PLAXIS

essential for geotechnical professionals

PLAXIS

Probabilistic analysis

2017

Edited by:

A. Laera
PLAXIS bv, The Netherlands

R.B.J. Brinkgreve
Delft University of Technology & PLAXIS bv, The Netherlands

Plaxis bv P.O. Box 572, 2600 AN DELFT, Netherlands

Fax: +31 (0)15 257 3107; E-mail: info@plaxis.nl; Internet site: www.plaxis.nl

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1 GENERAL INFORMATION

1.1 PREFACE

ProbAnalysis is a tool developed to perform probabilistic analysis on projects created in PLAXIS 2D. The tool combines the facilities offered by OpenTURNS, an open source library for the treatment of uncertainties, with the finite element software PLAXIS, used worldwide for geotechnical engineering and design.

ProbAnalysis is built to provide accurate results while maintaining easy use. The user simply needs to select and define the required input data. The application will execute a certain number of iterations in which ProbAnalysis automatically sets the corresponding input values for the selected parameters, performs the calculation of the project phases and retrieves the output results of interest. The results of the analysis are conveniently represented in charts and text.

The first ProbAnalysis tool is released in February 2017.

1.2 GETTING STARTED

To carry out a probabilistic analysis using ProbAnalysis, you need to extract the directory *ProbAna* from the distribution zip-file and put it in a recognizable local directory on your computer. You may consider adding a shortcut on the desktop referring to the file prob_gui_main.exe in this directory. Before starting this file, you need to open the PLAXIS project of interest and start the remote scripting server inside PLAXIS 2D. This can be done by opening the menu item *Expert* in PLAXIS and clicking *Configure remote scripting server*. The user should make sure that a working internet connection is available. After finding an available port, the server can be activated (Figure 1.1).

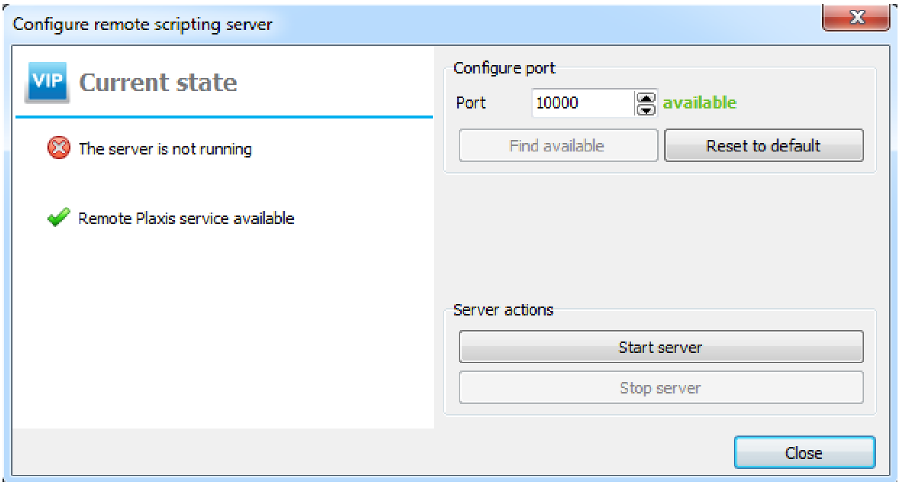

Figure 1.1 Configure remote scripting server.

After connecting to the server, ProbAnalysis can be started by double-clicking on the program icon or shortcut.

The tool is only supported by PLAXIS 2D 2016.01. You can check your version by clicking *About* in the PLAXIS *Help* menu.

1.3 PROBANALYSIS

In many geotechnical problems, most of the parameters, as for example the mechanical soil properties have a high uncertainty. It is possible to distinguish among two main categories of uncertainties: aleatory and epistemic. The first one is due to the natural randomness of a variable (as for example spatial and temporal variability), while the second one represents the uncertainty due to lack of knowledge on a variable (Nadim, 2007). To evaluate the combined effects of uncertainties, one can perform reliability analysis. A probabilistic calculation provides also the possibility to distinguish between conditions where uncertainties are particularly high or low (Duncan, 2000). In contrast to regular (deterministic) PLAXIS calculations, a reliability analysis uses stochastic distributions of model parameters, water levels or external loads, in order to determine the probability of failure, i.e. the probability that a particular output quantity (result) fails to meet a certain criterion.

The main steps of a probabilistic study are (Dutfoy, Dutka-Malen, Pasanisi, Lebrun, Mangeant, Gupta, Pendola & Yalamas, 2009):

- Specification of the case-study (uncertainty sources, model and criteria): identification of the sources of uncertainty (for example soil model parameters, water levels, forces), and the major characteristics of interest (results) that are influenced by these sources (e.g. displacements, structural forces, global stability);
- Quantification of the uncertainty sources: determination of the sources of uncertainty in a probabilistic framework, i.e. define the sources of uncertainty as stochastic variables according to the most suitable distribution;
- Uncertainty propagation: use of a numerical method to compute the uncertainty of the characteristics of interest of the system. In ProbAnalysis, methods based on the threshold exceeding approach are available: the event probability of exceeding a certain value is calculated.
- Interpretation / presentation of results: the results of a probabilistic analysis can be:
    - Distribution of output quantities, represented in a histogram;
    - Probability of failure, where failure is represented by all the events that exceed a given threshold, and reliability index;
    - Importance factors of the stochastic variables, i.e. the percentage by which each stochastic variable influences the probability of failure. The sum of all importance factors is 100%.

The methods that are based on the probability of exceeding a threshold are subdivided into approximation and sampling methods. An example of approximation method is FORM (First Order Reliability Method), where the limit state function is linearized but the results are quite accurate and calculated in reasonable time. On the other hand, Monte Carlo is a sampling method very accurate but that requires long computational time. Both FORM and Monte Carlo are supported in this version of ProbAnalysis.

In ProbAnalysis it is possible to model the material parameters, the load intensity and the water level position as stochastic variables. These variations do not require the regeneration of the mesh. As for the material parameters, some restrictions are applied based on the constitutive model and the selected parameter. More details can be found in the Reference chapter. Geometric changes in the model are not supported in this version of the tool.

The output quantities that can be selected to calculate the probabilistic results are the ones available in PLAXIS 2D, such as displacements in the soil, Cartesian stresses and strains, pore water pressures, forces and displacements in the plate elements, forces in the anchor elements, global safety factor. The results are retrieved in a preselected phase. Additional information, such as value type, position or material name, are needed based on the chosen quantity. More details can be found in the Reference chapter.

1.4 MANUAL STRUCTURE

This manual consists of four chapters:

- The General Information chapter gives an introduction about the ProbAnalysis application.
- The Tutorial chapter describes an elaborated example of a complete probabilistic analysis, in order to show how to define the input data and evaluate the results, pointing out the influence that some choices have on the analysis results.
- The Reference chapter explains the structure of the application, the function of the single features and their allowed range of values, and offers guidelines to help the users select the parameters values and read the results.
- The Verification chapter demonstrates the reliability of ProbAnalysis by means of the verification under different conditions.

2 TUTORIAL

2.1 INTRODUCTION

ProbAnalysis is an application that has been developed specifically to perform probabilistic analysis on finite element projects created in PLAXIS 2D. The simple input procedures enable a quick definition of stochastic variables, and the output facilities provide a clear presentation of computational results.

This Tutorial chapter is intended to help new users become familiar with ProbAnalysis. Note that the input properties used in the examples are fictitious and should therefore not be used as a basis for practical projects.

Users are expected to have a basic understanding of soil mechanics and statistics.

For detailed information on the available application features, the user is referred to the Reference chapter.

The following objectives will be achieved in the tutorial:

- Starting a new project.
- Selecting the parameters of interest.
- Defining the stochastic distribution for each variable.
- Selecting the probabilistic method.
- Defining the limit state function.
- Starting the calculation.
- Viewing the calculation results.

2.2 TUTORIAL 1

In this section a first application of ProbAnalysis is considered. This is the first step in becoming familiar with the practical use of the program. The general procedures for the selection and definition of stochastic variables, the choice of the probabilistic method and the output quantities of interest, the execution of a probabilistic analysis and the evaluation of the output results are described here in detail.

The PLAXIS reference project is created based on Tutorial 7 "Stability of a dam under rapid drawdown", as described in the PLAXIS 2D Tutorial Manual. The example evaluates the stability of a reservoir dam under different conditions: the water level is initially high and it is subsequently lowered considering two different time frames to simulate the rapid and slow drawdown; in the end the steady-state situation of the low reservoir level is considered. Each phase is followed by a safety phase to evaluate the stability of the model.

In this tutorial, the probability of failure with respect to the global stability after the slow drawdown process is calculated. The impact of the unit weight of soil, the stiffness and strength properties is evaluated.

2.2.1 SERVER CONNECTION

- Before launching ProbAnalysis, start PLAXIS 2D and open the reference project. You can create the project by following the description given in the PLAXIS 2D Tutorial Manual.
- Start the remote scripting server inside PLAXIS 2D, as indicated in Section 1.2 (Figure 2.1).

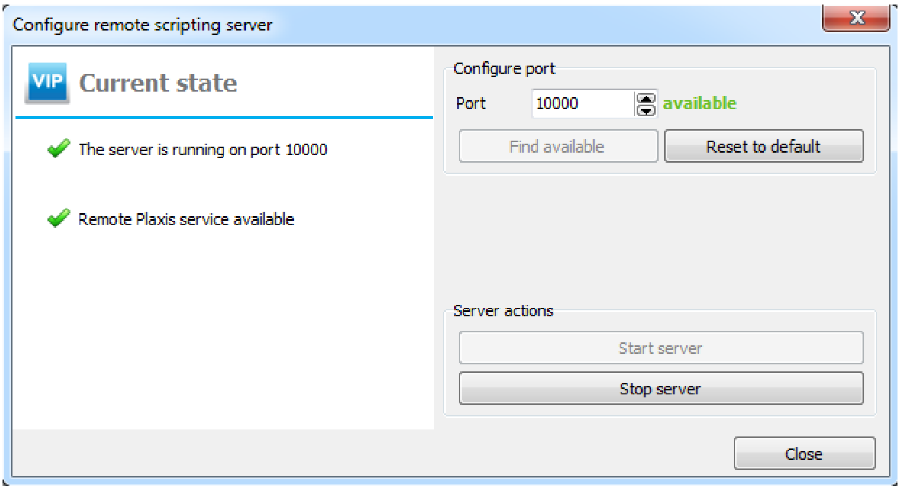

Figure 2.1 Started remote scripting server connection.

- While PLAXIS 2D remains open, start ProbAnalysis by double clicking the icon of the program in the directory where you installed it or by using the shortcut on the desktop. The *Settings* tabsheet appears in which you can establish the connection to PLAXIS (Figure 2.2). Note that the same *Input server port* number as the one used to activate the server in PLAXIS should be used.
- Click *New* on the Menu bar to load the characteristics of the reference project in ProbAnalysis. This may take a few seconds, depending on the complexity of your PLAXIS model.

2.2.2 INPUT DATA

Select parameters

The first step in every probabilistic analysis is to identify the sources of uncertainty. This is done in the *Select parameters* tabsheet.

- Click the *Select parameters* tab in the upper arrow bar. On the left side of the window the input parameters of the reference project are grouped in *Materials* and *Water levels*. No loads are defined in the reference project and therefore they do not appear in the *Select parameters* tabsheet.
- Expand the *Materials* group by clicking the corresponding arrow symbol. The group consists of *Soils* only, since no structural elements are present. By expanding the *Water levels* group, the *Borehole water levels* and *User water levels* sub-groups are shown (Figure 2.3)

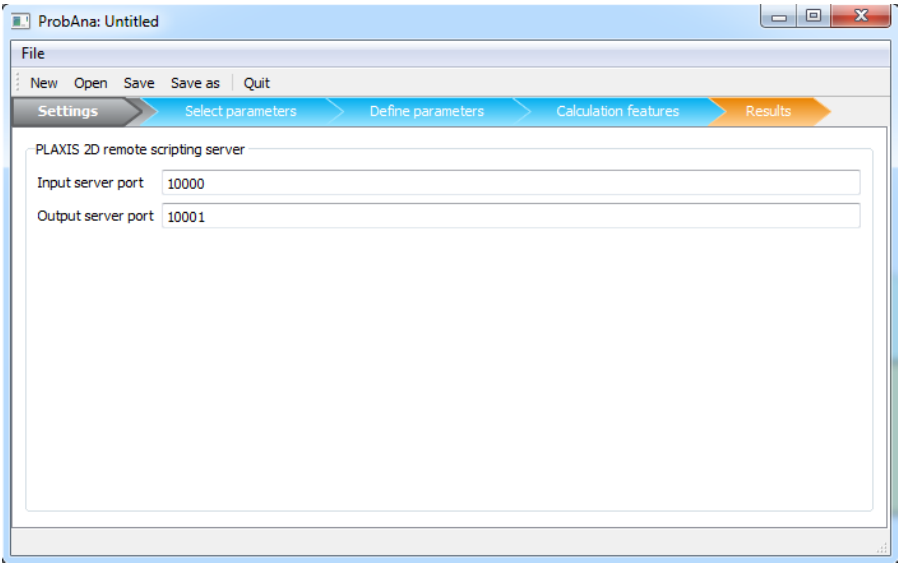

Figure 2.2 Starting window. *Settings* tabsheet.

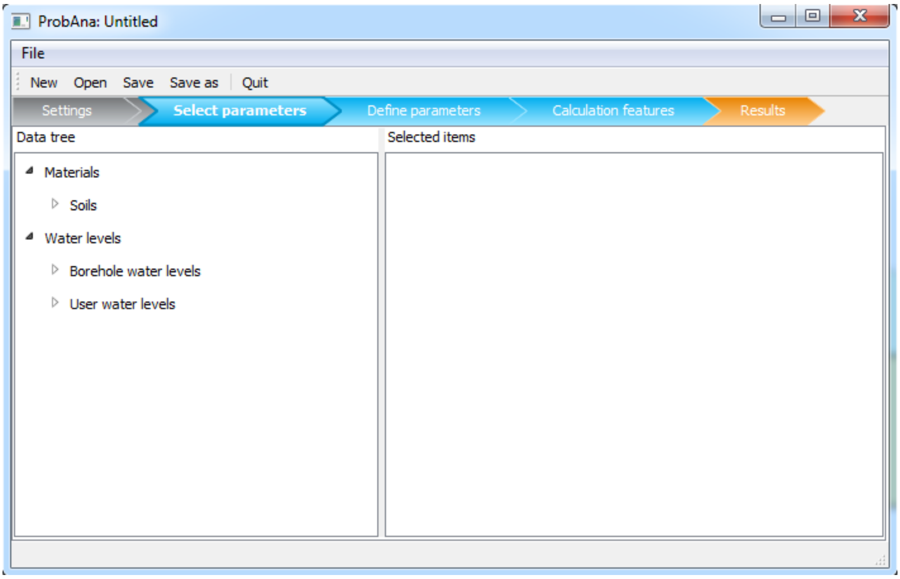

Figure 2.3 *Select parameters* tabsheet of Tutorial 1.

- Click the arrow next to *Soils*. The list of soil material data sets created in the reference project is shown, i.e. Core, Fill and Subsoil. By expanding each material data, a list of parameters and their corresponding check -box is displayed.
- For the material set Core select the saturated unit weight of soil γ_{sat}, for the Fill select the friction angle φ', while select the Young's modulus *E*' for the Subsoil material (Figure 2.4).

