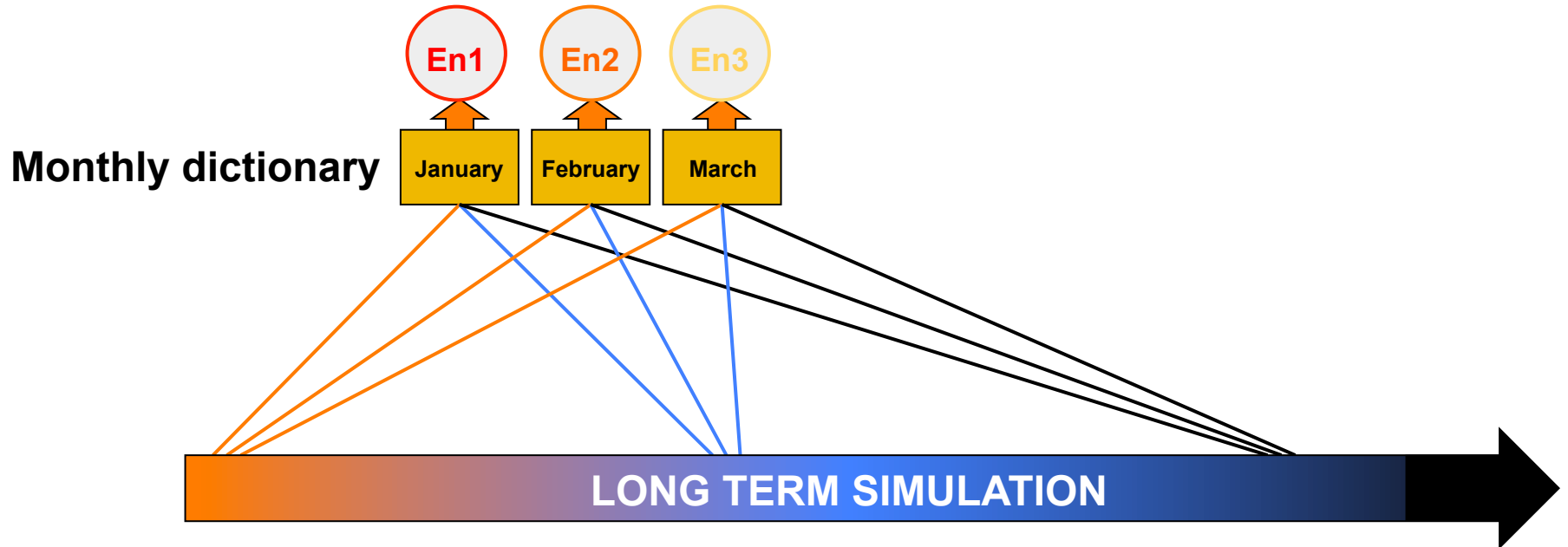
Seasonal EnOI



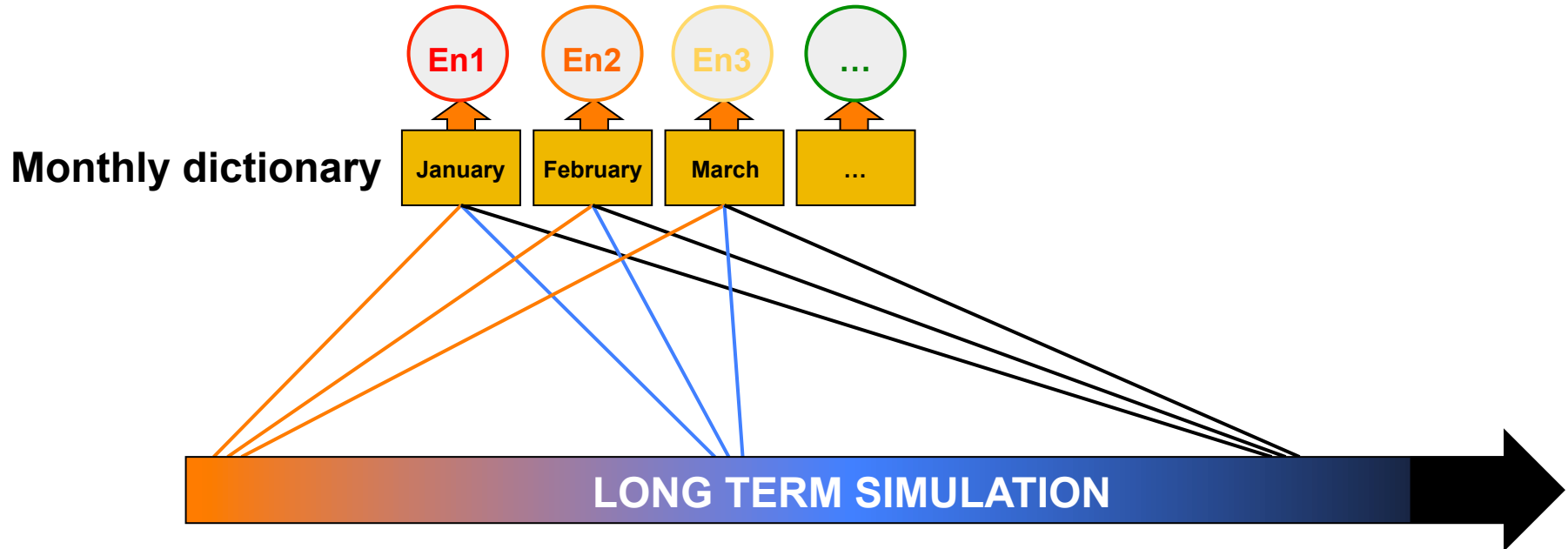
Seasonal EnOI



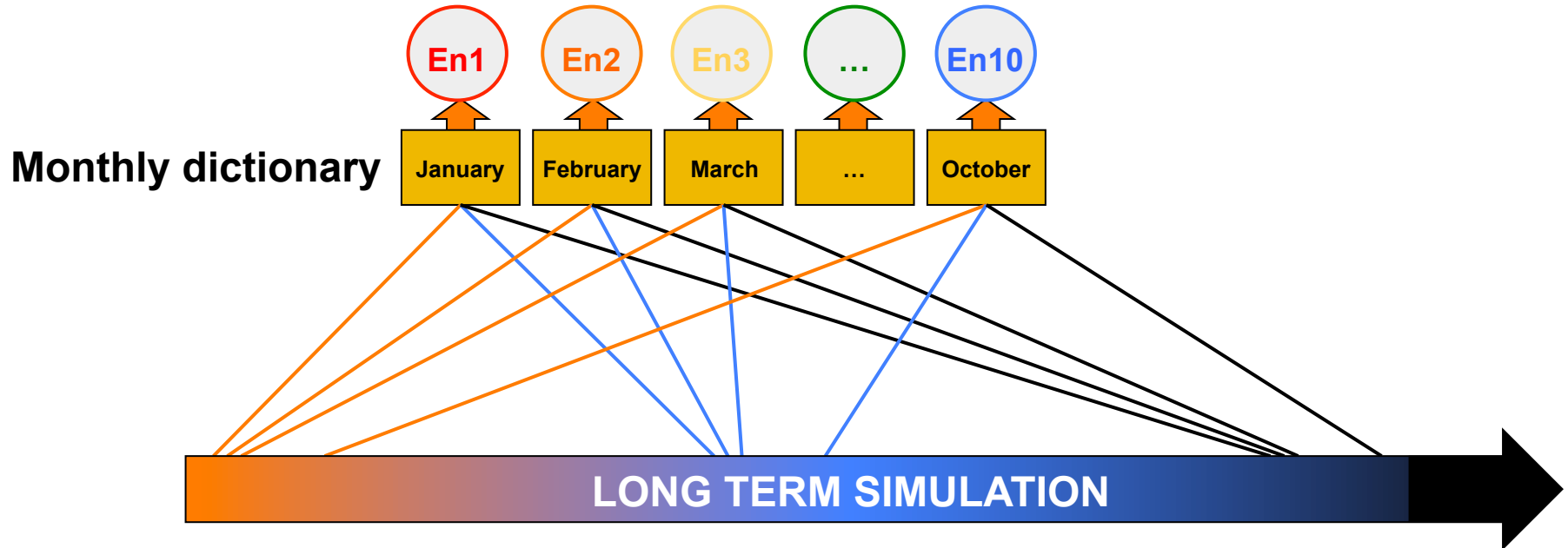
Seasonal EnOI



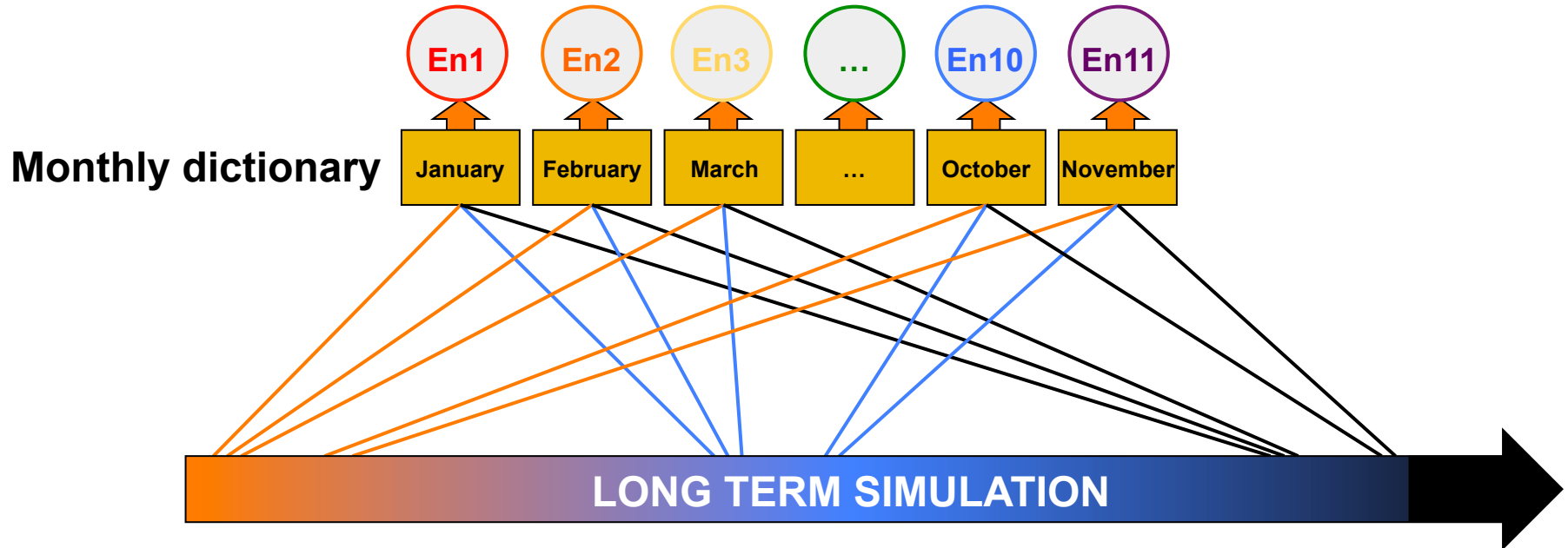
Seasonal EnOI



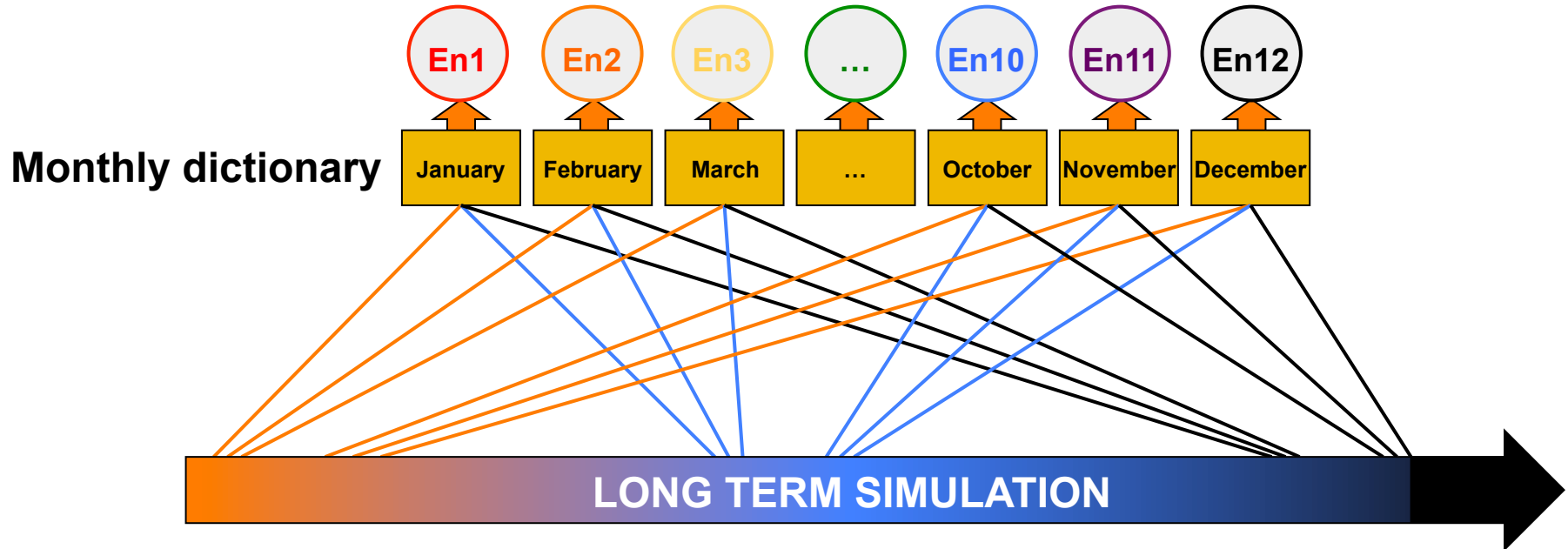
Seasonal EnOI



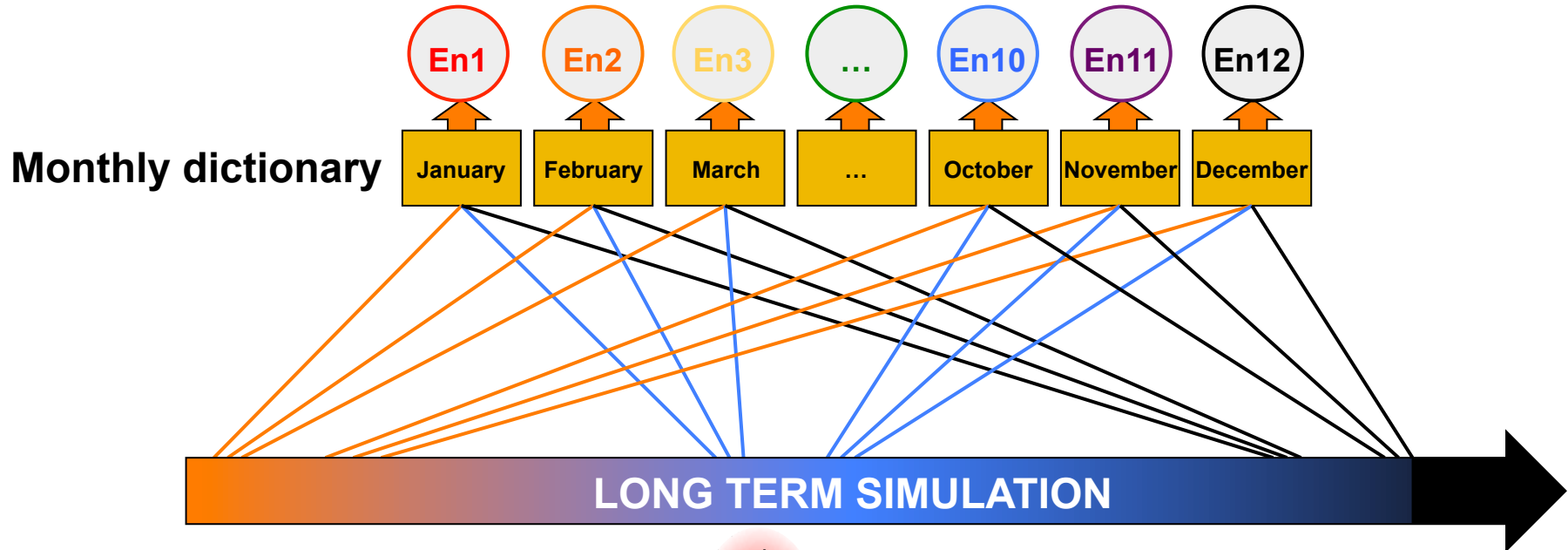
Seasonal EnOI



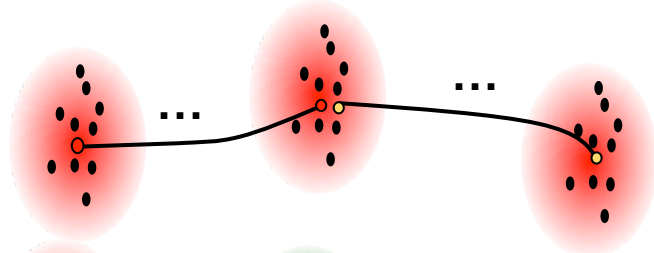
Seasonal EnOI



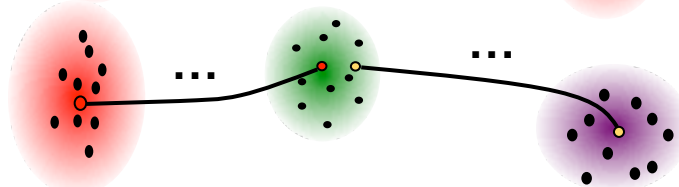
Seasonal EnOI



Conventional EnOI



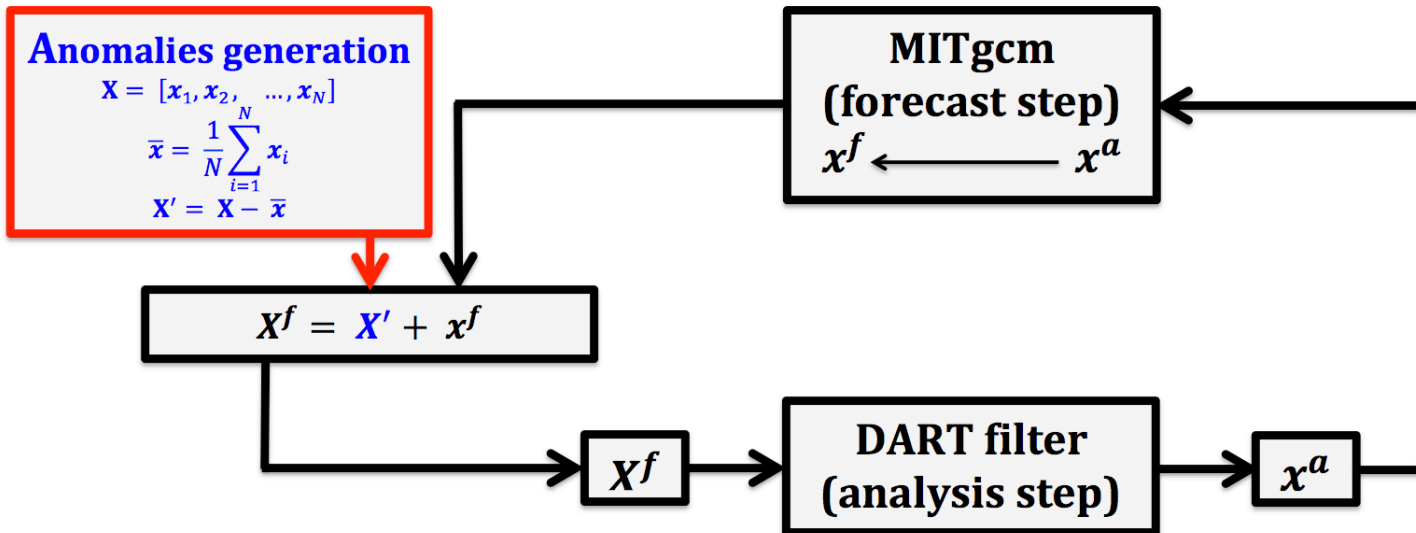
Seasonal EnOI



EnKF, EnOI, SEnOI

