Reliability analysis of quay walls

Exploring a metamodeling approach for more robustness and efficiency

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Summary

This report contains the results of the TKI-project GRAPA (Geotechnical Reliability Analysis for Practical Applications). The project aims to make full-probabilistic reliability analysis for geotechnical structures, and in this project particularly quay walls, more accessible to practicing engineers. Accessibility should be increased by improving the computational efficiency, while maintaining the robustness, of reliability analysis methods. A novel approach of adaptive metamodelling using Gaussian process regression is applied to a realistic case study using finite element modelling. This novel approach is referred to as 'ERRAGA', which is an abbreviation for Efficient and Robust Reliability Analysis for Geotechnical Applications.

The ERRAGA metamodeling approach to reliability analysis has been further developed to achieve such maturity that it is fit for use with (early adopter) engineering firms in projects. We believe that this goal has been achieved, though expert knowledge on reliability in first applications will remain necessary.

In the presented case study (and the accompanying MSc thesis) the ERRAGA approach has shown great potential:

- it can generate reliability indices and alpha values that are comparable to benchmark analyses with FORM, Monte Carlo Sampling (MCS) or Directional Sampling (DS).
- ERRAGA uses the same order of calculations as FORM and is orders of magnitude faster than MC-like approaches such as DS.
- ERRAGA can deal with noisy and incomplete data resulting from finite element calculations, e.g. PLAXIS, and as such generate results for complex (geotechnical) limit states where FORM cannot and where Monte Carlo approaches would take unpractically long calculation times (months).

The ERRAGA approach can be accessed through the Probabilistic Toolkit (PTK, freely available by Deltares[1]), which makes application easier for non-expert users through the graphical user interface (i.e. no programming skills required).

These developments should lead to better facilitating tailored reliability analyses of quay walls or other geotechnical structures. This will ultimately in turn facilitate improved assessment of existing structures and optimized design of new structures, resulting in extended life time and cost savings while meeting the reliability requirements of the relevant safety standards (e.g. Eurocodes).

[1] https://www.deltares.nl/en/software/probabilistic-toolkit-ptk/

Contents

1 Introduction

This report contains the results of the TKI-project GRAPA (Geotechnical Reliability Analysis for Practical Applications), which aims at making full-probabilistic reliability analysis for geotechnical structures, and in this project particularly quay walls, more accessible to practicing engineers by improving the computational efficiency and robustness of reliability analysis methods.

1.1 Why reliability analysis? (context and motivation)

There are numerous advantages of full-probabilistic reliability analysis for the assessment of geotechnical structures, both in design of new structures as well as assessment of existing ones:

- Avoiding inherent conservatism in one-size-fits-all partial factors (semi-probabilistic).
- Consistent treatment of uncertainties.
- Reducing uncertainties by incorporating additional data.
- Appropriate treatment of system reliability aspects.
- Etc.

Using these methodical advantages leads to more accurate reliability assessments, and hence, to more cost-effective designs of new structures; and it opens possibilities for life-time extension of existing structures as well as substantiating the safety of changes in functional use (e.g. admitting higher operational loads than designed for).

One of the applications of interest for reliability analysis of geotechnical structures are quay walls, especially the assessment of existing ones. In quay-wall engineering, many uncertainties must be taken into consideration in order to ensure the effective, safe and efficient handling of ships during their service life. Although new port infrastructure will still be developed, the focus is shifting towards the maintenance, repair, rehabilitation and adaptation of existing structures in fully up-and-running terminals (Roubos, 2019; Roubos & Grotegoed, 2014). In the coming years, thousands of quay walls will have to be reassessed as part of lifetime extension programmes throughout the world. Conventional, simplified design methods for assessing existing structures often lead to the conclusion that it is not possible to extend their service life. However, it is questionable if such simplified methods allow a realistic assessment. Experts in the field of marine structures expect that hidden capacities must be present in the failure modes of quay walls, but they are not yet able to explicitly identify and activate these. One solution to this situation is to evaluate the failure modes of critical structural members by performing reliability-based assessments (Phoon, & Retief, 2016).

The actual reliability level of most of the existing quay walls is still unknown; this is mainly because the practical applicability of reliability-based assessments in quay-wall engineering is rather low and a probabilistic framework that suits their specific risk profile is lacking. In the coming period, the demand for such advanced analyses is likely to increase, since many quay walls have to be reassessed and required computation times will further decrease.

1.2 Reliability analysis of quay walls (previous work)

Quay walls are marine structures that ensure safe and efficient handling of ships (Figure 1.1). Since they frequently have a complex soil-structure interaction (e.g. due to inclined retaining walls or relieving platforms), structural and geotechnical assessments are usually performed semi-probabilistically while modelling the quay wall based on finite elements. A more systematic way to account for uncertainties is to perform a reliability-based assessment (Phoon, & Retief, 2016). However, the efficiency and robustness of finite element-based reliability assessments in quay-wall engineering are rather low. In particular, it is still quite a challenge to achieve a robust coupling between probabilistic methods and finite element models, e.g. due to the highly complex and non-linear character of soil behaviour. Although a few studies (Rippi & Texeira, 2016; Schweckendiek et al., 2012; Teixeira et al., 2016; Wolters et al., 2012; Adel, 2018; Well, 2018; Roubos et al., 2019) show promising results for quay walls and other soil-retaining structures, most use simplified models in order to reduce calculation effort.

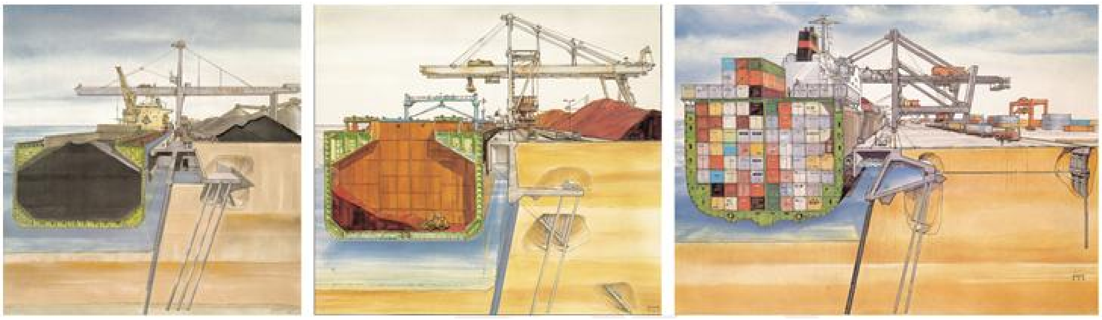

Figure 1.1. Typical quay walls equipped with a relieving platform in the Port of Rotterdam (De Gijt & Broeken, 2013). Used by permission of the Port of Rotterdam Authority (Roubos, 2019)

Three recent studies (Boero et al., 2012; Teixeira et al., 2016, Roubos et al., 2020) show that the uncertainty in material loss due to corrosion significantly influences the reliability level of soil-retaining walls. However, clear guidance on how to assess service-proven quay walls subject to corrosion-induced degradation is lacking. In the event of corrosion-induced degradation, the failure rate of a quay wall is expected to increase over time (Figure 1.2(B)). The extent of this effect will depend on the corrosion rate (Roubos et al, 2020). Only a few other studies have investigated the influence of corrosion on the reliability of steel soil-retaining walls (Houyoux et al., 2007; Osório et al., 2010; Schweckendiek et al., 2007), mainly using the first-order reliability method (*FORM*). None of these studies took successful past performance into account, however, and so they most likely overestimate the probability of failure of service-proven soil-retaining walls. This is because not all effects of the passage of time and service on structural reliability are negative (Hall, 1988). The beneficial effects of successful past performance can partly offset negative ones induced by degradation (Figure 1.2(B)). Roubos et al. (2020) showed that taking into account both the negative and positive effects can result in lifetime extension of existing quay walls. Since important assumptions in geotechnical engineering, such as characteristic strength properties of soil, are fraught with uncertainties (Fenton et al., 2016), it is expected that the annual failure rate (the frequency with which a structure fails, expressed in failures per year) of a service-proven and non-deteriorating quay wall will decrease during its early years of service and over time approach an almost constant value, since after a period of successful service only the uncertainty in time-dependent design variables, such as live loads, remains (Figure 1.2(A)).

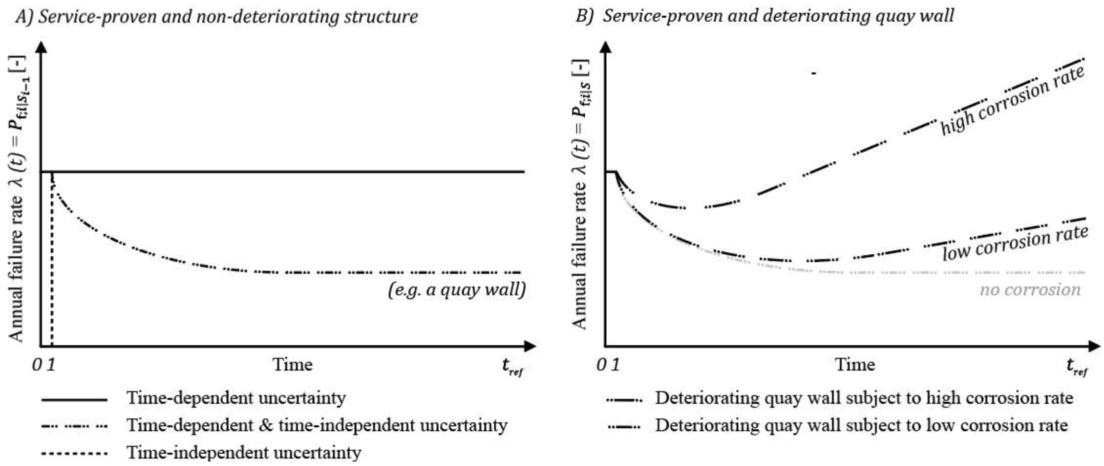

Figure 1.2. Failure rate of service-proven and non-deteriorating structures (A) and the effect of service-proven deteriorating quay walls (B) (Roubos, 2019)

1.3 Research objectives

The complexity of quay walls usually results in the need to model the structural behaviour, including the soil-structure interaction, with numerical methods such as finite elements (FE). In this report, the potential of a novel adaptive-metamodelling approach is investigated using Gaussian process regression (Van den Eijnden et al., 2021). This novel approach is hereafter referred to as 'ERRAGA', which is an abbreviation for Efficient and Robust Reliability Analysis for Geotechnical Applications.

The main research objectives for this research:

1. To test the ERRAGA metamodeling-method for reliability analysis of quay walls.
2. To formulate 'best-practices' reliability modelling recommendations for quay walls using the metamodeling approach.

1.4 Outline

Chapter 2 provides a description of the ERRAGA metamodelling approach to reliability analysis, as used in this study. Chapters 3 and 4 contain the setup and the result of the case study of a quay wall. Chapter 5 discusses the findings from applying the metamodelling-based approach to reliability analysis of quay walls based on the case study. Chapter 6 wraps up with concluding remarks.

Furthermore, appendix A contains best practices to carry out reliability analysis for quay walls. Appendix B gives more in-depth information on the parameters and (user-defined) settings of the ERRAGA approach. Finally, appendix C gives more background information on the case study as used in this study as well as a detailed description of the calculations performed on this study.

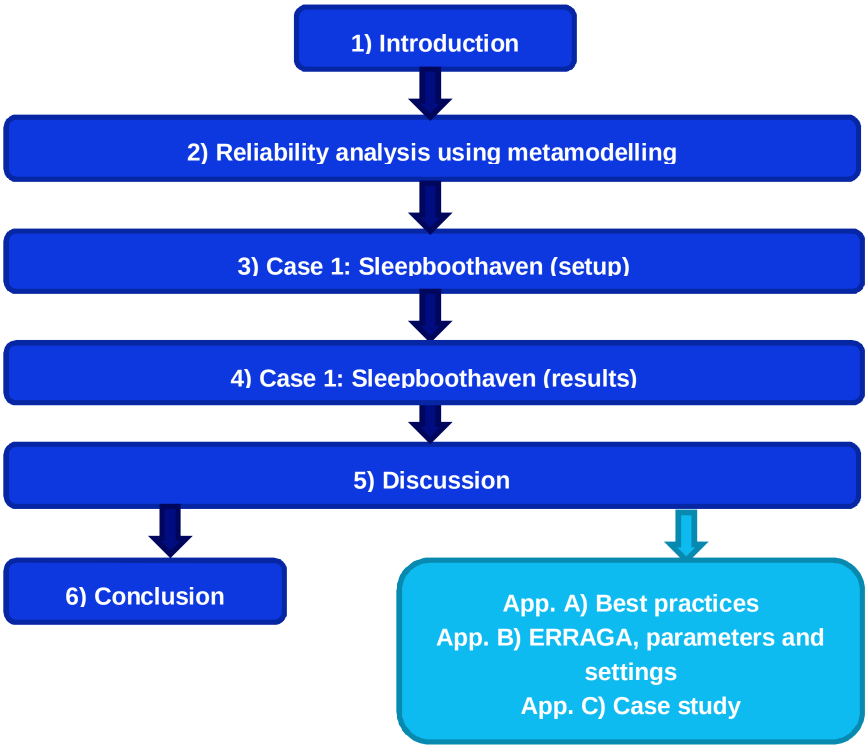

Figure 1.3 Visual outline

1.5 Acknowledgements

During this project dr. ir. Bram van den Eijnden of the TU Delft provided valuable insights into the workings and details of the ERRAGA approach. His contribution is gratefully acknowledged.

2 Reliability analysis using metamodelling

This chapter briefly discusses the fundamentals of reliability analysis (2.1) and metamodelling (2.2), to provide a general understanding for the ERRAGA approach. We focus on the basic notions, for more detailed information the reader is referred to relevant textbooks and literature. Furthermore, the ERRAGA approach is explained in more detail (2.3), followed by a description how the method can be used with existing software (2.4).

2.1 Reliability analysis

The goal of reliability analysis is the estimation of the probability of failure. Failure refers to an undesired event, not necessarily to the collapse of a structure, which we model by means of a (continuous) performance function *g(**X**)* such that the performance function assuming negative values represents failure (i.e. the failure domain F):

$$F = \{g(\boldsymbol{X}) < 0\}$$

where ***X*** is the vector of random variables. Often in geotechnical engineering, the result of an analysis is typically a safety factor *FS*, in which case the performance function can be expressed as $g = FS - 1$ (possibly complemented with a model factor). But more generally speaking, performance functions (or *limit state functions*) are defined in terms of resistance minus load.

Using the definitions above, the probability of failure (i.e., unwanted event) is given by:

$$P(F) = P(g(\boldsymbol{X}) < 0) = \int_{g(\boldsymbol{x})<0} f_{\boldsymbol{X}}(\boldsymbol{x})\, d\boldsymbol{x}$$

where $f_{\boldsymbol{X}}(\boldsymbol{x})$ is the joint probability density function (PDF) of $\boldsymbol{X}$.

Typical reliability methods currently used in practice are broadly categorized into (exact) level III methods and (approximate) level II methods, as characterized in Table 2.1.

Table 2.1 Overview of reliability methods popular in practice

	Methods (not exhaustive)	**Main features**	**Advantages and drawbacks**
Level III (exact)	Crude Monte Carlo Simulation (MCS) Importance Sampling (IS) Directional Sampling (DS) Numerical Integration (NI)	sampling-based methods; MCS: random sampling; IS: sampling focused on parameter space of interest; DS: more efficient sampling	robust methods; asymptotically exact; many realizations / model runs required, leading to long run times; IS requires previous knowledge of the problem; NI limited to very few dimensions (max. 5 random variables)
Level II (approximate)	FORM SORM FOSM	methods involving approximations (e.g. linearization of the limit state for FORM and FOSM)	require much less model runs than sampling-based methods for low-dimensional, high-reliability problems; convergence issues with erratic, non-continuous limit states

The characteristics of the reliability methods described in Table 2.1 imply that in practice, most problems with very fast model runs (e.g. closed-form equations) are analysed using level III methods, whereas heavier models with a modest number of random variables (<20) are analysed with FORM. Major challenges arise for heavier models (i.e. run times of minutes or more), which at the same time exhibit convergence problems with FORM, due to the non-linear nature of the performance function. It is this class of model the ERRAGA approach addresses, in geotechnical engineering specifically targeting FE-models.

2.2 Metamodelling

Metamodelling (surrogate modelling) strategies have been proposed to tackle both the computational expense and, to some extent, non-linear or erratic computational model responses for slope stability problems (see Li et al., (2016), for an overview), tunnel excavation (Mollon et al., 2009) and foundation footings (Sivakumar Babu and Srivastava, 2007). In this type of approach, the model performance function *g(x)* is replaced by an approximate model $\hat{g}(x)$ which provides a model prediction based on a concise set of model evaluations. This metamodel, after 'training', can then be used as a proxy for the true model response to make predictions of the reliability of the geotechnical structure.

There are different approaches to formulate the metamodel model, such as response surfaces, polynomial chaos expansion (PCE), support vector machines (SVM), Gaussian processes (GP) and artificial neural networks (ANN). The reader is referred to Teixeira et al. (2021), and the references therein, for an in-depth evaluation and comparison of the different methodologies in the context of reliability analysis for design.

A Gaussian process, in geotechnical engineering better known as a Kriging model, is one type of model that can be used as a metamodel. The method is particularly well-suited for strong non-linearities (Teixeira et al. 2021), giving it the advantage over classical polynomial response functions and polynomial chaos expansion when dealing with a strongly non-linear and noisy model response. In addition, the method is kernel-based and provides prediction uncertainty in a natural way, which is essential in the formulation of efficient strategies to improve the metamodel. To this end, Echard et al. (2011) linked Gaussian process metamodelling to MC integration for reliability analysis, outlining an iterative scheme for optimal sequential selection of new samples for model evaluations, known as the “Active learning reliability method combining Kriging and Monte Carlo Simulation” (AK-MCS). This scheme forms the blueprint for a series of active learning schemes for reliability analyses and is also underlying the ERRAGA approach.

2.3 ERRAGA approach

This section provides a high-level description of the ERRAGA metamodelling approach for reliability analysis, targeted at practitioners and specialists intending to apply the approach, not necessarily intending to understand all details or make changes. So, the target audience are rather potential users than developers. An in-depth description of ERRAGA can be found in Van den Eijnden et al. (2021).

2.3.1 ERRAGA workflow

The main ERRAGA workflow is depicted in Figure 2.1. The workflow starts by generating initial training data for the metamodels and evaluating the (limit state) model in the parameter combinations of the training data. In the current implementation, the initial training data are based on Crude Monte Carlo samples (with the option of increased variance sampling).

Subsequently, Crude Monte Carlo Samples (default value: $n = 10^5$) are generated to make an initial estimate of the probability of failure, and in order to feed the learning decision, namely whether to train the prediction model, the classification model (or both):

- The **prediction model** is a straightforward metamodel to replace actual model evaluations by Gaussian Process Regression (GPR; a.k.a. Kriging).

- The **classification model** is a binary metamodel, targeted at delineating the feasible from the infeasible parameter domain, the latter implying that the model does not give an answer (e.g. due to non-convergence).

