BAYESIAN ESTIMATION OF EXTREME RIVER DISCHARGES

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SUMMARY

Understanding and quantifying the behaviour of river floods at extreme discharges has important applications in design of civil structures such as river dikes. For design purposes, one is often interested in return periods that are substantially larger than the observation period. These estimates are often obtained using classical statistical methods. In this paper, a method based on Bayesian statistics is presented. This approach enables us to use all available sources of information, and to take statistical uncertainties into account as well.

Seven predictive probability distributions are considered for determining extreme quantiles of loads: the exponential, Rayleigh, normal, lognormal, gamma, Weibull and Gumbel. The presented method has been successfully applied to estimate extreme quantiles of discharges and their return periods. Prior information based on historical floods is represented in terms of censored data and is then used to determine informative prior distributions of the statistical parameters. This prior information can be updated with actual data to determine the posterior information, and provides a rational basis for extrapolation. As an example, a Bayesian analysis of annual maximum discharges of the river Rhine at Lobith is performed to assess extreme quantiles such as the design discharge.

Keywords: Bayesian analysis, informative prior distribution, non-informative prior distribution, posterior distribution, flood data.

1. INTRODUCTION

Understanding and quantifying the behaviour of river floods at extreme discharges has important applications in design of civil structures such as river dikes. For design purposes, one is often interested in extreme events with larger return periods than the observation period. Extreme discharges with very large return periods can be estimated by fitting various probability distributions to the available observations. See for example DH and EAC-RAND (1993) and Castillo (1988). Probability plots and goodness-of-fit tests, such as chi-square and Kolmogorov-Smirnov are commonly used to select an appropriate distribution.

A major practical difficulty in fitting probability distributions is that there is often a limited amount of observations for determining extreme quantiles and particularly extreme discharges. The associated return period is large compared with the length of the observation period. In the Netherlands, observed flood discharges are available for a period of 98 years only. Consequently, there is a large statistical uncertainty involved in estimating discharges with large return periods when using these observations. The maximum likelihood method has been recognized as one of the best parameter estimation methods (Castillo, 1988; Galambos et al., 1994); but it is especially suitable when there is a large number of observations. Furthermore, the method has the disadvantage that statistical uncertainties cannot be taken into consideration.

One consequence of sparse data is that different probability distributions seem to fit the observations and therefore only a few can be rejected. The different distributions involved usually lead to different extrapolated values and the goodness-of-fit tests for selecting an appropriate distribution are often inconclusive. The tests are more appropriate for the central part of the distribution than for the tail.

Recently, van Gelder (1999) presented an alternative based on a Bayesian approach for estimating extreme quantiles while statistical uncertainties are taken into account. Statistical uncertainty mainly occurs due to a lack of observations. This uncertainty can be subdivided into parameter uncertainty and distribution-type uncertainty. Bayesian estimates and so-called Bayes weights can then be used

to account for parameter uncertainty and distribution-type uncertainty, respectively. Using Bayes weights, it is possible to discriminate between different probability distributions and to quantify how well a distribution fits the observations. The Bayesian approach was successfully applied by van Noortwijk et al. (2001), Chbab et al. (2000) and van Gelder et al. (1999) for estimating extreme river discharges. Different distributions were investigated and therefore weights were determined corresponding to how well they fitted the observed data. In determining Bayes weights, so-called non-informative Jeffreys priors were used. The main disadvantage of non-informative priors is that they are often improper. Although this disadvantage can be overcome, the Bayesian approach is especially useful for combining different sources of information and for using informative priors.

This paper again addresses the Bayesian approach for estimating extreme river discharges. It differs from the results in van Noortwijk et al. (2001) and Chbab et al. (2000) in the sense that informative priors based on historical censored observations are used instead of non-informative priors. In Section 2 we briefly define statistical uncertainties. Bayes estimates of parameters and quantiles associated with large return periods are examined in Section 3. Non-informative Jeffreys priors, as well as informative priors based on historical censored observations are presented in Section 4. Bayes factors and Bayes weights will be treated in Section 5. In Section 6 we examine the river Rhine as a case study, and we end with conclusions in Section 7.