On the right side of the *Select parameters* window, a list of all the selected parameters is shown. The parameters are automatically added (or removed) when checking (or

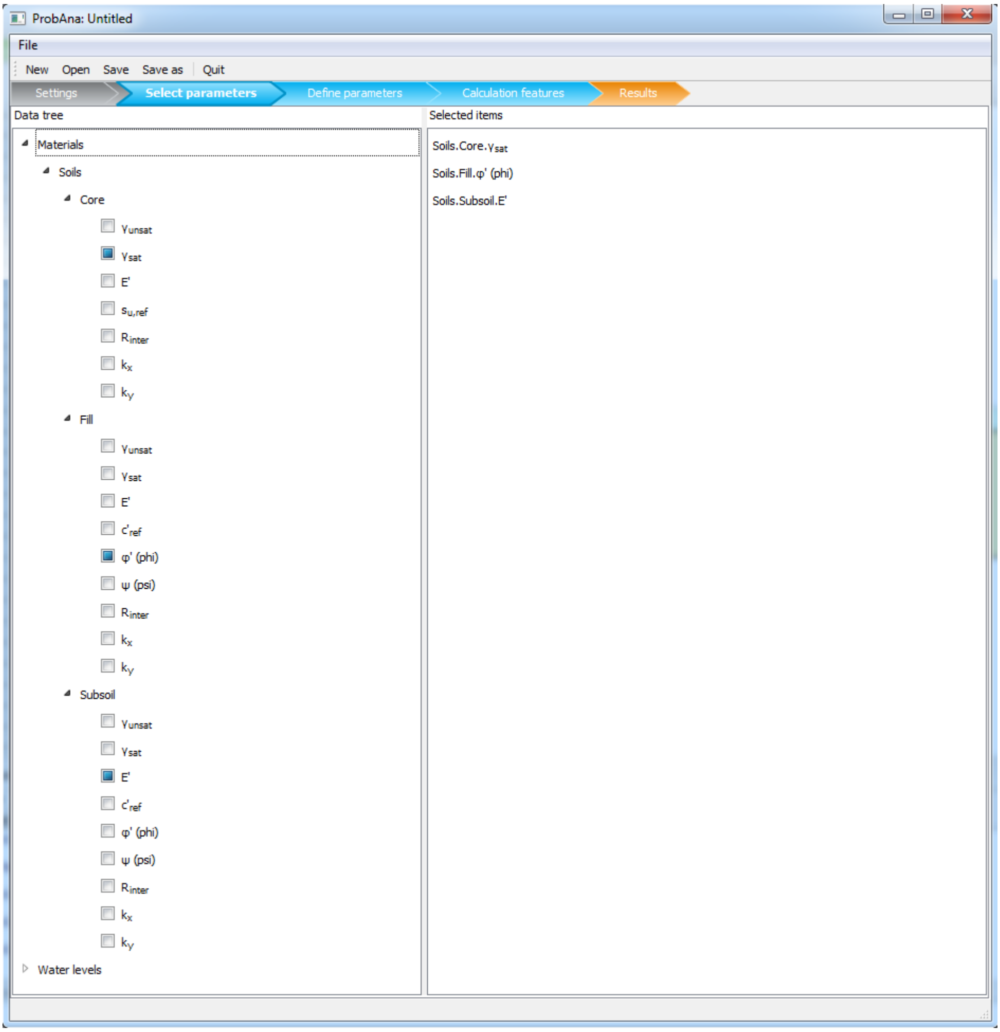

Figure 2.4 Selected parameters in *Select parameters*.

Hint: For all soil material models, you can select either the unsaturated or the saturated unit weight of soil (γ_{unsat} and γ_{sat}). More information about the restriction on the material parameters can be found in Section 3.5.1.

unchecking) the corresponding check-box.

The selected parameters represent the uncertainty sources in the reference project and need to be modelled as stochastic distributions.

- Click the *Define parameters* tab to proceed to the next step.

Hint: In the case of a mistake or for any other reason that the selected parameters need to be changed, you can access the *Select parameters* tabsheet by selecting the corresponding button from the upper bar.

Define parameters

The list of selected parameters is shown on the left side of the window (Figure 2.5). Each item can be expanded by clicking the corresponding arrow symbol.

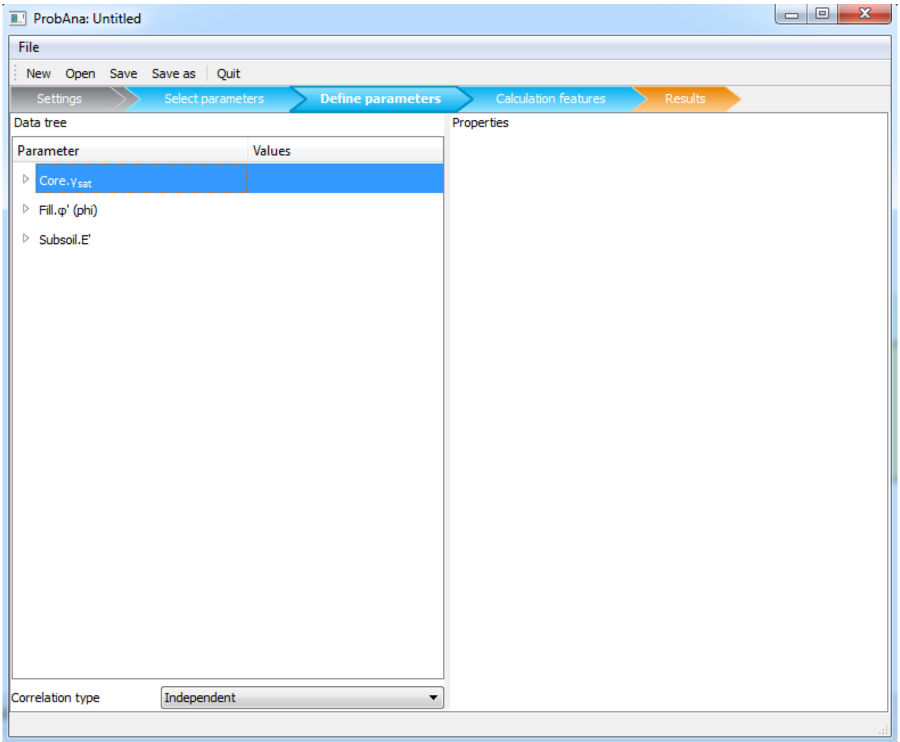

Figure 2.5 View of *Define parameters*.

- Select the distribution type and enter the corresponding parameters as in Table 2.1. The *Reference* value cannot be modified as it represents the value of the parameter in the reference project. While entering the values for each distribution, you can see that the corresponding distribution graph is drawn on the right side of the window (Figure 2.6).

Table 2.1 Distirbution properties of the selected variables

Parameter	Distribution	Lower bound	Upper bound	Mean value	Standard deviation
General					
Core.γ_{sat}	Truncated Normal	16.0	20.0	18.0	0.8
Fill.φ'	Log-Normal	-	-	31.0	2.5
Subsoil.*E*'	Log-Normal	-	-	50000	5000

- Keep the default options for *Correlation type*.

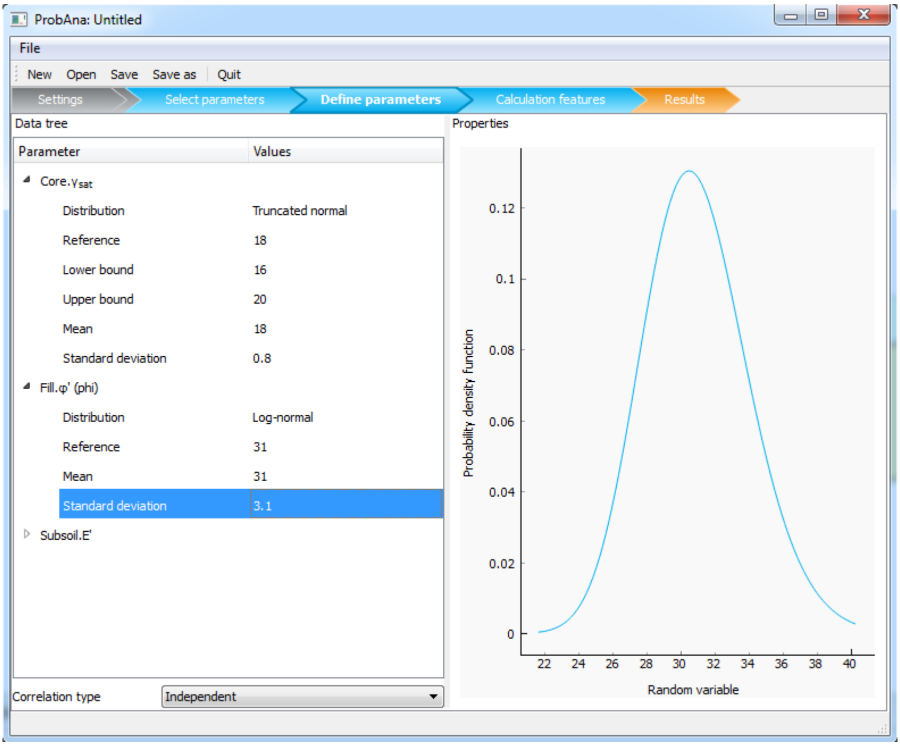

Figure 2.6 Distribution graph in *Define parameters*.

The identification and definition of the sources of uncertainty in the reference model is complete.

Calculation features

The next steps in a probabilistic analysis are the choice of a numerical method to compute the uncertainty propagation in the model and the identification of the main characteristics that are influenced by the uncertainties (among the available results in the PLAXIS 2D Output program).

- Click the *Calculation features* tab and keep the default selection for *Probabilistic method*, i.e. FORM.
- Click the arrow symbol next to *Probabilistic method* to define the output quantity of interest in the *Criterion* sub-tree. Select *Global safety factor* in the corresponding drop-down menu. The expanded view of *Criterion* shows two additional options: select *Slow drawdown - Safety* in the *Phase* menu and set 1.2 as the *Threshold* value (Figure 2.7).
- Keep the default values for the remaining parameters.

Hint: For a FORM analysis it is possible to select additional results and visualize their occurrence at the end of the calculation, but note that the FORM optimization process is governed by the selected criterion only.

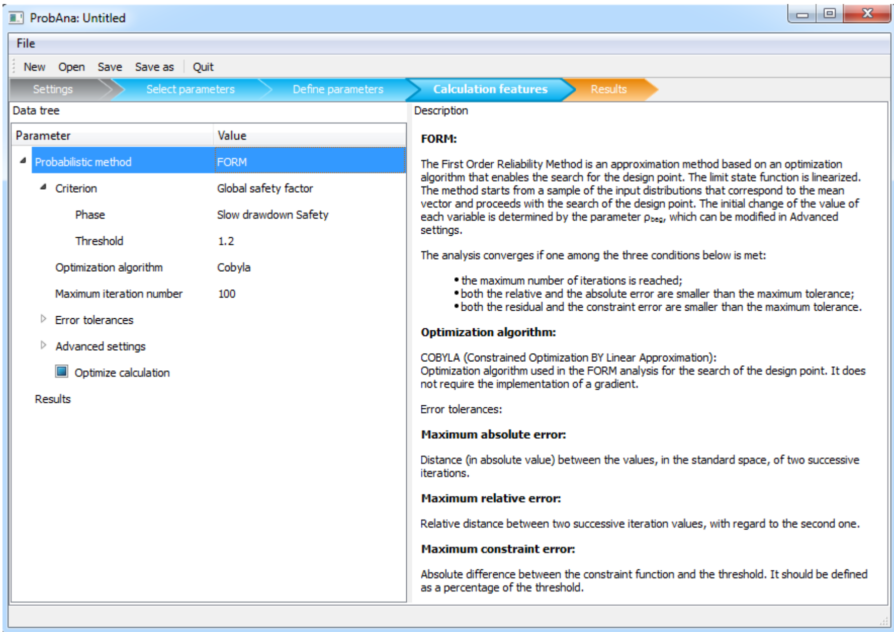

Figure 2.7 Distribution graph in *Define parameters*.

The input data are complete and the analysis can be started. Before proceeding to the next tab, it is advised to save the analysis configuration.

- Click the *Save* button on the Menu bar. A window pops up and you can enter a proper name and choose the location for the configuration file.

You can save the file in the same folder where the PLAXIS project file (<project>.p2dx) is, or in any other folder on your computer.

2.2.3 RESULTS

Once the calculation has been completed, the results are displayed in the *Results* tabsheet with an indication of the total amount of performed calculations. In this case about 53[*] calculations of the PLAXIS project are successfully completed. Note that this gives an indication only about the status of each calculation phase in the reference project but it is not related to the threshold value. The computational results are available in tables and charts, and they are grouped into the *Output results*, *Parameters* and *Convergence* tabsheets. You can save the complete project, including the results, by clicking the *Save* button.

Output results

Output results shows the probability of failure, the reliability index and a histogram with respect to the chosen criterion (Figure 2.8).

In this case, the probability of failure is about 0.015 and the reliability index is about 2.15. These values indicate that the model is safe in this configuration. In the histogram, the

[*] Because of the randomness of the statistical sampling method, a different number of calculations may be required even when running exactly the same project. This may result in a slightly different probability of failure.

values of the safety factor reached during the analysis is plotted on the x-axis, while the number of occurrences are shown on the y-axis. A vertical orange line indicates the threshold. In this analysis, all the values are above the threshold, even though most of them are very close to it.

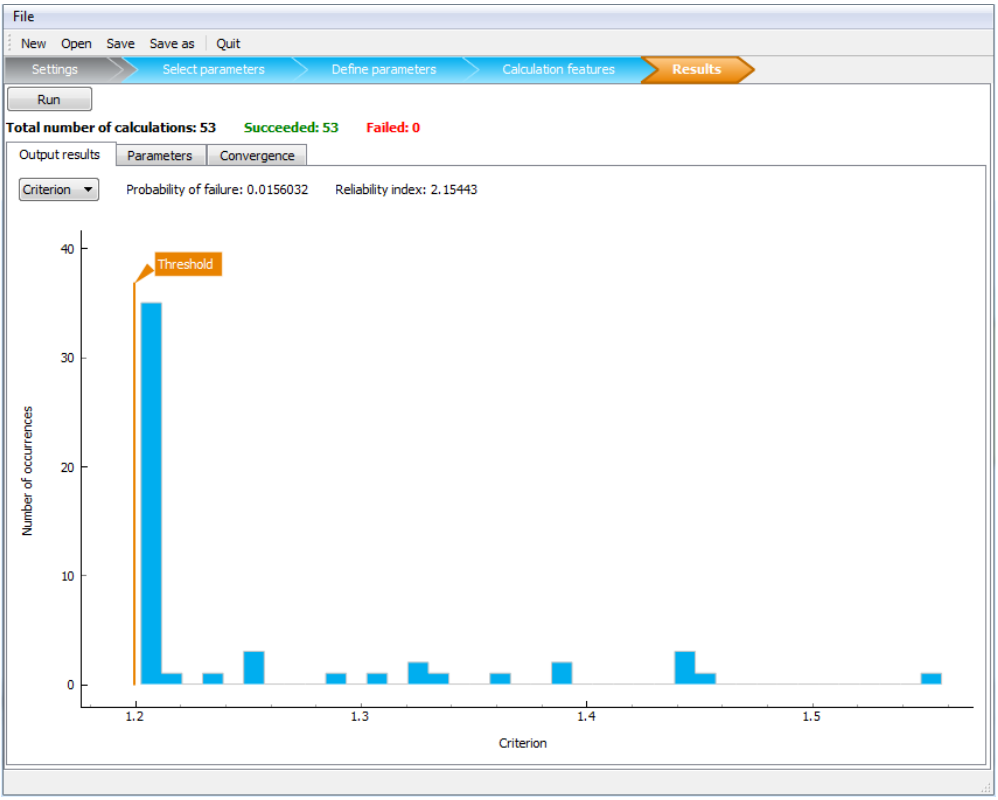

Figure 2.8 View of *Results*.