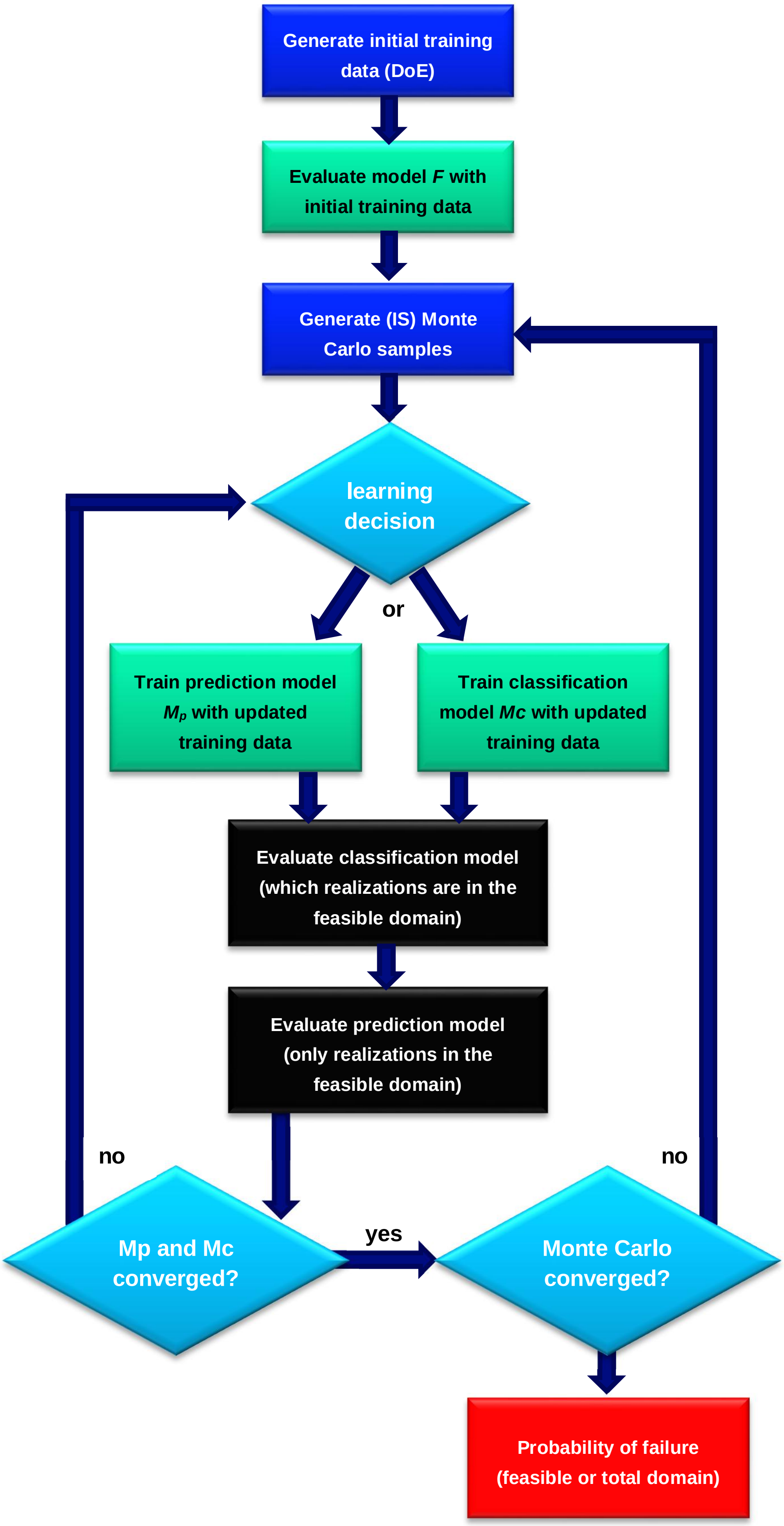

Figure 2.1 ERRAGA main workflow

Details on the learning criteria and learning decision can be found in appendix B and in Van den Eijnden et al. (2021). The essence is that the ERRAGA algorithm tries to reduce uncertainty in the estimate of the probability of failure as quickly as possible, by driving both the prediction model as well as the classification model towards convergence. For the prediction model, for example, the learning functions select the parameter combinations for the next model evaluations such that meta model is improved around the limit state in areas of high probability density.

After training (or updating) the prediction and/or the classification model, first the classification model is evaluated in the current pool of Crude Monte Carlo realizations, and subsequently the prediction model is evaluated for the realizations deemed in the feasible domain. Both, the prediction and the classification model are then checked for convergence. If the convergence is insufficient, a new training cycle is initiated.

If the meta-model do meet the convergence criteria, the convergence of the Monte Carlo sampling is checked both on the confidence bounds of the metamodel, as well as in terms of the coefficient of variation of the estimated probability of failure (see 2.3.2).

2.3.2 Reliability with Gaussian Process Regression

The metamodelling approach to reliability analysis involves replacing the computational model with an approximative metamodel when performing the time-consuming (Monte Carlo) integration in evaluating the probability of failure. To this end, the metamodel prediction $\hat{g}(\boldsymbol{u})$ is used in the MC integration to approximate P_f. As a result, any uncertainty in $\hat{g}(\boldsymbol{u})$ propagates through the MC integration and leads to uncertainty in the prediction of the probability of failure P_f. Conservative upper and lower bounds $P_f^{\pm}$ related to metamodel prediction uncertainty are given by:

$$\hat{P}_f^{\pm} = \mathbb{P}\left[\hat{g}(\boldsymbol{u}) \pm 1.96 \cdot \sigma_{\hat{g}}(\boldsymbol{u}) \leq 0\right]$$

Although the MC sample set typically contains more than 10^5 samples, uncertainty in the MC integration itself remains. This is quantified in terms of the coefficient of variation:

$$\delta_{\hat{P}_f,MC} = \frac{\sigma_{\hat{P}_f}}{\hat{P}_f} = \sqrt{\frac{1-\hat{P}_f}{N_{MC} \cdot \hat{P}_f}}$$

Both criteria above are used in the convergence criteria of the Monte Carlo integration. A maximum acceptable value can be set for:

- The coefficient of variation of the probability of failure (δ_{Pf}).
- The difference between the probability of failure estimated using the best estimate of the GPR and the lower bound estimate of the performance function (leading to the upper bound estimate of the Pf) based on the confidence intervals.

Details on the convergence criteria as settings for the algorithm can be found in Appendix B.

2.3.3 Probability of failure and infeasible domain

ERRAGA uses the classification metamodel to map the incompatible domain, both to avoid unnecessary computation and to improve the reliability estimate. The following probabilities are distinguished:

- Probability of failure (imputed) $\widehat{P}_f$: conventional P_f-estimate with MCS or IS.
- Probability of incompatibility $\widehat{P}_I$: probability density in the incompatible domain.

- Conditional probability of failure: probability of failure conditional on the compatible domain.

The latter can be interpreted as an updated probability of failure (see Van den Eijnden, 2021).

2.3.4 Simple Illustrative example

In this paragraph the described workflow will be illustrated using an example, anticipatory of the quay wall geometry presented in Figure 3.5. Two uncorrelated stochastic variables will be used of which the distribution parameters are given in Table 3.5: φ'_{clay} and $Q_{surface}$. Using two parameters allows for a complete description of the results while still being able to visualize them.

The iteration process is illustrated in Figure 2.2 in which the numbers indicate the successive realisations in PLAXIS 2D. Green dots imply non-failure (maximum displacement of the front wall, ux < 5 cm) and red dots imply failure (maximum displacement of the front wall, $u_x \geq$ 5 cm). The left plot shows the realisations in the physical space of φ'_{clay} and $Q_{surface}$. The right plot shows the same realisations in standard normal space.

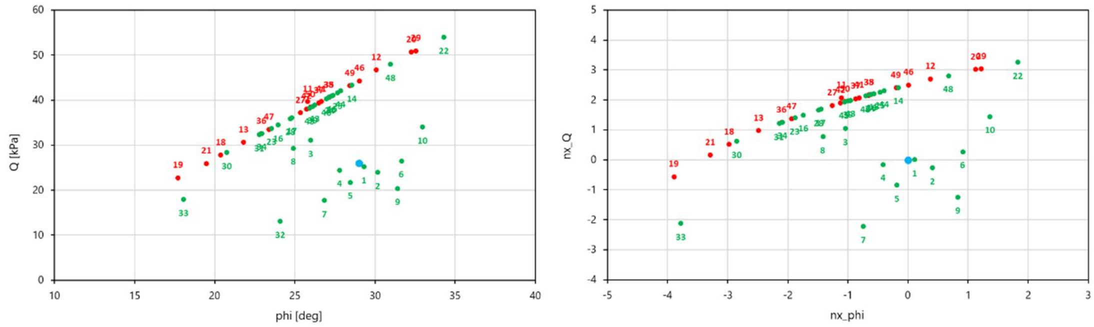

Figure 2.2 Illustration of ERRAGA iteration process for $Z = u_x - 0.05\,m$

It can be observed that the first 10 realisations are random around the mean. Based on the subsequent kriging field the 11th realisation is picked by the ERRAGA algorithm and this appears to be the only one with a visible overshoot (a Z value relatively far from 0). All subsequent realisations appear to be very close to the failure surface. In this procedure, a minimum number of 50 realisation was selected. Using an adequate convergence criterium, the required minimum number of realisations is likely lower than 50.

The reported reliability index by ERRAGA is β = 2.15. Using the resulting metamodel, it is possible to perform various probabilistic analyses, e.g. Monte Carlo. The result of a Monte Carlo analysis with sample size 100,000 is presented in Figure 2.3, yielding the same reliability index of β = 2.15. By comparing Figure 2.2 with Figure 2.3, it can be observed that the metamodel is exported correctly as the location and shape of the failure surface appear to be equal.

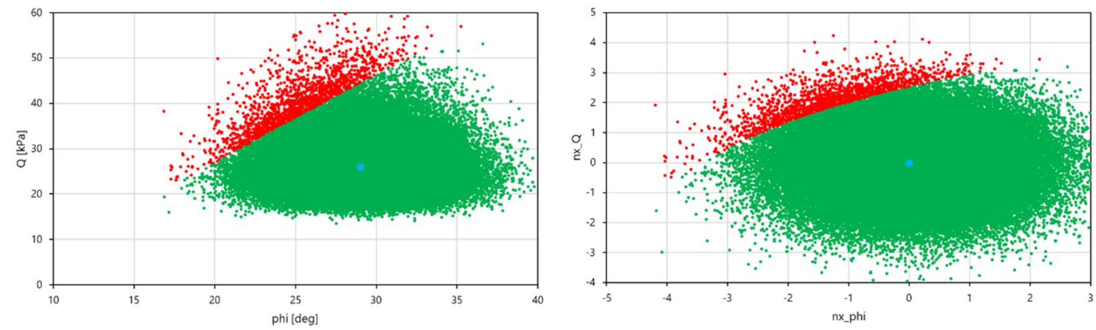

Figure 2.3 Monte Carlo analysis using the metamodel ($Z = u_x - 0.05\,m$)

Naturally, crude Monte Carlo is not the most efficient probabilistic method, but the comparison of Figure 2.2 with Figure 2.3 shows how potentially efficient ERRAGA can be. This becomes even more apparent if the failure criterium is less strict (and thereby β increases). Figure 2.4 shows the iteration process if failure is defined as an exceedance of the maximum displacement of the front wall being equal to 9 cm. ERRAGA requires 51 realisations eventually yielding β = 4.87. Again, a significant overshoot is observed at the 11$^{\text{th}}$ realisation, but realisations 14, 16 and 24 are relatively close to the failure surface, respectively having Z-values of -1.0, -3.1 and -5.5 mm. Although indicated with green dots, realisations 51, 13, 19 and 26 are also close to the failure surface and respectively have Z-values of +0.2, +1.2, +4.5 and +4.6 mm.

Thereby an adequate estimate of the failure surface is found using 51 realisations, whereas a crude Monte Carlo analysis of sample size of 100,000 is likely to result in 0 failure samples.

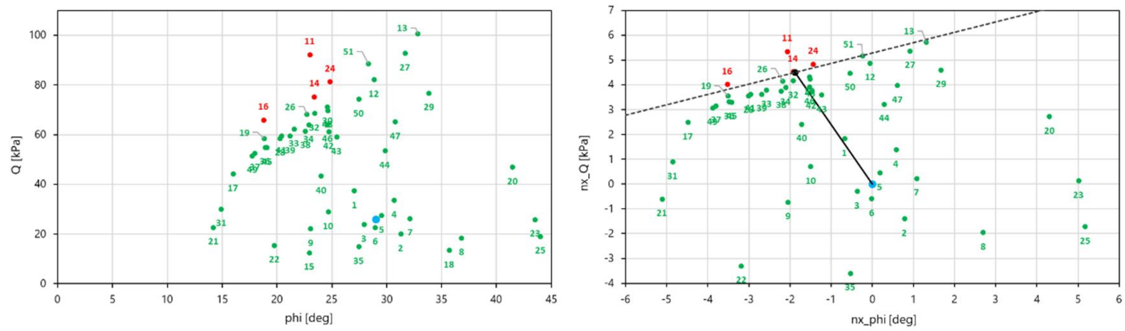

Figure 2.4 Illustration of ERRAGA iteration process for $Z = u_x - 0.09\ m$

Using the resulting metamodel, a FORM analysis can be performed yielding β = 4.88, α = 0.385 (α^2 = 0.148) for φ'_{clay} and $\alpha = -0.923$ (α^2 = 0.852) for $Q_{surface}$. The α-vector (solid), design point and the tangent line to the failure surface (dotted) are visualized in Figure 2.4.

2.4 Software set up

For the case study elaborated in this report a software set up was used in which a coupling was made between the Deltares software Probabilistic Toolkit, PTK (Deltares, 2020) and the commercial FE software PLAXIS 2D (Bentley, 2020). The PTK has several built-in reliability techniques and allows for controlling external packages such as the ERRAGA package. The principle of the coupling is shown in Figure 2.5.

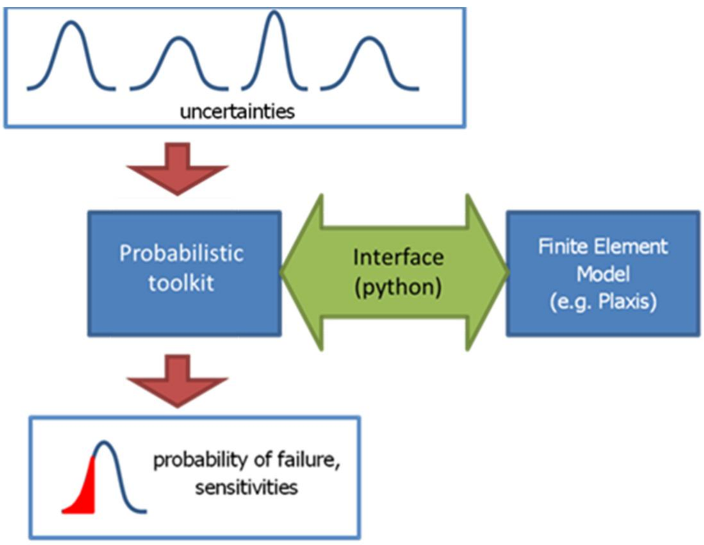

Figure 2.5 Principle of the coupling between PTK and PLAXIS 2D

An intermediate python interface is used to transfer input and output between the Probabilistic Toolkit and PLAXIS. The details of the intermediate python scripts are further elaborated in a separate Deltares report (Deltares, 2019).

The software versions used for preparing this report are the Probabilistic Toolkit v1.9.24, PLAXIS 2D 2019.00 and Python v3.

3 Case study – Sleepboothaven

3.1 Case description

In the Port of Rotterdam, as part of the Waalhaven port district, a smaller port basin exists which was used in the past to facilitate tugboats (Dutch name: Sleepboothaven). On the Northern side of the Sleepboothaven a quay wall is positioned and currently used for the transport of cargo. Also see Figure 3.1 for a top view of the Sleepboothaven.

Figure 3.1 Position of the quay wall within the Sleepboothaven. The quay wall is indicated with the red line

The current quay wall consists of a sheet pile front wall anchored to an anchor wall, see Figure 3.2 for a cross-section of the current quay wall structure.

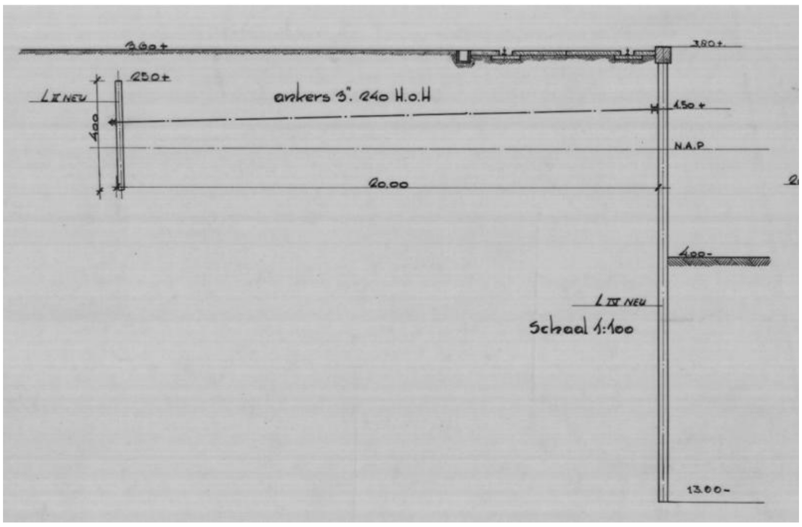

Figure 3.2 Cross-section of the current quay wall in the Sleepboothaven

It was concluded by the Port of Rotterdam authority that the current quay wall structure has reached the end of its service life. As such it was decided to replace the old quay wall structure. Engineering firm Witteveen+Bos was instructed to design a new quay wall structure. The final design of the new structure is reported in (Witteveen+Bos, 2019). This design is the basis for the in this document reported case study 1.

The new quay wall structure is shown in Figure 3.3. A summary of dimensions and levels is given:

- Front wall: NAP +3,6 m tot NAP -23,0 m.
- Level of connection grout anchors to front wall: NAP +1,5 m.
- Inclination of grout anchors to horizontal: 35°.
- Nautical Guaranteed Depth (NGD): NAP -5,2 m.
- Average dredging depth: NAP -7,05 m.
- Construction depth: NAP -7,8 m.

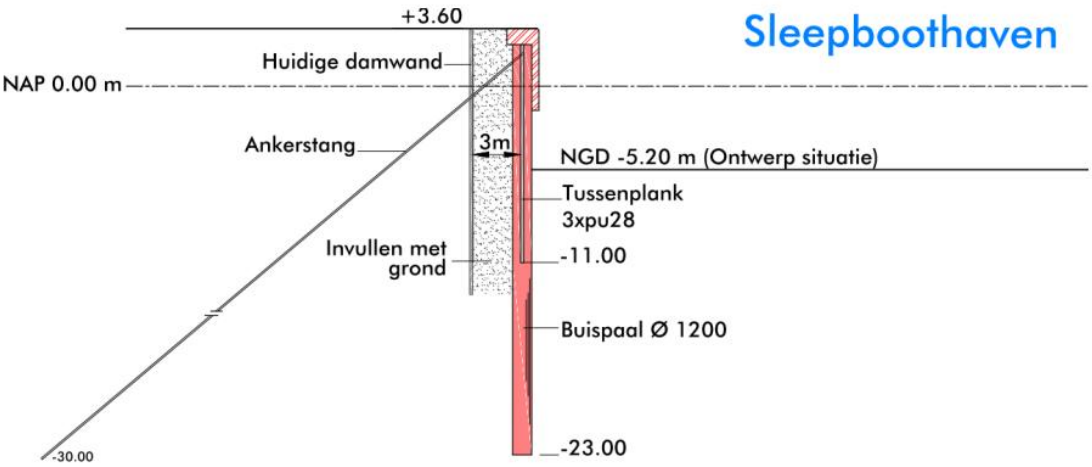

Figure 3.3 Cross-section of the new quay wall structure in the Sleepboothaven

The front wall consists of a so-called combined wall (combi-wall), i.e. a wall composed of primary tubular piles with a certain centre to centre (ctc) distance and with secondary intermediate sheet piles. The new soil-retaining wall is placed at a maximum distance of 3 m from the existing front wall. The existing front wall will be removed to a level such that the anticipated anchorage is able to pass above. The volume between existing and new front wall will be filled with soil.

The final design of the new quay wall structure anticipates on the connection of the quay wall to the existing situation on the east and west side. This connection requires some specific solutions, which are not a part of this case study. Basis for this case study is the cross-section as reported in Figure 3.3. There will be a complex interaction between existing structure and the new structure. Assumptions and simplifications are made to come to a realistic, but relatively simple structure, allowing for a relatively fast and efficient analysis for this case study. The modelling assumptions and simplifications made are further elaborated in the next paragraph.

3.2 Set up of the finite element model

3.2.1 Simplified model

For this case study Witteveen+Bos provided the PLAXIS 2D model as used for the final design of the new quay wall structure. A screenshot of the original model is shown in Figure 3.4. It was decided to simplify the situation while keeping the (expected) forces in the structure as close to reality as possible. The simplification (as elaborated in more detail in appendix A) was not necessary for tractability of the model, but meant to be able to focus on the essential behaviour of the model and not clutter the interpretation of results with particularities of the specific project.

In Figure 3.5 the simplified geometry is shown. In summary the adjustments and simplifications made are related to:

- Phasing.
- Soil layering.
- Geometry of the structure.
- Numerical settings per phase.