2. STATISTICAL UNCERTAINTIES

According to (amongst others) Slijkhuis et al. (1999) and Siu and Kelly (1998), uncertainties in risk analysis can primarily divided into two categories: inherent uncertainty and epistemic uncertainties; see Figure 2-1. Inherent uncertainties represent randomness or variability in nature. For example, even in the event of sufficient data, one cannot predict the maximum discharge that will occur next year. The two main types of inherent uncertainty are inherent uncertainty in time and inherent uncertainty in space. It is not possible to eliminate inherent uncertainty completely. Epistemic uncertainties represent the lack of knowledge about a (physical) system. The two main types of epistemic uncertainty are statistical uncertainty (due to lack of sufficient data) and model uncertainty (due to lack of understanding the physics). Statistical uncertainty can be parameter uncertainty (when the parameters of the distribution are unknown) and distribution-type uncertainty (when the type of distribution is unknown). In principle, epistemic uncertainties can be reduced as knowledge increases and more data becomes available.
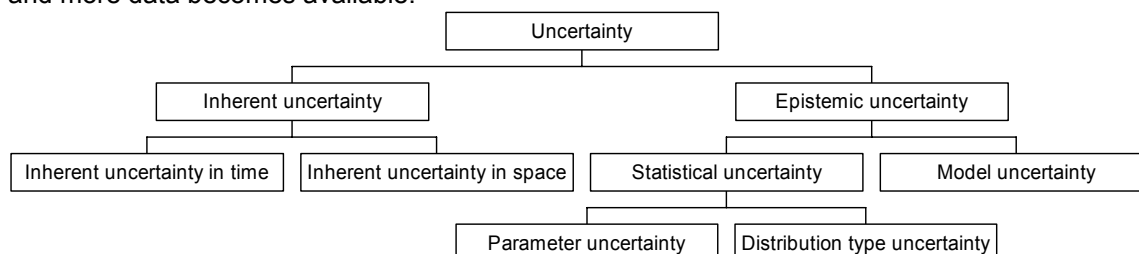
Figure 2-1: Types of uncertainty

3. BAYESIAN ESTIMATION

The only statistical theory that combines modelling inherent uncertainty and statistical uncertainty is Bayesian statistics. The theorem of Bayes (1763) provides a solution to how to learn from data. In the framework of estimating the parameters $\theta = (\theta_1,...,\theta_d)$ of a probability distribution $l(x|\theta)$, Bayes' theorem can be written as:

(1)
$$\pi(\theta \mid x) = \frac{l(x|\theta)\pi(\theta)}{\int_\theta l(x|\theta)\pi(\theta)d\theta} = \frac{l(x|\theta)\pi(\theta)}{\pi(x)}$$

with

$l(x|\theta)$ = the likelihood function of the observations $x = (x_1,....,x_n)$ when the parametric vector $\theta = (\theta_1,...,\theta_d)$ is given,

$\pi(\theta)$ =   the prior density of $\theta = (\theta_1,...,\theta_d)$ before observing data $\boldsymbol{x} = (x_1,....,x_n)$,

$\pi(\theta \mid \boldsymbol{x})$ =   the posterior density of $\theta = (\theta_1,...,\theta_d)$ after observing data $\boldsymbol{x} = (x_1,....,x_n)$,

$\pi(\boldsymbol{x})$ =   the marginal density of the observations $\boldsymbol{x} = (x_1,....,x_n)$.

The likelihood function $l(x \mid \theta)$ represents the inherent uncertainty of a random variable X when θ is given, whereas the prior density $\pi(\theta)$ and the posterior density $\pi(\theta \mid \boldsymbol{x})$ represent the statistical uncertainty in θ. This statistical uncertainty in θ is parameter uncertainty. Using Bayes' theorem, we can update the prior distribution to the posterior distribution as soon as new observations become available. When more observations become available, the parameter uncertainty gets smaller. If a random variable X has a probability density function $l(x \mid \theta)$ dependent on the parametric vector θ, then the likelihood function $l(x_1,...,x_n \mid \theta)$ of the independent observations $\boldsymbol{x} = (x_1,....,x_n)$ is given by:

$$(2) \qquad l(\boldsymbol{x} \mid \theta) = l(x_1,...,x_n \mid \theta) = \prod_{i=1}^{n} l(x_i \mid \theta)$$

The marginal density $\pi(\boldsymbol{x})$ is obtained by integrating the likelihood $l(x \mid \theta)$ over θ. Note that the maximum-likelihood estimate of the parameter vector θ is defined as the estimate $\hat{\theta}$, which maximizes the likelihood function $l(\boldsymbol{x} \mid \theta)$ as a function of θ.

