

Modelling

salt water intrusion density dependent groundwater flow

modelling

Why mathematical modelling anyway?

A model is only a schematisation of the reality!

modelling

Why mathematical modelling anyway?

+:

- · cheaper than scale models
- analysis of very complex systems is possible
- a model can be used as a database
- to increase knowledge about a system (water balances)

-

- simplification of the reality
- only a tool, no purpose on itself
- garbage in=garbage out: (field)data important
- · perfect fit measurement and simulation is suspicious

modelling

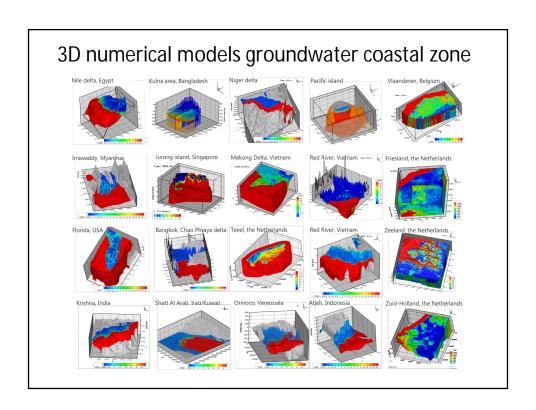
Numerical modelling variable density flow

Type:

- sharp interface models
- solute transport models

State of the art:

- · three-dimensional
- solute transport
- transient



modelling

Some existing 3D codes which simulate variable density groundwater flow in porous media:

SEAWAT (Guo & Bennett, 98) METROPOL (Sauter, '87) FEFLOW (Diersch, '94) MVAEM (Strack, '95) D3F (Wittum et al., '98) MOCDENS3D (Oude Essink, '98) HydroGeoSphere (Therrien, '92) SWICHA (Huyakorn et al., '87) SWIFT (Ward, '91) FAST-C 3D (Holzbecher, 98) MODFLOW+MT3D96 (Gerven, '98) HST3D (Kipp, '86) SUTRA (beta-version, Voss, '02)

Restrictions 3D salt water intrusion modelling

- the data problem:
 - -not enough hydrogeological data available
 - -e.g. the initial density distribution
 - -especially important issue in data-poor countries
- the computer problem:
 - -modelling transient 3D systems: computer only good enough at high costs
- the numerical dispersion problem:
 - -numerical dispersion is large in case of coarse grid

Restrictions 3D salt water intrusion modelling now

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modelling transient of bitsems: computer only good solution is at high costs

• the numerical dispersion problem solvers
-numerical dispersion is large in case of coarse grid

variable density

Stability criteria for solute transport equation (I)

1. Neumann criterion:

$$\frac{D_{xx}\Delta t_s}{\Delta x^2} + \frac{D_{yy}\Delta t_s}{\Delta y^2} + \frac{D_{zz}\Delta t_s}{\Delta z^2} \le 0.5$$

$$\Delta t_s \le \frac{0.5}{\frac{D_{xx}}{\Delta x^2} + \frac{D_{yy}}{\Delta y^2} + \frac{D_{zz}}{\Delta z^2}}$$

variable density

Stability criteria for solute transport equation (II)

2. Mixing criterion:

$$\Delta t_s \le \frac{n_e b_{i,j,k}^k}{Q_{i,j,k}'}$$

Change in concentration in element is not allowed to be larger than the difference between the present concentration in the element and the concentration in the source

variable density

Stability criteria for solute transport equation (III)

3. Courant criterion:

$$0 < \xi < = \sim 1$$

$$\Delta t_s \le \frac{\xi \Delta x}{V_{x,\text{max}}}$$

$$\Delta t_s \le \frac{\xi \Delta y}{V_{y,\text{max}}}$$

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 $\Delta t_s \le \frac{\xi \Delta z}{V_{z,\text{max}}}$

Modelling fresh-salt groundwater on different scales

Sub-local: fingering, salty sand boils Sri Lanka (Tsunami 2004), Zandmotor

cell size=1cm-1m









Local: rainwaterlenses, heat-cold Tholen, Schouwen-Duiveland cell size=5-25m







Regional:

Zeeland, Gujarat/India, Philippines cell size=100m







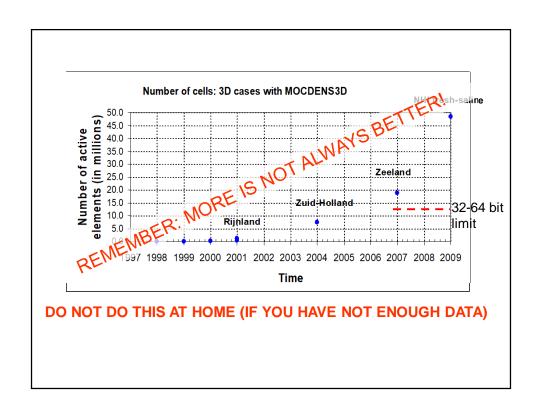
National: salt load Bangladesh, Zuid-Holland, NHI cell size=250m-2km

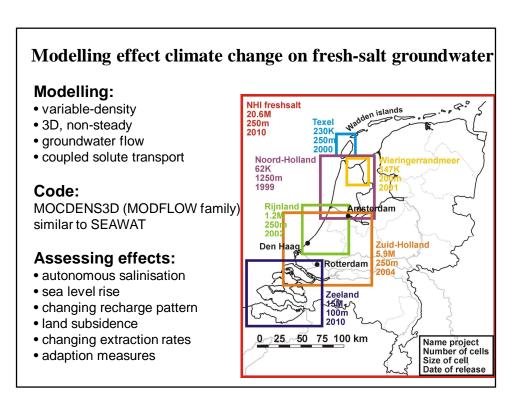






To take largest cell size possible to accurately model relevant salinisation processes