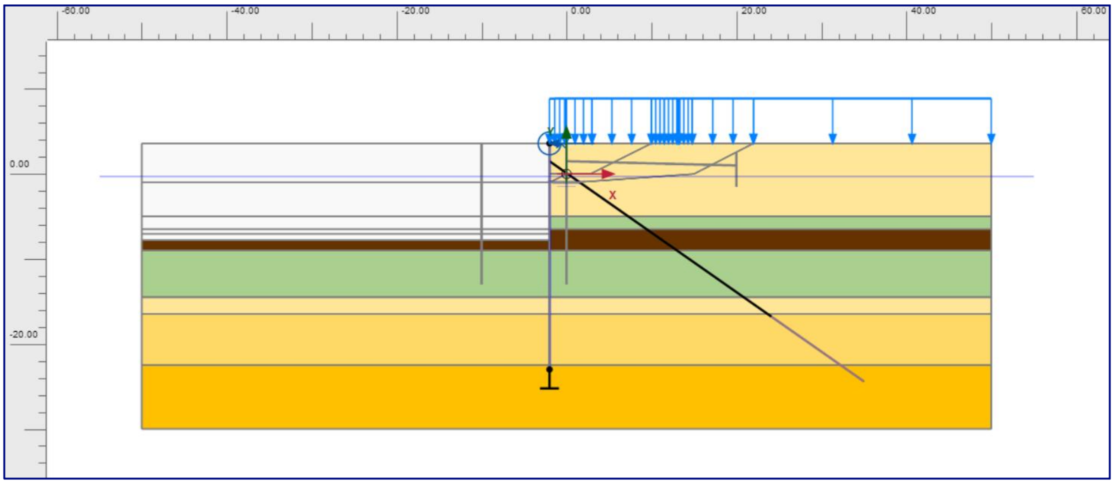

Figure 3.4 Original geometry used in the PLAXIS model for final design by Witteveen+Bos

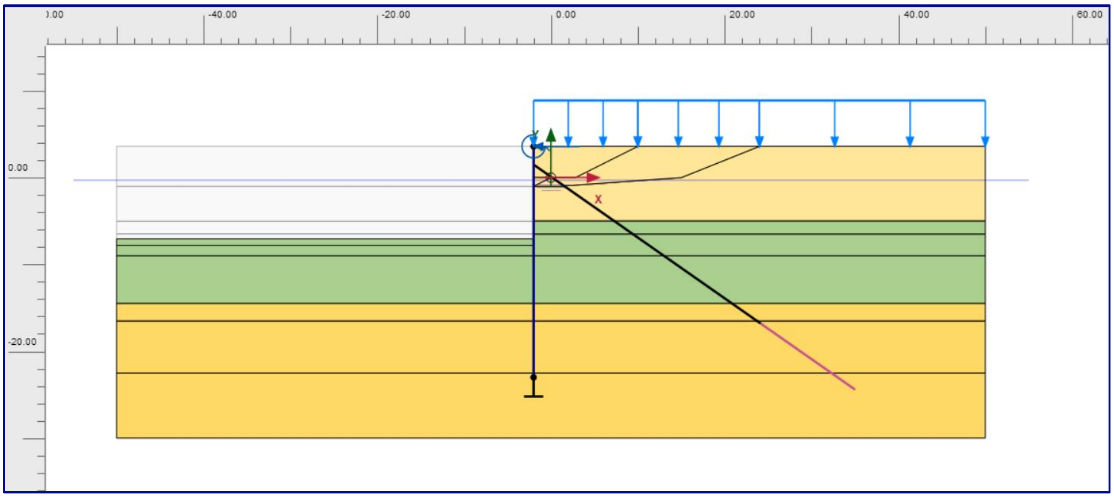

Figure 3.5 Simplified geometry used in this case study

It is concluded that a reasonable resemblance is found between the models. The bending moments do show a relatively large difference. This is a consequence of the simplifications especially in the phasing and it does not seem possible to further reduce these differences. The differences however are accepted for this case study.

3.2.2 Constitutive soil model

In the finite element model, use is made of the Hardening Soil (HS) model. The HS model is an advanced soil model which considers stress- and strain-path dependent stiffness behaviour of soils. This more realistic formulation of soil behaviour, in relation to simpler models such as the Mohr Coulomb model, are relevant when modelling soil-structure interaction problems such as quay walls. For further details on the HS model reference is made to the PLAXIS manuals (Bentley, 2020).

Challenge when working with the HS model in (automated) reliability analyses is handling the consistency of the parameter set. The HS model cannot allow any parameter combination in a set. Examples are the ratios between the independent stiffness parameters and the ratio between the strength parameters and $K_{0;nc}$. The approach chosen here is to analyse the (im)possible combinations of parameters using the option 'Soiltest' in PLAXIS. Based on this analysis fixed ratios between the stiffness parameters are chosen as well as minimum values for the $K_{0;nc}$ values such that no consistency issues are to be expected. The derived conditions are imposed through the intermediate python script.

Notice that this type of parameters consistencies could in principle also be tackled by the classification model as outlined in 2.3. In the implementation, however, we chose to follow known best practices for reliability analysis with Plaxis as much as possible and only leave the unresolved issues to be improved with metamodeling.

3.3 Limit state functions

Table 3.1 Formulas used for assessing the different limit states

LSF formulation	#
$Z_{SLS} = \delta_{x;max} - \theta_u \delta_{x;actual}$	(1)
$Z_{STR;yield;land} = \mathbf{f_y} - \max\left(\frac{\theta_M \mathbf{M'}_{\mathbf{plaxis;land}}(z)}{W'_{combi-wall;land;corr}} + \frac{\theta_N \mathbf{N'}_{\mathbf{plaxis;land}}(z)}{A'_{tube;corr}}\right)$	(2)
$Z_{STR;yield;water} = \mathbf{f_y} - \max\left(\frac{\theta_M \mathbf{M'}_{\mathbf{plaxis;water}}(z)}{W'_{combi-wall;water;corr}} + \frac{\theta_N \mathbf{N'}_{\mathbf{plaxis;water}}(z)}{A'_{tube;corr}}\right)$	(3)
$Z = \theta_B 1.573 e^{\frac{-0.0021 D_{tube} f_y}{t_{tube} 235000}} f_y \left(1 - \left(\frac{N'_{plaxis} L_{sytem}}{A_{tube;corr} f_y}\right)^{1.7}\right) - \frac{M'_{plaxis} L_{sytem}}{W_{tube;land;corr}}$	(4)
$Z_{STR;anchor} = f_{y;a} - \theta_F \mathbf{F}_{\mathbf{anchor}} / \mathbf{A}_{\mathbf{anchor;corr}}$	(5)
$Z_{GEO;global} = \theta_{Soil} \max(\Sigma\mathrm{Msf}, \Sigma\mathrm{Mstage}) - 1.0$	(6)

3.3.1 Front wall failure

In this paragraph, the limit state functions (LSF) for the soil-retaining walls are formulated. Three possible ways of failure of the front wall are considered in this case study:

- Excessive deformations: it is assumed that there is a certain deformation threshold that when crossed leads to 'failure'. Also see the accompanying LSF formulation #1 in Table 3.1.
- Yielding: it is assumed that upon reaching the yield stress in the outer fibre of the combi-wall has failed. In the LSF formulation we include the effects of the bending moment and the normal stress. Effects of the shear forces are assumed to be negligible. For efficiency the LSF are calculated for the land and water side separately. Also see the accompanying LSF formulation #2 and #3 in Table 3.1.

- Local buckling: depending on the slenderness of the tubular pile (i.e. dependency on diameter, wall thickness and allowable yield stress) local buckling may become decisive for the compressed parts of the combiwall. Upon reaching a certain compressive stress the wall is assumed to fail. Also see the accompanying LSF formulation #4 in Table 3.1. Note that this formulation is to be used for empty tubular piles, i.e. no dense sand fill.

Note:

- In red stochastic variables are indicated.
- In orange mechanical properties dependent on stochastic variables are indicated.
- In green deterministic values are indicated.
- In blue PLAXIS output values are indicated.

The symbols used in Table 3.1 are further explained and elaborated in the following sub-paragraphs 3.3.3, 3.3.4, 3.3.5 and 3.4.1.

3.3.2 Anchor rod failure

For the anchor rod failure, it is assumed that upon reaching the yield stress in the rod the anchor has failed, i.e. excessive deformations will occur. In the LSF formulation we include the effects of the normal force. Second order effects such as lateral soil loads on the anchor leading to bending moments in the rod are neglected. Also see the accompanying LSF formulation #5 in Table 3.1. The anchor limit state has ultimately not been examined in the case study.

3.3.3 Geotechnical failure (global instability)

Formulating a, for all cases general applicable, limit state for geotechnical failure is challenging. Insufficient experience has been gathered so far with a (physical) PLAXIS output parameter that can be selected with a clear threshold that indicates geotechnical failure, such as for instance the steel stress. The deformation of one or multiple well-chosen points on the structure of course gives information, however in general sense it is not directly clear if this will always indicate geotechnical failure in a part of the model. Furthermore, it is debatable which deformation threshold to take as failure. Alternatively, one may for instance also consider mobilised shear strengths of characteristic points in the model.

In this case study, it was decided to make use of the PLAXIS output parameters SUMMstage and SUMMsf. The SUMMstage parameter is a measure of how UNSTABLE the structure is. The parameter SUMMsf is a measure of how STABLE the structure is. If the structure is unstable, SUMMstage is smaller than 1 and SUMMsf is not assessed. If the structure is stable SUMMstage is equal to 1 and SUMMsf is calculated. The larger SUMMsf, the more stable the structure is. By combining SUMMstage and SUMMsf (taking the maximum and subtracting 1) a continuous limit state function is obtained, potentially leading to a faster and more efficient reliability analysis.

3.3.4 Mechanical properties used in LSF

As can be seen in Table 3.1 several mechanical properties (e.g. cross-sectional area A, section modulus W) are used to evaluate the LSF. Some of these mechanical properties are taken as deterministic values in this study, see Table 3.2. Some of these mechanical properties are to be calculated based on these deterministic values and the stochastic variables. An overview of these dependent mechanical properties is given in Table 3.3. These mechanical properties are calculated, in the intermediate python script, using the standard mechanical formulas.

Table 3.2 Deterministic parameters used as input for calculating the mechanical properties and the LSF

Deterministic parameter	Unit	Value	Explanation
L_sheet	[m]	1.8	Horizontal length of the sheet pile infill, 3 * PU28 (Witteveen+Bos, 2019)
I_sheet	[m4]	106490.0E-8	Moment of inertia of sheet pile infill, 3 * PU28
E_steel	[kN/m2]	2.1E8	Youngs modulus of steel
δ_x;max	[m]	varied (sensitivity analysis)	Deformation threshold at which failure is assumed, user defined choice
A_anchor	[m2]	6442E-6	Cross-sectional area anchor, anchor 101,6x28mm (Witteveen+Bos, 2019)

Table 3.3 Overview of relevant mechanical properties used in the LSF and in the PLAXIS calculations

Mechanical property	Unit	Explanation
$EI'_{combi;plaxis}$	[kNm2/m]	Bending stiffness of combi wall per m1 quay. No corrosion used.
$EA'_{tube;plaxis}$	[kN/m]	Normal stiffness of the pipe pile per m1 quay. No corrosion used.
EA'_{anchor}	[kN/m]	Normal stiffness of the anchors per m1 quay. No corrosion used.
$A'_{tube;corr}$	[m2/m]	Corroded cross sectional area pipe pile per m1 quay
$W'_{combi\text{-}wall;land;corr}$	[m3/m]	Section modulus of corroded combi wall per m1 quay on the land side
$W'_{combi\text{-}wall;water;corr}$	[m3/m]	Section modulus of corroded combi wall per m1 quay on the water side
$A_{anchor;corr}$	[m2/anchor]	Corroded cross sectional area per anchor

3.3.5 PLAXIS output for LSF

To be able to evaluate the LSF's output is needed from the PLAXIS model. The relevant output used in this case study is extracted from the decisive phase for deformations and structural forces (i.e. phase_33, see appendix C.1) using an intermediate python script. Exception is the SUMMsf value which is extracted from the safety phase after phase_33. The relevant output parameters are shown in Table 3.4.

Table 3.4 Output parameters of PLAXIS used in the LSF

Output parameter	Unit	Explanation
$\boldsymbol{\delta_{x;actual}}$	[m]	Maximum absolute horizontal deformation of front wall
$M'_{plaxis;land}$ (z)	[kNm/m]	Bending moment that, in combination with $N_{plaxis;land}$, causes the maximum steel stress to occur at land side in the combiwall at a certain level (z)
$N'_{plaxis;land}$ (z)	[kN/m]	Bending moment that, in combination with $M_{plaxis;land}$, causes the maximum steel stress to occur at land side in the combiwall at a certain level (z)
$M'_{plaxis;water}$ (z)	[kNm/m]	Bending moment that, in combination with $N_{plaxis;water}$, causes the maximum steel stress to occur at water side in the combiwall at a certain level (z)
$N'_{plaxis;water}$ (z)	[kN/m]	Bending moment that, in combination with $M_{plaxis;water}$, causes the maximum steel stress to occur at water side in the combiwall at a certain level (z)
F_{anchor}	[kN/anchor]	Anchor force
SUMMstage	[-]	The reached SUMMstage value, also see paragraph 3.3.3
SUMMsf	[-]	The reached SUMMsf value, also see paragraph 3.3.3

3.4 Statistical distributions

3.4.1 Stochastic variables

Based on expert judgement a first choice was made for a number of influential stochastic variables. The number of variables chosen is in the order of 10 to 15 which is believed to be representative for a real case. However, to keep the case relatively simple, known influential variables as water levels and water bottom levels were not selected. Ideally sensitivity analyses are made to select the most important/influential parameters. The selected variables with their distribution, mean and standard deviation are presented in Table 3.5.

Table 3.5 Stochastic variables

Variable	Unit	Distribution	μ	σ	V	Explanation
$\Delta t_{water;tube}$	[mm]	Normal	varied	varied	-	Corrosion value for combi wall. Not applied to anchor Set to deterministic at first. Values used are varied to see impact on calculation process
t_{tube}	[m]	Uniform	16E-3	0.8E-3	0.05	Wall thickness tube μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019)
D_{tube}	[m]	Normal	1219E-3	60.95E-3	0.05	Diameter tube μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019)
f_y	[kN/m2]	Lognormal	337E3	13.5E3	0.04	Yield stress tube Quality X46 from (Witteveen+Bos, 2019) Shift used: 275E3 μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019)
$f_{y;a}$.	[kN/m2]	Deterministic	5.15E5	-	-	Yield stress anchor μ taken from (Witteveen+Bos, 2019)
$\gamma_{unsat,sandloose}$	[kN/m3]	Normal	17	0.85	0.05	Unsaturated volumetric weight sand, loose Fully correlated with $\gamma_{sat,sandloose}$ μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019)
$\gamma_{sat,sandloose}$	[kN/m3]	Normal	19	0.95	0.05	Saturated volumetric weight sand, loose Unsaturated volumetric weight sand, loose Fully correlated with $\gamma_{sat,sandloose}$ μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019)
$\varphi'_{sandloose}$	[degrees]	Normal	35	3.5	0.1	Friction angle sand, loose μ estimated from (Witteveen+Bos, 2019): μ = 30 + 1.64 * 3 ≈ 35 V taken from (Roubos, 2019)
$E_{50;sandloose}$	[kN/m2]	Lognormal	15000	3000	0.2	Secant stiffness for sand, loose. Use fixed ratio for E50ref/Eoedref/Eurref = 1/1/3. Reference pressure is 100 kPa. No shift applied. μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019)
φ'_{clay}	[degrees]	Normal	29	2.9	0.1	Friction angle clay μ estimated from (Witteveen+Bos, 2019): μ = 25 + 1.64 * 2.5 ≈ 29 V taken from (Roubos, 2019)
c'_{clay}	[kN/m2]	Lognormal	5	1.0	0.2	Cohesion clay μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019
$E_{50;clay}$	[kN/m2]	Lognormal	2000	400	0.2	Secant stiffness clay. Use fixed ratio for E50ref/Eoedref/Eurref = 1/0.526/4. Reference pressure is 100 kPa. μ taken from (Witteveen+Bos, 2019) V taken from (Roubos, 2019
$\varphi'_{sandmedium}$	[degrees]	Normal	38	3.8	0.1	Friction angle sand medium μ estimated from (Witteveen+Bos, 2019): μ = 32.5 + 1.64 * 3.25 ≈ 38 V taken from (Roubos, 2019)

Variable	Unit	Distribution	μ	σ	V	Explanation
$\boldsymbol{\theta}_{u;M;N;F;M;Soil}$	[-]	Normal	1	0.1	0.1	Factors to account for model uncertainty. Values based on expert judgement
$Q_{surface}$	[kPa]	Gumbel	26	5.2	0.2	Annual expected surcharge load on surfacelevel Values taken from (Roubos, 2019)

3.4.2 Correlations

Correlations are taken between the stochastic variables according Table 3.6 and are taken from (Roubos, 2019). Essentially correlations are taken between stochastic variables of the same soil layer. All other stochastic variables are assumed to be uncorrelated.

Table 3.6 Correlations between the stochastic variables

#	Variable	1	2	3	4a	4b	5	6	7	8	9	10	11	12	13
1	$\Delta t_{water;tube}$	1													
2	t_{tube},		1												
3	D_{tube}			1											
4a	f_y.				1										
4b	$\gamma_{unsat,sandloose}$					1	1	0.5	0.5						
5	$\gamma_{sat,sandloose}$					1	1	0.5	0.5						
6	$\varphi'_{rep;sandloose}$					0.5	0.5	1	0.25						
7	$E_{50;sandloose}$					0.5	0.5	0.25	1						
8	$\varphi'_{rep;clay}$									1	-0.65	0.25			
9	$c'_{rep;clay}$									-0.65	1	0.12			
10	$E_{50;clay}$									0.25	0.12	1			
11	$\varphi'_{rep;sandmedium}$												1		
12	$\theta_{u;M;N;F;M;Soil}$													1	
13	$Q_{surface}$														1

4 Results

This paragraph presents the main results per limit state function (LSF). For more detailed results the reader is referred to Appendix C.2.

To be able to have a better oversight over the various calculations runs a coding was used. In Table 4.1 the numbering of the different calculations is presented. Additionally, letters are used to identify variations within each main calculation, for example run #2d.

Table 4.1 Main numbering of different calculations.

Run #	Explanation
1	Used for evaluating front wall failure, LSF 1, (excessive displacements)
2	Used for evaluating front wall failure, LSF 2, 3 and 4, (yielding land, yielding water and local buckling)
3	Used for evaluating soil failure, LSF 6, (geotechnical)
4	Used for evaluating soil failure, LSF 6, (geotechnical but with an alternative approach by means of displacements)

4.1 Front wall failure due to excessive displacements, LSF 1

For the limit state function 1 (LSF 1) a front wall displacement larger than 0.02 m ([2]) is considered as failing. The relevant settings used for the ERRAGA approach for the calculations presented in this paragraph are shown in Table 4.2. A summary of the numerical results of the reliability calculations for run #1 are presented in Table 4.3.

Table 4.2. Settings for ERRAGA for run #1 (see appendix B)

Method	ERRAGA
Convergence criterium	PfStop
Classification model	No
Noise term	Off
Beta prior	0

Table 4.3 Summary of numerical results of run #1 for evaluating LSF 1

Method	Beta	CoV (Pf)	Nr of calculations	Calculation time (*)
ERRAGA	1.38	Converged (0.05)	91	~2.5 hours
Directional Sampling	1.43	0.10	3008	~3 days

(*) calculation times reported are based on single PLAXIS 2D calculations after each other. Computer uses an Intel Xeon CPU 3.50 GHz processor with 16 GB RAM.

[2] Note that after these runs an error was found in the determination of one of the mechanical properties. Causing the required unrealistic low value for dx_max = 0.02 m to be able to generate results. However, the error has no effect on the comparison of the results between these runs. The issue was solved at the start of evaluating LSF 2 and 3.