The cumulative distribution function and the survival function of the random variable X are denoted by $F(x \mid \theta)$ and $\bar{F}(x \mid \theta) = 1 - F(x \mid \theta)$, respectively. The posterior predictive probability of exceeding a certain x_0 is given by:

$$(3) \qquad Pr\{X > x_0 \mid \boldsymbol{x}\} = \int_{\theta} Pr\{X > x_0 \mid \theta\} \pi(\theta \mid \boldsymbol{x})\, d\theta = \int_{\theta} \bar{F}(x_0 \mid \theta)\pi(\theta \mid \boldsymbol{x})\, d\theta$$

Besides representing parameter uncertainty on the basis of Bayesian statistics, distribution-type uncertainty can also be taken into account using so-called Bayes factors or Bayes weights. This will be treated in Section 5.

4. PRIOR CHOICE

To obtain Bayes estimates, a prior distribution of the parameters of the assumed likelihood function and some observations (sample information) are required. The prior distribution and the sample information contained in the likelihood function are then combined using Bayes' theorem to determine the posterior distribution of the parameters. The key question is how to choose the prior distribution.

4.1 Non-informative priors

The Bayesian approach is especially useful for combining prior subjective information with actual observations. Prior information can be represented in terms of a prior distribution, which can be either non-informative or informative. There might be situations in which we would like the observations to 'speak for themselves', especially in comparison to the prior information. This means that the prior distribution should describe a certain 'lack of knowledge' or, in other words, should be as 'vague' as possible. For this purpose, so-called non-informative priors have been developed. An advantage of using non-informative priors is that it provides us with a methodology to perform statistical inference in situations where 'little is known a priori'.

According to (amongst others) Box and Tiao (1973), Kass and Wasserman (1996) and van Noortwijk et al., (2002), the best and most widely used method for determining non-informative priors is that of Jeffreys (1961). This method chooses the non-informative prior to be proportional to the square root of the expected Fisher information measure. In mathematical terms, the non-informative Jeffreys prior is given by:

$$(4) \qquad J(\theta) \propto \left[\mathrm{I}(\theta) \right]^{1/2}$$

where $I(\theta)$ is the expected Fisher information for a single observation of the probability distribution involved; that is:

$$(5) \qquad I(\theta) = -E_{X|\theta}\left(\frac{\partial^2}{\partial\theta^2} log \ l(x|\theta)\right)$$

The non-informative Jeffreys priors can be easily extended to the multivariate case. The Jeffreys prior is then taken proportional to the square root of the determinant of the expected Fisher information matrix for a single observation (Box and Tiao, 1973; van Noortwijk et al., 2002). An excellent overview on the selection of non-informative priors is presented by Kass et al. (1996). The main advantage of the Jeffreys prior is that it is always both invariant under transformations and dimensionless.

A disadvantage of most non-informative priors is that they can be improper; that is, they often do not integrate to one. This disadvantage can be resolved by focussing on the posterior distributions rather than on the prior distributions. As a matter of fact, formally carrying out the calculations of Bayes' theorem by combining an improper prior with observations often results in a proper posterior.

4.2 Informative priors

The Jeffreys priors considered in the previous section are useful in situations in which we would like the observations to 'speak for themselves'. This means that a non-informative prior distribution should describe a certain 'lack of knowledge'. On the other hand, an informative prior distribution is one that reflects 'subjective' knowledge concerning the unknown statistical parameters.

For estimating extreme flood quantiles, a useful informative prior is a distribution based on historical floods, which occurred before the period of systematic gauging. For the river Rhine this could be the period 1800-1900. Although the real historical discharges are generally unknown and the river geometry has changed, reference books on storm surges and river floods form a valuable source of additional information. These books mention, for example, whether there was flood damage or even a catastrophe occurred involving drowned people, dike bursts, flooded polders, dislodged houses and collapsed bridges. Their main sources of information are old newspapers, chronicles, letters, diaries, memories, legends, government or business records, and even folk songs (Chen et al., 1975). In addition to these written records, man made high water marks and memorials, as well as geomorphologic and botanical evidence of large and catastrophic floods can be used. As opposed to the systematic gauging records, these floods are generally referred in the hydrological literature as 'historical floods' (see, e.g., Stedinger and Cohn (1986) and Hirsch and Stedinger (1987)). The main aim is to approximately assess the number and magnitudes of the largest pregauge historical floods, which occurred within a period, which is generally larger than the systematic record.