Fields of application of fresh-saline groundwater models

- Water system analysis in brackish-saline environments (salt loads, salt boils, freshwater lenses)
- Quantifying effects of climate change & sea level rise
- Drinking water issues: upconing saline groundwater under extraction wells
- Developing measurements to stop salinization groundwater systems (e.g. fresh keeper, coastal collectors, freshwater storage underground)
- Impact of the disasters as tsunamis on fresh groundwater resources
- Submarine Groundwater Discharge (marine water pollution, Harmful Algae)

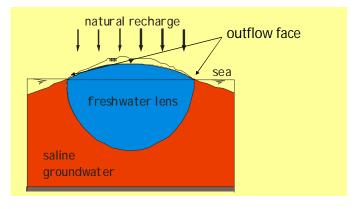
variable density

Difficulties with variable density groundwater flow

- Initial density distribution (effects on velocity field)!
- Velocities freshwater lens at the outflow face near the sea
- Boundary conditions (especially concentration boundaries)
- Choice of element size
- Length of flow time step to recalculate groundwater flow

problems

Outflow face at the coast is difficult to model



Flow converges and thus velocities are very high at the outflow face

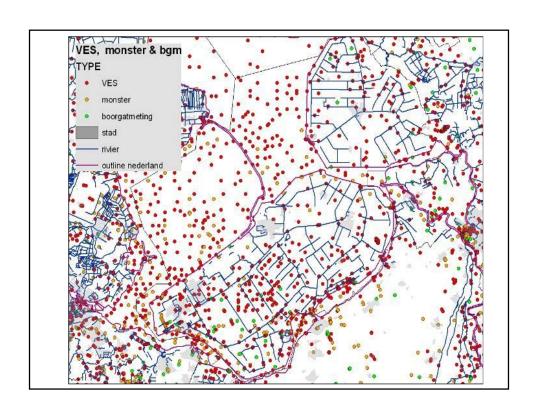
This is numerically difficult to handle

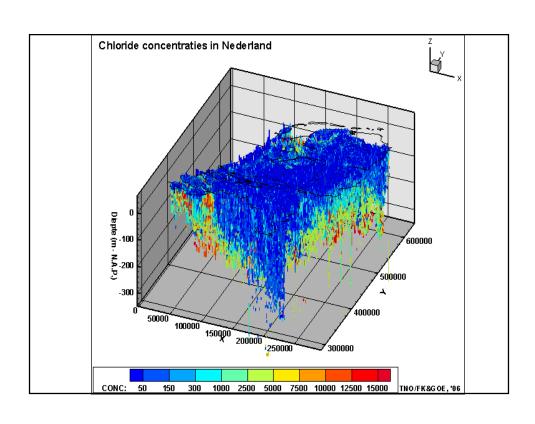
A good initial density distribution is essential

- Because groundwater and solute transport are coupled, the density influences grondwater velocities
- Numerous density measurements are necessary to get a reliable 3D density matrix

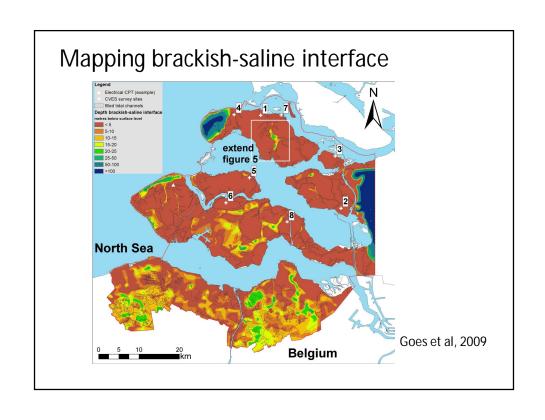
'Procedure' to improve initial density distribution

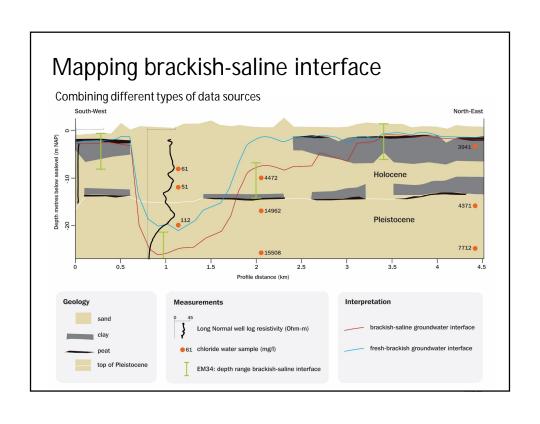
- Implement all chloride data
 - Analyses, Borehole, VES, Airborne techniques (HEM, SkyTem)
 - Better old then nothing
 - Better VES then nothing
- Interpolate and extrapolate
 - Sea = easy (salt)
 - Inland = fresh?
- Start with simulation (10/20/30 years) with mol.diffusion*1000 to smooth out artificial densities