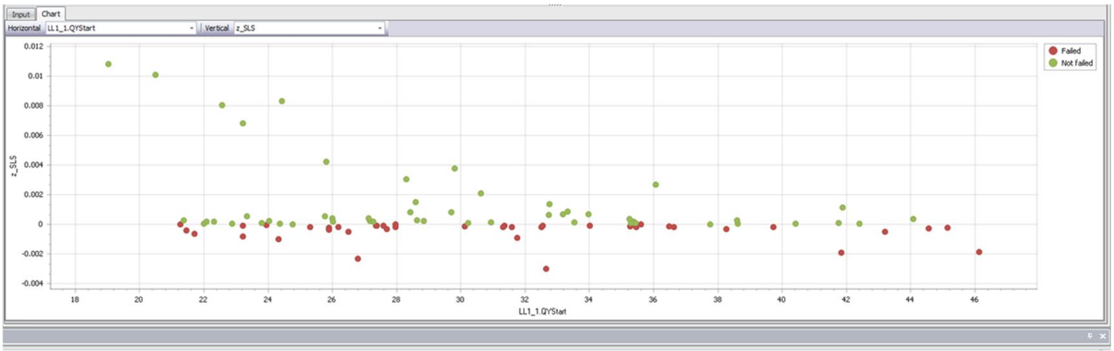

Figure 4.1. Visualisation of results of run #1 with ERRAGA (z_SLS vs surcharge load)

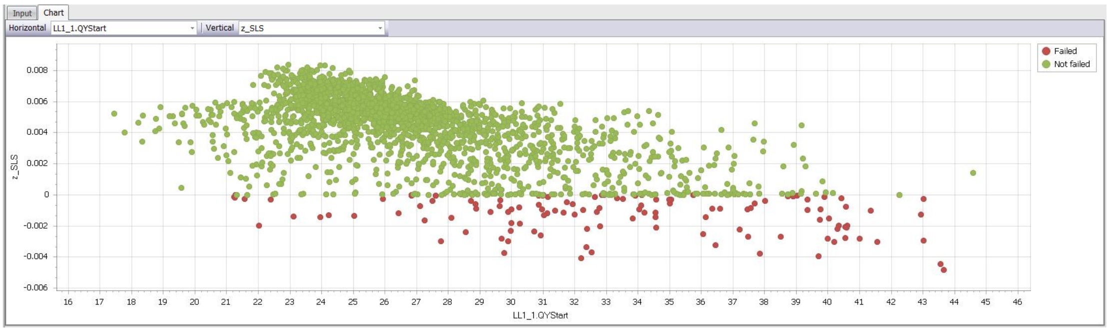

Figure 4.2. Visualisation of results of run #1 with DS (z_SLS vs surcharge load)

The following main observations were made:

- The reliability indices (beta) obtained by ERRAGA and Directional Sampling (DS) are comparable.
- Large reduction of calculation time when using ERRAGA compared to DS.
- Further investigation needed into alpha values (and hence design point values) for ERRAGA as they seem to differ from DS.

4.2 Front wall failure due to yielding, LSF 2 and 3

At the start of evaluation of limit state 2 and 3 (LSF 2 and 3) a number of issues were handled:

- An error was corrected in the formulas for determining the mechanical properties causing amongst others the unrealistic low dx_max value as reported in the previous paragraph.
- An error was adjusted in the coupling between ERRAGA and the PTK causing a sign inversion (+/-) in the calculated alpha values in the PTK, resulting in a wrong design point.
- The first calculations for run #2 were made for with a simplified limit state formulation using the maximum bending moment from the front wall in combination with the maximum normal force from the front wall.

For the first calculations of run #2 reported in this paragraph the ERRAGA settings as presented in Table 4.4 were used.

Table 4.4 Settings for ERRAGA for run #2.

Method	ERRAGA
Convergence criterium	PfStop
Classification model	No
Noise term	Off
Beta prior	0

The numerical results of run #2 are shown in Table 4.5. The influence factors (alpha squared) are compared in Table 4.6.

Table 4.5 Numerical results of run #2

Method	Run	Beta	CoV (Pf)	Nr of calcs	Calculation time
ERRAGA	GRAPA #2	3.50	Converged (0.05)	50 (min)	~1.5 hour
FORM	GRAPA #2_FORM	3.46	0.05	48 (less strict precision reliability)	~1.5 hour
Directional Sampling	GRAPA #2_DS	3.40	0.302	14546	~10 days
IS on converged meta model	GRAPA #2b_AI	3.50	0.031	-	~ minutes

Table 4.6 Comparison of Influence factors (alpha squared, u-space) of run #2

variable	X (in AI som)	ERRAGA Influence factor [%]	FORM Influence factor [%]	DS Influence factor [%]	IS op AI Influence factor [%]
ZANDSchoonLos.gammaunsat	x1	0.7	1.4	1.0	2.4
ZANDSchoonLos.phi	x2	6.2	1.1	1.0	0.0
ZANDSchoonLos.ERef	x3	2.6	0.1	0.3	0.0
KLEISiltigHumeus.phi	x4	23.7	18.2	28.7	17.4
KLEISiltigHumeus.ERef	x5	5.0	3.5	3.6	3.0
KLEISiltigHumeus.cref	x6	4.8	0.2	0.2	0.0
ZANDSchoonMatig.phi	x7	2.8	1.2	0.4	1.0
LL1_1.QYStart	x8	34.1	52.1	44.5	56.1
Steel.fy	x9	3.3	6.4	5.6	6.1
D_tube.Dt	x10	13.0	12.1	12.6	11.4
t_tube.tt	x11	3.9	3.6	2.1	2.6

After a meeting with the ERRAGA developers the following adjustments were made:

- For the ERRAGA convergence line shown in the PTK now the ERRAGA parameter 'Ucrit' is shown instead of the parameter 'Pf_cov'.
- The limit states were reformulated into a more realistic (less conservative) form that uses the combination of bending moment and normal force per node for land or water side leading to the maximum stresses.
- For the additional calculations of run #2 reported in this paragraph the ERRAGA settings as presented in Table 4.7 were used.

Table 4.7 Settings for ERRAGA for additional calculations of run #2

Method	ERRAGA
Convergence criterium	PfStop
Classification model	No
Noise term	On, NVRR = 0
Beta prior	1

Table 4.8 Numerical results of additional calculations for run #2

Method	Run	Beta	CoV (Pf)	Nr of calculations	Calculation time
ERRAGA	GRAPA #2d	3.78	Converged (0.05)	50 (min)	~1.5 hours
FORM	GRAPA #2e	3.73	0.008	348	~ 9.5 hours
Directional Sampling	GRAPA #2f	3.57	0.246	11986	~ 10 days

In addition to the observations already made in the previous paragraph the following new main observations are made:

- Reliability indices obtained by ERRAGA, FORM, DS and by applying IS directly on the, from ERRAGA extracted, meta model are comparable
- The number of calculations required by ERRAGA is in the same order as FORM. For both ERRAGA and FORM the number of calculations may be optimised by applying different settings
- Comparable, same order of magnitude, alpha values (and influence factors) are found between ERRAGA, FORM and DS. The influence factors calculated with a separate IS run directly on the, from the ERRAGA approach extracted, converged meta model show a good match with FORM
- The calculated reliability index Beta has become somewhat higher in run #2d compared with run #2 (3.78 vs 3.50) which seems logical due to the updated (less conservative) LSF using the bending moment and normal force per node instead of simply using the max bending moment in combination with the maximum normal force from the front wall
- A different convergence criterium is now shown for ERRAGA in the PTK which however is still not very logical/intuitive and as such would require more investigation
- Activating the ‘noise’ option in ERRAGA does not seem to give an improved performance for this situation, it seems however the number of calculations is still governed by the (user-specified) minimum of 50 calculations.
- Reliability indices found with DS and FORM seem to be in the same order of magnitude as ERRAGA. The DS value is somewhat lower, but the calculation is stopped prematurely.

4.3 Front wall failure due to yielding and local buckling, LSF 2, 3 and 4

With the experiences of the previous runs it was decided to try and further complicate the LSF for the front wall by adding local buckling (LSF 4) as well. As such the LSF for the front wall was specified as the minimum of LSF 2, 3 and 4. By this change it was hoped to create a more non-linear LSF such that a situation was reached were FORM would no longer work. Additionally, the situation was further complicated by first adding the model factors (theta factors) into the LSF and later also by adding corrosion as a stochastic variable.

The following observations were made:

- Although the LSF were further complicated it was not possible to create a situation where FORM could no longer converge (and ERRAGA could).
- In all the situations analysed FORM and ERRAGA came to same beta values while using the same order of magnitude of calculations.

4.4 Geotechnical failure, LSF 6

At the start of evaluation of limit state 7 it became clear that the reliability, regarding geotechnical failure, of the used PLAXIS model was so high it complicated the calculation. The high reliability is a result of the large wall length and the elastic (infinite strong) anchor rod used. It was decided to make several (unrealistic) adjustments to the case to allow for geotechnical failure to occur more easily. Effectively the embedment depth of the front wall was lowered, and the anchor rod has been set from elastic to elasto-plastic so it may yield.

At first the following ERRAGA settings were used:

Table 4.9 ERRAGA settings used for LSF 6.

Method	**ERRAGA**
Convergence criterium	PfStop
Classification model	No
Noise term	On, NVRR = 0
Beta prior	2

After several attempts, calculation #3n showed a stable beta value. It however did not seem to have converged correctly as it stopped due to reaching the maximum specified number of calculations (400). Also see Figure 4.3. The calculated beta value was doublechecked with an alternative approach based on LSF 1 and seemed to be realistic for this case. This alternative approach is further elaborated in appendix C.2.5.

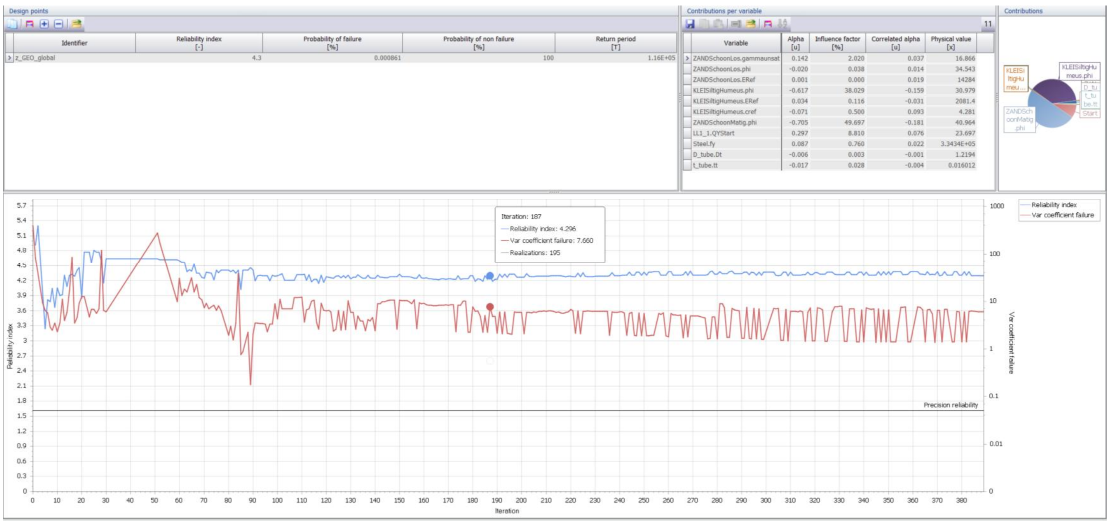

Figure 4.3 Graphical result of the non-converged run #3n. Note that in these results still the issue with sign (+/-) convention is present in the alpha values, also leading to wrong design point values shown in the PTK. This issue has been resolved during the project

The following observations were made:

- The reliability analysis of the geotechnical limit state is challenging. The difficulties relate to several issues such as the specifics of the chosen Case study, the robustness of the intermediate python scripts in relation to PLAXIS specifics/details and the observed noise in results of the SUMMstage and the SUMMsf chosen here are as output values.
- Nevertheless, it is promising that even for such a challenging situation ERRAGA was able to generate realistic results for the beta.
- It is furthermore concluded that good insight is lacking into the several calculation options of ERRAGA and the optimal (default) settings of these calculation options. Research into this calculation options and its settings is recommended.

MSc thesis by (Van der Werf, 2021)

During the GRAPA project a MSc thesis student joined the project to further investigate the potential of ERRAGA, it's calculation options and the recommended settings. The MSc thesis was not finished at the time of writing. However, some of the preliminary results of this research are included hereafter.

In (van der Werf, 2021) two Case Studies have been analyzed using the ERRAGA approach. One of the Case Studies is a continuation of the Case Study presented in this report. Starting point for the research was run #3n as presented before. After several trials some adjustments were made to the approach and some recommendations are presented for the (default) settings of the ERRAGA settings:

- Van der Werf (2021) introduced an alternative convergence criterium 'BetaAbsStop' that uses an absolute criterium instead of a relative criterium which can be too strict in situations preventing convergence.
- It was recommended to always activate a classification model that helps convergence in case incompatible model results (i.e. no results) are found. In case no incompatible results are found the activation of the classification model does not affect convergence.
- It is recommended to always activate the noise term. The corresponding NVRR value is to be further investigated. The higher the value the more noise is accepted and hence the faster the convergence. The impact on accuracy is to be further investigated.

Table 4.10 Updated settings of the ERRAGA approach for run #3y and run #3z after (van der Werf, 2021)

Method	**ERRAGA**
Convergence criterium	BetaAbsStop
Classification model	Yes, SVM
Noise term	On, NVRR = 0.2 (run #3y) / NVRR = 0.75 (run #3z)
Beta prior	4

Using the updated ERRAGA approach and the updated settings with an NVRR = 0.2 (run #3y) convergence was found after 306 realizations, leading to a beta value of 4.39. Using an NVRR = 0.75 convergence was found after 193 realizations (run #3z) leading to a beta value of 4.38. Also see Figure 4.4 and Figure 4.5.

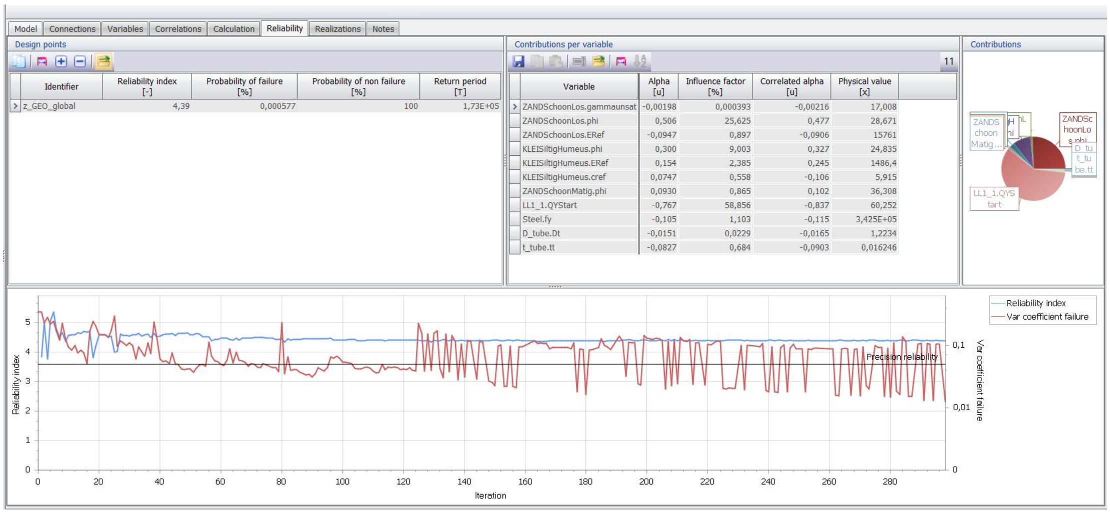

Figure 4.4. Graphical result of run #3y. Note that in these results the issue with sign (+/-) convention of the alpha values is resolved

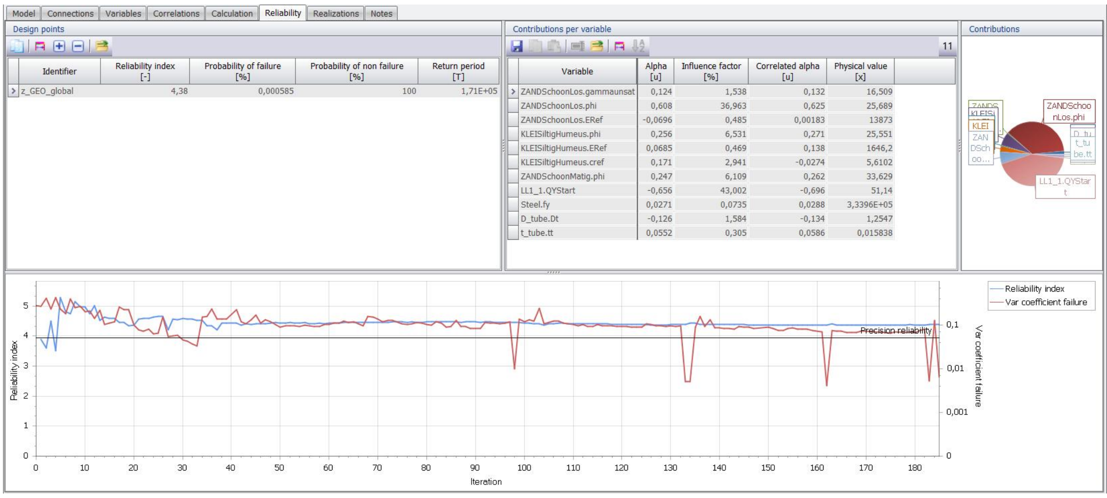

Figure 4.5. Graphical result of run #3z. Note that in these results the issue with sign (+/-) convention of the alpha values is resolved

Using the updated settings, the following numerical results are found for run #3y and #3z compared to run #3n.

Table 4.11 Numerical results of run#3n compared with run #3y and #3z using updated settings of the ERRAGA approach after (Van der Werf, 2021)

Method	Run	Beta	Converged	Nr of calculations	Total calculation time
ERRAGA	GRAPA #3n	4.30	No	400 (max)	~48 hours
ERRAGA	GRAPA #3y	4.39	Yes	306	~36 hours
ERRAGA	GRAPA #3z	4.38	Yes	193	~20 hours

Comparisons with Directional Sampling (DS) will be included in the MSc thesis, but based on the reliability level and the number of random variables, a DS analysis will likely require in the order or 10^5 calculations to achieve the same accuracy.

5 Discussion of results

Is this chapter the results obtained from the case study are reflected upon. In this discussion both the experiences of the authors of this report are included as well as the experiences from the accompanying master thesis by Van der Werf (2021). In Chapter 4 the interaction between this study and the master thesis is explained in more detail.

Discussed are the following issues:
- Plausibility of results.
- Performance (robustness and efficiency).
- Default settings.
- Attention points.

Plausibility of results
The relevant output of any reliability method is the calculated reliability expressed as a beta value and the alpha values (or influence factors = alpha squared) with which the design point values are calculated.

The calculated value of the reliability indices with ERRAGA were, for all cases analysed here, within a 0.1 to 0.2 accuracy compared to FORM and DS, with DS being the actual benchmark as a level III method. Such an accuracy is considered to be sufficient for practical applications.

The alpha values of influence factors found for the one situation analysed here were quite similar to FORM and DS. For the dominant variable (surcharge load) a difference was found of roughly 20% for the calculated influence factor compared to FORM. Such a result is as expected for the current ‘nearest to mean’ method used by ERRAGA. Using an alternative approach, by running IS directly on the extracted meta model, the difference was within 5% compared to FORM.

Performance (robustness and efficiency)
For the first LSF analysed in this report (LSF 1, 2 and 3) the robustness of the ERRAGA approach was good. Without any further adjustments to the settings of ERRAGA convergence was found. The required number of calculations are in the same order as FORM and hence can be considered efficient. Since FORM was also able to converge for these LSF the problem can be considered relatively simple / linear.

For the more complex geotechnical limit state (LSF 6) ERRAGA was expected to distinguish oneself as FORM is not converging anymore and MC-like approaches would require an unpractical high amount of calculations. In this study and the accompanying master thesis it was shown that, while a significant amount of noise was present in the output of the calculation model, ERRAGA could give plausible and converged results for the geotechnical limit state within a reasonable amount of calculations (hundreds). As such the ERRAGA approach seems to have potential to deal with these kind of complex limit states in a robust and efficient manner. Attention points are:
- the robustness of the intermediate python scripts and the finite element (FE) model also plays a significant role in the robustness of the method;
- the (user-defined) settings of ERRAGA have a significant effect on the robustness and efficiency of the method. Van der Werf presented some first recommendations for the ERRAGA settings (see next heading).

Default settings of ERRAGA variables

The robustness and efficiency of the ERRAGA approach are dependent on the user-defined settings. The recommendations presented by (Van der Werf, 2021) are:

- Make use of the additionally added stop criterium 'BetaAbsStop' as this effectively is a less strict criterium and prevents overfitting.
- Always activate a classification model to allow the method to deal with incompatible data.
- Always activate the noise term and set a value for NVRR in the range of 0.75 (seems to increase convergence while hardly affecting the final value of the reliability index based on experience).
- Set the BetaPrior and use a value in the order of the result expected.

Attention points

Main attention point of the method is that the algorithm is complex using many non-intuitive user-specified settings. Furthermore, it requires several, also non-intuitive, parameters to converge before a converged result is obtained. If the (verified) method would always work in any situation with defaults settings this would not be an issue. This however is not clear yet.