To view the other results, follow these steps:

- Clik the *Parameters* tab (Figure 2.9). The table shows the design value (i.e. the value in the design point) and the importance factor of each parameter. It can be seen that the friction angle of the Fill material is predominant compared to the others. The sum of all the importance factors is equal to 100%.
- On the right side, the scatter plot of two input variables is shown. In the corresponding drop-down boxes you can select the two parameters of interest and the output quantity. In this case, only the one related to the defined criterion is available. The different colours of the points indicate that the selected result goes from the lowest to the highest value (from dark blue to light blue).
- Click the *Convergence* tab. The evolution of each type of error during the analysis is plotted in the graph shown in Figure 2.10. The trial number is on the x-axis and the error value is on the y-axis. The legend enables to distinguish among the errors. It can be seen that, after an initial fluctuation, all the errors are very small at the end of the analysis. See Section 3.7 of the Reference chapter of this manual for more details on the various types of error.

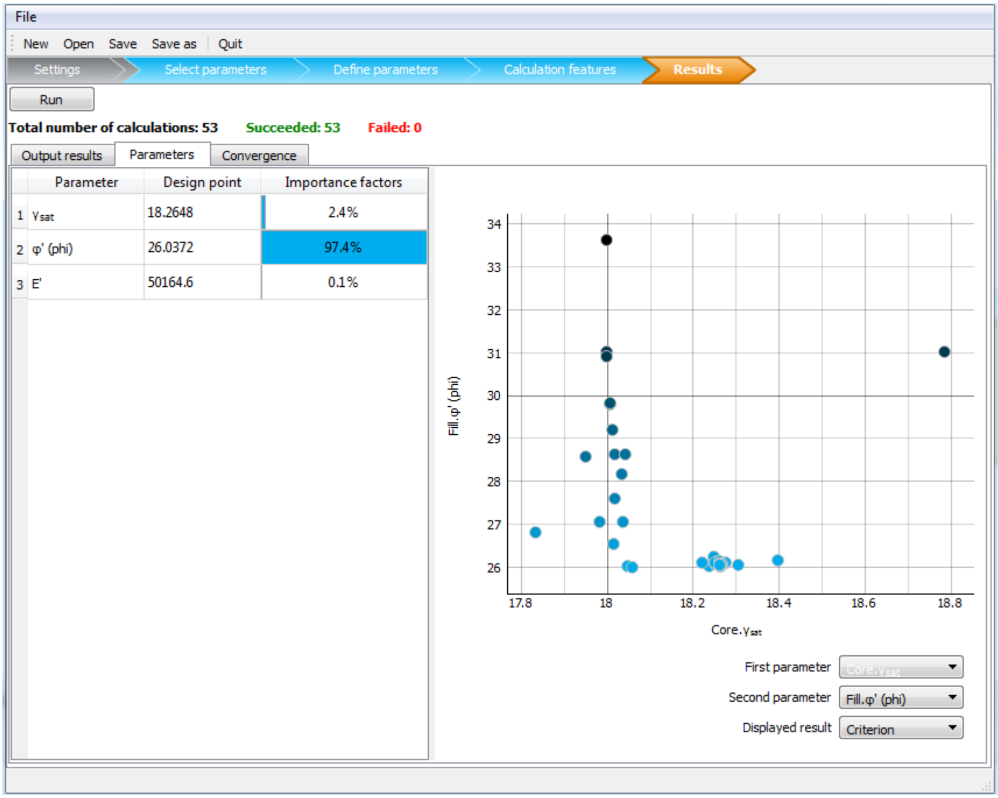

Figure 2.9 View of *Parameters* in the *Results* tabsheet.

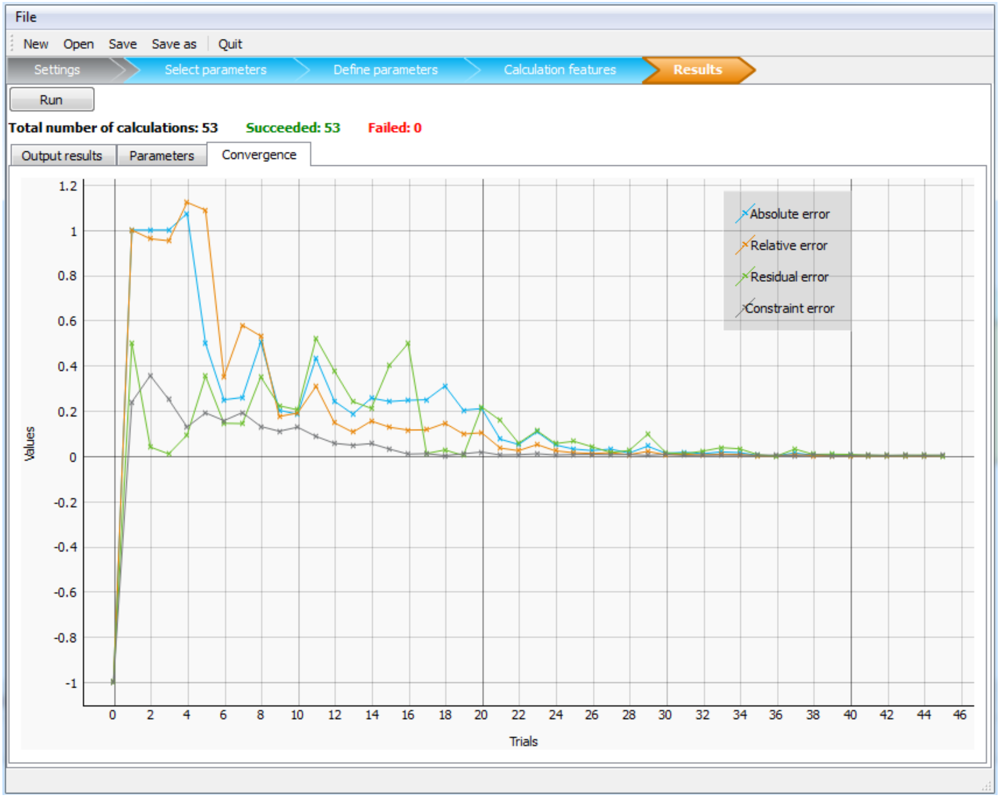

Figure 2.10 View of *Convergence* in the *Results* tabsheet.

3 REFERENCE

3.1 INTRODUCTION

ProbAnalysis is a special purpose application to perform probabilistic analysis in PLAXIS 2D by means of the Python wrapper for the PLAXIS HTTP REST API. Both PLAXIS 2D Input and PLAXIS 2D Output support the usage of a Remote Scripting server. The application uses a convenient graphical user interface that enables users to easily select and define the quantities to be treated as stochastic variables for the situation at hand, and inspect the results. To obtain a quick working knowledge of the main features of ProbAnalysis, users should work through the example problem elaborated in the Tutorial chapter.

The Reference chapter is intended for users who want more detailed information about application features. The section covers topics that are not covered exhaustively in the Tutorial chapter.

The user interface consists of five tabsheets:

- The *Settings* tabsheet enables the connection to the server.
- The *Select parameters* tabsheet enables to select the quantities to treat as stochastic variables, such as material properties, load intensity and water levels.
- The *Define parameters* tabsheet is used to define the distribution of each selected quantity and to establish the correlation between the selected parameters, when applicable.
- The *Calculation features* tabsheet allows to select the output quantities of interest, the failure criterion to be evaluated and the probabilistic method to use.
- The *Results* tabsheet is used to inspect the results of each analysis. It displays the number of performed calculations and different types of charts plotting the output quantities distribution, the input parameters scatter plot and the convergence process. The most relevant information are highlighted, as for example the probability of failure and reliability index.

The contents of this Reference chapter are arranged according to the tabsheets and their respective options as listed in the corresponding sections and boxes. This manual does not contain detailed information about the approaches used to account for the solution algorithms used in the program. For detailed information on these and other related subjects, users are referred to the papers and documents listed in the Reference chapter.

3.2 GENERAL INFORMATION

Before describing the specific features in the different parts of ProbAnalysis user interface, some general information is described here. The information given in this section applies to all parts of the application.

3.2.1 UNITS

The application does not show units. The user should input values that are consistent with the system of units used in the reference project. To help the user, the reference values of the parameters in the selected project are displayed (Figure 3.1). The output data should be interpreted in terms of the same system.

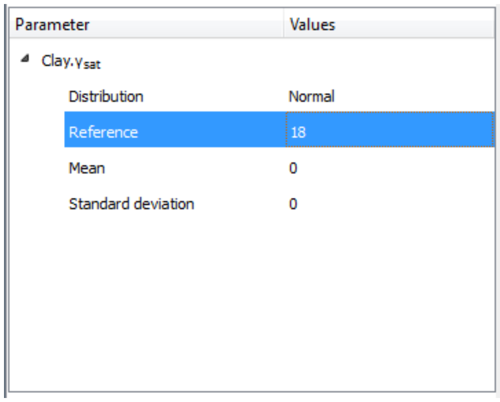

Figure 3.1 View of the *Reference* in *Define parameters*.

3.2.2 SIGN CONVENTION

In ProbAnalysis it is required to specify quantities to be used both for input and output values. For input quantities, the same sign convention as used in PLAXIS 2D is applied, e.g. a load with negative reference value will be represented by negative quantity/ies in the corresponding distribution. For the output values, the results are retrieved in terms of absolute values. This means that if the output result of interest is a settlement (i.e. a displacement in the vertical direction), the threshold must be defined as a positive number.

3.2.3 HELP FACILITIES

In the *Define parameters* and *Calculation features* tabsheets, when selecting specific characteristics, some explanatory text is displayed in the right panel, giving indications about the available options and helping the user making the most suitable choice with respect to the situation at hand.

3.3 GENERAL OVERVIEW

To carry out a probabilistic analysis using ProbAnalysis, the user has to identify and select the sources of uncertainty in the list of available input parameters, define the corresponding representative stochastic distribution and the correlation between the selected variables, specify the results that are influenced by the sources of uncertainty and the method to be used for the uncertainty propagation. This is done in the tabsheets *Select parameters*, *Define parameters* and *Calculation features* of the application. The calculation can be started and the results are displayed in the last tabsheet (*Results*). Note that the project of interest should be successfully calculated and the connection to the input port of the remote scripting server should be activated before starting ProbAnalysis.

3.3.1 APPLICATION STRUCTURE: MENU BAR AND MODE TABS

After starting the ProbAnalysis application, the user can perform an analysis by entering the necessary input information in the corresponding tabs. The user can save the full analysis (i.e. input configuration and results) or a partial configuration (i.e. any modification in the input tabs) by clicking the *Save* or *Save as* button in the menu bar. To load an existing analysis or configuration, the user needs to click the *Open* button in the menu bar. A window pops up where the user can select the saved file. When the *New* button is clicked, the application resets all the tabsheets and a new analysis can be defined.

In the upper part of the window, below the menu bar, you can find the mode tabs (Figure 3.2).

Figure 3.2 Mode tabs.

The mode tabs are used to separate different modelling modes. The grey color of the Settings tab indicates that this involves settings irrespective of any particular PLAXIS project. The blue color indicates that the user must enter some input data before performing a calculation. The orange color is used for the tab in which all the results can be visualized. For the tabs corresponding to the blue color, the area below the mode tabs is subdivided into two parts: on the left side the user can select and modify the input parameters, while the right side is used as a summary of the selected parameters or can contain explanation text, charts and tables.

The following tabs are available:

- *Settings*: the input and output port are specified and the connection with the PLAXIS project is done. More information on the *Settings* tabsheet can be found in Section 3.4.
- *Select parameters*: the stochastic variables must be selected to be able to model the distributions in the subsequent tabsheets. Note that the input parameters can only be selected in this mode. The *Select parameters* tabsheet is fully described in Section 3.5.
- *Define parameters*: each selected variable needs to be modeled as a stochastic distribution and can be correlated to a certain degree to other selected parameters by means of a correlation matrix. If no parameters are selected in the previous tabsheet, *Define parameters* does not contain any data. A detailed description on how to model the parameter distribution and define the parameters correlation is given in Section 3.6. Note that stochastic distributions can only be defined in this mode.
- *Calculation features*: the output quantities of interest, the failure criterion and the probabilistic method are chosen. In this mode other input parameters cannot be selected and no modification can be introduced to the parameters distributions. Note that the output result used as the analysis criterion and the additional output quantities can be chosen and defined only in this mode. The definition of the calculation options depends on the selected probabilistic method. A detailed description of the tabsheet is given in Section 3.7.

- *Results*: the calculation can be started and the results of the performed calculation are displayed. When the analysis is completed, the *Results* tabsheet displays a feedback about the number of performed calculations, with an indication of the number of successful and failed runs. Information about the output quantities, the input parameters and the convergence process are shown in tables and charts. The procedure to be followed is explained in Section 3.8. When an existing complete analysis is opened, the user can directly display the corresponding results in the *Results* tabsheet. If the user modifies some input data in one of the previous tabsheets, a consistency check is performed to warn the user that the displayed results do not correspond to the new configuration.

3.4 SETTINGS MODE

The *Settings* tabsheet is automatically displayed when the program is started (Figure 3.3).

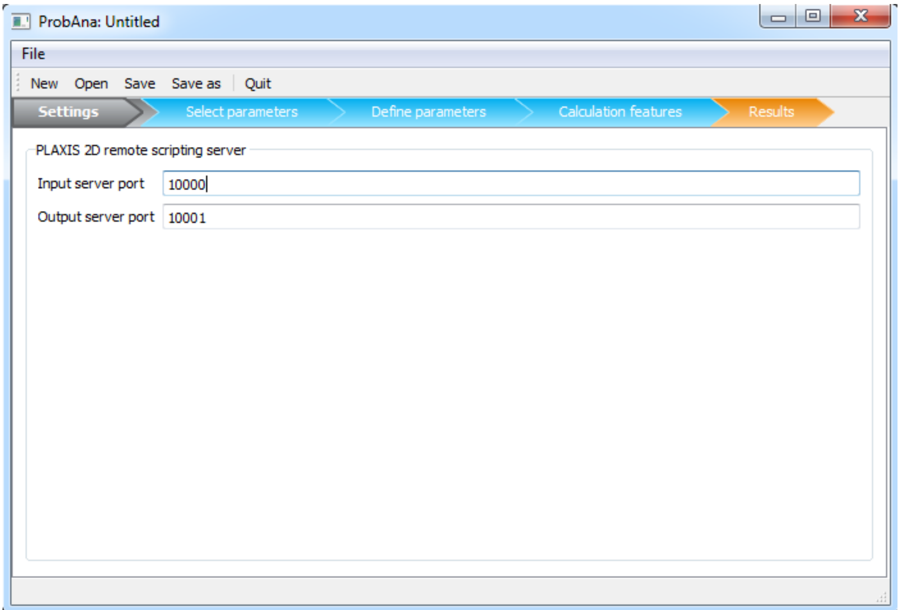

Figure 3.3 View of the *Settings* mode.

It is required to specify the Input server port and the Output server port. By default, these numbers are 10000 and 10001, respectively. Note that the input server port should be the same as the one used to activate the server in PLAXIS. If a different number is given for the Input server port, ProbAnalysis performs a consistency check and fails when trying to connect to PLAXIS 2D (Figure 3.4).

When launching ProbAnalysis, it is advised to open only the program PLAXIS 2D Input and specify an Output server port number equal to the Input server port incremented by 1.