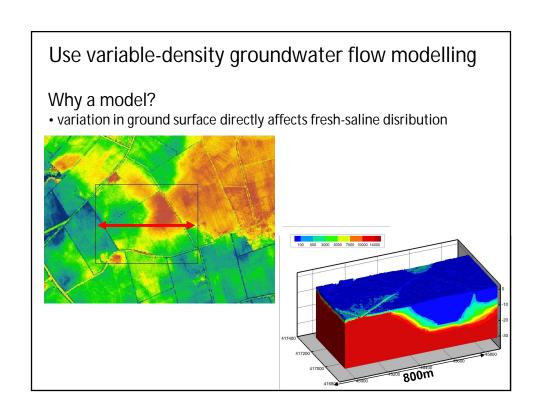


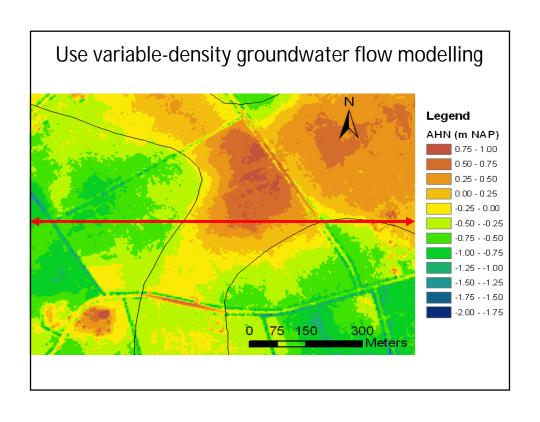


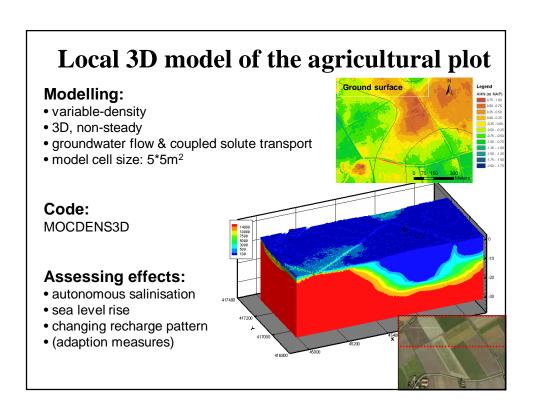
Mapping brackish-saline interface Zeeland Combining different types of data sources: Accuracy depth of interfaces Characteristics Data type # Data Determined of measurement 0D in situ Depends on positions of Groundwater 721 Chloride concentration Samples Geo-electrical 1D in situ 149 1D chloride profile, Depth borehole logs fresh-brackish and brackishsaline interface, Inversions. Electrical CPT 1D in situ (max. Borehole log ±1 m depth 50 m) VES 1D from surface 1113 Depth brackish-saline ±20% of depth interface, Major inversions, (1D chloride profile). EM34 1D from surface 3251 Depth brackish-saline ranges of 7.5, 15 or 30 m (accuracy decreases with depth) interface Groundwater 0D in situ 716 Depth brackish-saline a range depending Unique locations 6021