Ideally a set of default settings is found for the ERRAGA approach allowing it to converge in most of the situations. Furthermore, the convergence (and the corresponding thresholds) of all the relevant parameters should be visualised.

6 Conclusion

6.1 Concluding remarks

In the present project on Geotechnical Reliability Analysis for Practical Applications (GRAPA), the ERRAGA metamodeling approach to reliability analysis has been further developed to achieve such maturity that it is fit for use with (early adopter) engineering firms in real life projects. We believe that this goal has been achieved, though expert knowledge on reliability in first applications will remain necessary.

In the presented case study (and the accompanying MSc thesis) ERRAGA approach has shown great potential:

- ERRAGA generates reliability indices and alpha values that are comparable to benchmark analyses with FORM, MCS or DS.
- ERRAGA uses the same order of calculations as FORM and is orders of magnitude faster than MC-like approaches such as DS.
- ERRAGA can deal with noisy and incomplete data resulting from PLAXIS calculations and as such generate results for complex (geotechnical) limit states where FORM cannot and where Monte Carlo approaches would take unpractically long calculation times (months).

With this potential comes the higher complexity of the algorithm. Several settings must be specified by the user. In this study, experience has been gained with these settings in order to provide default values for efficient and robust analyses (appendix B), though we believe that these can still be improved.

The ERRAGA approach can be accessed through the Probabilistic Toolkit (PTK, freely available by Deltares[3]), which makes application easier for non-expert users through the graphical user interface (i.e. no programming skills required).

These developments should lead to better facilitating tailored reliability analyses of quay walls or other geotechnical structures. This will ultimately in turn facilitate improved assessment of existing structures and optimized design of new structures, resulting in extended life time and cost savings while meeting the reliability requirements of the relevant safety standards (e.g. Eurocodes).

6.2 Recommendations

Our main recommendations for further development are:

1. Further test the practical application of ERRAGA in other realistic case studies.
2. The convergence criteria and learning functions can still be improved to further improve performance.
3. The integration with the Probabilistic Toolkit can be made more user-friendly to lower the bar for non-expert users.
4. Further testing should reveal robust and efficient user settings (see appendix B).

More detailed suggestions can be distilled from the discussion in chapter 5.

[3] https://www.deltares.nl/en/software/probabilistic-toolkit-ptk/

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A Setting up the reliability analysis

The following components play a role in the, Finite Element (FE) based, reliability analysis:

- The set-up and interaction of the software tools.
- The stochastic variables and their correlations.
- The limit state formulations.
- The set-up of the finite element model.
- Consistent parameter set for the constitutive soil model.
- Verification of results.

These components are further elaborated hereafter.

Software tools
The following software components are used in this study:

- The Probabilistic Toolkit (PTK). The Probabilistic Toolkit serves as central program and steers the ERRAGA package and the python coupling script (which controls PLAXIS). In the PTK the stochastic variables and their correlations are defined as well as the limit states.
- The reliability method, i.e. ERRAGA (python) package. ERRAGA is the (external) approach for determining the reliability index and alpha values.
- Python coupling script (to couple PLAXIS to the PTK). The python coupling script handles the data input and output of the FE program PLAXIS 2D. To allow for efficient, unmanned, calculation this script needs to be robust in the sense that it can handle the different kind of warnings and errors of the FE code as well as errors that occur due to external sources such as network, computer, license, etc.
- The finite element software, i.e. PLAXIS 2D.

More detailed information on the software set-up used here can be found in (Deltares, 2019).

Stochastic variables and their correlations
The most influential (uncertain) parameters need to be selected. The selection can be done by means of engineering judgement and/or in combination with a sensitivity analysis. Next a suitable distribution needs to be selected with a mean and standard deviation or coefficient of variation. Use here can be made of existing literature. Furthermore, a correlation needs to be specified between the stochastic variables.

Limit state formulation
To perform a reliability analysis limit state functions (LSF) must be formulated. The LSF are a set of analytical formulations of the boundary between failure and non-failure. As such the LSF formulation is an explicit formulation of what is considered ‘failure’ and what is ‘non-failure’. It should be kept in mind that a ‘good’ LSF formulation is needed to allow for fast and efficient analysis. Ideally:

- LSF formulations are smooth and continuous functions, with a low degree of non-linearity.
- In case of complex functions consider if it is possible to split the LSF into multiple ones.
- The LSF gives results in the failure and non-failure domain.
- In case of analysis per LSF/failure mechanism: results of the LSF formulations have a physical meaning, this way allowing for interpretation by the user/engineer.

- In case of a system analysis (taking all LSF/failure mechanisms into account): results should be normalised to allow for 'equal' assessment of all LSF, i.e. '$Z = 1 - S/R$'.

In this case study the focus is on the separate LSF/failure mechanisms. As such, the formulations are not normalised.

The set-up of the finite element model
Finite element calculations are in potential more powerful than analytical or limit equilibrium type of calculations. However due to the increased complexity of the method the calculations also become less robust, i.e. errors or warnings will occur more frequently. Normally these must be addressed by the user but for reliability analyses these have to be handled by automated procedures (python coupling script). To reduce the number of exceptions that must be handled and to reduce calculation times is it highly recommended to invest in the set-up of the finite element model prior to starting a reliability analysis. Issues that should be given attention are:

- Simplifying the geometry and the phasing as much as possible without compromising the main results.
- Testing behaviour of the model by manually entering different (more extreme) parameter combinations. Try to prevent/reduce (numerical) issues by adjusting for instance: tension cut-off, adding small amount of cohesion, adjusting numerical settings (e.g. switching off arc length control).

Consistent parameter set for constitutive model
When using a more advanced constitutive soil model it should be considered if any parameter combination is acceptable. For quay walls frequent use is made of the Hardening Soil model. For this model it may happen that inconsistent parameter sets are defined resulting in an error. As such a procedure must be used to prevent these situations.

The approach used here is relatively straight forward. A priori the possible parameter sets are explored using the *Soiltest* functionality in PLAXIS. For the more sand like soils restrictions are placed on the $K_{0;nc}$ parameter that is entered into the program, enforcing the parameter set to be accepted in these cases. For the more clay like soils restrictions are placed on the $E_{oed;ref}$ parameter, enforcing the parameter set to be accepted in these cases. This method is not generic however, there may be possibilities to make this generic in the future.

Verification of results
When results are obtained with the reliability calculation, i.e. a reliability index beta and alpha values in combination with a design point, the user should verify that realistic results are obtained. There is no straightforward way of doing this, but things that can be considered:

- Inspect the convergence plot. Note: this is currently still a challenge for the ERRAGA approach as not all relevant convergence criteria (and their thresholds) are easily inspectable.
- Inspect the behaviour of the FE model around the design point, i.e. run the FE model using the found design point parameters as input. The FE model should be on or around the specified limit state.
- Run an additional importance sampling calculation (with a limited number of realisations) around the design point.

B ERRAGA method parameters and settings

This appendix gives more in-depth information on the ERRAGA implementation, more specifically on the method parameters, which can be adjusted by the user for better performance. A detailed description of the ERRAGA approach can be found in Van den Eijnden et al. (2021).

B.1 Software interaction

In the current implementation, the Probabilistic Toolkit (PTK) is the central tool, interfacing with the finite element software PLAXIS (PLX), which performs the geotechnical calculations. The PTK has multiple reliability methods at its disposal (e.g. FORM, MCS), the metamodelling method ERRAGA has been added to its inventory. In the PTK the user defines the variables, the limit state function (LSF) and settings for the reliability method. The PTK communicates with PLX by using Python scripts, in this way the variables (of a deterministic sample) are sent to PLX. With the results from PLX the LSF is solved. ERRAGA determines the metamodel (also called surrogate model) by using these data.

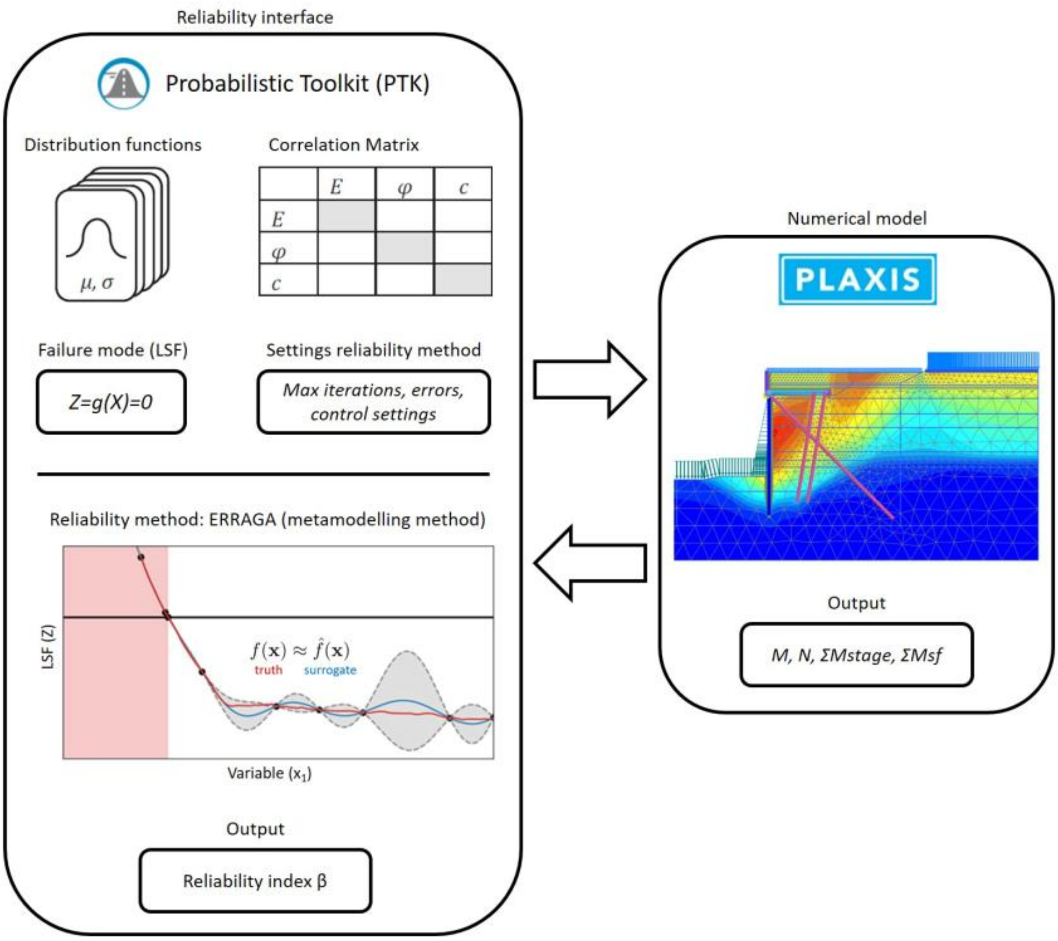


Figure B.1 Overview ERRAGA setup with PTK and PLAXIS

B.2 ERRAGA algorithm

Figure B.2 gives an overview of the ERRAGA algorithm, which in its core trains the prediction model M_p and the classification model M_c to replace the actual model (e.g. FEM) in the estimation of the probability of failure.

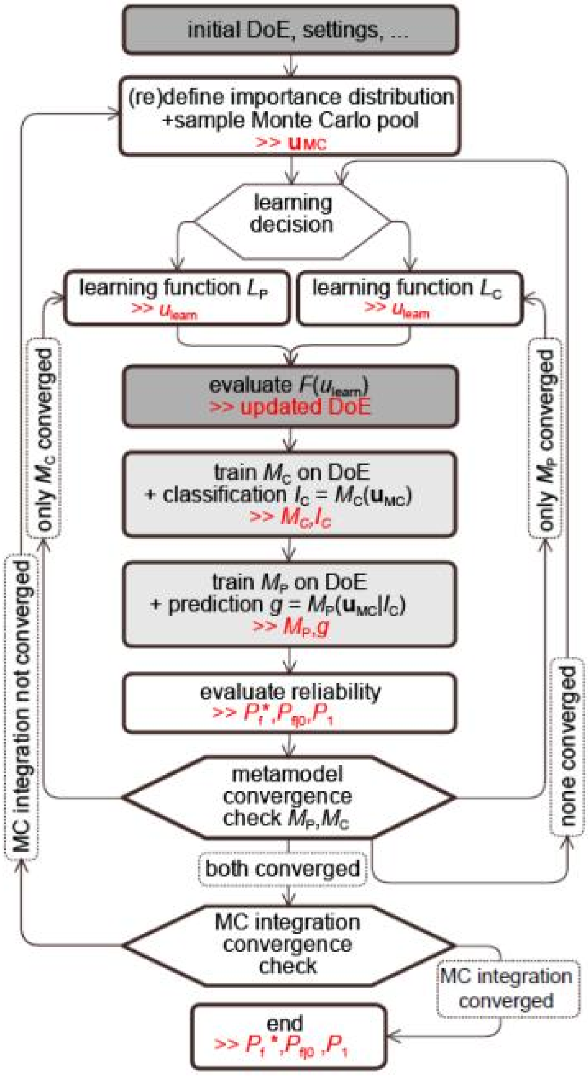


Figure B.2 Overview ERRAGA algorithm

The classification model is used to map the parameter space, by distinguishing the feasible and the infeasible parameter space. The definition of infeasible for our purposes is parameter combinations that lead to the model not being able to produce an answer. The classical example is loss of equilibrium, a situation in which an FEM model does not converge, and hence will not produce a (useful) result for use in the performance function.

B.3 Initialization

For initializing the ERRAGA procedure the computational model is evaluated for an, in principle arbitrary, set of parameter combinations. The initialization can be done in a random fashion, using any kind of design of experiments (DoE) or prior knowledge. In the current implementation, the initial sample is a random Monte Carlo sample with ‘NinitED’ samples. The variance can be increased by ‘BetaPrior’ in order to increase the likelihood of producing failure samples.

Initial Realizations
Attribute: *NinitED* Class: *learn*
Number of realizations (training samples) for the initialization. These samples are random realizations and form the initial Design of Experiment (DoE). The amount of training samples taken initially can influence the convergence. It is important to produce ‘failure points’ in this phase for the metamodel to be trained effectively.

Beta Prior
Attribute: *beta* Class: *prior*
If BetaPrior is not zero, ERRAGA starts directly with IS with increased variance sampling instead of crude MCS. By giving an estimate of the reliability index beta, the variance increase factor U_{amp} is determined as:

$$U_{amp} = \sqrt{1 + \frac{BetaPrior}{n_{dimensions}}}$$

Roughly speaking, the standard deviations are multiplied with U_{amp} in the initiation phase then. This increases the chance that ERRAGA finds failure samples during the first learning

B.4 Learning decision

The classification metamodel M_c and the prediction metamodel M_p are combined into a two-staged metamodel M. Both the metamodels have to be trained so active learning includes a learning function for the classification metamodel L_c and for the prediction metamodel L_p. This also mean that a learning decision has to be made. If one of the metamodels is converged, the learning function of the not-converged metamodel is applied. If none of the metamodels are converged, the learning decision is made based on priority (see van der Eijnden, 2021).

B.5 Metamodel training and convergence

B.5.1 Number of realizations

The metamodel is a Gaussian process (Kriging) model which is fit through the model evaluations of the training samples, for which a minimum and a maximum number can be set.

Minimum Realizations
Attribute: *NminED* Class: *learn*
A minimum can be set to the number of training samples which have to be taken before the calculation can stop. This minimum number of realizations includes the initial realizations (*NinitED*).

Maximum Realizations
Attribute: *NmaxED* Class: *learn*
The maximum number of realizations determines how may training samples are taken during each learning cycle before the MCS/IS sampling set is updated. The user must take into consideration that a lower amount of maximum realizations will force MCS/IS updating faster, which can lead to faster convergence. However, if the amount of maximum realizations is too low, it can hamper convergence because the model simply needs more

B.5.2 Convergence
The convergence of the metamodel is assessed based on the (relative) difference in probability of failure or reliability index produced by the upper and or lower bound estimates of the metamodel:

PfStop
As default *PfStop* is used as convergence criterion based on the probability of failure:

$$\epsilon_{P_f,M} = \frac{\hat{P}_f^+ - \hat{P}_f^-}{\hat{P}_f} < \text{ConvReq} = 0.05\ \text{(default)}$$

BetaAbsStop
This criterion is based on the difference between the lower bound estimate of the reliability index value (β^-) and the best estimate of the reliability index value (β^0):

$$\epsilon_{\beta,M} = \beta^0 - \beta^- < \text{ConvReq} = 0.05\ \text{(default)}$$

Only the lowest possible reliability index is interesting when assessing the reliability of a structure, that is why the difference with the lower bound is most interesting for engineering problems.

Stopping Criterion Prediction Metamodel
Attribute: *ConvCrit* Class: *learn*
The convergence of the prediction metamodel M_p is determined by a stopping criterion, it judges whether the predicted metamodel is accurate and reliable enough. The two main (recommended) options are *PfStop* and *BetaAbsStop* as discussed above.

Convergence Requirement
Attribute: *ConvReq* Class: *learn*
The Convergence Requirement (*ConvReq*) is a convergence limit for the prediction model and for one convergence criterion of the classification model. As default, a value of 0.05 is used for *ConvReq*.

B.5.3 Learning functions
The purpose of the learning function is to select the most informative next realization.

Learning Function
Attribute: *LearnFnc* Class: *learn*
The options for learning functions are U-learn and UNIS-learn (see descriptions below). UNIS-learn is better suited for noisy limit state functions and when using Importance Sampling (IS).

U-learn

The default learning function for the prediction metamodel *Lp* is U-learn. It selects the next realization at the location with the highest probability of misclassification (i.e. find the largest uncertainty with respect to the absolute predicted value (i.e. $|\hat{g}(\boldsymbol{u})|$). U-learn function according to Echard et al. (2011):

$$U(u) = \frac{|\hat{g}(\boldsymbol{u})|}{\sigma_{\hat{g}}(\boldsymbol{u})}$$

$$u_{learn} = L_p(\boldsymbol{u}_{MC}) = \underset{\boldsymbol{u} \in \boldsymbol{u}_{MC}}{\arg\min}\,(U(\boldsymbol{u}))$$

The probability of misclassification is reduced to zero at the location of the training sample (see Figure 3.7.a). However, this is only the case when the training data is noise-free and when the training data has equal sample weights.

UNIS-learn

When dealing with noise, there is a noise term added to the kernel and therefore the probability of misclassification of the training sample will not reduce to zero. If IS is used, the unequal sample weights should be included in the learning function. To deal with these effects a learning function is developed which uses the concept of maximizing utility. The utility function is based on the most likely difference in probability of misclassification before and after taking a learning sample u_i:

$$U_{NIS}(u_i) = \left(\Phi\left(-\frac{|\hat{g}(u_i)|}{\sigma_{\hat{g}}}\right) - \Phi\left(-\frac{|\hat{g}(u_i)|}{\sigma_{\hat{g}+1}}\right)\right) w_{imp,i}$$

Where the left-hand probability is the current probability of misclassification based on the current prediction variance. The right-hand probability is the probability of misclassification based on the prediction variance after taking a learning sample u_i:

$$u_{learn,P} = L_p(\boldsymbol{u}_{MC}) = \underset{\boldsymbol{u} \in \boldsymbol{u}_{MC}}{\arg\max}\,(U_{NIS}(\boldsymbol{u}))$$

This function does not account for reduction of uncertainty in adjacent points, therefore it underestimates the total reduction in uncertainty.

B.6 Classification model

A classification model is used to classify the incompatible domain as is explained in subsection 3.1.1. In the incompatible domain is no data response from the computational model, no unnecessary samples are taken from this domain when it is classified.

Classification model
Attribute: *Classifier* Class: *learn*
ERRAGA has access to two types of classification models: 1.) Gaussian process classification (GPC) that uses the logistic regression function and 2.) Support vector machine (SVM) for classification. Based on the experience so far, no clear preference can

B.7 Noise in metamodel

The limit state function can be noisy (e.g. due to numerical procedures), see Figure below. In order to avoid over-fitting, the metamodel contains a white-noise component that is automatically fit based on the data. The noise-fitting can be influenced by some user settings as detailed below.