The features of the project opened in PLAXIS 2D Input can be loaded in ProbAnalysis by clicking the *New* button in the Menu bar. Note that the *Open* button is used to load an existing analysis configuration but not a PLAXIS project. The *New* button also enables to

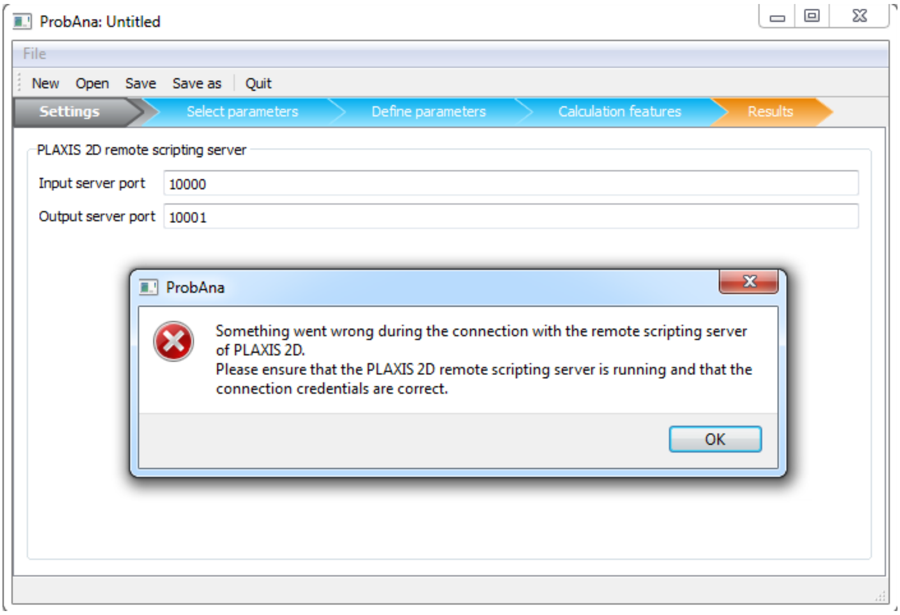

Figure 3.4 Error in the *Settings* mode.

completely reset all the tabsheets. Note that if the *New* button is used before saving the analysis configuration, any modification will be lost and cannot be recovered. If PLAXIS 2D Input has been connected to the server but no project has been opened, clicking the *New* button in the ProbAnalysis application results in an error.

3.5 SELECT PARAMETERS MODE

You can access the *Select parameters* tabsheet by simply clicking the *Select parameters* mode in the upper bar (Figure 3.5). The *Select parameters* tabsheet enables to select the parameters that the user wants to model as stochastic variables. The window consists of two main parts: on the left side a tree containing the parameters and the corresponding check-box is shown, on the right side a summary list of the selected parameters is visualized.

The content of the tree depends on the reference project.

In the most complete configuration, the tree consists of three main categories: Materials, Water levels and Loads. If no loads are defined in the reference project, this item is not present.

Each category contains a certain number of sub-categories that can be visualized by clicking the arrow button that precedes the category name. The Materials group can be expanded showing a tree of the second order with the material types: Soils, Plates and Anchors. The Water levels group is subdivided into Borehole water levels and User water levels. The Loads group can be expanded showing Point loads and Line loads.

Each sub-category collects the corresponding objects created in PLAXIS 2D Input, e.g. Soils collects all the soil material data sets created by the user.

Each object is defined by several features that represent the parameters that can be

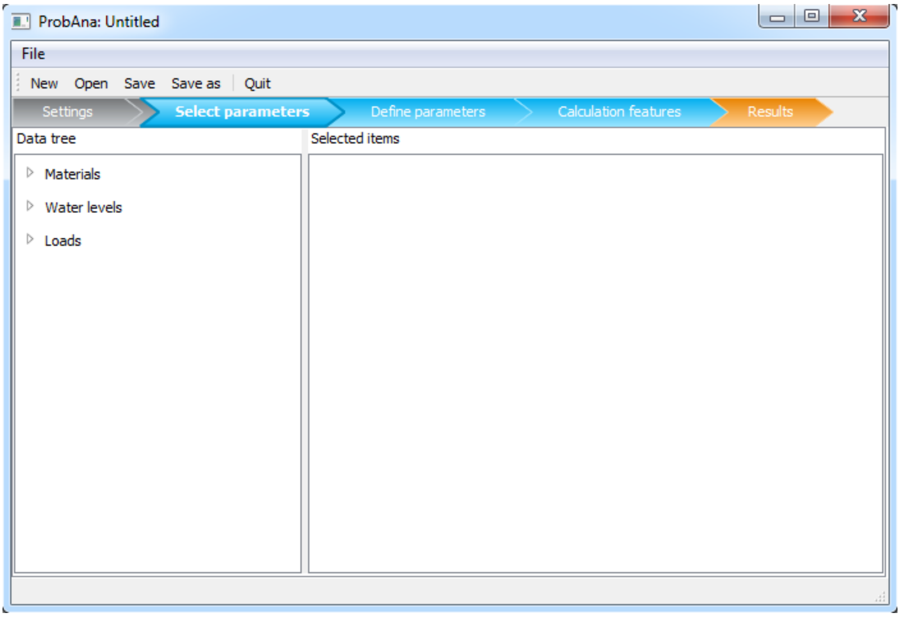

Figure 3.5 View of the *Select parameters* mode.

selected by the user.

When a parameter is selected, it is shown in *Selected items* with the name consisting of three parts separated by a dot:

$$[sub - category\ name].[PLAXIS\ object].[parameter] \quad (3.1)$$

As an example, if we select the unsaturated unit weight of a certain soil material with name "Sand", in *Selected items* it is shown as $Soils.Sand.\gamma_{unsat}$. The order of representation of the selected parameters is the same as the one in which they are shown in the tree. *Selected items* has the function of a summary of all the selected parameters and cannot be used to modify the selection. To remove a selection, the corresponding parameter check-box should be unchecked.

After selecting the parameters of interest, their stochastic distribution can be defined in the next tabsheet. In the case of a mistake or for any other reason that the selected parameters need to be changed, the user can access the *Select parameters* tabsheet by clicking the corresponding tab on the bar at the top of the window. The modification automatically applies to the content of *Define parameters*, so there is no need to save the settings before changing tabs.

3.5.1 MATERIALS

The Materials group can be expanded showing a tree of the second order with the types of material present in the reference project, as for example Soil, Plate and Anchor (Figure 3.6).

By clicking the arrow symbol, each item can be expanded showing the material or the list of the material data sets created by the user for the project. All the materials defined in the Material sets window are shown even if they are not used in the project. For each

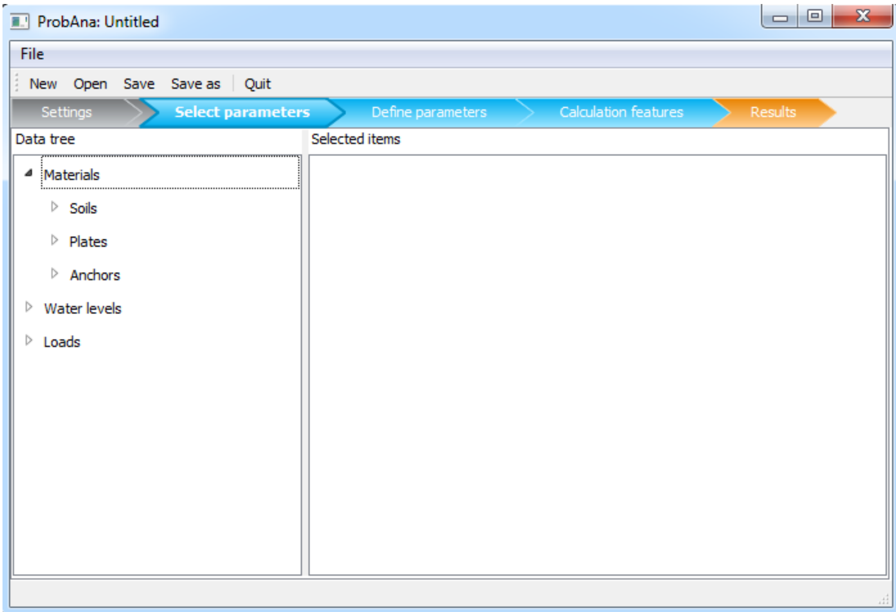

Figure 3.6 View of the *Materials* sub-categories.

material data set, the user can select one or more material parameters. The parameters depend on the chosen constitutive model. ProbAnalysis currently supports Linear elastic, Mohr Coulomb and Hardening Soil material models.

For soils the list of selectable parameters is the following:

- Common to all constitutive models:
  - γ_{unsat}: unsaturated unit weight of soil
  - γ_{sat}: saturated unit weight of soil (only for drainage type Drained, Undrained (A) and Undrained (B), when possible)
  - R_{inter}: interface strength reduction factor
  - k_x: permeability along the horizontal axis
  - k_y: permeability along the vertical axis
- Stiffness for Linear Elastic and Mohr Coulomb model:
  - E': Young's modulus (for drainage type Drained, Undrained (A) and Undrained (B), when possible)
  - E_u: Young's modulus (for drainage type Undrained (C))
  - E: Young's modulus (for drainage type Non-porous)
- Stiffness for Hardening soil model:
  - E_{50}^{ref}: Secant stiffness in standard drained triaxial test
  - m: power
- Strength for Mohr Coulomb and Hardening Soil model:
  - c'_{ref}: cohesion (for drainage type Drained, Undrained (A) and Non-porous,

when possible)

- $s_{u,ref}$: undrained strength (for drainage type Undrained (B) and Undrained (C), when possible)
- φ': friction angle (for drainage type Drained, Undrained (A) and Non-porous, when possible)
- ψ: dilatancy angle (for drainage type Drained, Undrained (A) and Non-porous, when possible)

Only one of the three stiffness parameters that describe the Hardening Soil model is shown and can be selected in the analysis. In general, the material parameters can be completely independent from each other or have a certain degree of correlation. Some correlation are automatically implemented in ProbAnalysis to make sure that meaningful parameters are input into the program, leading to a robust calculation process. When E_{50}^{ref} is selected and modeled according to a certain distribution, E_{oed}^{ref} and E_{ur}^{ref} (the tangent stiffness for primary oedometer loading and the unloading/reloading stiffness, respectively) are modified such that the initial ratios E_{oed}^{ref} /E_{50}^{ref} and E_{ur}^{ref} /E_{50}^{ref} are kept constant. The stiffness properties cannot be modified if they are defined through the alternative parameters given in the Parameters tabsheet of the material set.

For all soil material models, either the unsaturated or the saturated unit weight of soil (γ_{unsat} and γ_{sat}) can be selected; the other one will vary together with the one selected by keeping constant the difference between γ_{unsat} and γ_{sat}. When the user selects one of the two parameters and clicks the check-box of the other one, the first one is automatically unchecked. The saturated unit weight of soil is not displayed when the drainage type of the material is of type Undrained (C) or Non-porous.

When modifying the friction angle φ' or the dilatancy angle ψ, the condition for which ψ cannot be larger than φ' must be satisfied:

- if φ' is changed and the new value is less than ψ, then ψ is set equal to the new value of φ'.
- if ψ is changed and the new value is larger than φ', then ψ is set equal to φ'.

In the case of the Hardening Soil model, the change in the friction angle determines also a change in the value of K_0^{nc} (K_0 value for normal consolidation) according to the formula:

$$K_0^{nc} = (1 - sin\varphi') \tag{3.2}$$

The friction angle and the dilatancy angle are not displayed in the list if the drainage type is Undrained (B) or Undrained (C), since their value is zero and cannot be changed.

For plates, the list of selectable parameters is the following:

- EA_1: normal stiffness
- EA_2: normal stiffness in the out of plane direction, if the material is not isotropic
- EI: bending stiffness
- w: distributed weight
- υ(*nu*): Poisson ratio

For anchors, the list of selectable parameters is the following:

- *EA*: anchor axial stiffness
- $L_{spacing}$: out of plane spacing
- $|F_{max,comp}|$: Maximum compression force (if the material type is elastoplastic)
- $|F_{max,tens}|$: Maximum tension force (if the material type is elastoplastic)

For plate and anchor material types no implicit correlations have been implemented in the tool. The user may correlate the parameters in the next tabsheet.

3.5.2 WATER LEVELS

The Water levels category may be subdivided into Borehole water levels and User water levels. If at least one borehole has been defined in the project, the sub-category Borehole water levels is present. By clicking the arrow symbol that precedes the sub-category, the list of borehole water levels defined in the project is shown, with *Head* as selectable parameter.

If one or more user water levels have been created in the PLAXIS project, they are grouped under the User water levels sub-category. Each user water level is defined by one or more water segments, which represent the selectable parameters. By selecting a water segment, it is possible to model its offset variation in the vertical direction.

No restriction are applied on the selection of the water segments but note that the selection of two contiguous water segments (i.e. with one extreme point in common) is not advised: the offset of each water segment is applied one after the other and the end result may be different than the desired one.

In Figure 3.7, the top picture represents an example of a water level that consists of three segments (*WaterSegment_1*, *WaterSegment_2* and *WaterSegment_3*). Each water segment is defined by two points at the extremities of the line, i.e. *Point_1* and *Point_2* for *WaterSegment_1*, *Point_2* and *Point_3* for *WaterSegment_2*, *Point_3* and *Point_4* for *WaterSegment_3*. The picture below shows the case in which *WaterSegment_1* and *WaterSegment_3* are chosen as stochastic variables and they are moved in the vertical direction of a given quantity (see dashed lines). Note that the two selected water segments have no points in common, while *WaterSegment_2* moves based on the new position of the points that are shared with the other water segments.

Figure 3.8 shows the case in which *WaterSegment_1* and *WaterSegment_2* are both selected as variables. These two segments have in common *Point_2*. During one iteration, *WaterSegment_1* is moved first (for example an offset below the original position as indicated by the dashed line), then *WaterSegment_2* is moved based on another offset value starting from the new position of *Point_2*. A possible configuration would be the one shown in the picture. Note that also *WaterSegment_3* is moved based on the new position of *Point_3*.

3.5.3 LOADS

The Loads category contains two sub-categories: Point loads and Line loads. Each of them consists of all the point and line loads created in the model.

When expanding the single load, a list of the load characteristics in terms of intensity is shown, with check boxes to select the one of interest. Note that the load intensity of each load object must be specified in the Structures mode in the reference project in PLAXIS

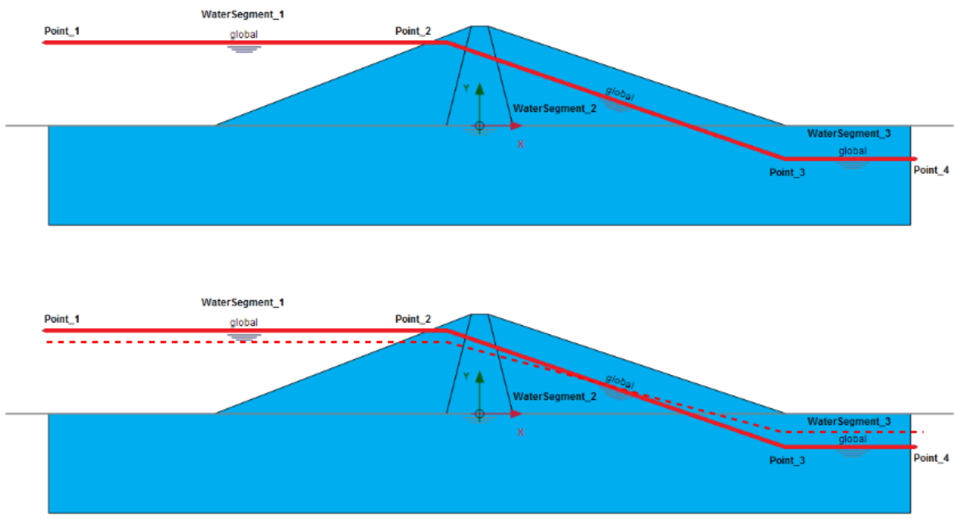

Figure 3.7 An example of a change in the user water levels.