Initial ensemble is selected from January dictionary

Adaptive EnOI



Dynamic update of the anomalies for each assimilation cycle, while keeping only 1 forecast member at the forecast step

Adaptive EnOI – selection algorithm



Adaptive EnOI – selection algorithm



1. Inputs:

- A dictionary $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_L]$ of model outputs
- The desired ensemble size N (with $L \gg N$ and at least $L \geq N$)
- The forecast \mathbf{x}^f

Adaptive EnOI – selection algorithm



1. Inputs:

- A dictionary $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_L]$ of model outputs
- The desired ensemble size N (with $L \gg N$ and at least $L \geq N$)
- The forecast \mathbf{x}^f

2. Iterate through the dictionary to apply the related selection algorithm.

Adaptive EnOI – selection algorithm



1. Inputs:

- A dictionary $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_L]$ of model outputs
- The desired ensemble size N (with $L \gg N$ and at least $L \geq N$)
- The forecast \mathbf{x}^f

2. Iterate through the dictionary to apply the related selection algorithm.

3. List and/or sort the obtained elements based on the selection ordering criteria:

$\mathbf{d}_{j_1}, \mathbf{d}_{j_2}, \dots, \mathbf{d}_{j_N}, \dots, \mathbf{d}_{j_L}$

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Adaptive EnOI – selection methods



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- Through correlations:
 - Keep the most correlated members with the forecast



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- Through correlations:
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- Through l_1 and l_2 norms:
 - Keep the dictionary members that are closest to the forecast



Adaptive EnOI – selection methods

- Through correlations:

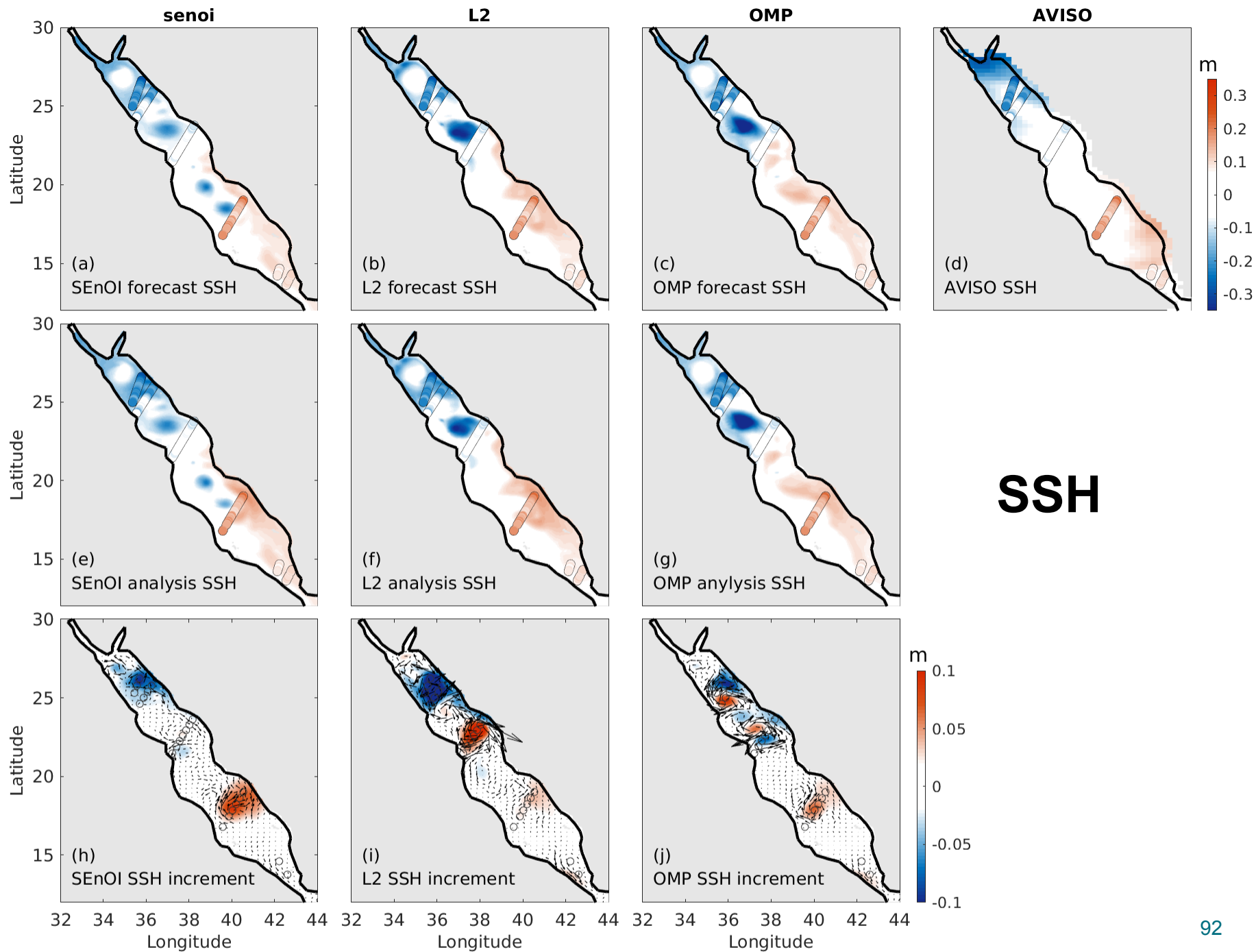
Keep the most correlated members with the forecast

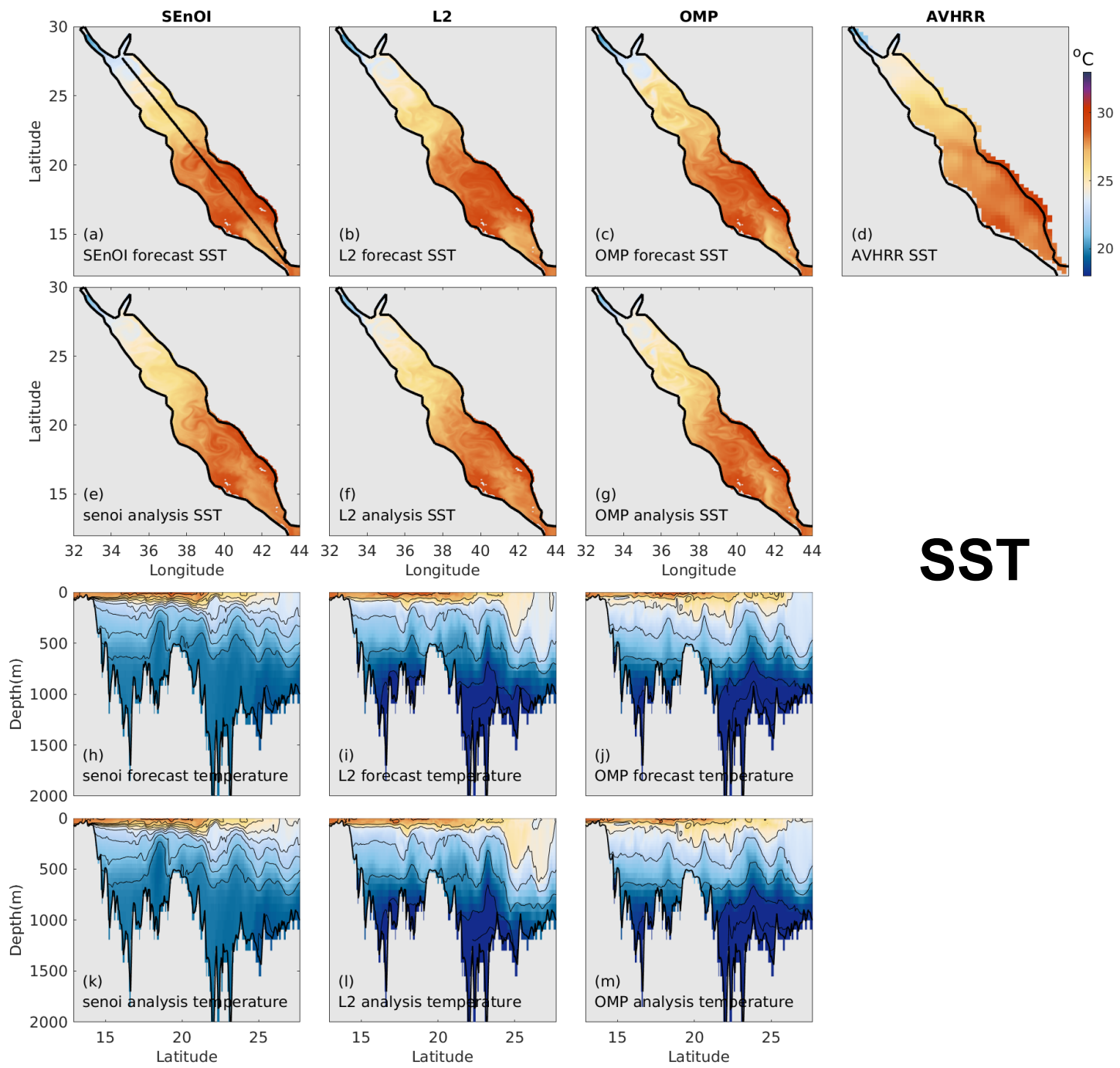
- Through l_1 and l_2 norms:

Keep the dictionary members that are closest to the forecast

- Through orthogonal matching pursuit (OMP):

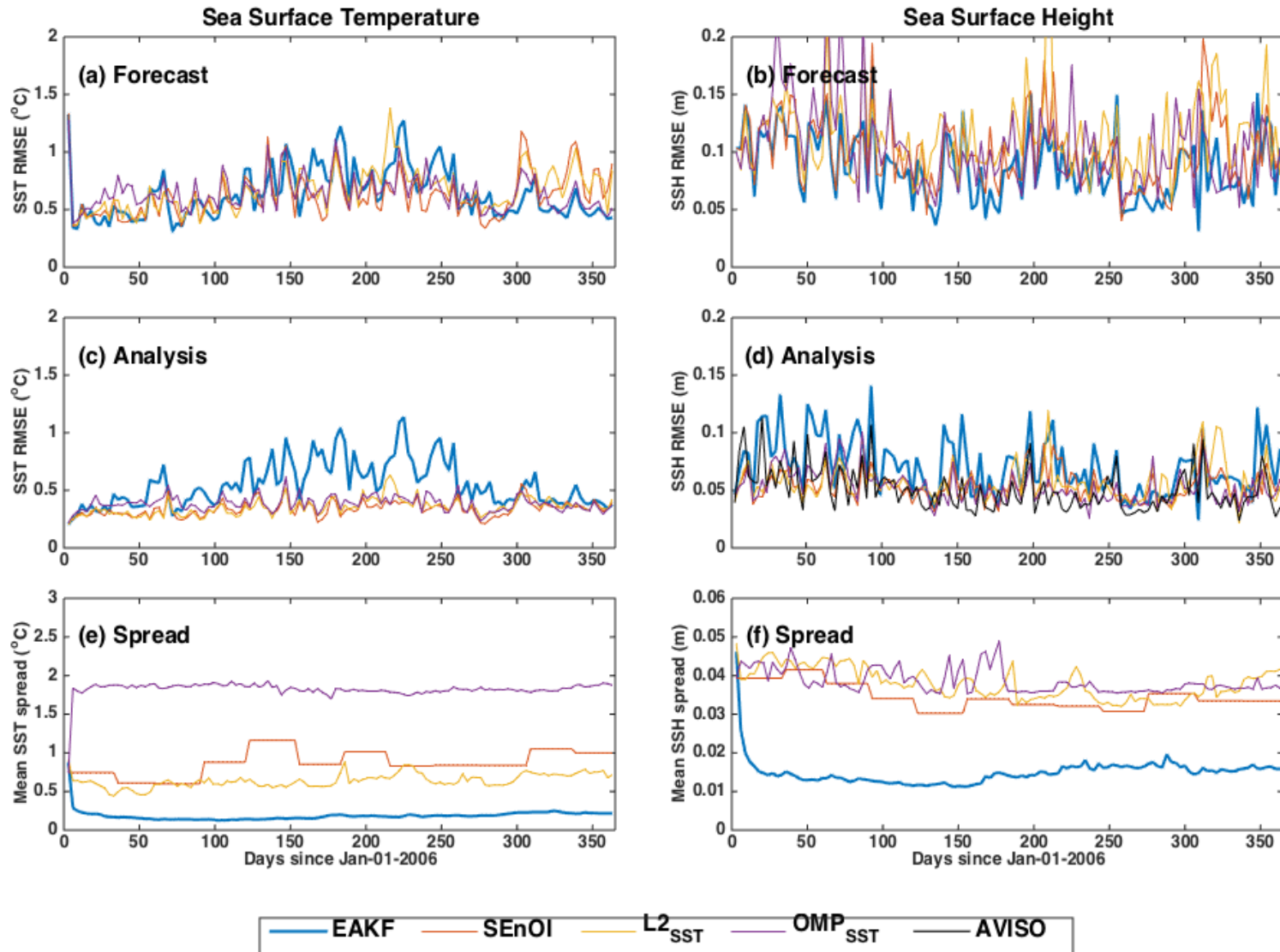
Perform a decomposition of the forecast based on the OMP algorithm and keep the members that represent the forecast