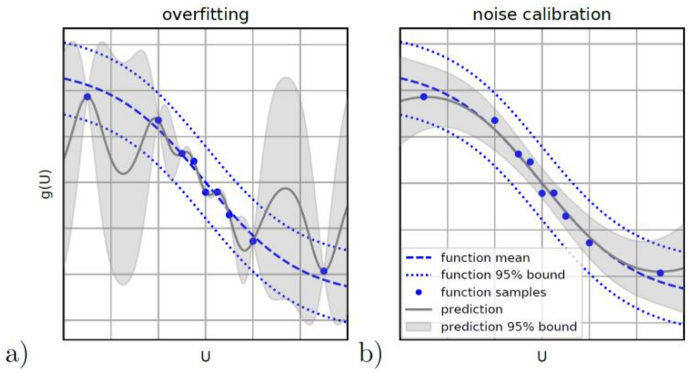


Figure B.3: Illustration of overfitting versus noise calibration

Noise Term
Attribute: *Noise* Class: *learn*
This switch can turn the noise component of the metamodeling kernel on or off.

Noise Bounds
Attribute: *NoiseBounds* Class: *learn*
The noise term represents the variance of white noise σ^2_{wn}. The user can influence this solution by adjusting the upper- and lower bound values of the noise term, the so-called noise bounds. The noise term will be between these bounds and this feature comes into play when ERRAGA finds an optimal solution which is lower (or higher) than the actual noise in the model response. If the noise term is too low it can cause overfitting and this can slow down the convergence drastically, what will result in an increase in the amount

Noise Variance Reduction Ratio - NVRR
Attribute: *NoiseVarianceReductionRatio* Class: *learn*
The fitted noise has an influence on the confidence bounds of the metamodel, and hence on the convergence criteria. The noise can deliberately (partially) be ignored in determining the confidence bounds by setting the NVRR. The NVRR can range from 0 to 1, the reduction of uncertainty is shown by:
NVRR = 0.0 No noise ignored
NVRR = 0.75 Standard deviation reduced by ignoring 50% of σ^2_{wn}
NVRR = 1.00 All noise ignored

B.8 Reliability evaluation

There are convergence criteria for both the estimate of the probability of failure, as well as for the probability of incompatibility (classification model).

B.8.1 Probability of failure

The convergence criterion on the probability of failure is straightforwardly the conventional coefficient of variation (both for MCS and IS), which must be lower than *ConvReq*.

Size of MCS/IS Pool
Attribute: *Nlearn* Class: *learn*
The size of MCS/IS pool for sampling domain integration can be adjusted by the user, as default this is set to 100 000 samples.

B.8.2 Probability of Incompatibility (Classification Model)

The convergence of the classification model is reached when satisfying at least one of the two following criteria:

1. The maximum relative difference in the probability of incompatibility, P_1, out of the last 4 iterations is less than the convergence requirement (*ConvReq*):

$$\max_{i\in\{1,2,3,4\}}\left(\frac{\left|\hat{P}_1^{(N_c-i)}-\hat{P}_1^{(N_c)}\right|}{\hat{P}_1^{(N_c)}}\right)<\epsilon_{P_1}=\text{ConvReq}=0.05\text{ (default)}$$

2. The relative importance of the incompatible domain with respect to the failure domain is less than 1%: $\frac{\widehat{P_1}}{\widehat{P_f}}<0.01$

B.9 Summary of the ERRAGA parameters

A summary of the parameters of ERRAGA is given in the table below.

Table B.1 Parameters ERRAGA

Parameter	Description	Default value
Beta Prior	Initial estimate of the reliability index. The value is used to determine the variance increase factor for the initial Monte Carlo sampling of the training data in u-space.	2
Initial Realizations	Initial model runs (training samples) to start with before fitting the metamodel.	10
Minimum Realizations	Minimum model runs (training samples) before the calculation can stop.	10
Maximum Realizations	Maximum model runs (training samples) for the calculation to stop.	200
Stopping Criterion Prediction Metamodel	The criterium for convergence. It can be based on the probability of failure (PfStop) or on the reliability index (BetaAbsStop).	PfStop
Convergence Requirement	Convergence criteria for the stopping criterion.	0.05
Learning Function	Function that selects the most informative point for the next realization to fit the metamodel. Choice is amongst others U-learn and UNIS-learn.	U-learn
Classification model	Model to classify the incompatible domain. Choice is GPC or SVM.	GPC
Noise Term	Add a noise component to the model. Options are YES or NO.	Yes
Size of MCS Pool	Size of the MCS or IS pool. This is the number of samples used to integrate a probability of failure from the metamodel.	1E5

C Case study

C.1 Setting up a simplified model

For this case study Witteveen+Bos provided the PLAXIS 2D model as used for the final design of the new quay wall structure. A screenshot of the original model is shown in Figure C.1. In Figure C.2 a screenshot is shown of the construction phasing and design phases. From these figures it becomes clear that there is a complex interaction between existing and new structure as well as a complex construction phasing. For this case study this complexity was decided undesired and it was decided to simplify the situation. Aim however was to keep the (expected) forces in the structure as close to reality as possible.

In Figure C.3 the simplified geometry is shown while in Figure C.4 the simplified phasing is shown. In summary the adjustments and simplifications made are related to:

- Phasing
- Soil layering
- Geometry of the structure
- Numerical settings per phase

In more detail adjustments are made to:

- Decreased slope steepness of initial excavation, to prevent local instability of the soil layer ‘zand schoon los’. This layer has also been given a cohesion value of 1 kPa and the tension cut-off is disabled. This was done to reduce local slope instability during construction stages and reduce the number of tension cut-off points (to improve numerical stability).
- The soil layering has been adjusted to a three-layer system:
  - Zand schoon los.
  - Klei siltig humeus.
  - Zand schoon matig.
- The existing quay wall structure is fully removed from the model.
- The separate surcharge loads on surface level are combined into a uniform load.
- Some changes are made to the numerical settings:
  - Max load fraction = 0.2 in each phase.
  - Deactivated arc length control in a number of phases to speed up calculation.
  - Run calculation always on 1 core (force_single_thread).

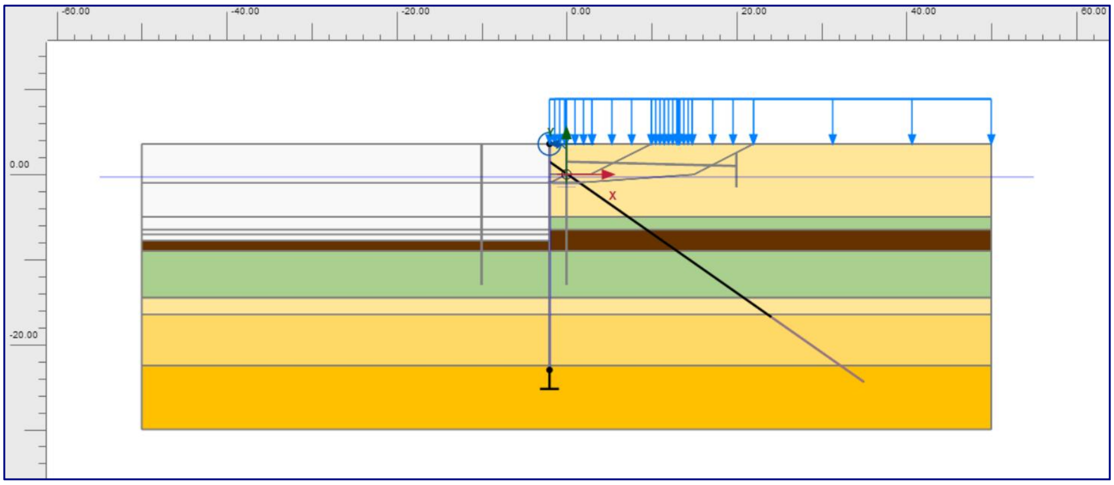
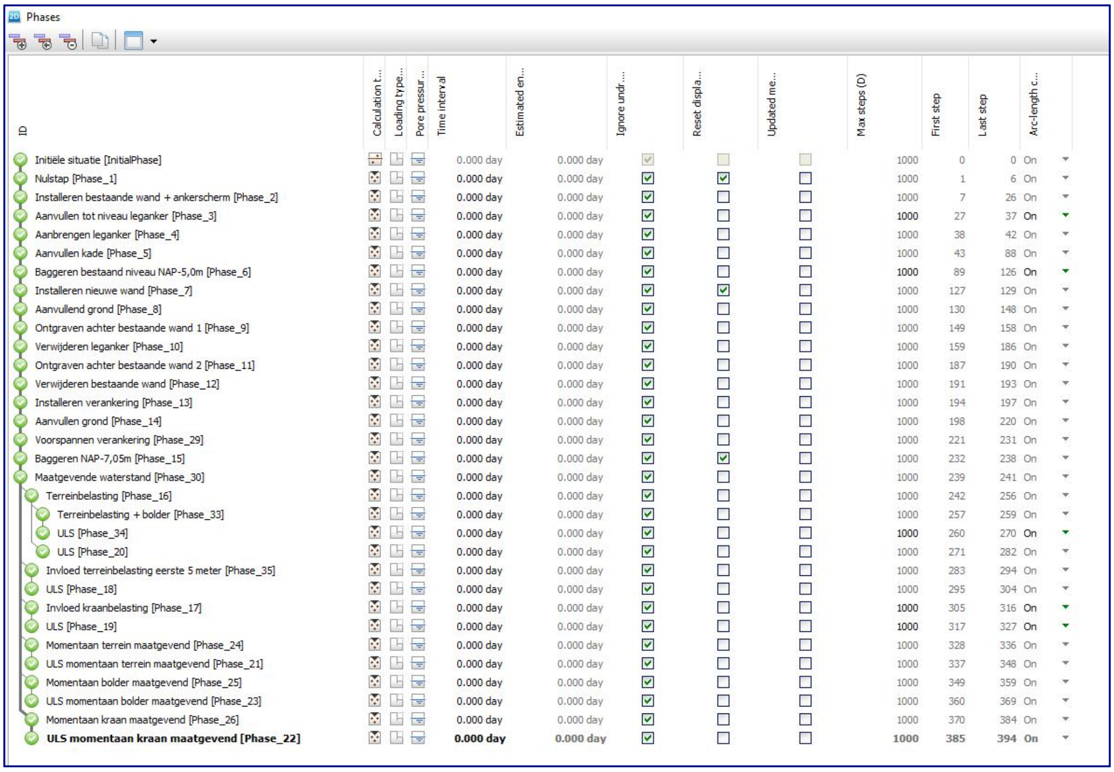


Figure C.1 Original geometry used in the PLAXIS model for final design by Witteveen+Bos



Phases

ID	Calculation t...	Loading type...	Pore pressur...	Time interval	Estimated en...	Ignore undr...	Reset displa...	Updated me...	Max steps (D)	First step	Last step	Arc-length c...
Initiële situatie [InitialPhase]				0.000 day	0.000 day	☑	☐	☐	1000	0	0	On
Nulstap [Phase_1]				0.000 day	0.000 day	☑	☑	☐	1000	1	6	On
Installeren bestaande wand + ankerscherm [Phase_2]				0.000 day	0.000 day	☑	☐	☐	1000	7	26	On
Aanvullen tot niveau leganker [Phase_3]				0.000 day	0.000 day	☑	☐	☐	1000	27	37	On
Aanbrengen leganker [Phase_4]				0.000 day	0.000 day	☑	☐	☐	1000	38	42	On
Aanvullen kade [Phase_5]				0.000 day	0.000 day	☑	☐	☐	1000	43	88	On
Baggeren bestaand niveau NAP-5,0m [Phase_6]				0.000 day	0.000 day	☑	☐	☐	1000	89	126	On
Installeren nieuwe wand [Phase_7]				0.000 day	0.000 day	☑	☑	☐	1000	127	129	On
Aanvullend grond [Phase_8]				0.000 day	0.000 day	☑	☐	☐	1000	130	148	On
Ontgraven achter bestaande wand 1 [Phase_9]				0.000 day	0.000 day	☑	☐	☐	1000	149	158	On
Verwijderen leganker [Phase_10]				0.000 day	0.000 day	☑	☐	☐	1000	159	186	On
Ontgraven achter bestaande wand 2 [Phase_11]				0.000 day	0.000 day	☑	☐	☐	1000	187	190	On
Verwijderen bestaande wand [Phase_12]				0.000 day	0.000 day	☑	☐	☐	1000	191	193	On
Installeren verankering [Phase_13]				0.000 day	0.000 day	☑	☐	☐	1000	194	197	On
Aanvullen grond [Phase_14]				0.000 day	0.000 day	☑	☐	☐	1000	198	220	On
Voorspannen verankering [Phase_29]				0.000 day	0.000 day	☑	☐	☐	1000	221	231	On
Baggeren NAP-7,05m [Phase_15]				0.000 day	0.000 day	☑	☑	☐	1000	232	238	On
Maatgevende waterstand [Phase_30]				0.000 day	0.000 day	☑	☐	☐	1000	239	241	On
Terreinbelasting [Phase_16]				0.000 day	0.000 day	☑	☐	☐	1000	242	256	On
Terreinbelasting + bolder [Phase_33]				0.000 day	0.000 day	☑	☐	☐	1000	257	259	On
ULS [Phase_34]				0.000 day	0.000 day	☑	☐	☐	1000	260	270	On
ULS [Phase_20]				0.000 day	0.000 day	☑	☐	☐	1000	271	282	On
Invloed terreinbelasting eerste 5 meter [Phase_35]				0.000 day	0.000 day	☑	☐	☐	1000	283	294	On
ULS [Phase_18]				0.000 day	0.000 day	☑	☐	☐	1000	295	304	On
Invloed kraanbelasting [Phase_17]				0.000 day	0.000 day	☑	☐	☐	1000	305	316	On
ULS [Phase_19]				0.000 day	0.000 day	☑	☐	☐	1000	317	327	On
Momentaan terrein maatgevend [Phase_24]				0.000 day	0.000 day	☑	☐	☐	1000	328	336	On
ULS momentaan terrein maatgevend [Phase_21]				0.000 day	0.000 day	☑	☐	☐	1000	337	348	On
Momentaan bolder maatgevend [Phase_25]				0.000 day	0.000 day	☑	☐	☐	1000	349	359	On
ULS momentaan bolder maatgevend [Phase_23]				0.000 day	0.000 day	☑	☐	☐	1000	360	369	On
Momentaan kraan maatgevend [Phase_26]				0.000 day	0.000 day	☑	☐	☐	1000	370	384	On
ULS momentaan kraan maatgevend [Phase_22]				**0.000 day**	**0.000 day**	☑	☐	☐	**1000**	**385**	**394**	**On**

Figure C.2 Original phasing used in the PLAXIS model for final design by Witteveen+Bos

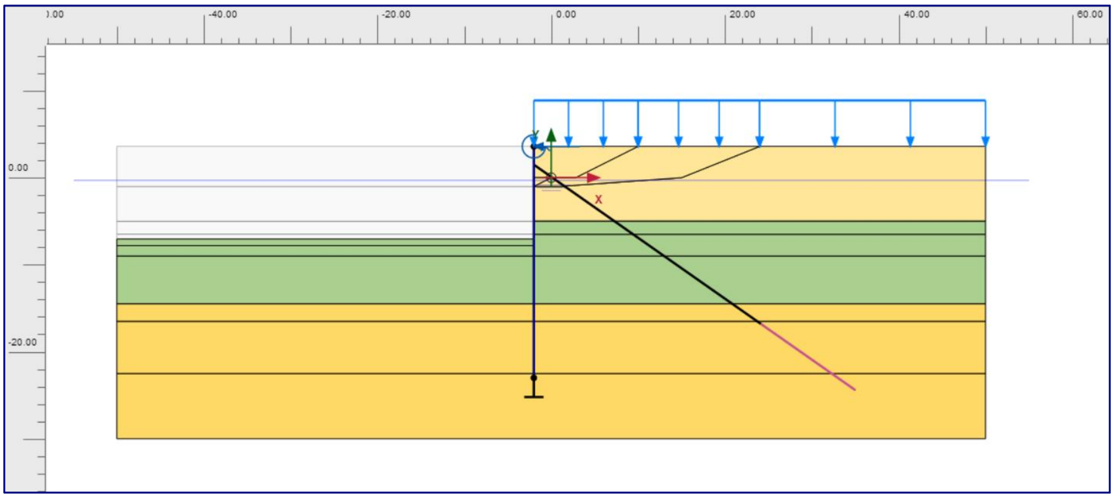
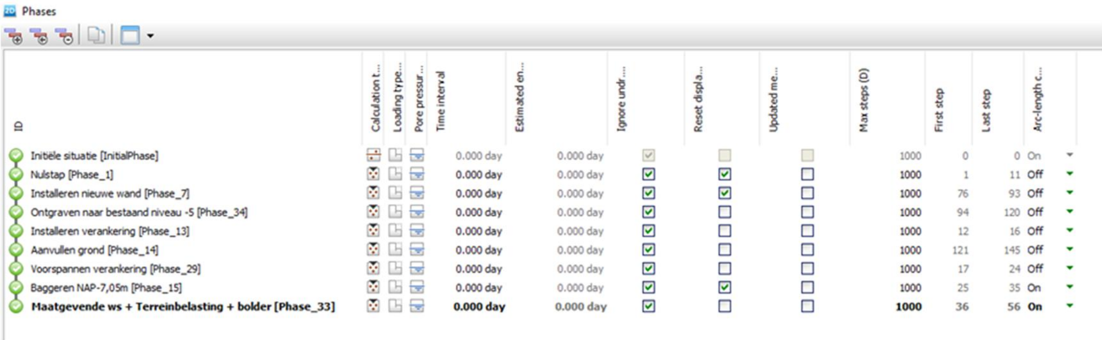


Figure C.3 Simplified geometry used in this case study


Phases

ID	Calculation t...	Loading type...	Pore pressur...	Time interval	Estimated en...	Ignore undr...	Reset displa...	Updated me...	Max steps (D)	First step	Last step	Arc-length c...
Initiële situatie [InitialPhase]				0.000 day	0.000 day	☑	☐	☐	1000	0	0	On
Nulstap [Phase_1]				0.000 day	0.000 day	☑	☑	☐	1000	1	11	Off
Installeren nieuwe wand [Phase_7]				0.000 day	0.000 day	☑	☑	☐	1000	76	93	Off
Ontgraven naar bestaand niveau -5 [Phase_34]				0.000 day	0.000 day	☑	☐	☐	1000	94	120	Off
Installeren verankering [Phase_13]				0.000 day	0.000 day	☑	☐	☐	1000	12	16	Off
Aanvullen grond [Phase_14]				0.000 day	0.000 day	☑	☐	☐	1000	121	145	Off
Voorspannen verankering [Phase_29]				0.000 day	0.000 day	☑	☐	☐	1000	17	24	Off
Baggeren NAP-7,05m [Phase_15]				0.000 day	0.000 day	☑	☑	☐	1000	25	35	On
Maatgevende ws + Terreinbelasting + bolder [Phase_33]				**0.000 day**	0.000 day	☑	☐	☐	**1000**	36	56	**On**

Figure C.4 Simplified phasing used in this case study 1. Note: not shown in this figure but added during the project is an additional 'safety' phase after Phase_33. This phase is used to calculate the so-called SUMMsf

To get a feeling for the differences in results between the original model and the simplified model some results were compared for the decisive SLS situation (no partial factors applied to loads and strengths), see Table C.1. Results compared are:

- M_max: the maximum bending moment in the front wall
- F_max: the maximum anchor force
- U_x max: the maximum horizontal deformation of the front wall

Table C.1 Comparing some results of the original and simplified FE model

Model	M_max [kNm/m]	F_max [kN]	U_x max [m]
Original	1131 (@ NAP -5 m)	2043	-0.051
Simplified	1546 (@ NAP -5 m)	1817	-0.062

It is concluded that a reasonable resemblance is found between the model. The bending moments do show a relatively large difference. This is a consequence of the simplifications especially in the phasing and it does not seem possible to further reduce these differences. The differences however are accepted for this case study.

C.2 Results

In this paragraph the calculation results are shown in detail per limit state function (LSF). To be able to have a better oversight over the various calculations runs a coding was used. In

Table C.2 the main numbering of the different calculations is presented. Additionally, letters are used to identify different sub-runs, for example run #1_DS.

Table C.2 Main numbering of different calculations

Run #	Explanation
1	Used for evaluating front wall failure, LSF 1
2	Used for evaluating front wall failure, LSF 2, 3 and 4
3	Used for evaluating soil failure, LSF 6
4	Used for evaluating soil failure, LSF 6, with an alternative approach by means of displacements

C.2.1 Front wall failure due to excessive deformations, LSF 1

The calculation log is presented in Table C.3.

Note that after the calculations reported in this paragraph an error was found in the determination of one of the mechanical properties. Causing the required unrealistic low value for dx_max to be able to generate results. However, the error has no effect on the comparison of the results between these runs.