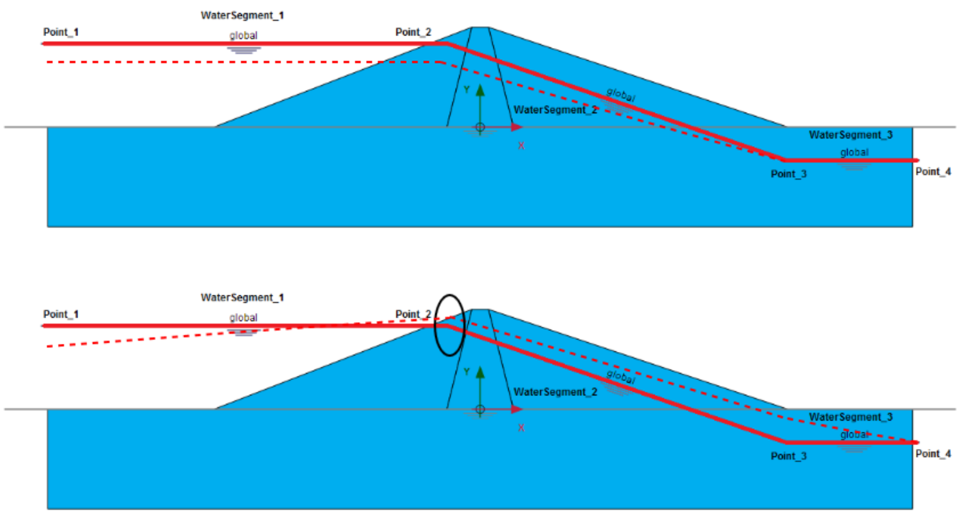

Figure 3.8 Possible unexpected behaviours when modifying the user water levels.

2D. If this is done in Staged construction, the new value that is assigned during the probabilistic analysis will be overwritten by the one specified in Staged construction.

For a point load, the parameters that can be selected are:

- $F_{x,ref}$: horizontal load component
- $F_{y,ref}$: vertical load component
- M: bending moment

For a line load, the list of parameters depends on the distribution type:

- if the type is *Uniform*:
  - $q_{x,start,ref}$: horizontal force per length
  - $q_{y,start,ref}$: vertical force per length
- if the type is *Linear*:
  - $q_{x,start,ref}$: horizontal force per length in the first point
  - $q_{y,start,ref}$: vertical force per length in the first point
  - $q_{x,end,ref}$: horizontal force per length in the second point
  - $q_{y,end,ref}$: vertical force per length in the second point
- if the type is *Perpendicular*:
  - $q_{n,ref,ref}$: perpendicular force per length
- if the type is *Perpendicular, vertical increment*:
  - $q_{n,ref,ref}$: reference value of erpendicular force per length
  - $q_{n,inc,ref}$: increment of the perpendicular force
  - y_{ref}: position where the reference load is defined

Note that the reference position y_{ref} does not refer to the geometrical position of the load.

3.6 DEFINE PARAMETERS MODE

You can access the *Define parameters* tabsheet by simply clicking the *Define parameters* mode in the upper bar (Figure 3.9).

In *Define parameters*, for each parameter that has been selected as a stochastic variable in the previous tabsheet, the user has to choose the distribution type and define the required parameters.

The window is subdivided into three main areas. On the left side, the upper part shows the list of selected parameters in a tree view. Below, the user can choose the type of parameters correlation via the drop-down menu. The right side panel can have different functions, based on which item of the expanded tree is selected on the left side. When the user selects *Distribution*, an explanation text regarding the available distributions is visualized. If one of the features that defines the distribution is selected, the corresponding distribution graph is drawn (Figure 3.10). Finally, if the correlation type is not set to *Independent*, a table is displayed where the user can input the correlation coefficients.

3.6.1 PARAMETERS TREE

The items selected in the previous tabsheet are listed in the *Define parameters* tabsheet. If no selection has been made, *Define parameters* is empty. Each item can be expanded by clicking the arrow button. Under the *Parameters* column, the *Distribution* and *Reference* features are always present, while the number of the additional parameters depends on the chosen distribution. By default, the distribution type is set to *Normal*, with

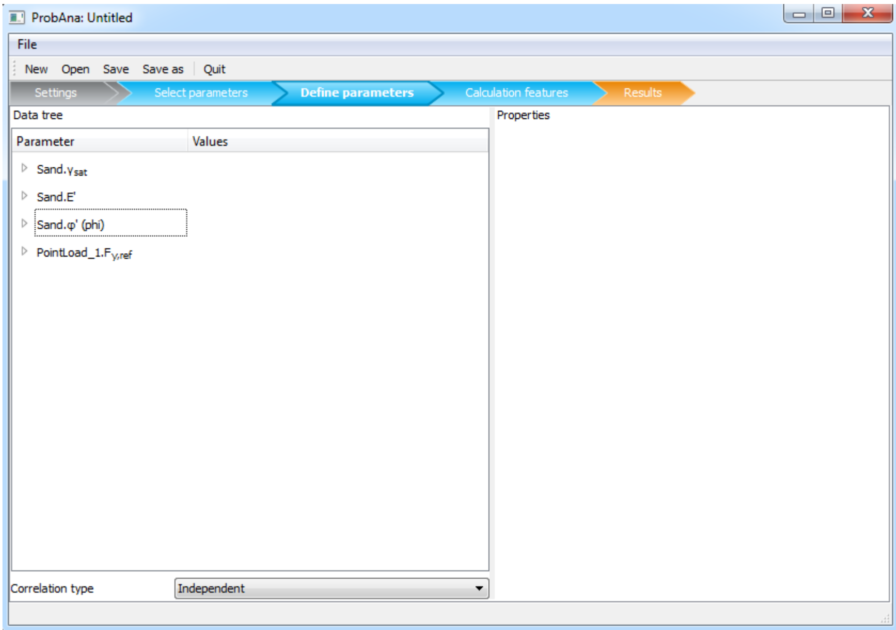

Figure 3.9 View of the *Define parameters* mode.

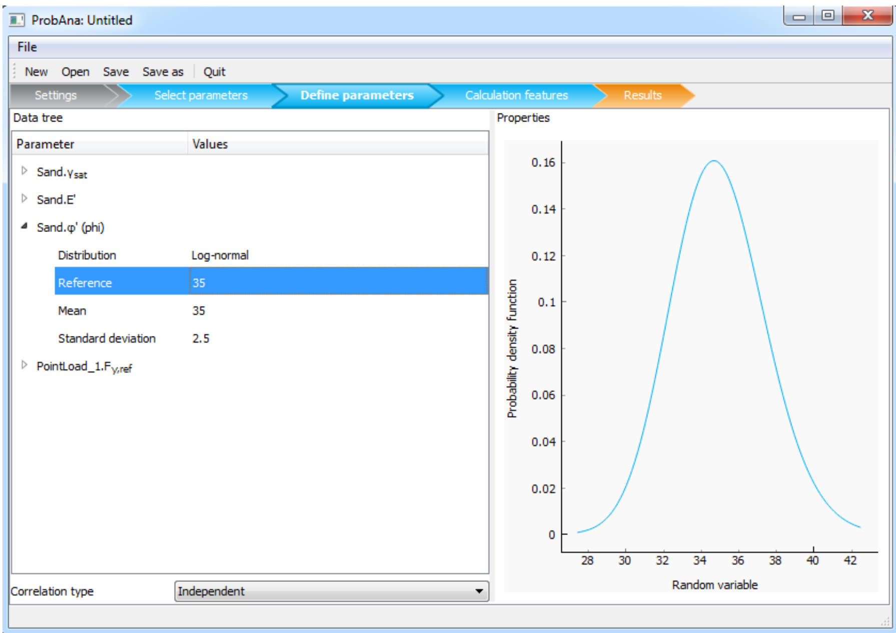

Figure 3.10 Distribution graph.

Mean and *Standard deviation* as characterizing parameters.

The distribution type can be changed by selecting one of the available options in the corresponding drop-down menu (Figure 3.11), such as:

- Normal

- Truncated Normal
- Log-Normal
- Uniform

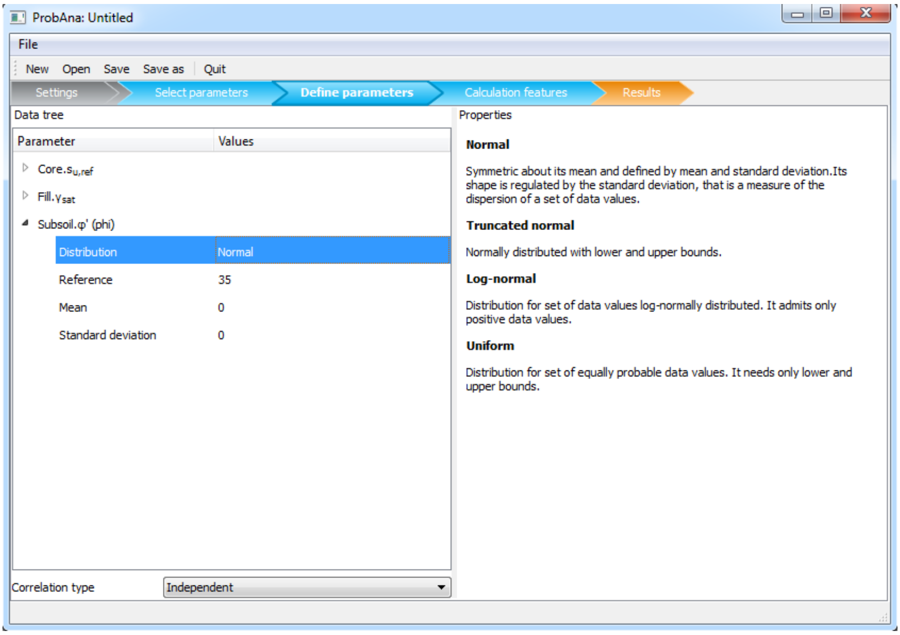

Figure 3.11 Distribution types in *Define parameters*.

The Normal distribution is a function characterized by a symmetrical bell-shaped graph. Its shape depends on two factors: the mean and the standard deviation. The mean of the distribution determines the location of the center of the graph, while the standard deviation is a measure of the data dispersion: it determines if the curve is tall and narrow (for low standard deviation) or short and wide (for large standard deviation).

The Truncated normal distribution is the probability distribution of a normally distributed random variable whose value is bounded between two points. For this reason, in addition to the mean and standard deviation, it is required to specify the lower and upper bound. Outside the range defined by the lower and upper bound, the probability density function is zero.

The Log-Normal distribution describes the probability distribution of a random variable whose logarithm is normally distributed. It is defined by mean and standard deviation, and admits only positive data values.

The Uniform distribution is a probability distribution described by a lower and upper bound, for example *a* and *b*, where $a < b$. All values of the variate between the lower limit *a* and the upper limit *b* are equally likely to occur (Kottegoda & Rosso, 2008).

The integral under any distribution function is equal to unity (100%).

After choosing the distribution type, the user must enter a meaningful value for the parameters characterizing the distribution. Below the distribution type, the reference value of the variable in the original project is shown. This can give an useful indication about the order of magnitude to use for the stochastic parameters and their sign. Under the *Values* column, by double-clicking the box next to the parameters, you can enter a number. Note that the standard deviation must be always positive while mean, lower bound and upper bound should have a sign consistent with the project at hand. This is especially important for parameters belonging to the Water levels and Loads categories. In fact, if a certain load is characterized by negative quantities (e.g. a line load with $q_{y,ref,start}$ equal to -10), this should be taken into account when specifying the mean and/or the lower and upper bounds (e.g. mean could be -10, lower and upper bounds could be -15 and -5, respectively). In the case of Truncated normal and Uniform distributions, the lower bound must be defined by a number that is smaller than the upper bound, otherwise ProbAnalysis will crash when launching the analysis through the *Run* button in the *Results* tabsheet.

Hint: The choice of the distribution is very important since it may prevent to set unrealistic values for the parameters. It should be noted that the stiffness and strength soil parameters cannot be negative or, in some cases, equal to zero. For these parameters a truncated normal or log-normal distribution may be chosen to make sure that the parameters remain positive even in extreme cases.

3.6.2 PARAMETERS CORRELATION

The selected parameters can be treated as independent from each other or bounded with a certain degree of correlation. At the bottom of *Define parameters*, the *Correlation type* option is set to Independent by default. The user can click the box and select Normal. In that case, the user can specify the correlation coefficients between the parameters in the corresponding table on the right panel. The table is a user-friendly representation of the correlation matrix, i.e. a square symmetric diagonal matrix of n x n dimension, where n is the number of stochastic variables in the analysis.

The correlation matrix for parameters x_1, x_2, x_3, ..., x_n is:

$$\begin{bmatrix} 1 & R_{12} & R_{13} & \dots & R_{1n} \\ R_{21} & 1 & R_{23} & \dots & R_{2n} \\ R_{31} & R_{32} & 1 & \dots & R_{3n} \\ & & \dots\dots\dots & & \\ R_{n1} & R_{n2} & R_{n3} & \dots & 1 \end{bmatrix}$$

with $R_{ij} = R_{ji}$, for i = 1, ..., n and j = 1, ..., n, and $i \neq j$

In ProbAnalysis, the table is the matrix column corresponding to each parameter. For example, the first column corresponds to x_1:

$$\begin{bmatrix} 1 \\ R_{21} \\ R_{31} \\ ... \\ R_{n1} \end{bmatrix}$$

The second column corresponds to x_2:

$$\begin{bmatrix} R_{12} \\ 1 \\ R_{32} \\ ... \\ R_{n2} \end{bmatrix}$$

The table has only one column and is shown when clicking one of the listed parameters on the left side. There are as many rows as the number of variables (excluding the one selected for which the correlation coefficient is always 1) and the header of each row is the name of each variable. By default the coefficients are set to zero but they can assume values from -1 to 1. When a different value is entered, the cell is highlighted in orange, to help the user easily visualize the dependencies in the case of many parameters. Since the matrix is symmetric, it is not required to enter the same correlation coefficient for the other parameter when changing the value selected on the left. This is automatically done in ProbAnalysis, as shown in Figure 3.12 and Figure 3.13.

The options Independent and Normal refer to the copula, that is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform (Nelsen, 2007). Any multivariate joint distribution can be written in terms of univariate marginal distribution functions and a copula which describes the dependence structure between the variables independently of the marginal laws of the variables involved (Kottegoda & Rosso, 2008). The choice of the dependence structure is disconnected from the choice of the marginal distributions. While the Independent copula represents the case of all the input variables are independent the ones from the others, the Normal copula is a function of the correlation matrix R, that must be symmetric, definite and positive (Openturns, 2010). To ensure these conditions, a consistency check is performed when clicking the *Run* button to start the analysis. A warning message will be shown if the matrix is non definite and/or non positive.