Let the random quantity X be defined as the annual maximum river discharge and let us consider a time period of r years. We propose to quantify historical information as follows: if historical references do not mention any flood damage then $X \leq y_1$. If historical references mention flood damage but no catastrophe, then $y_1 \leq X \leq y_2$. If historical references mention a catastrophe then $y_2 < X$.

On the basis of expert judgement, the number of years falling in each of the above three categories denoted by r_i, $i = 1,2,3$, must be assessed.

The subjective estimates of the proportions of historical flood per discharge category can be regarded as censored observations. An observation x can be censored on the left, doubly censored and censored on the right, when $x \leq y_1$, $y_1 < x \leq y_2$ and $y_2 < x$ respectively. For more details and examples of data censoring, see Kaczmarek (1977).

When considering historical observations, the censoring is due to the absence of systematic measurements, changes in river and meteorology, as well as unreliable reports of historical floods. Under the assumption of the historical censored observations being independent, the likelihood function can be formulated as:

$$(6) \qquad \begin{aligned} l(r_1,r_2,r_3 \mid \theta) &= [Pr\{X \leq y_1 \mid \theta\}]^{r_1} [Pr\{y_1 < X \leq y_2 \mid \theta\}]^{r_2} [Pr\{y_2 < X \mid \theta\}]^{r_3} = \\ &= [F(y_1 \mid \theta)]^{r_1} [F(y_2 \mid \theta) - F(y_1 \mid \theta)]^{r_2} [1 - F(y_2 \mid \theta)]^{r_3} \end{aligned}$$

Given the likelihood function of the historical censored observations $l(r_1,r_2,r_3 \mid \theta)$ and a non-informative Jeffreys prior $\pi(\theta)$, Bayes' theorem (1) can be used to obtain the posterior density $\pi(\theta \mid r_1,r_2,r_3)$. This posterior can then be considered as the informative prior in a subsequent Bayesian analysis for combining historical information with recent systematic uncensored and reliable observations $\boldsymbol{x} = (x_1,...,x_n)$. The final result is the posterior density of θ when both historical censored and systematic uncensored and reliable observations are given: that is, $\pi(\theta \mid r_1,r_2,r_3,\boldsymbol{x})$.

In the absence of doubly censored historical observations, likelihood function (6) reduces to:

$$(7) \qquad\qquad l(r_1,r_2 \mid \theta) = [F(y_1 \mid \theta)]^{r_1} [1 - F(y_1 \mid \theta)]^{r_2}$$

According to Stedinger and Cohn (1986), Cohn and Stedinger (1987) and Hirsch and Stedinger (1987), a record of historical flood peaks is generally of this form. To apply Eq. (7), we should be able to define a time period of r years, where $r = r_1 + r_2$, and a threshold level y_1 such that over that period floods greater than y_1 left a record which is still available today. Kaczmarek (1998) used Eq. (7) in a maximum-likelihood analysis to fit a gamma distribution to historical and systematic annual maximum discharges of the Polish river Warta.

When censored observations are used as an informative prior distribution, an important point is how much information is contained in the sample of censored observations. In a Bayesian analysis, the posterior density is formed as a combination of prior information on the one hand and actual observations on the other hand. The more information is contained in the historical censored observations, the more weight they get in comparison with the systematic uncensored observations.

5. BAYES FACTORS AND BAYES WEIGHTS

The Bayesian approach to hypothesis testing originates from the work of Jeffreys (1961). He developed a methodology for quantifying the evidence in favour of a scientific theory using the so-called Bayes factors. This factor is the posterior odds of the null hypothesis when the prior probability on the null is one-half. A recent overview on Bayes factors can be found in Kass and Raftery (1995).