Table C.3 Calculation log for GRAPA run #1.

Run	Calculation log	Results
GRAPA #1	Using dx_max = 0.10 m no results found with LSF yield. When setting dx_max to 0.02(?) the LSF deformations did give a result for ERRAGA.	YES
GRAPA #1_DS	Rerun v1 with DS. Comparable results found for Beta.	YES

Graphical results of the runs are presented in the figures below.

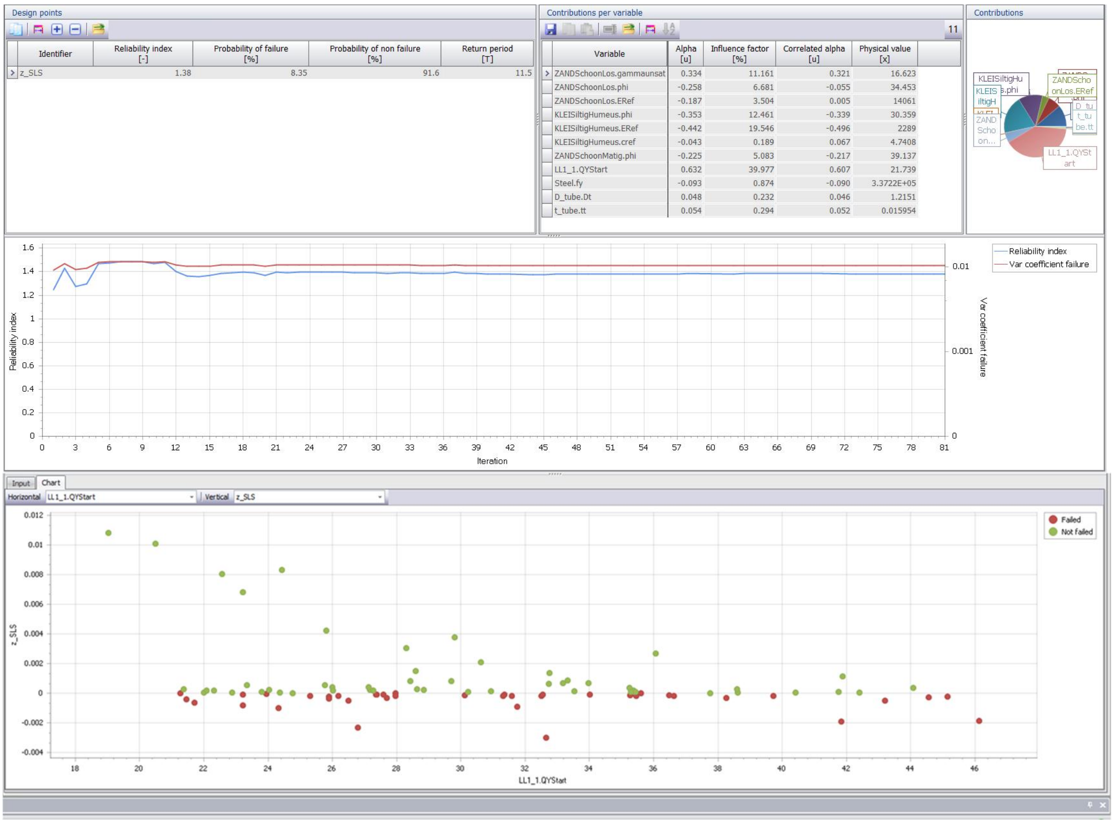


Figure C.5 Results of run #1 with ERRAGA

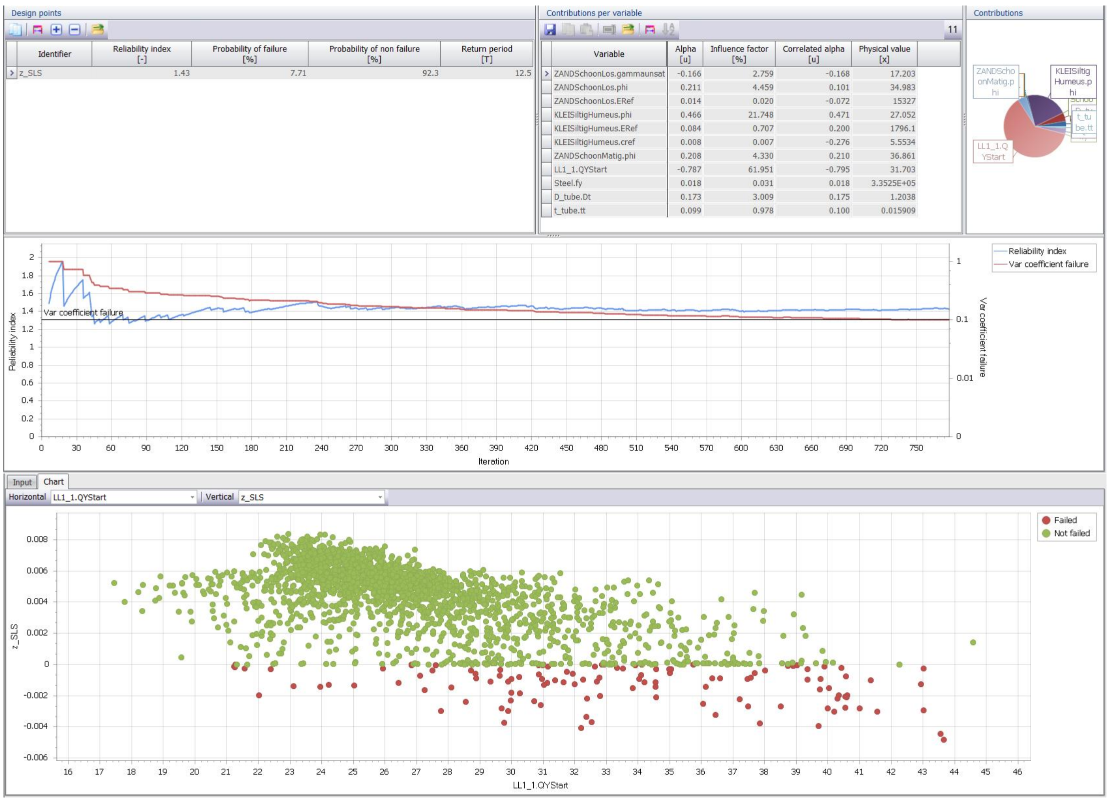


Figure C.6 Results of run #1 with DS

The numerical results are summarised in Table C.4.

Table C.4 Numerical results of run #1

Method	Beta	CoV (Pf)	Nr. Of calculations	Calculation time
ERRAGA	1.38	Converged (0.05)	91	~2.5 hours
Directional Sampling	1.43	0.10	3008	~3 days

Observations made:

- Reliability indices (Beta) obtained by ERRAGA and DS are similar.
- Large reduction of calculation time when using ERRAGA compared to DS.
- Further investigation needed into convergence value of ERRAGA as the graphical values seems strange (constant for a long time).
- Further investigation needed into alpha values (and hence design point values) for ERRAGA as they seem to differ from DS.

C.2.2 Front wall failure due to yielding, LSF 2, 3

The calculation log is presented in Table C.5. Note that at the start of run #2 the observed error in the determination of the mechanical properties of the front wall was corrected.

Table C.5 Calculation log of run #2

Run	Comments log	Results?
GRAPA #2	Adjustments made: use floats in division, edited formula for l_shell_tube_land, dx_max = 0.10m. Using Erraga	YES
GRAPA #2_FORM	Same as #2, using FORM, diff failure definition set to 0.05 instead of 0.01 to speed up	YES
GRAPA #2_DS	Same as #2, now using DS	YES
GRAPA #2b	use different metamodelling.py (to create AI.pkl)	-
GRAPA #2b_AI	run Importance Sampling directly on obtained metamodel to see resulting Beta and Influence factors	YES

The graphical results of the runs are presented in the figures below.

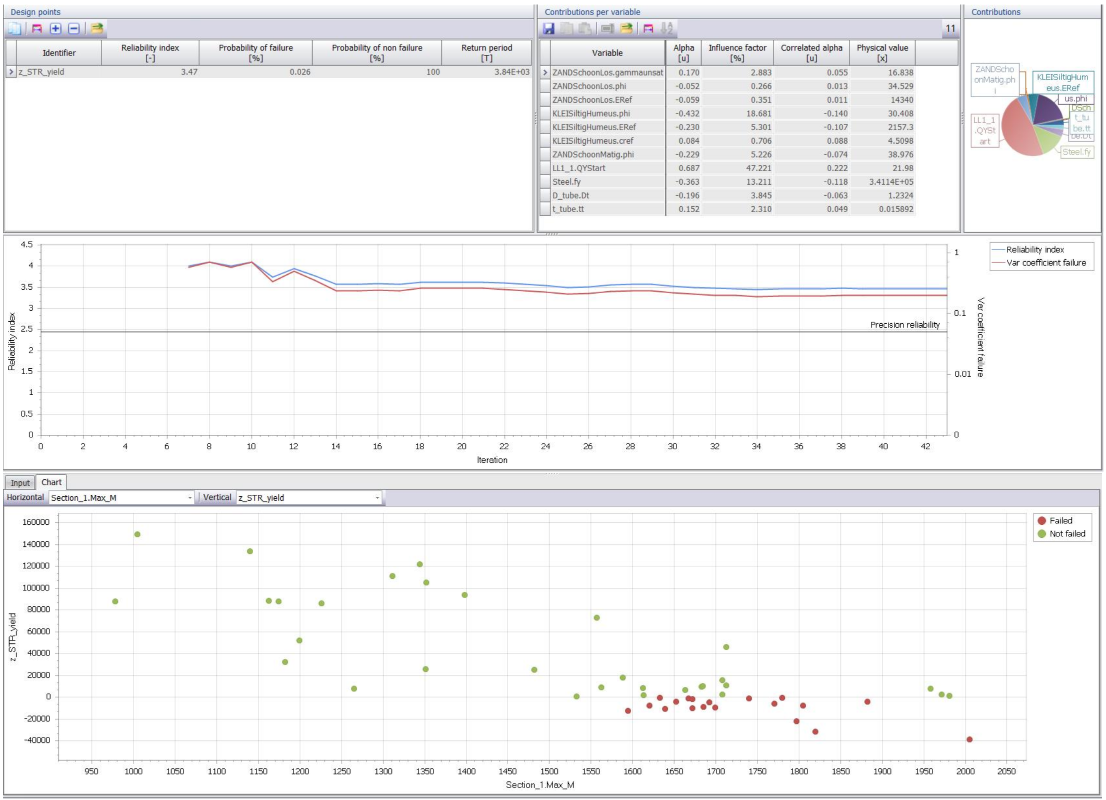


Figure C.7 Graphical results run #2 with ERRAGA

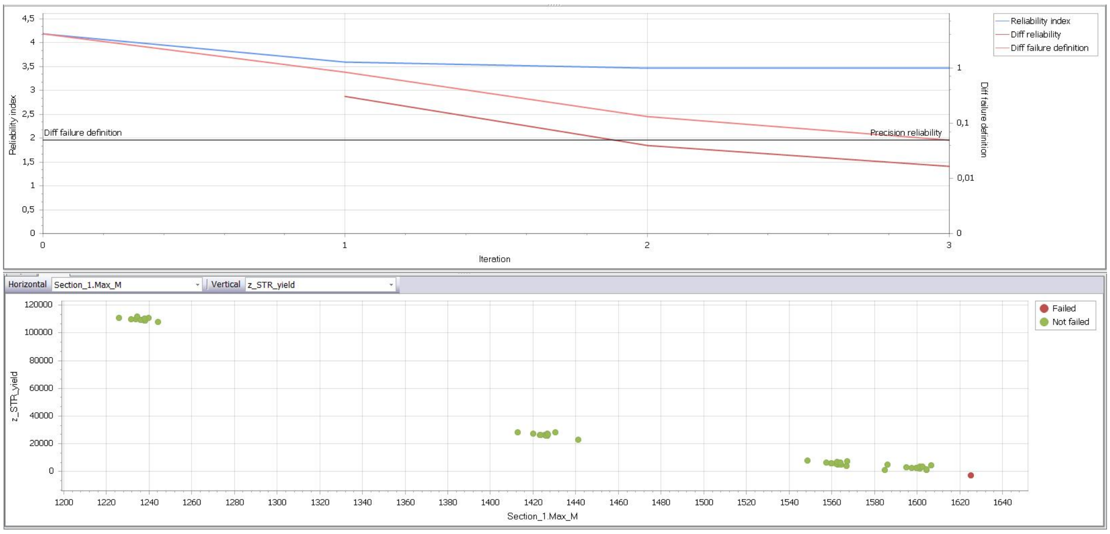


Figure C.8 Graphical results run #2 with FORM

Figure C.9 Graphical results run #2 with DS

The numerical results of the calculations are shown in Table C.6.

Table C.6 Numerical results of run #2

Method	Run	Beta	CoV (Pf)	Nr of calcs	Calculation time
ERRAGA	GRAPA v2	3.50	Converged (0.05)	50 (min)	~1.5 hour
FORM	GRAPA_FORM	3.46	0.05	48 (increased precision rel)	~1.5 hour
Directional Sampling	GRAPA v2_DS	3.40	0.302	14546	~10 days
IS on converged meta model (AI)	GRAPA v2b_AI	3.50	0.031	-	~ minutes

For comparison reasons also the influence factors (alpha squared) are compared in Table C.7.

Table C.7 Comparison of Influence factors (alpha squared) of run #2

Variable	X (in AI som)	ERRAGA Influence factor [%]	FORM Influence factor [%]	DS Influence factor [%]	IS on AI Influence factor [%]
ZANDSchoonLos.gammaunsat	x1	0,687	1,447	0.965	2.365
ZANDSchoonLos.phi	x2	6,165	1,122	0.989	0.040
ZANDSchoonLos.ERef	x3	2,557	0,148	0.259	0.001
KLEISiltigHumeus.phi	c4	23,720	18,205	28.741	17.431
KLEISiltigHumeus.ERef	x5	4,987	3,452	3.615	2.971
KLEISiltigHumeus.cref	x6	4,835	0,184	0.162	0.000
ZANDSchoonMatig.phi	x7	2,762	1,211	0.393	1.040
LL1_1.QYStart	x8	34,062	52,090	44.545	56.072

Variable	X (in AI som)	ERRAGA Influence factor [%]	FORM Influence factor [%]	DS Influence factor [%]	IS on AI Influence factor [%]
Steel.fy	x9	3,299	6,354	5.619	6.115
D_tube.Dt	x10	12,980	12,144	12.611	11.405
t_tube.tt	x11	3,946	3,644	2.100	2.560

Observations:

- Reliability indices obtained by ERRAGA, FORM, DS and by applying IS on the meta model directly are similar.
- Large reduction of number of calculations when using ERRAGA compared to DS.
- Same number of calculations as FORM. Looking at the development of Beta over the different calculations it seems convergence of ERRAGA could be maybe be reached in less than 50 calculations (which is now specified as minimum nr. of calculations), this has not been investigated further.
- Further investigation is needed into the convergence value of ERRAGA as shown in the PTK. The graphical value seems strange (the convergence criteria is not yet met but the calculation stops).
- During the project it appeared there was a sign confusion in the communication between PTK and ERRAGA causing alpha values to be used with a wrong (+/-) sign in the PTK. After solving this issue, the results of run #2 with ERRAGA were updated here. This update shows that comparable, same order of magnitude, alpha values (and influence factors) are found between ERRAGA, FORM and DS.
- The influence factors calculated with a separate IS run directly on the, from the ERRAGA approach extracted, converged meta model shows a good match with FORM.

Meeting November 2019

After these initial runs a meeting was held with the ERRAGA development team. Conclusions from this meeting:

- Default the option 'noise' is deactivated in ERRAGA. Activating this option may result in faster convergence.
- It seems the PTK currently displays the convergence criterium for the MC/IS sampling done on the meta model. This seems not correct as the convergence of the meta model is desired to show. An updated 'metamodelling.py' is created which shows the desired convergence criterium.
- ERRAGA also has an option for increased variance, allowing the first random 10 samples to be taken to a larger extend.

With the above knowledge and update a new series of runs were made. The calculation log of the additional runs for #2 are shown in Table C.8.

Table C.8 Calculation log of additional runs made for #2

Run	Comments log	Results
GRAPA #2c	like v2, LSF = z_yield, but now with updated metamodelling.py, noise term ON and showing Ucrit instead of pf_cov. changed increased variance in metamodelling.py instellen in class Frozen, parameter beta (regel 1111) to 1	NO
GRAPA #2d	like 2c, but new LSF2, based on updated LSF using max sigma M and N per node for either land or water side	YES
GRAPA #2e	like 2d, but using FORM instead of ERRAGA	YES
GRAPA #2f	like 2d, but using DS instead of ERRAGA	YES

The graphical results are shown in below figures.

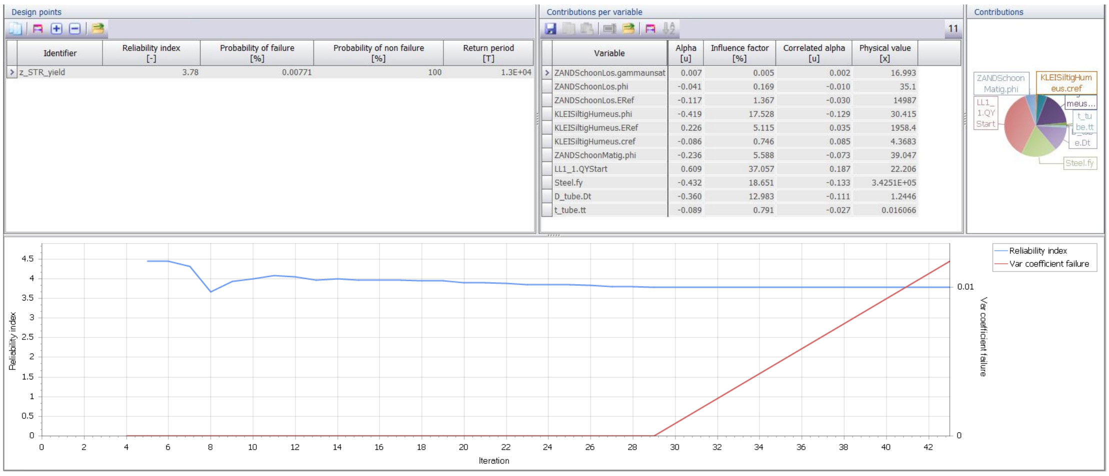


Figure C.10 Graphical results for run #2d using ERRAGA. Note that in these results still the issue with sign (+/-) convention is present in the alpha values, also leading to wrong design point values shown in the PTK. This issue has been resolved during the project

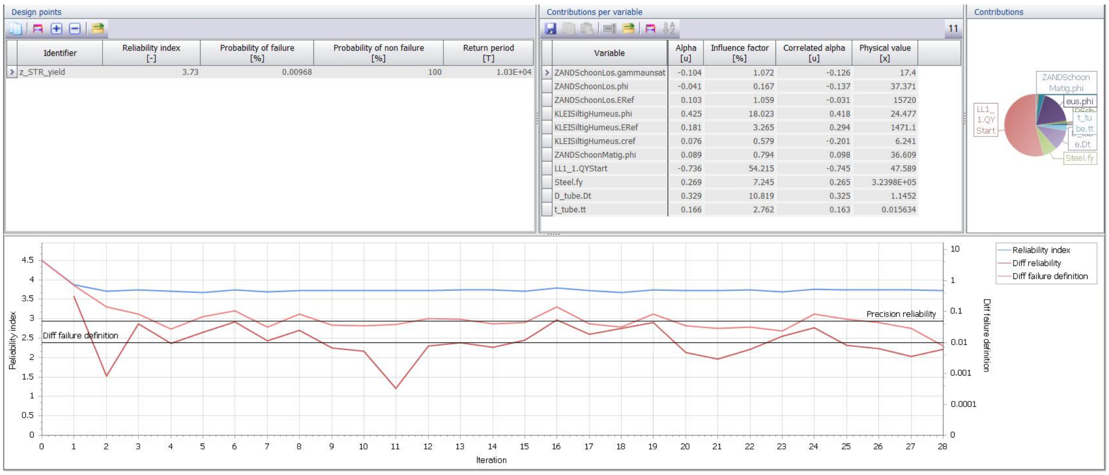


Figure C.11 Graphical results for run #2e using FORM (default convergence criteria)

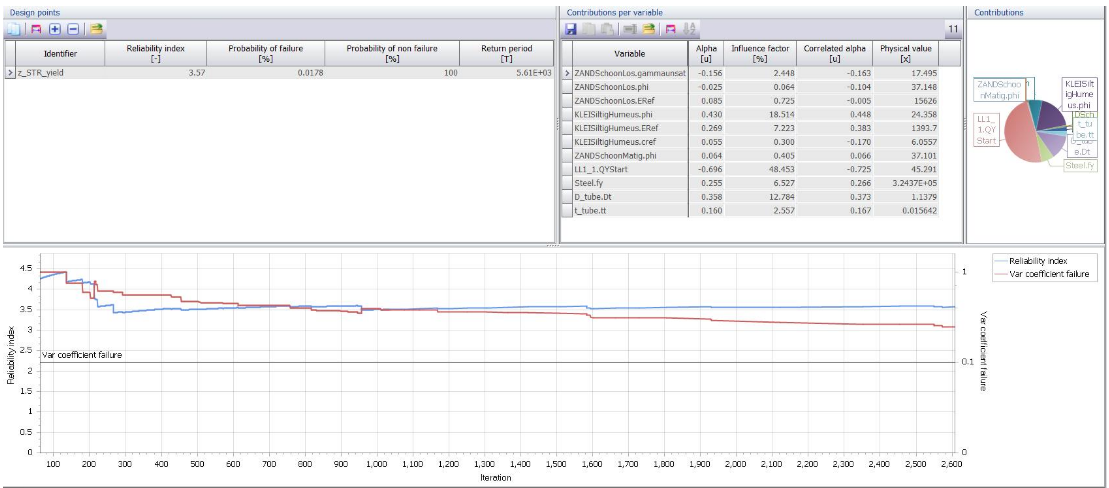


Figure C.12 Graphical results for run 2f using DS

The numerical results of the additional runs for #2 are shown in Table C.9.