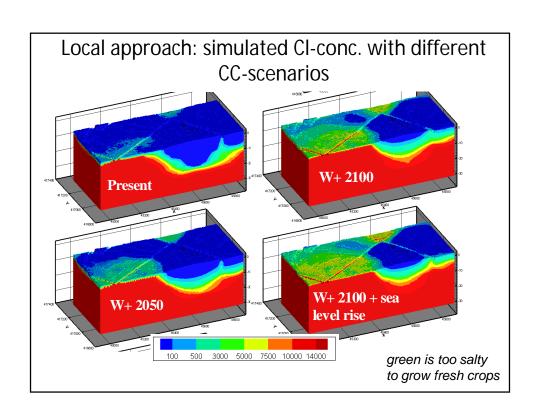


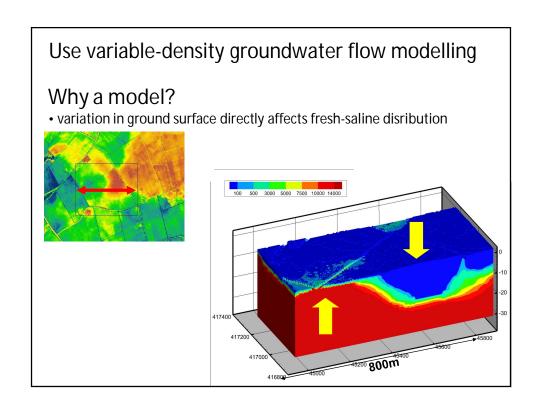


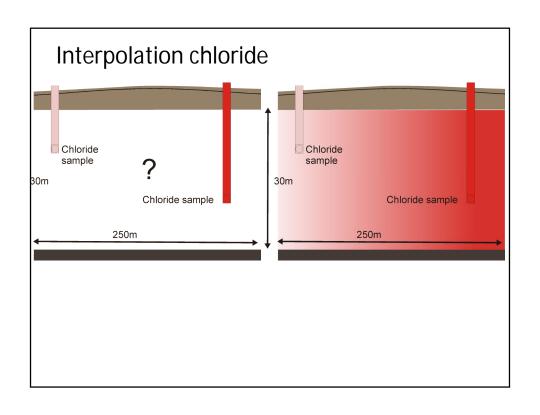


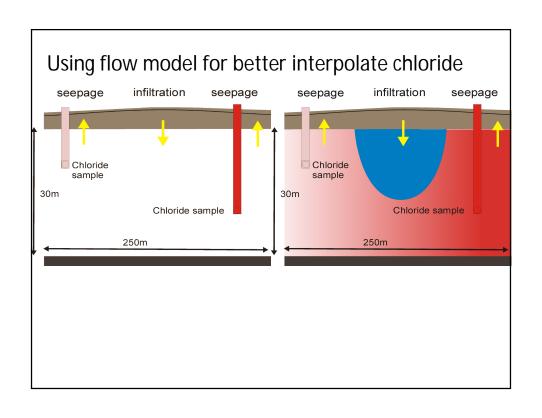


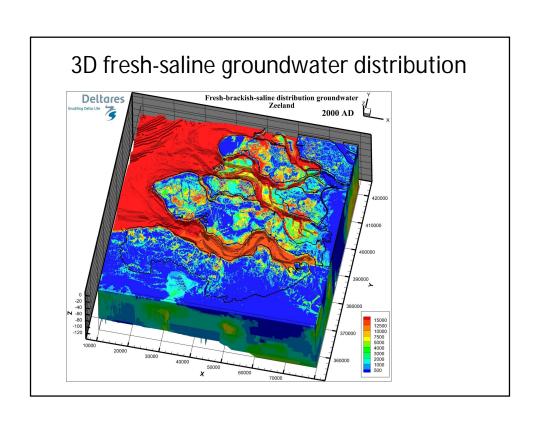


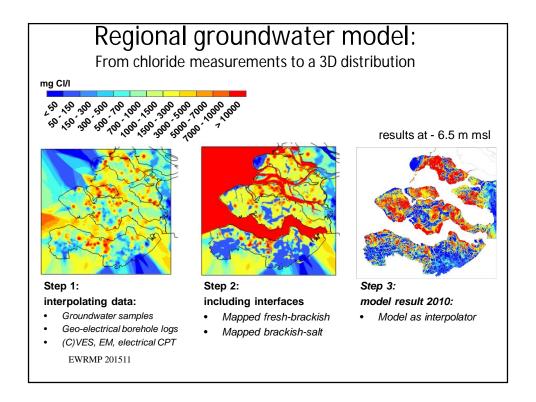










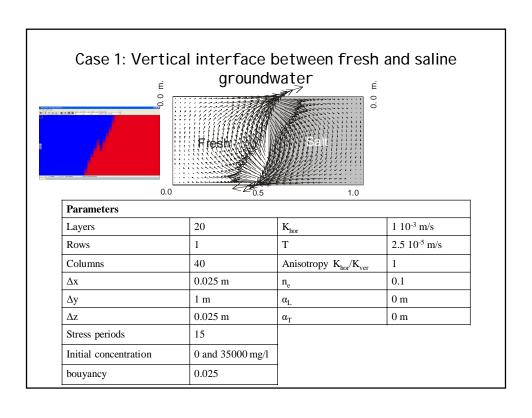


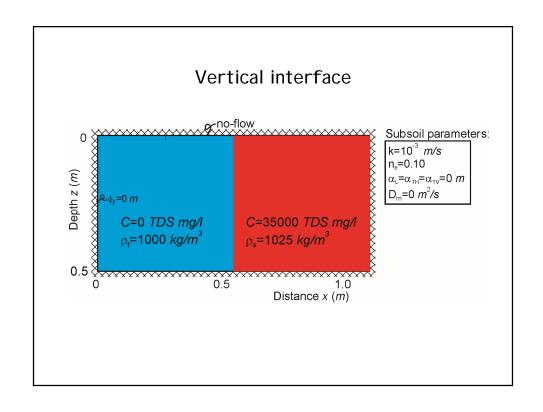
modelling

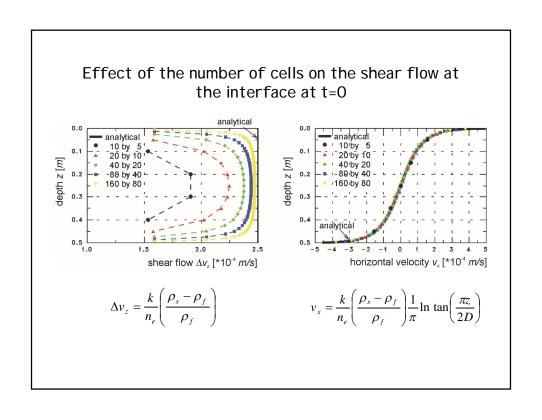
Examples of variable-density groundwater flow

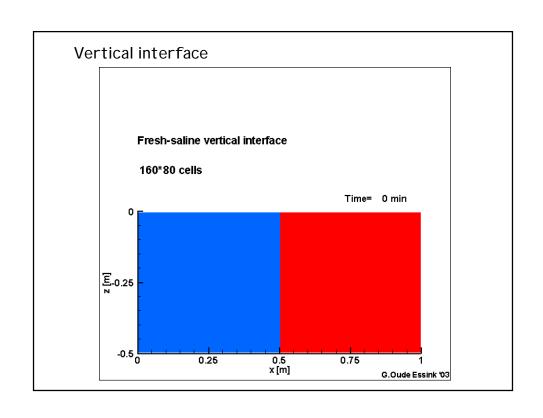
- Rotating immiscible interface
- Henry's problem
- Evolution freshwater lens
- Hydrocoin
- · Salt water pocket
- Broad 14 Basin, North Sea
- Heat transport: Elder and Rayleigh=4000
- 5 Dutch 3D cases
- Freshwater lenses
- Effect of Tsunami on groundwater resources