Assume the data $\boldsymbol{x} = (x_1,....,x_n)$ to have arisen under one of the m models H_k, $k = 1,...,m$. These hypotheses represent m marginal probability densities $\pi(\boldsymbol{x} \mid H_k)$, with prior probabilities $p(H_k)$, $k = 1,...,m$, where $\sum_{j=1}^{m} p(H_j) = 1$ and $\sum_{j=1}^{m} p(H_j \mid x) = 1$. These posterior probabilities can be obtained using Bayes' theorem as follows:

$$(8) \qquad\qquad p(H_k \mid \boldsymbol{x}) = \frac{\pi(\boldsymbol{x} \mid H_k)p(H_k)}{\sum_{j=1}^{m} \pi(\boldsymbol{x} \mid H_j)p(H_j)}, \quad k = 1,...,m$$

where:

$$(9) \qquad\qquad B_{jk} = \frac{\pi(\boldsymbol{x} \mid H_j)}{\pi(\boldsymbol{x} \mid H_k)}, \quad j,k = 1,...,m$$

is denoted by the Bayes factor. The marginal densities of the data under H_k, $\pi(\boldsymbol{x} \mid H_k)$, can be obtained by integrating with respect to the probability distribution of the uncertain parametric vector $\theta_k = (\theta_{1k},....,\theta_{dk})$ with number of parameters d:

$$(10) \qquad\qquad \pi(\boldsymbol{x} \mid H_k) = \int l(\boldsymbol{x} \mid \theta_k,H_k) \, \pi(\theta_k \mid H_k) \, d\theta_k$$

where $\pi(\theta_k \mid H_k)$ is the prior density of θ_k under H_k and $l(\boldsymbol{x} \mid \theta_k,H_k)$ is the likelihood function of the data $\boldsymbol{x}$ given θ_k and H_k.

A difficulty in using non-informative improper priors for calculating Bayes factors is that the prior odds, and thus the Bayes factors, may be undefined. The reason for this is that strictly speaking, the prior probability $p(H_k)$ is defined as:

(11)
$$p(H_k) = w(H_k) \int J(\theta_k \mid H_k) \, d\theta_k$$

where the integral over the non-informative Jeffreys prior $J(\theta_k \mid H_k)$ is often infinite and $w(H_k)$ is the prior weight. However, according to Dawid (1999), this problem can be resolved by redefining the posterior odds as:

(12)
$$\frac{p(H_j \mid \boldsymbol{x})}{p(H_k \mid \boldsymbol{x})} = \frac{\pi(\boldsymbol{x} \mid H_j)}{\pi(\boldsymbol{x} \mid H_k)} \times \frac{w(H_j)}{w(H_k)}, \quad j,k = 1,....,m$$

These posterior odds are well defined so long as both integrals in it converge, which will typically be the case so long as the sample size n is large enough. For the seven probability distributions considered in this paper, the marginal densities of the data do indeed converge (Chbab et al., 2000). Using Eqs. (8) and (12), the posterior probability of model H_k being correct can now be rewritten as:

(13)
$$p(H_k \mid \boldsymbol{x}) = \frac{\pi(\boldsymbol{x} \mid H_k) w(H_k)}{\sum_{j=1}^{m} \pi(\boldsymbol{x} \mid H_j) w(H_j)}, \quad k = 1,...,m$$

It remains to choose the prior weights $w(H_k)$. For formal model comparison, we propose to use equal prior weights. Using the Bayes weights $p(H_k \mid \boldsymbol{x})$, $k = 1,...,m$, the weighted predictive probability of exceeding x_0 is defined by:

(14)
$$Pr\{X > x_0 \mid H_k, \boldsymbol{x}\} = \sum_{k=1}^{m} p(H_k \mid \boldsymbol{x}) Pr\{X > x_0 \mid H_k, \boldsymbol{x}\}$$

where $Pr\{X > x_0 \mid H_k, \boldsymbol{x}\}$ is the predictive probability of exceeding x_0 under likelihood model H_k, $k = 1,...m$.