The effect of the correlation matrix on the probability of failure depends on the correlated parameters and the corresponding coefficients. If all the variables in the project are considered independent, a certain probability of failure is found and the importance factors can give an idea of the influence of each variable on the failure. If two variables with a very low importance factor are correlated, you can assign the correlation coefficient any value between [-1, 1]; this will have an extremely small (or null) effect on the probability of failure. On the other hand, if failure is driven mainly by two parameters and failure occurs when both of them are large, a positive correlation coefficient increases the probability of failure, while a negative coefficient decreases it. In general,

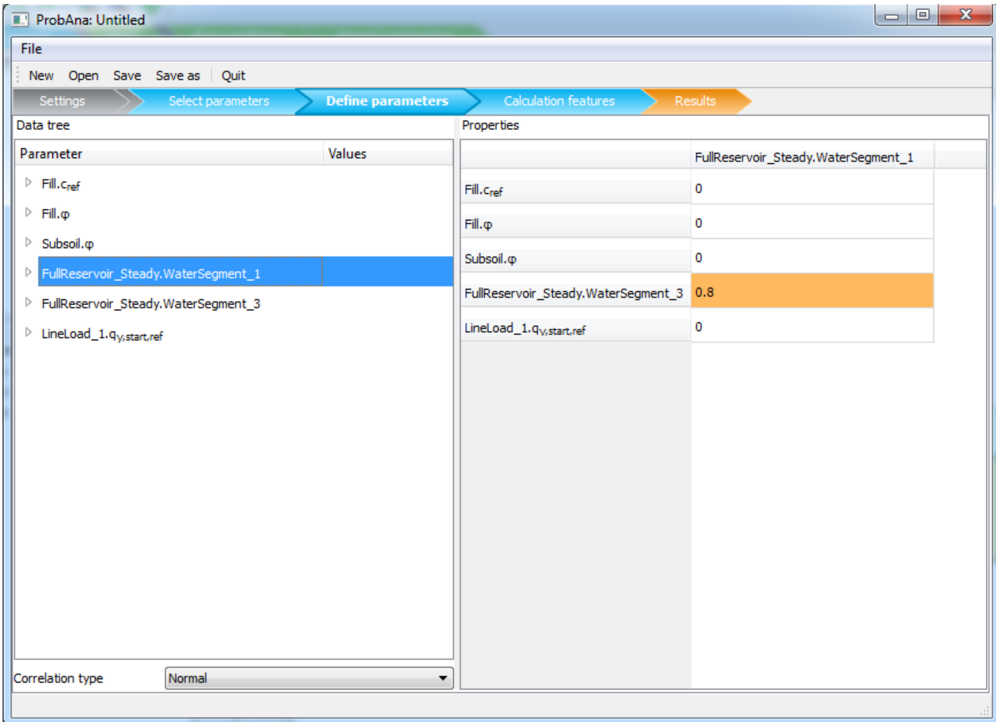

Figure 3.12 Symmetric correlation matrix (a).

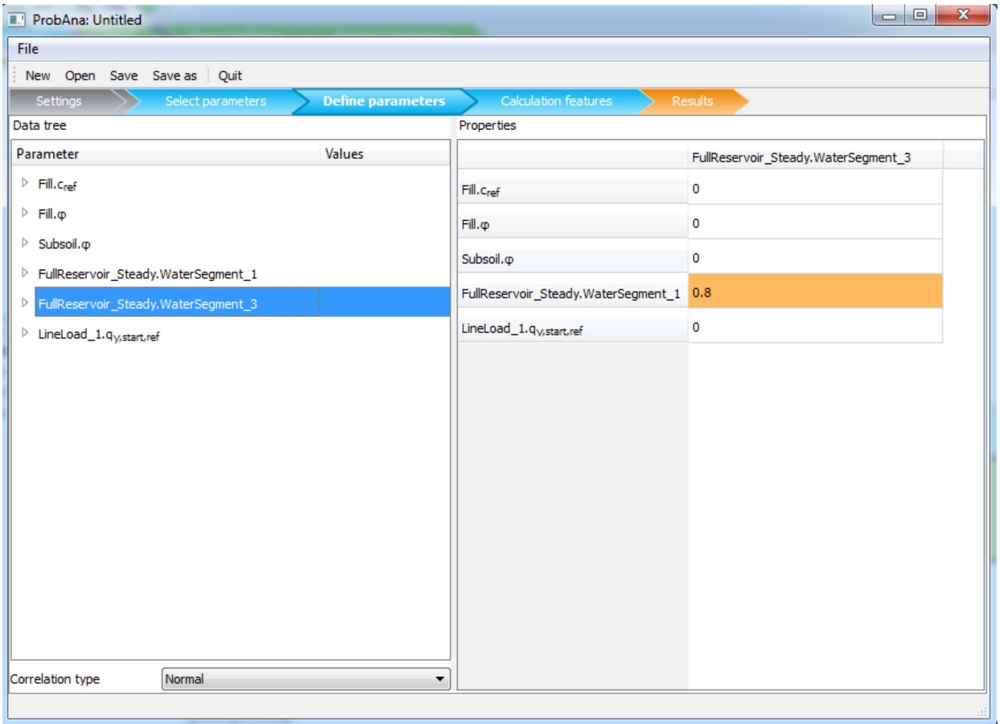

Figure 3.13 Symmetric correlation matrix (b).

given two variables x_i and x_j, a positive coefficient R_{ij} is representative of a higher probability of having x_j large when also x_i is large, compared to the case in which the variables are independent. This probability increases for larger values of R_{ij}.

The correlation coefficient (also known as Pearson product-moment correlation coefficient) is a direct measure of the degree of linear dependence between two variables. When two variables are perfectly linearly related, ρ_{ij} is either 1 or -1. In general, given two variables x_i and x_j, a positive coefficient ρ_{ij} means that when x_j is large also x_i tends to be large, compared to the case in which the variables are independent. This probability increases for larger values of ρ_{ij}. Similarly, if ρ_{ij} is negative, large values of x_j tends to give small values of x_i (Fenton & Griffiths, 2007). If ρ_{ij} is equal to zero, it does not mean that the two variables are independent, but only that they are not linearly correlated. In order to define independence, the Independent copula should be used. Note that the correlation matrix to be used for the Normal copula corresponds to the Pearson correlation matrix if and only if all the marginal laws are normal. Otherwise, the coefficient must be modified. For more information you can refer to the OpenTURNS Reference Manual (Openturns, 2010).

3.7 CALCULATION FEATURES MODE

You can access the *Calculation features* tabsheet by simply clicking the corresponding mode tab (Figure 3.14). The input data required in this window is independent from the previous two.

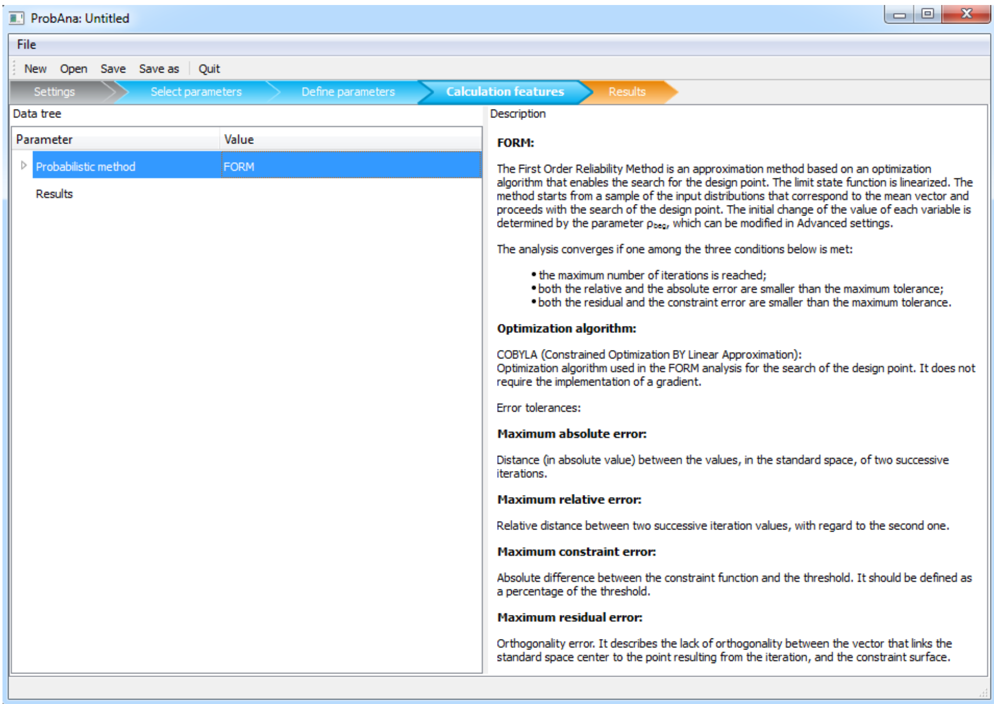

Figure 3.14 View of a new project in the *Calculation* mode.

The *Calculation features* tabsheet consists of two parts: on the left side the user can enter the required input, while the right side shows some explanation text.

In this window, the probabilistic method for the uncertainty propagation and the output quantities of interest may be defined.

Before running the calculation it is advised to save the analysis configuration. This allows you to quickly recover your input data in the case of errors. If the input data related to other tabsheets have been already saved, you can update the existing file by clicking the *Save* button on the Menu bar. If no data have been saved until that moment, a window pops up and you can specify name and location of the file. If you want to save the configuration on a new file, you can click the *Save as* button on the Menu bar.

The calculation can be started in the *Results* tabsheet.

3.7.1 PROBABILISTIC METHOD

The user may choose between FORM and Monte Carlo by clicking the *Probabilistic method* drop-down menu.

FORM is a First Order Reliability Method (Hasofer & Lind, 1974) where the limit state function $g(\mathbf{X})$ (with **X** representing the vector of the random variables $\mathbf{X} = (X_1, X_2, ..., X_n)$) is linearized by the first order Taylor expansion. The method starts from a sample of the input distributions that correspond to the mean vector, i.e. a vector where all the variables are represented by their mean value. The analysis proceeds with the search of the design point (or MPP, Most Probable Point), which represents the point with the highest probability density, i.e. the point on the limit state function that is the nearest to the origin of the standard space. This is an optimization problem where the result of each iteration is used to determine the new set of input parameters.

Monte Carlo is a method that uses repeated random sampled values from the input probability distributions as described in the random vector **X**. For each set of samples a calculation is performed (iteration) and the resulting outcome from that sample is recorded. The method can deal with non regular limit state function and does not apply any approximation but, for this reason, many iterations are generally needed.

Both methods require the definition of a criterion, which represents the limit state function (Figure 3.15). The result can be selected for the soil, plate and anchor elements, or for the global safety factor.

When the user selects the type of interest in the drop-down menu next to *Criterion*, the additional required input for each type will be different. The criterion type can be chosen among one of the output quantities that can be visualized in PLAXIS 2D Output at the end of the calculation. This version of ProbAnalysis supports:

- Soil
- Plate
- Anchor
- Global safety factor

All the types have in common the selection of the phase. By clicking the drop-down menu of the *Phase* item, you can select one of the calculation phases in the project. Each phase is identified with the corresponding phase ID. The selected phase represents the phase in which the result of interest will be retrieved.

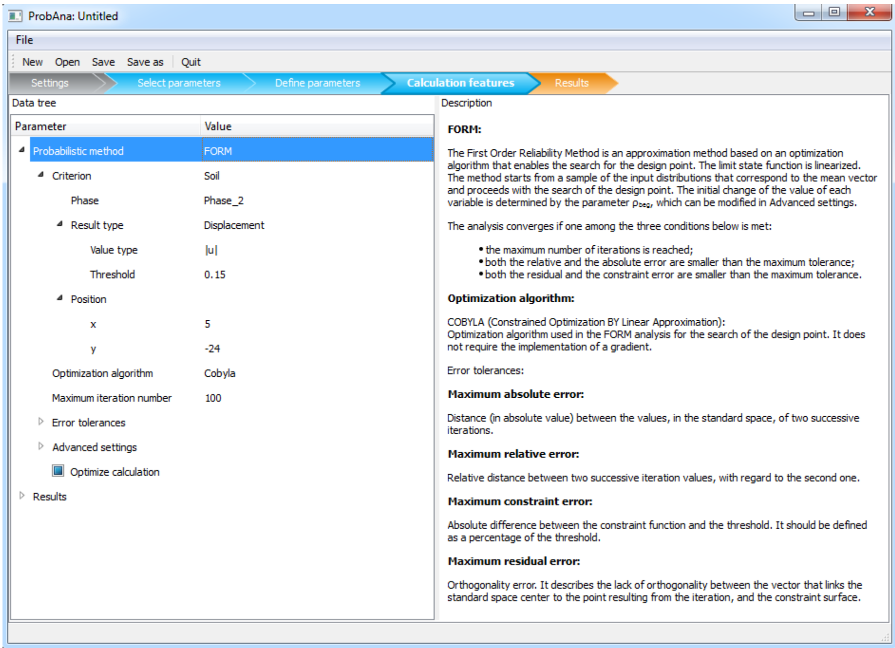

Figure 3.15 View of the criterion sub-tree in the case of a Soil type.

For all types it is required to specify the threshold. For Global safety factor, it is an item under the Criterion tree, while for the others it is an item under the *Result type* subtree. The threshold is a number. It is always defined as a positive quantity and can be typed in the corresponding editable box. For FORM and Monte Carlo, the threshold is used to calculate the probability of failure. In particular, for the *Global safety factor* criterion, failure is represented by all the cases in which the safety factor is less than a defined value (e.g. 1.5), while for all the other criterion types, failure is associated to all the events in which the result is larger than the given threshold.

For Soil, Plate and Anchor, the *Result type* option enables the selection of the type of the output quantity.

For Soil, the available options are:

- Displacement
- Strain
- Stress
- Pore pressure

When selecting one of them, *Result type* can be expanded showing two required information types:

- Value type
- Threshold

Value type enables the selection of an output quantity, different for each available option:

- Displacement

  - absolute displacement, $|u|$
  - horizontal displacement, u_x
  - vertical displacement, u_y
- Strain
  - horizontal deformation, ϵ_{xx}
  - vertical deformation, ϵ_{yy}
  - shear deformation, γ_{xy}
- Stress
  - effective horizontal stress, σ'_{xx}
  - effective vertical stress, σ'_{yy}
  - total horizontal stress, σ_{xx}
  - total vertical stress, σ_{yy}
  - shear stress, σ_{xy}
- Pore pressure
  - excess pore pressure, p_{excess}
  - active pore pressure, p_{active}
  - steady-state pore pressure, p_{steady}

These quantities are retrieved in a specific position, that can be entered in the Position sub-tree in terms of x and y coordinate of the point of interest. Note that no units are shown and the same as the ones used in the reference project should be considered. The point of interest does not need to correspond to an existing node or stress point, but it can be an arbitrary point in the soil. The result of interest in this point is interpolated using the element shape functions.

For Plate and Anchor, the result represents the maximum quantity in the structural element with a given material set. The user may choose the material set in the Material drop-down menu. Note that if the same material data set (with the same name) has been assigned to multiple plates (or anchors), the output quantity is the maximum in all the plates (or anchors) with that material name. If it is intended to be evaluated for one particular plate or anchor only, then another material data set must be assigned to the other plates or anchors in the reference project.

For Plate, the available options for *Result type* are:

- Displacement
- Force

with options:

- Displacement
  - absolute displacement, $|u|$
  - horizontal displacement, u_x
  - vertical displacement, u_y

- Force
  - Maximum normal force, *N*
  - Maximum shear force, *Q*
  - Maximum bending moment, *M*

The result type available for anchors is the Force, i.e. the maximum anchor force *N*.

In the case of the Global safety factor, only *Phase* and *Threshold* are the required input items. Note that you need to select a safety phase to be able to retrieve the safety factor.