Table C.9 Numerical results of additional calculations for run #2

Method	Run	Beta	CoV (Pf)	Nr of calculations	Calculation time
ERRAGA	GRAPA v2d	3.78	Converged (0.05)	50 (min)	~1.5 hours
FORM	GRAPA v2e	3.73	0.008	348	~ 9.5 hours
Directional Sampling	GRAPA v2f	3.57	0.246	11986	~ 10 days

Observations:

- The calculated reliability index Beta has become somewhat higher (3.7 vs 3.5) which seems logical due to the updated (less conservative) LSF using the bending moment and normal force per node instead of simply using the max bending moment in combination with the maximum normal force from the front wall.
- A different convergence criterium is now shown for ERRAGA which however is still not very intuitive and as such would require more investigation.
- Activating the 'noise' option does not seem to give an improved performance in this case, it seems however the calculation time is still governed by the (user-specified) minimum of 50 calculations.
- Reliability indices found with DS and FORM seem to be in the same order of magnitude as ERRAGA. The DS value is somewhat lower, but the calculation is stopped prematurely.
- The problem can still be handled by FORM, however takes a higher number of calculations using the default convergence settings in the PTK compared to ERRAGA. Loosening the (relatively strict) convergence criteria for the FORM calculations would probably result in less required calculations.

C.2.3 Front wall failure due to yielding and local buckling, LSF 2,3 and 4

With the experiences of the previous runs it was decided to further complicate the LSF for the front wall by adding local buckling (LSF 4) as well. The z function for the front wall was specified as the minimum of LSF 2, 3 and 4. By this change it was hoped to create a more non-linear z function such that a situation was reached were FORM would no longer work. A number of approaches were used for the formulation of the LSF, the situation was further complicated by first adding the model factors (theta factors) into the LSF, and later on also by adding corrosion as a stochastic variable. The calculation log for these additional calculations is shown in Table C.10.

Table C.10 Calculation log of additional calculations for run #2 with LSF 2, 3 and 4

Run	Calculation log	Results
#GRAPA v2g	like 2c, using ERRAGA, reformulated LSF yield, splitted into land and water	YES
#GRAPA v2h	like 2g, using ERRAGA, added LSF buckling, seems buckling never decisive	NO
#GRAPA v2i	like 2h, using ERRAGA, added LSF buckling, reset buckling factor from 1.573 to 1.3	NO
#GRAPA v2j	like 2h, using ERRAGA, added LSF buckling, reset buckling factor from 1.573 to 1.2	YES
#GRAPA v2k	like 2j, using FORM (manually stopped before fully converged)	YES
#GRAPA v2l	like 2h, using ERRAGA, set power of exp from -0.0021 to -0.0051 to make buckling more decisive	NO

Run	Calculation log	Results
#GRAPA v2m	like 2h, using ERRAGA, set power of exp from -0.0021 to -0.0061 to make buckling more decisive (not correctly saved...)	YES
#GRAPA v2n	like 2m, using FORM	YES
#GRAPA v2o	like 2h, using FORM, activated theta values M N B	YES
#GRAPA v2p	like 2o, using ERRAGA, activated theta values M N B	YES
#GRAPA v2q	like 2h, using ERRAGA, activated theta values, reset buckling factors	NO
#GRAPA v2r	like 2h, using ERRAGA, activated theta values, reset buckling factors, added theta factors in buckling formula	NO
#GRAPA v2s	like 2r, added corrosion as stochast	NO
#GRAPA v2t	like 2r, added corrosion as stochast, but smaller values, also corrected error (2 / (3.0 * math.pi)) in one of the mechanical properties, relevant when using non-zero corrosion. Using ERRAGA.	YES
#GRAPA v2u	like 2t, using FORM	YES
#GRAPA v2v	like 2t, using FORM, increased amount of corrosion	NO

For these calculations only the reliability indices were reported in Table C.11.

Table C.11 Calculated reliability indices for the additional calculations for run #2 with LSF 2, 3 and 4

	Beta - ERRAGA	Beta - FORM
GRAPA v2g	3.78	-
GRAPA v2j/k	3.70	3.70
GRAPA v2m/n	3.06	2.97
GRAPA v2o/p	2.27	2.28
GRAPA v2t/u	2.93	2.93

Observations:

- Although the LSF were further complicated it has not been possible to create a situation in which ERRAGA was able to converge where FORM could not. After the reported runs efforts were stopped.
- In all cases ERRAGA outperformed FORM (using default convergence criteria) regarding required number of calculations and hence calculation times.

C.2.4 Geotechnical failure, LSF 6

After working on the failure mechanisms of the front wall it was decided to explore the limit state related to geotechnical failure. Several attempts were made, all with ERRAGA, and are reported in the calculation log in Table C.12. Some trial runs were tried with FORM but it was clear that FORM was not able to generate results for this situation.

Table C.12 Calculation log of calculations for run #3

Run	Comments log	Results
GRAPA #3	added two phases in PLX model, phase_35 = turn off fixed anchor, phase_3 = safety. Added command to InputCommands to extract final SUMMsf. Adjusted LSF soil. Swicthed to LSF soil.	NO
GRAPA #3b	changed increased variance in metamodelling.py instellen in class Frozen, parameter beta (regel 1111) to 1 (was 0)	NO
GRAPA #3c	changed increased variance in metamodelling.py instellen in class Frozen, parameter beta (regel 1111) to 3 (was 0)	NO
GRAPA #3d	removed phase_35, set puntveer as elasto-plastic with max compression is 2000 kN	NO
GRAPA #3e	set anchor to elastoplastic with max tension 1500 kN, changed increased variance in metamodelling.py instellen in class Frozen, parameter beta (regel 1111) to 1 (was 0)	NO
GRAPA #3f	prestress to 450 kN, set anchor to elastoplastic with max tension 800.1 kN, arc-length auto in phase_33 (to detect soil failure automatically)	NO
GRAPA #3g	reduced wall length, prestress to 1000 kN/anchor, max tension anchor 1300 kN, arc-length auto in phase_33 (to detect soil failure automatically), if sumMstage > 0.995 AND Mresult ==1, no recalculate with arc length off	NO
GRAPA #3h	like 3g but recalculate last two phases with arc length off in secondlast phase when necessary	NO
GRAPA #3i	like 3h but with Forcesinglethread on, readtimeout = 8000	NO
GRAPA #3j	like 3i, but now lowered tolerance in last 2 phases to 0.1%, timesleep = 1e6, all phases loadstep_max=0.1 and max iter=100	NO
GRAPA #3k	like 3j swicthed off option "stop if run not succeeded", timesleep=100, readtimeout=1800	NO
GRAPA #3l	like 3k but set tolerated error back to 1% in last two phases	NO
GRAPA #3m	like v3l but now with updated metamodelling.py, noise term ON and showing Ucrit instead of pf_cov. Does not work	NO
GRAPA #3n	like v3l but now with updated metamodelling.py, noise term ON and showing Ucrit instead of pf_cov. changed increased variance in metamodelling.py instellen in class Frozen, parameter beta (regel 1111) to 2 (was 0, also already tried 1). Gives realistic result, but high CoV.	YES?

During the several attempts it became clear that the reliability, regarding geotechnical failure, of the used PLAXIS model was so high it complicated the calculation. The high reliability is a result of the large wall length and the elastic (infinite strong) anchor rod used. It was decided to make several (unrealistic) adjustments to the case to allow for geotechnical failure more easily. Effectively the embedment depth of the front wall was lowered, and the anchor rod has been set from elastic to elasto-plastic so it may yield. See Figure C.13.

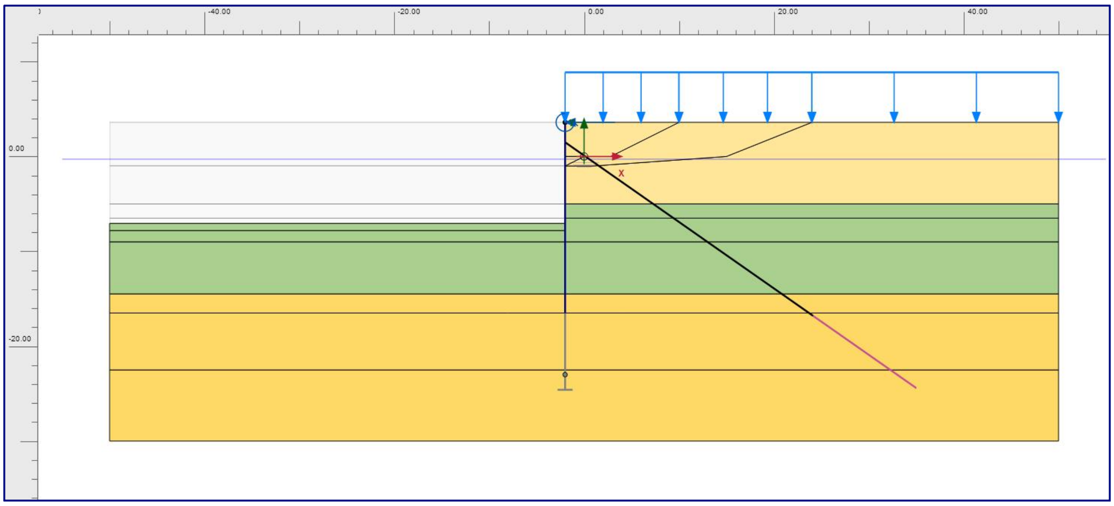


Figure C.13 Adjusted PLAXIS model. Frontwall has been shortened. Sheet pile tip from NAP -23 m to NAP - 16.5 m. Anchor rod has been set from elastic to elastic-plastic properties so it may yield

In the figures below results are shown for 2 calculations:

- Run #3g: from the convergence plot it becomes clear that ERRAGA is not converging.
- Run #3n: from the convergence plot it appears a stable Beta value is found, however the CoV is very unstable suggesting that ERRAGA has trouble converging.

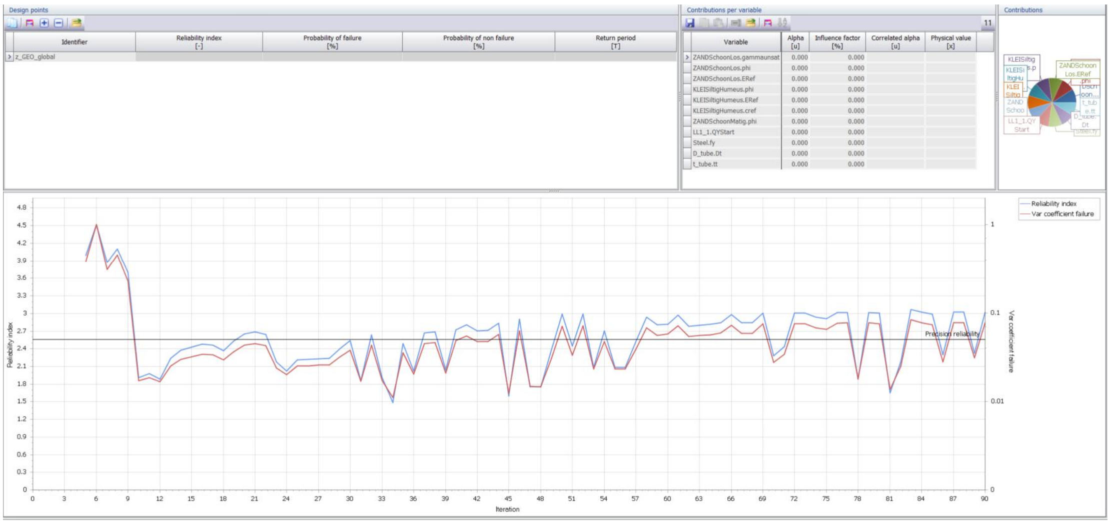


Figure C.14 Graphical results of run #3g

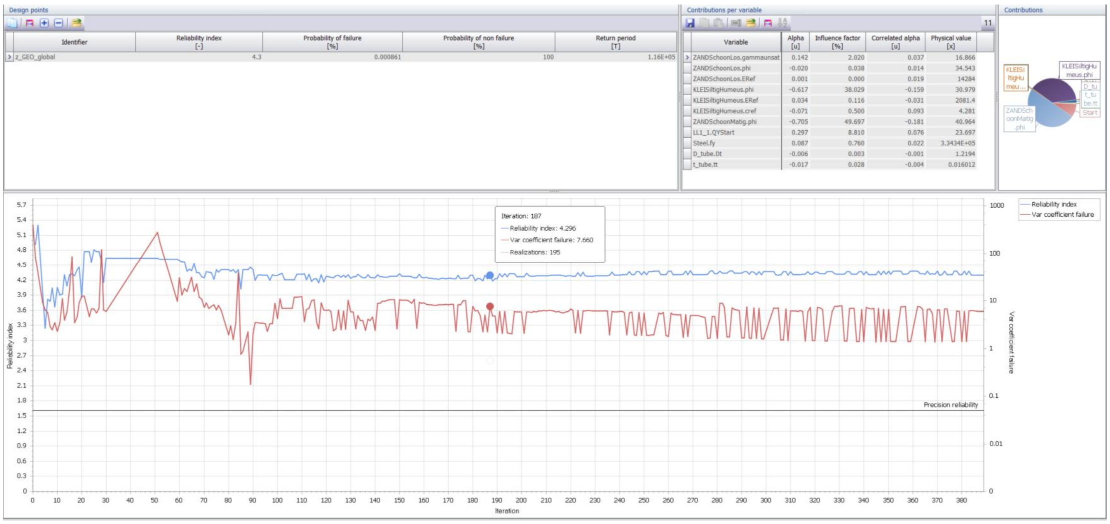


Figure C.15 Graphical results of run #3n

Observations:

- Several adjustments were made to the PLAXIS model to reduce the reliability regarding geotechnical failure and as such allow for a reliability analysis using ERRAGA.
- It has been difficult to find a reliable beta value for the LSF geotechnical failure using ERRAGA. The difficulties relate to several issues such as the specifics of the Case study, the robustness of the intermediate python scripts in relation to PLAXIS specifics/details, the observed noise in results. Furthermore, it is concluded that insight is lacking in the several calculation options of ERRAGA and the optimal (default) settings of these calculation options.
- Run #3n seemed to give a beta value that could be representative for this case. The convergence however seems questionable. A doublecheck was made by applying an alternative approach, see paragraph C.2.5. The results of this alternative approach suggest that the calculated beta value, with ERRAGA, seems reliable.
- It seems that combining results from SUMMstage and SUMMSf has some potential for defining a geotechnical limit state. At the same time, it seems that there is a significant noise present in these PLAXIS output values.

C.2.5 Geotechnical failure using alternative approach using LSF 1

To be able to get a feeling if the calculated beta values in run #3n are reliable an alternative approach was tried. Instead of using LSF 6 the LSF 1 was used with different thresholds for the allowable deformations dx_max. This way the reliability of the structure can be calculated at increasing thresholds, the results should converge to a reliability index belonging to the geotechnical limit state. The calculation log for run #4 is shown in Table C.13.

Table C.13 Calculation log for run #4

Run	Calculation log	Results?
#GRAPA v4	like v3l but now evaluate soil failure by looking at max displacement of front wall, create Beta vs ux_max line, dx_max = 0.3m	YES
#GRAPA v4b	like v3l but now evaluate soil failure by looking at max displacement of front wall, create Beta vs ux_max line, dx_max = 0.5m	YES
#GRAPA v4c	like v3l but now evaluate soil failure by looking at max displacement of front wall, create Beta vs ux_max line, dx_max = 0.1m	YES
#GRAPA v4d	like v3l but now evaluate soil failure by looking at max displacement of front wall, create Beta vs ux_max line, dx_max = 0.7m	YES

Run	Calculation log	Results?
#GRAPA v4e	like v3l but now evaluate soil failure by looking at max displacement of front wall, create Beta vs ux_max line, dx_max = 0.9m	YES
#GRAPA v4f	like v3l but now evaluate soil failure by looking at max displacement of front wall, create Beta vs ux_max line, dx_max = 1.1m	YES
#GRAPA v4g	like v4f but now with noise term ON, metamodelling.py, regel 840	YES

The calculated reliability indices of the 6 runs (#4 to #4f) are plotted in Figure C.16 together with the calculated beta value in run #3n.

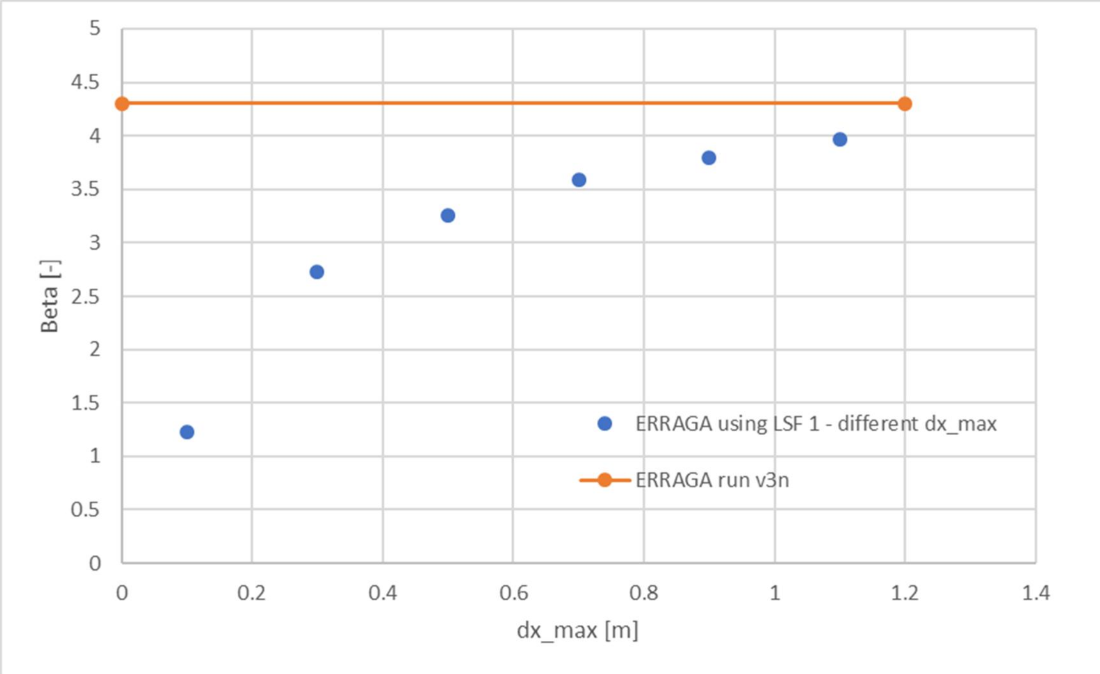


Figure C.16 Plot of different calculated Beta values using different values for dx_max vs. result of run v3n

Observations:

- Using the trend of the results of run #4 it can be confirmed that the calculated beta value of run v3n seems reliable.
- Using the ‘noise’ term resulted in only half the required model evaluations (200 instead of 400) for run v4g with ERRAGA.

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