6. CASE STUDY: THE RIVER RHINE AT LOBITH

Bayesian analysis has been applied to the annual maximum discharges of the river Rhine at Lobith during the period 1901-1998. These observations have been corrected for non-homogeneities as changes in river geometry and suchlike. A non-informative Jeffreys prior as well as an informative prior based on historical censored observations in the period 1800-1900 was used. Statistical analysis of the seven distributions based on the non-informative Jeffreys priors can be found in Chbab et al. (2000) and van Noortwijk et al. (2001). In the former paper an approximate Jeffreys prior for the gamma distribution was used, whereas in the latter paper the exact Jeffreys prior was used. The Bayes weights corresponding to these seven distributions were determined. It was found that the Bayes weights depend largely on the location parameter. For proper model selection we then proposed to use the same location parameter for all seven distributions. On the basis of a statistical analysis the location parameter was chosen to be equal to 2,125 m^3/s for all the seven probability distributions. This location parameter was derived by maximising the weighted marginal density of the observations, where Bayes weights have been attached to the seven individual probability distributions, see Table 6-1. The Rayleigh and Weibull distribution appeared to fit best with Bayes weights of 57% and 32%, respectively. The Bayes estimate of the discharge with an average return period of 1,250 years (the standard in the Netherlands) was 15,860 m^3/s.

Table 6-1: Prior and posterior Bayes weights (van Noortwijk et al., 2001)

Bayes Weights	Exponential	Rayleigh	Normal	lognormal	Gamma	Weibull	Gumbel
Prior	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
Posterior	0.0000	0.5718	0.0481	0.0000	0.0072	0.3173	0.0555

New Bayesian calculations have been made using an informative prior for the parameters of the seven distributions stated above, whereas the location parameter was still chosen to be equal to 2,125 m^3/s. The informative prior is based on historical censored observations in the period 1800-1900. Two

methods concerning data censoring, presented in subsection 4.2, have been used. They are indicated as Method 1 and Method 2, respectively. More information about censored observations that was used to define an informative prior is summarised in Table 6-2. The obtained Bayes weights of the seven probability distributions can be found in Table 6-3. Again the Rayleigh and Weibull distributions appear to fit best when an informative prior has been used. The corresponding Bayes weights are 69% and 19% by Method 1 and 67% and 21% by Method 2, respectively. The Gumbel distribution makes a score of circa 10% by both methods. The Bayes estimates of the discharge with an average return period of 1,250 years are 15,290 m^3/s by method 1 and 15,224 m^3/s by method 2, respectively. The difference is comparatively speaking, small.

Table 6-2: Subjective estimates of the number of historical floods of the Rhine River spread over two or three discharge categories

	Period	r	r_1	r_2	r_3	y_1 [m^3/s]	y_2 [m^3/s]
Method 1	1800-1900	100	75	20	5	7,000	10,000
Method 2	1800-1900	100	75	25	-	7,000	-

Table 6-3: Prior and posterior Bayes weights with informative prior based on censored observations. (Posterior 1 and Posterior 2 correspond with Method 1 and Method 2, respectively).

Bayes Weights	Exponential	Rayleigh	Normal	lognormal	Gamma	Weibull	Gumbel
Prior	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429	0.1429
Posterior 1	0.0000	0.6929	0.0040	0.0000	0.0148	0.1870	0.1012
Posterior 2	0.0000	0.6688	0.0089	0.0000	0.0144	0.2143	0.0937