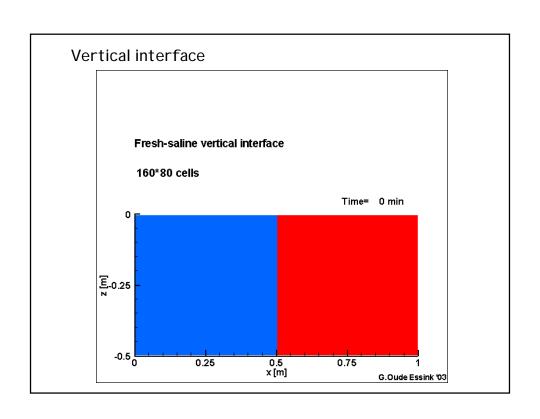








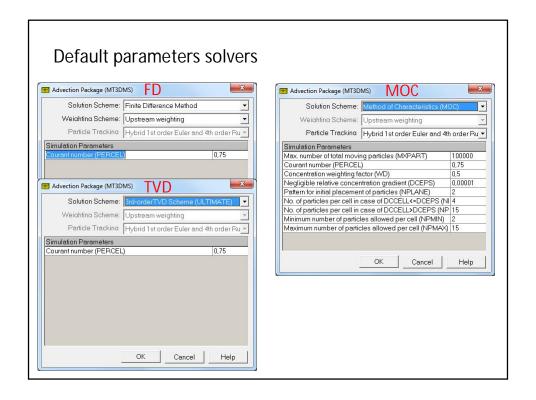


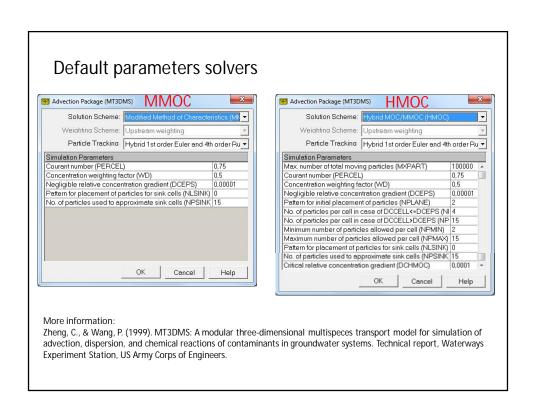


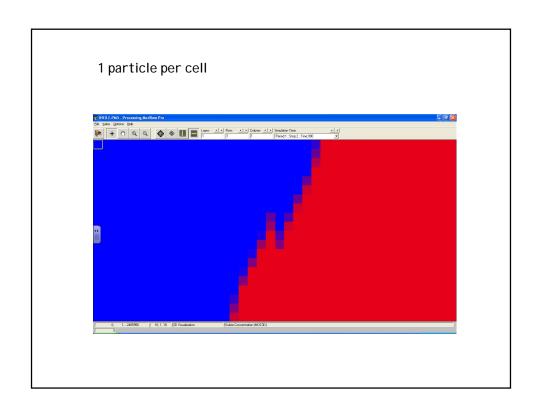
The effect of numerical solvers on the salt transport