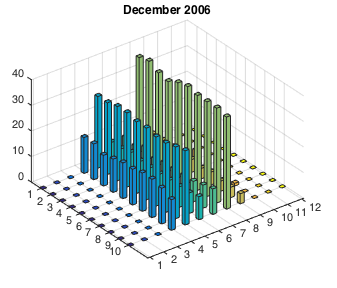
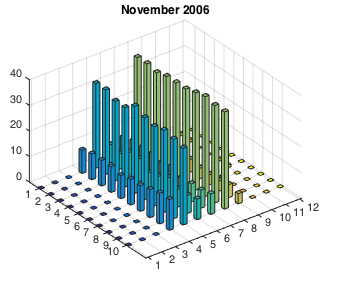
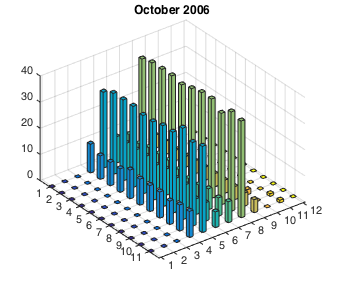
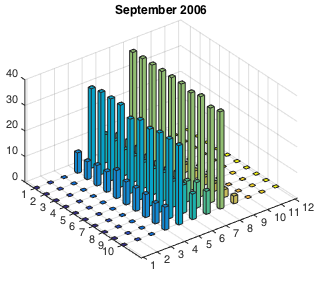
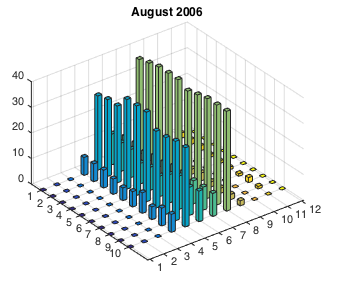
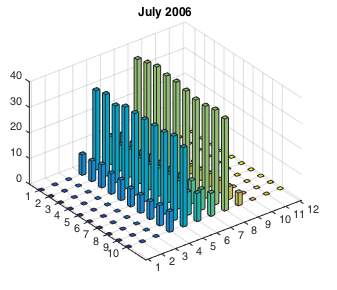
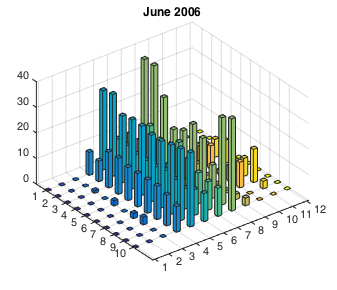
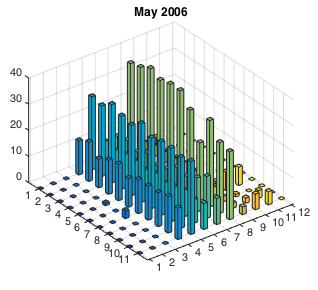
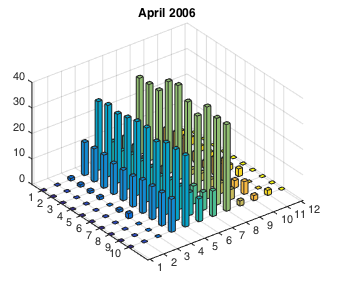
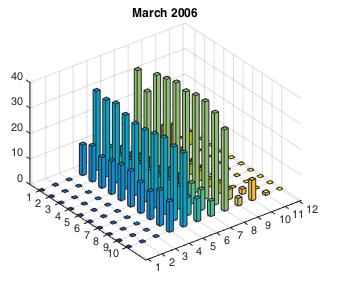
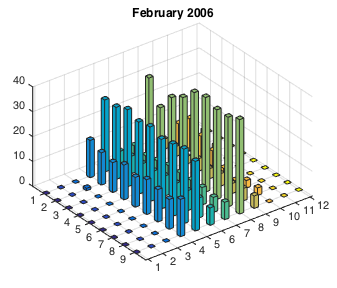
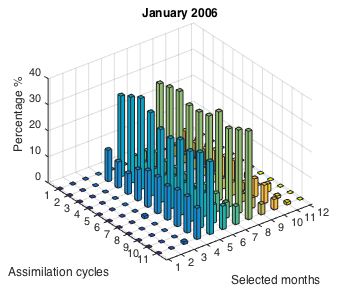
SST