The additional input is different in the case of a FORM or Monte Carlo method.

FORM method

FORM is an approximation method based on an optimization algorithm that enables the search for the design point (Figure 3.16). The implemented algorithm is called COBYLA (Constrained Optimization BY Linear Approximation) (Powell (1994) and Powell (2007)) and it does not require the implementation of a gradient.

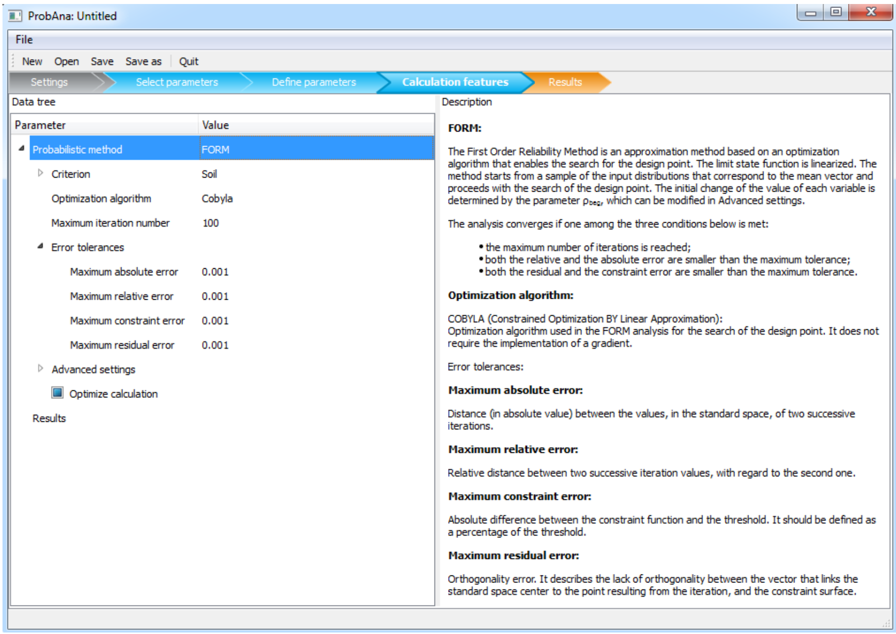

Figure 3.16 Features of the FORM method.

The user should specify the desired *Maximum iteration number* (set to 100 by default) and the *Error tolerances*. By default, the error tolerances (Openturns, 2010) are set to the following values:

- Maximum absolute error = 0.001, i.e. the distance (in absolute value) between the values, in the standard space, of two successive iterations;
- Maximum relative error = 0.001, i.e. the relative distance between two successive iteration values, with regard to the second one;
- Maximum constraint error = 0.001, i.e. the absolute difference between the

constraint function and the threshold;

- Maximum residual error = 0.001, i.e. the orthogonality error, that describes the lack of orthogonality between the vector that links the standard space center to the point resulting from the iteration, and the constraint surface.

It is advised to set the maximum constraint error equal to the criterion threshold divided by 100.

The analysis converges if one among the three conditions below is met:

- the maximum number of iterations is reached;
- both the relative and the absolute error are smaller than the maximum value set in the settings;
- both the residual and the constraint error are smaller than the maximum value set in the settings.

It is suggested to keep maximum absolute and relative error very small compared to 1, since the optimization problems are solved in the standard space. The maximum constraint error depends on the chosen criterion: at the end of the analysis it is equal to the difference between the threshold and the result in the design point.

The user may expand the *Advanced settings* option. For FORM analysis, you may enter a different value for the coefficient ρ_{beg}, that represents the initial change of each variable. By default it is set to 1.

Monte Carlo

In the case of a Monte Carlo analysis, it is required to specify the number of iterations to perform in *Sampling points* (Figure 3.17). By default it is set to 1000. The number of iteration should be chosen, such that stability of the results is reached.

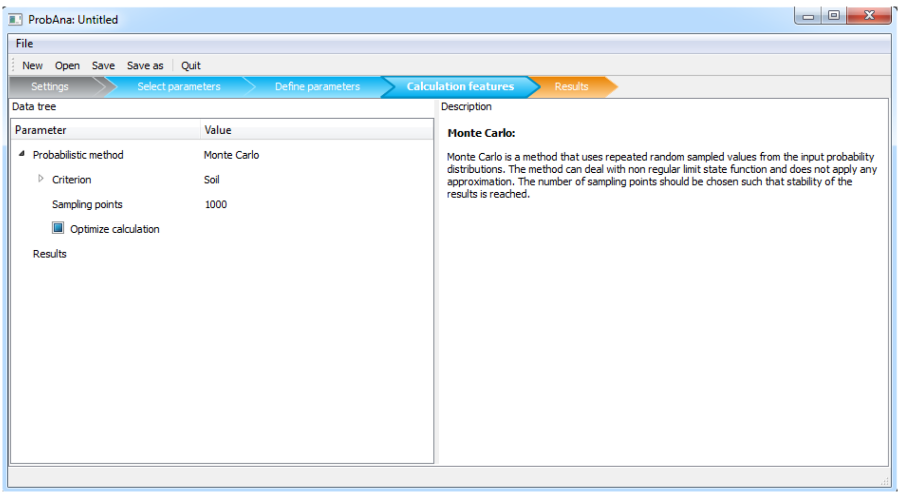

Figure 3.17 Features of the Monte Carlo method.

Optimize calculation

The last item listed under *Probabilistic method* is the *Optimize calculation* option. By default, the box is always checked. When the user launches a probabilistic analysis, the calculation of the reference project is performed many times with a different set of input parameters. The user is interested in the results in the calculation phase selected in the *Criterion* option. If the project consists of multiple phases and the calculation fails in a phase that follows the one of interest, the failure is ignored and the results in the phase of interest can be retrieved.

Several strategies are implemented to make the calculation more robust and to deal with failed calculations. At the beginning, some numerical control parameters are changed by default for all the phases (excluding the initial phase and the safety phases) to improve the calculation robustness:

- the maximum number of unloading steps is set equal to 10;
- the maximum number of steps is changed based on the value in the original project: at first, the maximum between 5 times the initial Max steps value and 1000 is taken and then, the minimum between that value and 10000 is set.

For a detailed explanation of the numerical control parameters in PLAXIS 2D you can refer to the PLAXIS Reference Manual (Brinkgreve, Kumarswamy & Swolfs, 2016).

If the option *Optimize calculation* is unchecked or if the failed phase is the parent phase of the phase selected in *Criterion* several actions are taken. At first, the number of maximum steps is set to 10000 (if the actual value is less than 10000), then the value of the option *Max Load Fraction Per Step* is set to 0.1 (if the actual value is larger than that) and the option Use gradual error is enabled. At last, the maximum number of iterations is set to 90 (if the actual value is less than 90). If the calculation fails even after these modifications, in the case the criterion is the evaluation of the safety factor and the failed phase is the parent phase of the safety phase of interest, the strength parameters of the soil materials (cohesion and friction angle) are increased by 20%. If the calculation succeeds, the resulting safety factor is decreased by 20%. In all the other cases in which the calculation fails, the strategy to deal with that is different based on the chosen method.

In the case of a FORM analysis, it is necessary to define a criterion with a threshold value that represents failure. Based on the output result of one iteration, the set of parameters for the next iteration is defined. The output result can be a safety factor, but also the displacement in a point or in a structural element, the forces in the structures, the stresses and strains in the soil. The value of the output result determines the new set of values given by the optimization algorithm for the next iteration. For this reason, if the calculation of the PLAXIS project fails (after trying to improve the performance by applying the actions described above), the FORM analysis is stopped. Assigning an arbitrary value in the case of failure will influence the next set of values, and the complete analysis. A new FORM analysis needs to be started, improving the input configuration to avoid failing situations. A FORM analysis needs usually less iterations than a Monte Carlo analysis, even though the computational time depends on the reference project and its complexity.

In the case of a Monte Carlo analysis, the output result of the criterion result does not have an influence on the new set of values assigned at each iteration. For this reason, in the case a calculation fails, a None value is returned and the number of failed calculations will be indicated in the *Results* tabsheet.

3.7.2 ADDITIONAL RESULTS

The output result that will be shown at the end of the analysis is the one selected in the Criterion option, in the Probabilistic method section. More results can be displayed by right-clicking the Results feature and selecting *Add new result*. An item with the name *New result* is created under the Results sub-tree. It is advised to change the name of the item to a more meaningful description, to be able to easily recognize it in the *Results* tabsheet (Figure 3.18). The name can be modified by double-clicking *New result*.

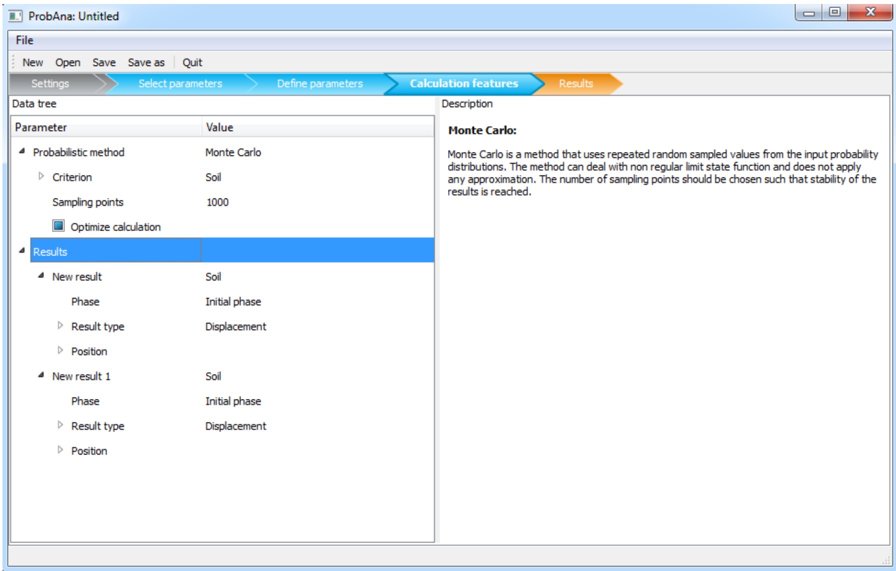

Figure 3.18 Add more results to the analysis.

The additional results may be added for both FORM and Monte Carlo methods. Note that, in the case of FORM, the optimization algorithm determines the new set of input parameters based on the output result of the quantity selected as the method criterion, for each iteration. This means that the distribution of the additional quantities should be considered dependent on the selected criterion. A new FORM analysis with another criterion may lead to a different result distribution.

For each added item, the user may select the type of result, the phase, the threshold and the necessary additional information, if present. The description of the *New result* item is the same as the *Criterion* option. For FORM the threshold of the additional results are not taken into account since the probability of failure is calculated with respect to the selected criterion. For this reason, the definition of the threshold can be ignored in the case of FORM.

Each additional result can be duplicated or removed by simply right-clicking the result and selecting the desired option in the menu.

3.8 RESULTS

You can access the *Results* tabsheet by simply clicking the *Results* mode in the arrow bar (Figure 3.19).

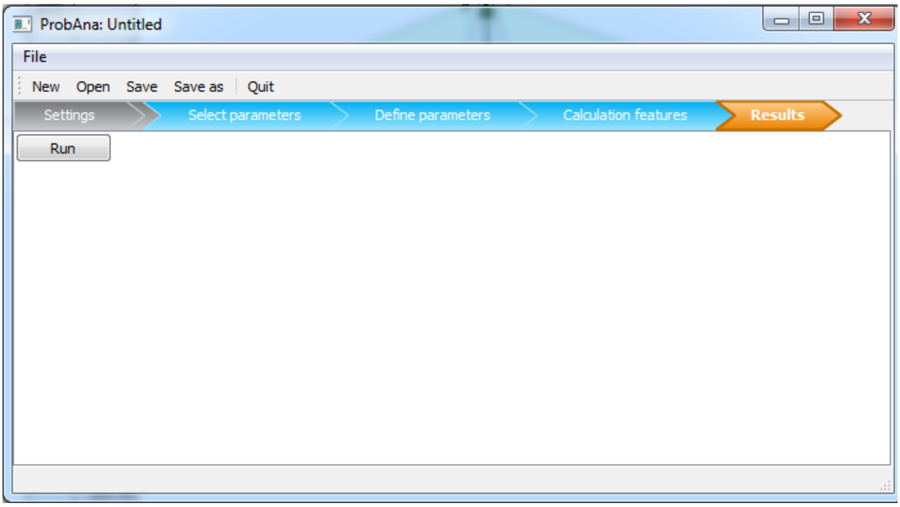

Figure 3.19 View of the *Results* mode.

If no results have been calculated for the current analysis configuration, only the *Run* button is shown in the upper part of the *Results* tabsheet. Click *Run* to start the calculation. Be aware that the analysis may require some time before it is completed, depending on the reference project, the chosen probabilistic method and its characteristics.

During the analysis, a certain amount of calculations are performed: a new value is assigned to the selected stochastic variables in PLAXIS 2D Input, the phases are calculated and the output results of interest are retrieved. At the end of the analysis, the computational results are displayed in the *Results*.

You can save the results by clicking the *Save* button (the existing configuration file of this analysis is updated) or the *Save as* button (you can specify name and location of the new file to save). In both cases the file consists of the complete analysis configuration from the *Settings* to the *Results* tabsheets.

If you want to load an existing completed analysis, click the *Open* button and select the appropriate file. Results can be accessed by simply selecting the *Results* tab in the upper arrow bar.

If any modification is done in one of the previous tabsheets, *Results* shows the results of the original configuration and a warning message at the top of the window to inform the user that there are no results for the modified analysis configuration because they have not yet been calculated.

At the end of the analysis the amount of performed calculations is displayed below the *Run* button. The number of calculations that have been successfully completed in PLAXIS is indicated in green with the label *Succeeded*, while the number of calculations that have failed is indicated in red with the label *Failed*. The sum of them corresponds to the *Total number of calculations*. Note that a failed calculation is any PLAXIS analysis that could not be performed due to invalid parameter input value/s or that could not be completed successfully in *Staged Construction*.

In the case of FORM, if the calculation fails, the analysis stops since it is not possible to retrieve the results that are used to determine the new set of input values for the stochastic variables. It is advised to check the PLAXIS command line to identify the possible error due to an invalid input parameter (as for example a negative material stiffness) and the status of the calculation phases in *Staged Construction*. In the first case, the user may consider to use a different distribution that does not allow for negative values, such as a Log-normal distribution or a Truncated normal distribution with a positive lower bound. In the other case, the user may check the *Log info* box of the failed phase and refer to Appendix G of the PLAXIS 2D Reference Manual (Brinkgreve, Kumarswamy & Swolfs, 2016) where a table with the error codes and a corresponding hint has been created.

In the case of Monte Carlo, if the calculation fails, the next iteration can be performed with a new set of parameters since it does not depend on the output results. A *None* value is returned and it is not taken into consideration in the computation of the probability of failure (as explained in Section 3.8.1)

For a completed analysis, the results are organized into three main groups: *Output results*, *Parameters* and *Convergence*.

3.8.1 OUTPUT RESULTS

Output results shows the probability of failure, the reliability index and a histogram of the defined criterion.

The probability of failure is defined as $P\{g(\mathbf{X}) < 0\}$, i.e. the probability that the random variables $\mathbf{X} = (X_1, X_2, ..., X_n)$ are in the failure region ($g(\mathbf{X}) < 0$):

$$P_f = P\{g(\mathbf{X}) < 0\} = \int_{g(\mathbf{x})<0} f_x(\mathbf{x})d\mathbf{x} \tag{3.3}$$

where $g(\mathbf{X})$ is known as limit state function and $f_x(\mathbf{x})$ is the joint density probability function (Nadim, 2007). The probability of failure can assume all the values between 0 and 1 (for a situation from completely safe to completely unsafe).