Figures 6-1 and 6-2 show both the empirical exceedance probabilities, based on complete and censored observations, and the predictive exceedance probabilities. In these figures asterisks indicate the censored observations.
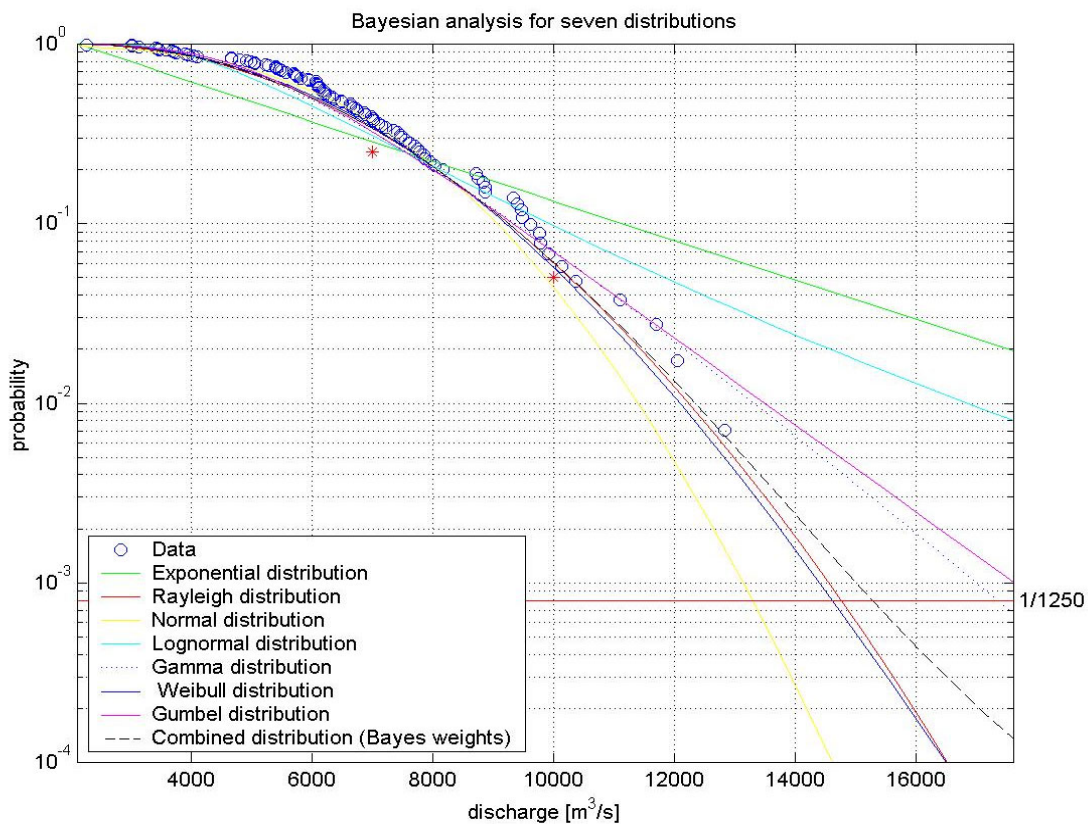

Figure 6-1: Predictive exceedance probability of annual maximum river Rhine discharge using censored observations by Method 1.
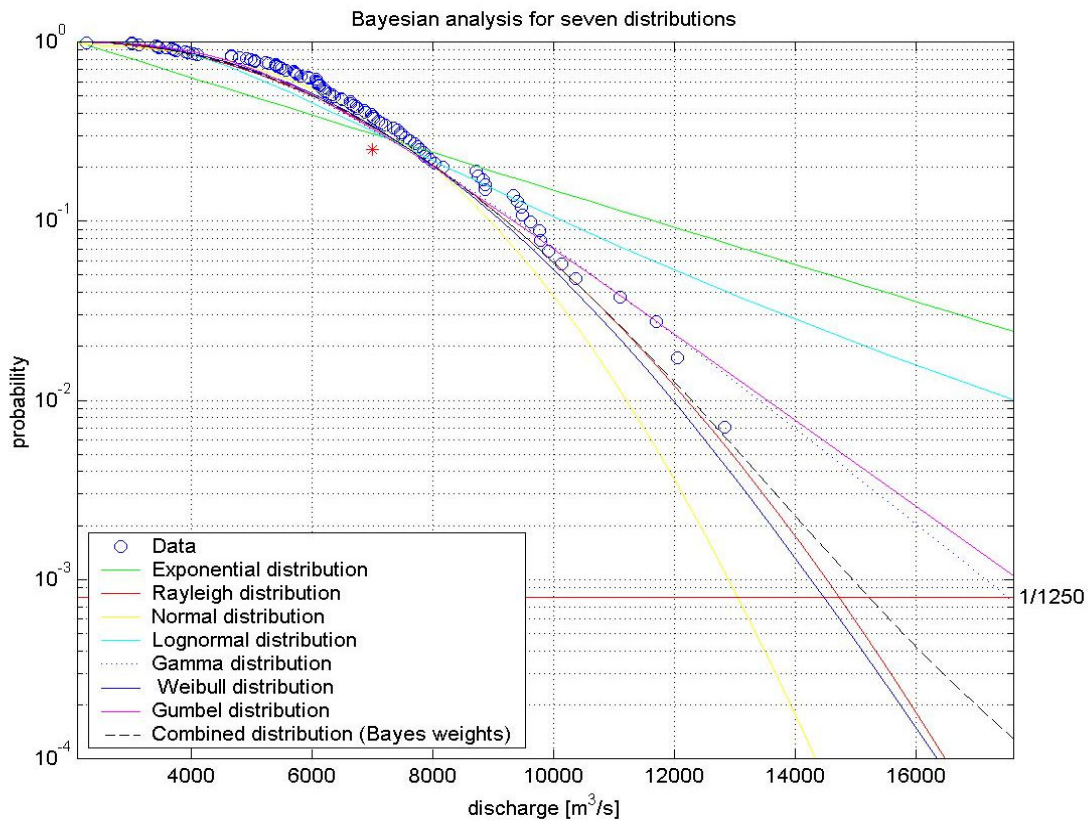
Figure 6-2: Predictive exceedance probability of annual maximum river Rhine discharge using censored observations by Method 2.