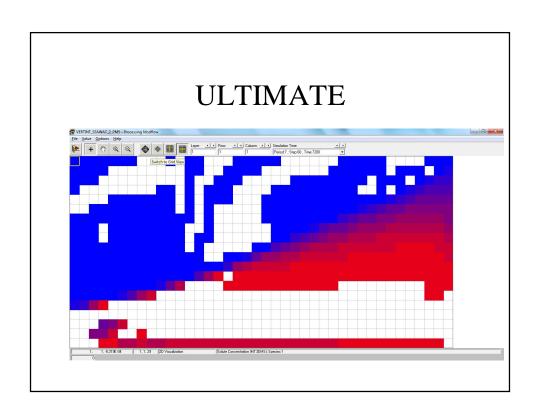
Examples

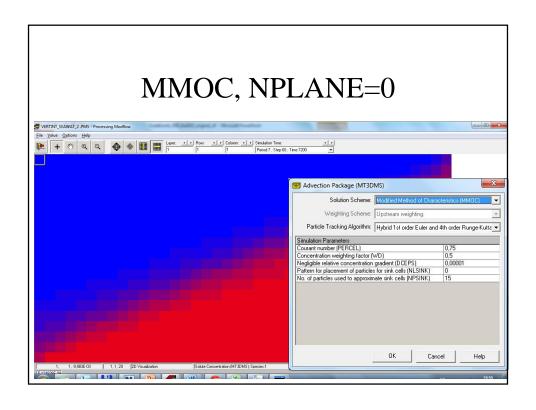
EWRMP 201511

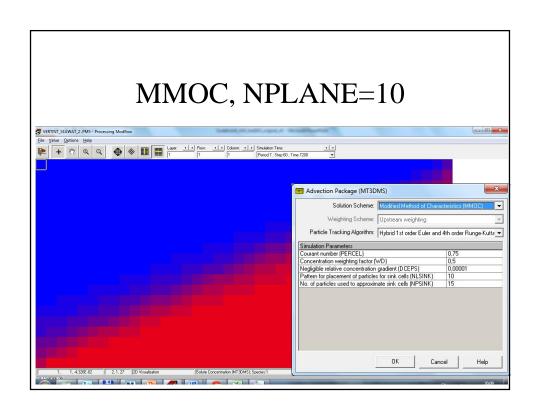


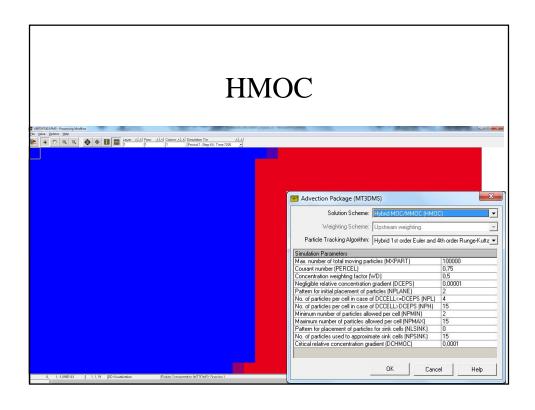


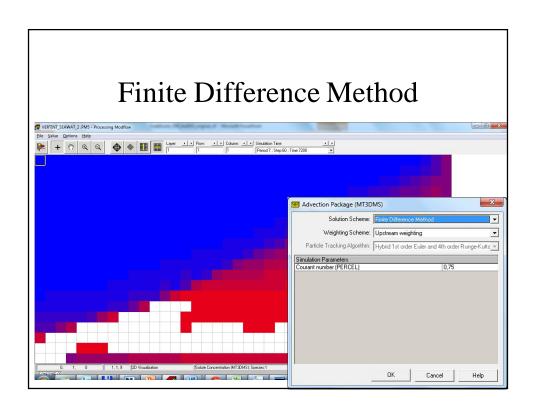


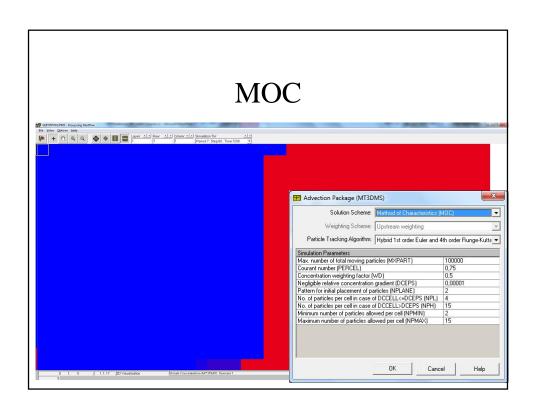


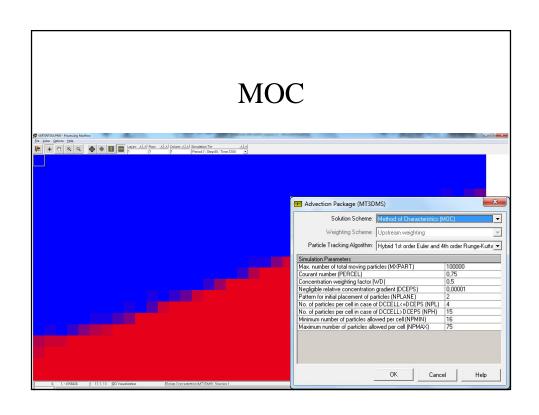


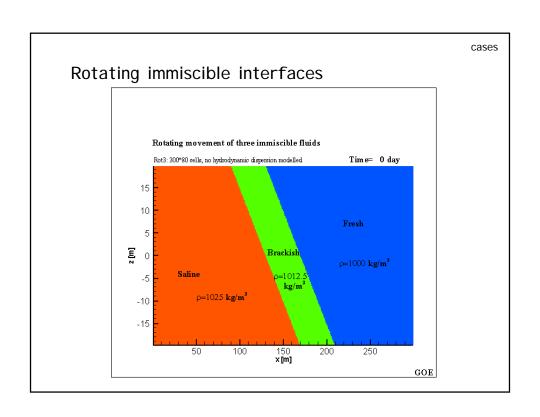


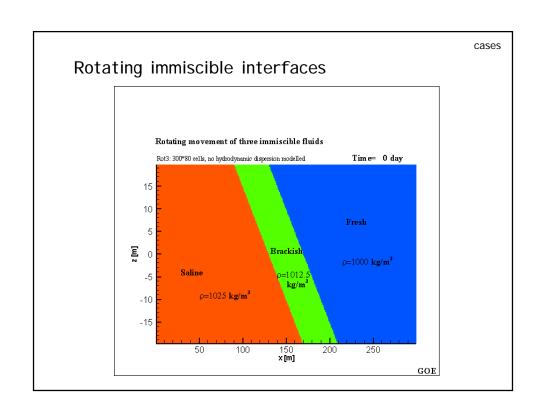


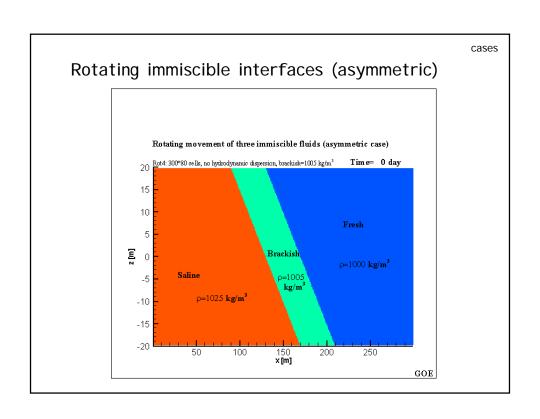


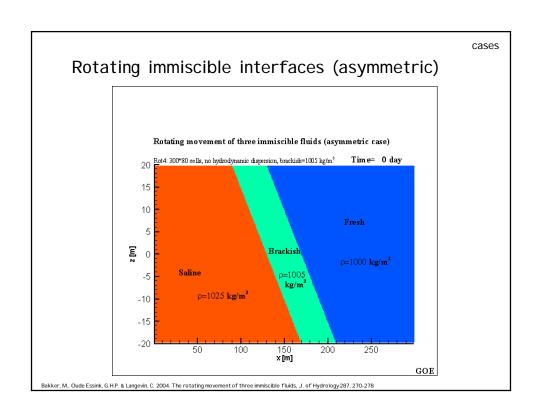


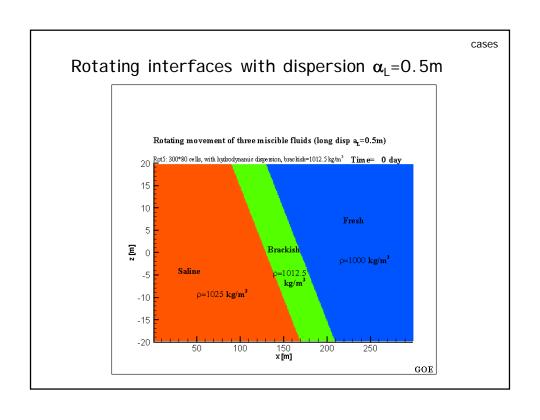