Schemes comparison



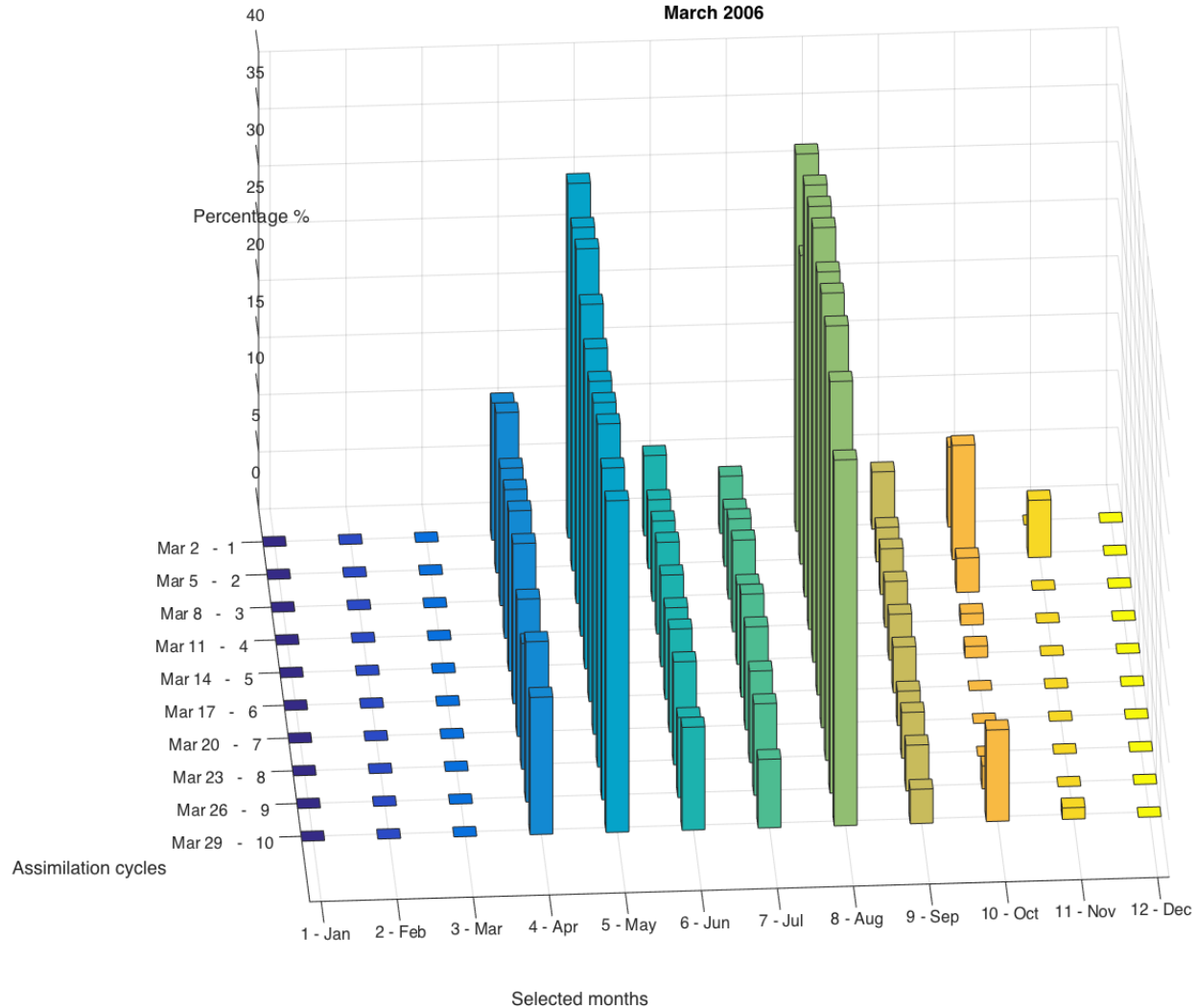


Months of OMP selected members



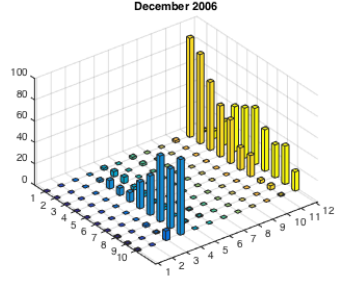
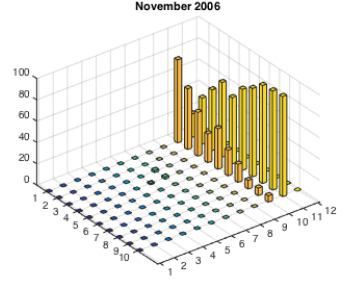
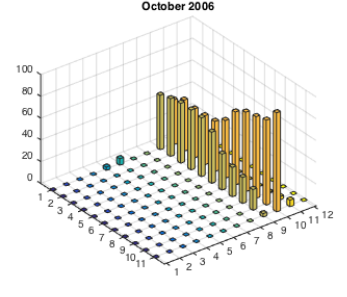
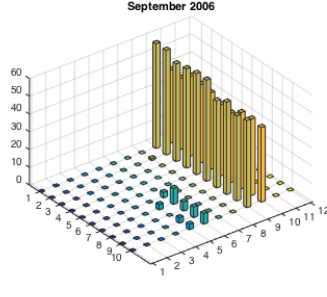
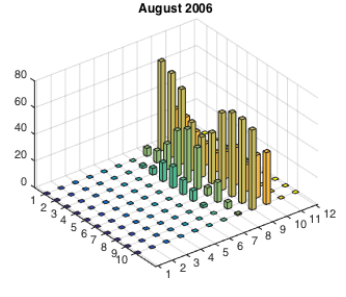
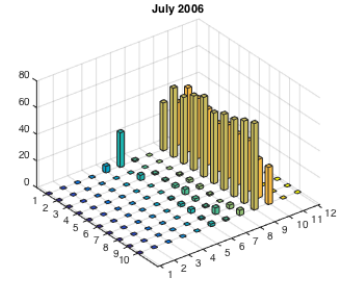
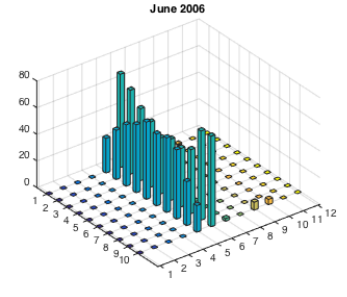
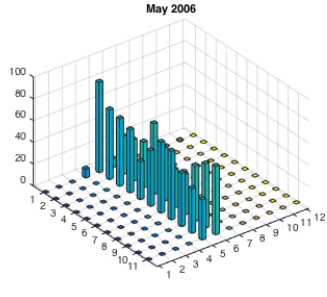
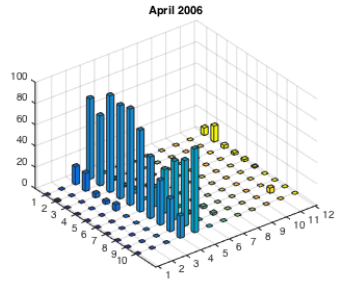
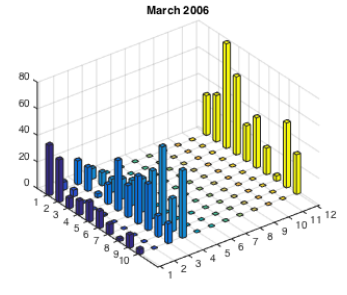
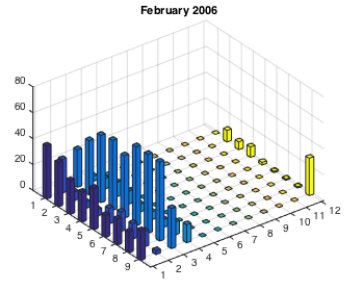
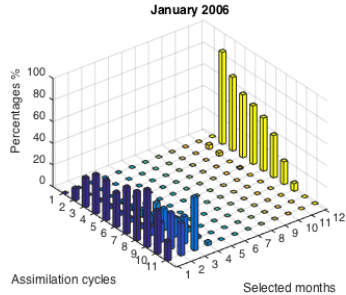


Months of OMP selected members





Months of I2-norm selected members



Conclusion



- The different OI schemes results are comparable
- The adaptive schemes require more computation
- The Seasonal EnOI seems to be the best choice

Acknowledgment



This research work was supported by King Abdullah University of Science and Technology (KAUST), Saudi Arabia and the Saudi ARAMCO-KAUST Marine Environmental Research Center (SAMERCK). The research made use of the resources of the Super computing Laboratory and computer clusters at KAUST.



Thank you!