Reliability can be defined as the probability that the random variables described by the vector $\mathbf{X}$ are in a safe region defined by $g(\mathbf{X}) > 0$. The reliability index β can be negative or positive, and should be interpreted considering that the probability of failure decreases as the reliability increases.

In the case of a FORM or a Monte Carlo analysis, the reliability index is calculated in a different way.

In a FORM analysis, the first step consists of transforming the random variables from the original space (X) to the standard normal space (U). The transformed variables are described by the vector $\mathbf{U} = (U_1, U_2, ..., U_n)$ of standard normal distributions. This is automatically done in the probabilistic tool by means of an isoprobabilistic transformation (Openturns, 2010). The design point (or Most Probable Point) is represented by the point on the g($\mathbf{U}$) function closest to the origin of the standard space. The measure of this distance describes the FORM reliability index β (Hasofer & Lind, 1974). The reliability index can be negative or positive, based on the fact that the standard space center lies in the failure space or not.

In the standard space, the limit state surface is approximated by a linear surface at the design point. The probability of failure is obtained as a function of the marginal cumulative density function E of the spherical distributions in the standard space and the reliability index β as defined above (Openturns, 2010).

$$P_{f,FORM} = E(\beta) \tag{3.4}$$

In the case of a Monte Carlo analysis, the probability of failure is calculated as:

$$P_f = \frac{n_f}{N} \tag{3.5}$$

where n_f is the number of occurrences of results in the failure domain and N is the total amount of trials (i.e. the number of succeeded calculations). This value should be considered as an estimation of the probability of failure that converges to the exact P_f as the sample size N tends to infinity.

Supposing that the result can be described by a normal distribution with mean value μ and standard deviation σ, the reliability index β is calculated as:

$$\beta = \frac{\mu - threshold}{\sigma} \tag{3.6}$$

In the case the standard deviation is null, the reliability index cannot be calculated and the message "zero standard deviation!" is shown.

Below the values that indicate the probability of failure and the reliability index, an histogram is plotted. The graph can be visualized for the defined criterion and for the additional results, if present. The user may select the output quantity of interest in the corresponding drop-down menu.

The histogram shows the distribution of the result (for the selected output quantity): the values reached by the output quantity of interest are plotted on the x-axis, while the number of occurrences are shown on the y-axis (Figure 3.20). For the criterion and, in the case of Monte Carlo only, for the additional results, the threshold value is indicated by an orange vertical line.

3.8.2 PARAMETERS

In *Parameters* the user may find useful information regarding the input variables. An example of the results in this tabsheet is shown in Figure 3.21.

On the left side, a table consisting of 3 columns is shown. Each row of the table corresponds to one input parameter, identified by its name (as shown in the *Define parameters* tabsheet), the value of the parameter in the design point and the associated importance factor. The *Design point* and *Importance factors* column of the table can be sorted, showing the numbers in ascending or descending order. The *Importance factors* column can be sorted for a quick visualization of the most relevant variables with respect to the realization of g(**x**)<0. The importance factors are represented in percentage values. The sum of all the importance factors is 100%.

The importance factors are calculated based on the value of the design point in the

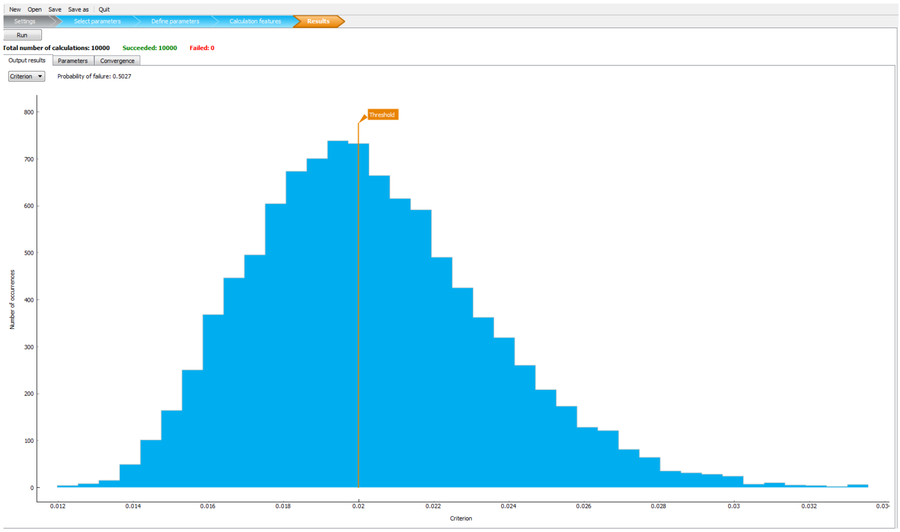

Figure 3.20 Representation of the output result.

Figure 3.21 Parameters result.

standard space (Openturns, 2010). Each factor is calculated as:

$$\alpha_i^2 = \frac{(u_i^*)^2}{\beta^2} \tag{3.7}$$

where u_i^* is the value of the i-parameter in the design point and β is the reliability index.

On the right side, a scatter diagram is shown. The user can select two parameters of interest among the ones defined for the analysis, and the criterion or one of the additional results, if present. The chart gives an indication of the correlation between the two parameters and on how they are spread within the computational domain. The color of the points varies from dark blue to light blue, from the lowest to the highest result values; the white color indicates the points where failure has occurred during the analysis.

3.8.3 CONVERGENCE

In the *Convergence* tabsheet, a chart enables to visualize the convergence quality of the algorithm. The chart is different for FORM and Monte Carlo analysis, but in both cases it gives an indication of the amount of trials necessary to reach a stable value.

In the case of FORM, the four types of error (absolute error, relative error, residual error and constraint error) are visualized with different colors, as indicated in the legend (Figure 3.22).

Figure 3.22 Convergence graph of a FORM analysis.

On the x-axis, the iteration number is shown, while the corresponding error value is plotted on the y-axis. The analysis converges either if the maximum number of iteration has been reached or if one between the two couples of errors (absolute and relative error, or residual and constraint error) are below the maximum tolerance defined in *Calculation features*.

In the case of Monte Carlo, the history of the probability of failure during the sampling process is reported (Figure 3.23). Each curve corresponds to the single selected criterion and additional result. The number of trials is shown on the x-axis while the probability of failure is shown on the y-axis. The Monte Carlo analysis stops when the defined number of sampling points is reached.

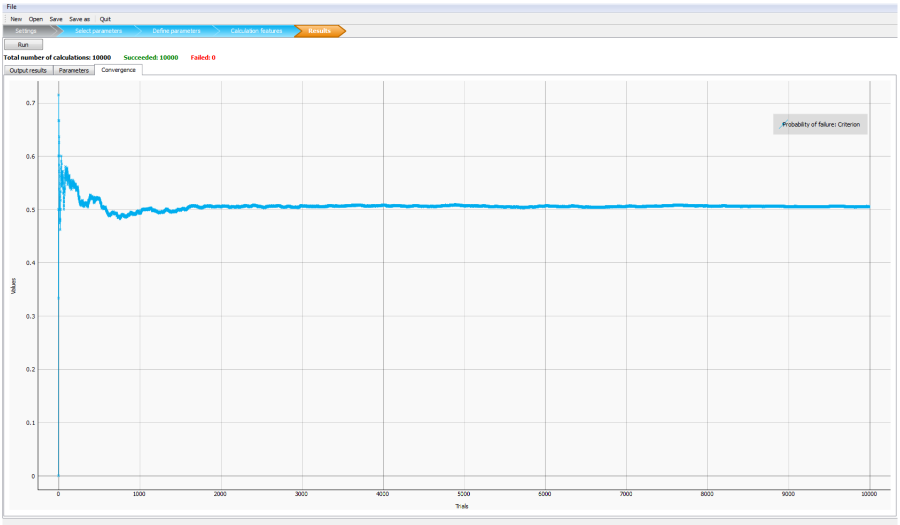

Figure 3.23 Convergence graph of a Monte Carlo analysis.

4 VERIFICATION

4.1 VERIFICATION CASE

In this verification example, a probabilistic analysis is performed on a very simple project. The mean value of each variable is equal to the reference value (i.e. the deterministic value used in the reference project) and the threshold is equal to the result obtained from the PLAXIS calculation. A probability of failure of 50% is expected.

The PLAXIS reference project consists of a cluster 1 meter wide and 1 meter high (Figure 4.1) with a linear elastic material with the following properties (Table 4.1):

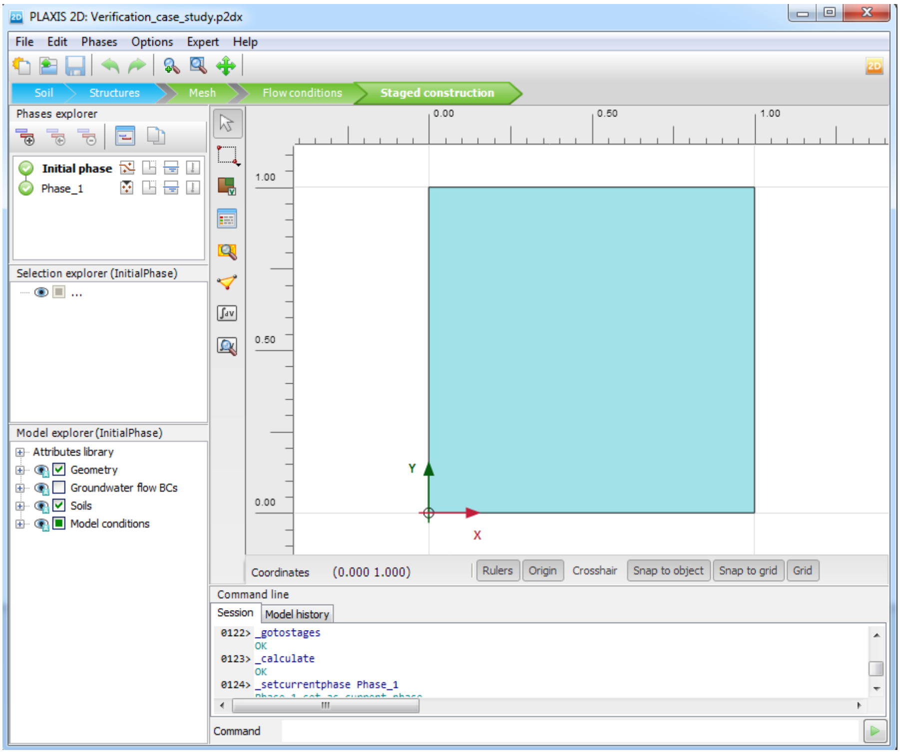

Figure 4.1 PLAXIS model for the verification case study.

Table 4.1 Material properties.

Parameter	Name	Value	Unit
General			
Name identification	Identification	LE	-
Material model	*Model*	Linear Elastic	-
Type of material behaviour	*Type*	Drained	-
Soil unit weight above phreatic level	γ_{unsat}	20.0	kN/m^3
Soil unit weight below phreatic level	γ_{sat}	20.0	kN/m^3
Parameters			
Young's modulus (constant)	*E*'	333.0	kN/m^2
Poisson's ratio	ν'	0.3333	-

The coarseness factor is very high (5.0) and the calculation consists of two phases[†]: an Initial Phase and a safety phase.

In ProbAnalysis, the unsaturated unit weight of soil and the Young's modulus are selected and defined according to the properties described in Table 4.2. The mean values are equal to the reference value in the PLAXIS project.

Table 4.2 Distribution properties of the selected variables

Parameter	Distribution	Lower bound	Upper bound	Mean value	Standard deviation
General					
LE.γ_{unsat}	Truncated Normal	15.0	25.0	20.0	2.0
LE.E'	Truncated Normal	233.0	433.0	333.0	40.0

The criterion is the soil vertical displacement u_y in the top middle point of the model at the end of the Initial Phase. The threshold is set to 0.02, as it is the resulting value of the deterministic calculation.

A FORM and a Monte Carlo analysis are performed. After 24 runs, the FORM analysis converges with a probability of failure of 0.500399, as expected (Figure 4.2). In the case of Monte Carlo, the number of *Sampling points* is set equal to 10000. The probability of failure is 0.5051 (Figure 4.3).

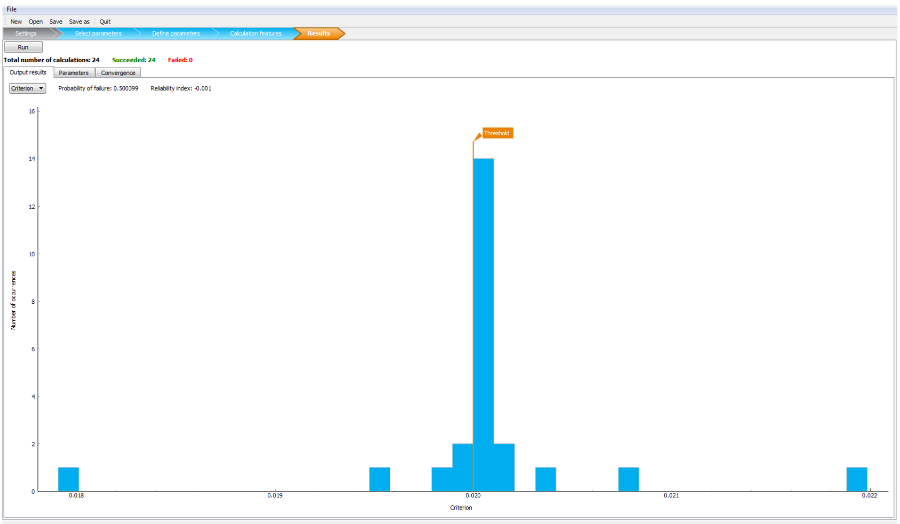

Figure 4.2 Results of the FORM analysis for the verification case study.

[†] To run a probabilistic analysis it is required to specify at least two phases

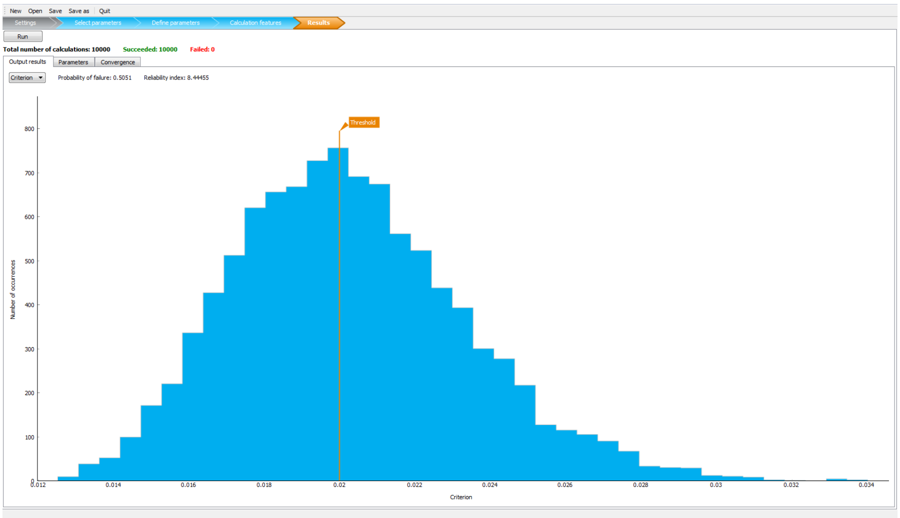

Figure 4.3 Results of the Monte Carlo analysis for the verification case study.

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