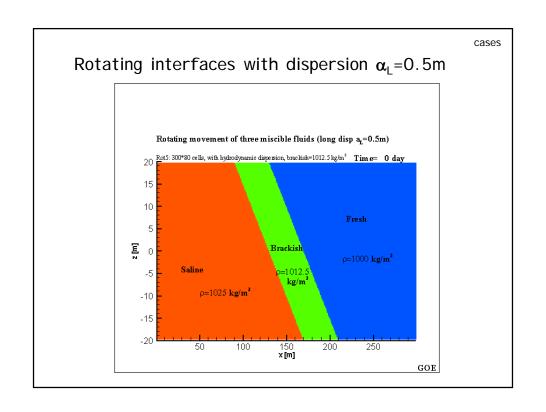


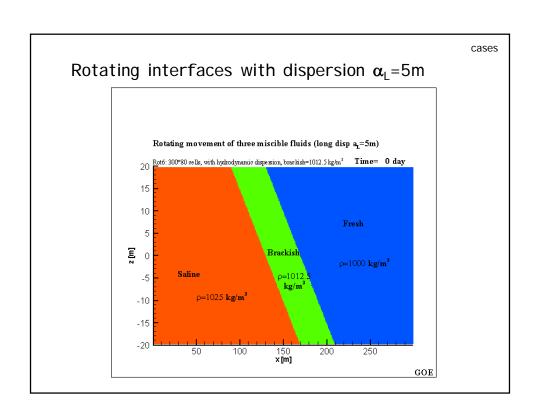


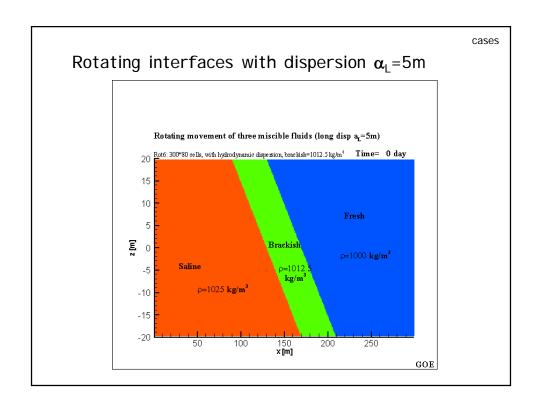










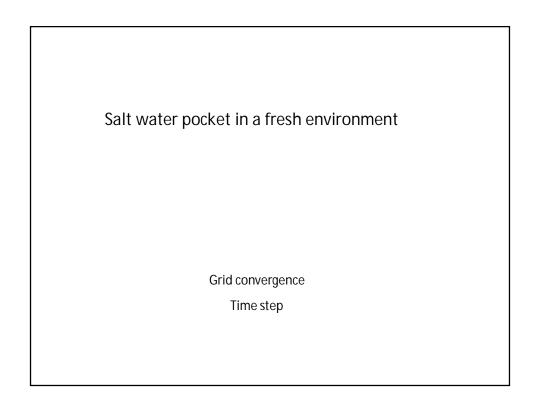


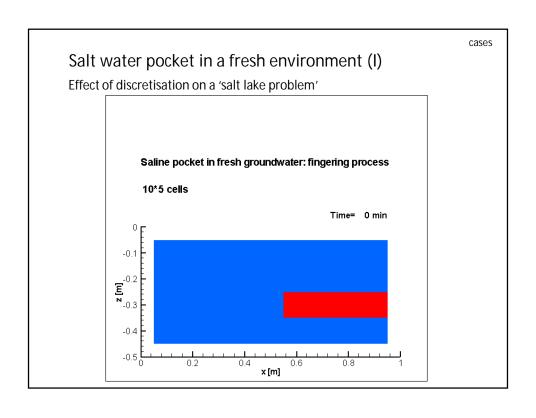
cases

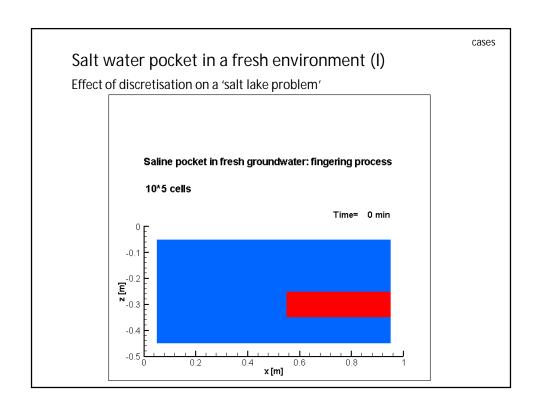
Rotating immiscible interfaces

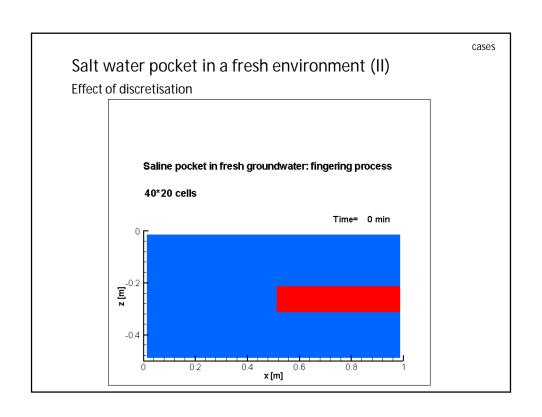
Conclusion:

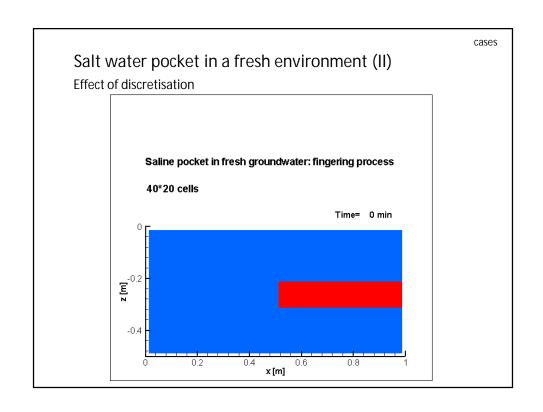
To check the variable-density component of your code, this immiscible interface benchmark can be used.