7. CONCLUSIONS

A method, based on Bayesian approach, for estimating extreme values of river discharges is presented in this paper. The method deals with inherent uncertainties as well as statistical uncertainties. Bayesian parameter estimates and Bayes weights can be used to account for parameter uncertainty and distribution type uncertainty, respectively. Using Bayes weights, it is possible to discriminate between different probability distributions and to quantify how well a distribution fits the data. For formal distribution comparison, either non-informative or informative priors can be used. Furthermore, the present paper explores the use of informative priors, which are determined using historical censored floods. These floods occurred before the period of systematic gauging.

A Bayesian analysis is carried out on the river Rhine to determine the discharge at station Lobith with an average return period of 1,250 years. From the analysis with both historical floods from the period 1800-1900 and reliable and homogeneous observations from the period 1901-1998, we conclude that a Bayesian approach seems to be very promising. However, we remark that the use of informative priors based on historical floods, instead of non-informative priors results in a lower discharge with a return period of 1,250 years. It is not known yet whether this is due to unreliable information about historical floods, which are generally not homogeneous, or to an actual lower occurrence rate of these floods. Furthermore, it must yet be investigated what perception thresholds can best be chosen in order to classify historical floods.

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9. APPENDIX

This appendix contains the probability distributions, which are considered in the statistical analysis of the annual maximum discharges, as well as their non-informative Jeffreys priors.

Distribution	Probability density function	Jeffreys prior
Exponential	$\frac{1}{\theta} exp\left\{\frac{-x}{\theta}\right\} I_{(0,\infty)}(x),\ \theta > 0$	$J(\theta) = \frac{1}{\theta}$
Rayleigh	$\frac{2x}{\theta} exp\left\{\frac{-x^2}{\theta}\right\} I_{(0,\infty)}(x),\ \ \theta > 0$	$J(\theta) = \frac{1}{\theta}$
Normal	$\left(\frac{r}{2\pi}\right)^{\frac{1}{2}} exp\left\{\frac{-r}{2}(x-m)^2\right\} I_{(0,\infty)}(x),\ \ r > 0$	$J(m,r) = \frac{1}{\sqrt{2r}}$
Lognormal	$\left(\frac{r}{2\pi}\right)^{\frac{1}{2}} \frac{1}{x} exp\left\{\frac{-r}{2}(log(x)-m)^2\right\} I_{(0,\infty)}(x),\ \ r > 0$	$J(m,r) = \frac{1}{\sqrt{2r}}$
Gamma	$\frac{b^a}{\Gamma(a)} x^{a-1} exp\{-bx\} I_{(0,\infty)}(x),\ \ a,b > 0$	$J(a,b) = \frac{\sqrt{a\psi'(a)-1}}{b}$
Weibull	$\frac{a}{b}\left[\frac{x}{b}\right]^{a-1} exp\left\{-\left[\frac{x}{b}\right]^a\right\} I_{(0,\infty)}(x),\ a,b > 0$	$J(a,b) = \frac{1}{b}\frac{\pi}{\sqrt{6}}$
Gumbel	$\frac{1}{b} exp\left\{-\frac{x-a}{b}\right\} exp\left\{-exp\left\{\frac{x-a}{b}\right\}\right\},\ \ b > 0$	$J(a,b) = \frac{1}{b^2}\frac{\pi}{\sqrt{6}}$

Remark with respect to the Jeffreys prior of the gamma distribution: The trigamma function $\psi'(a)$ is the first derivative of the digamma function:

$$\psi'(a) = \frac{\partial \psi(a)}{\partial a} = \frac{\partial^2 log\,\Gamma(a)}{\partial a^2}$$

for $a > 0$, where $\Gamma(a) = \int_{t=0}^{\infty} t^{a-1} e^{-t} dt$ is the gamma function for $a > 0$.