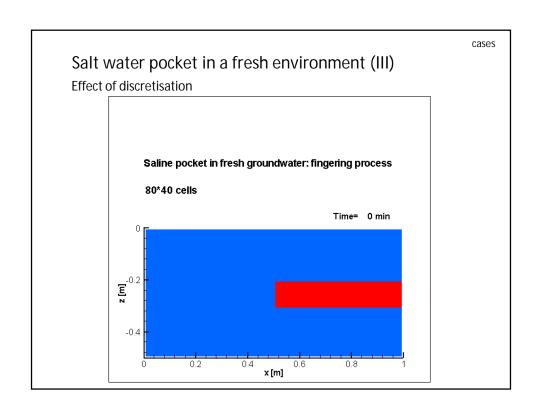


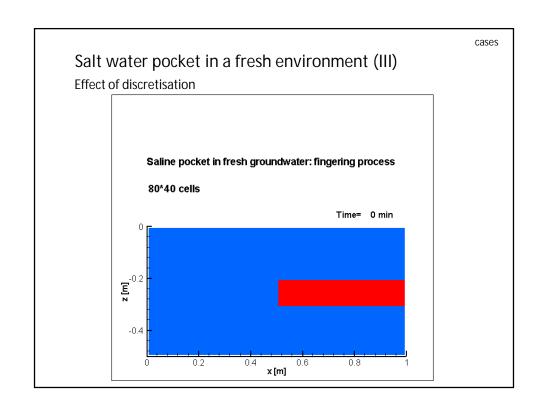


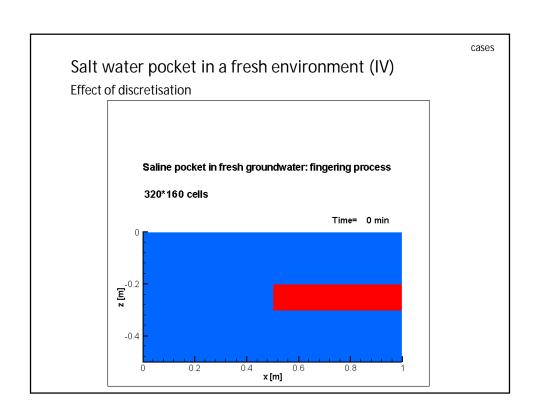


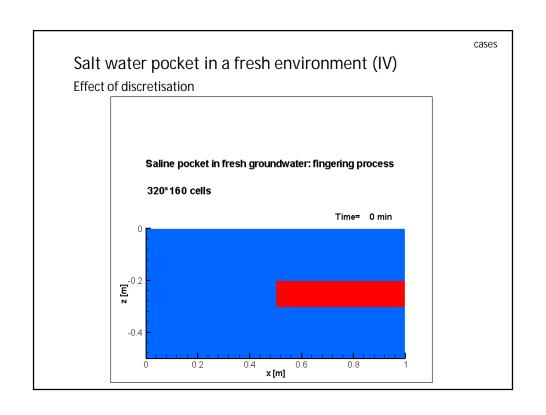


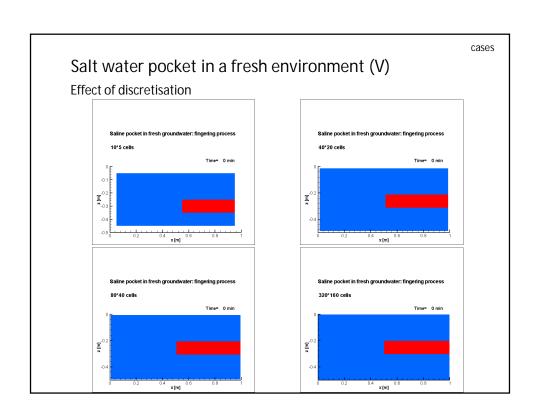


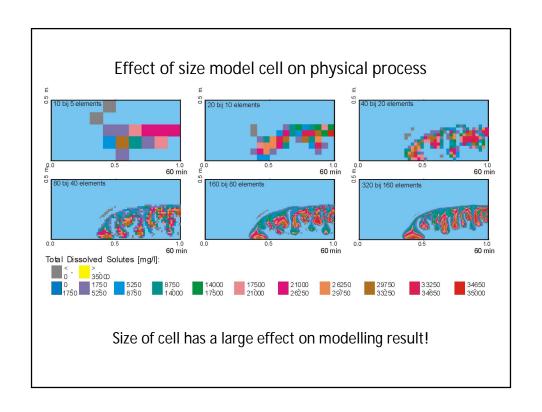


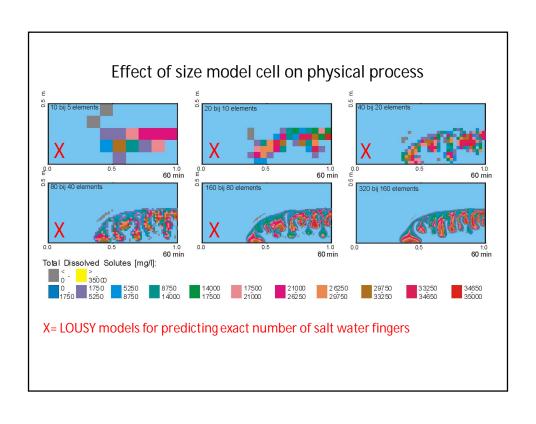


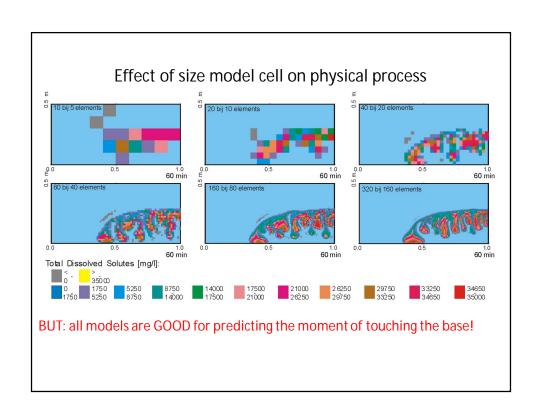










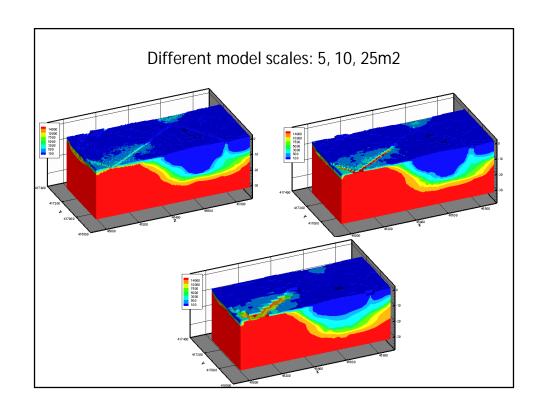


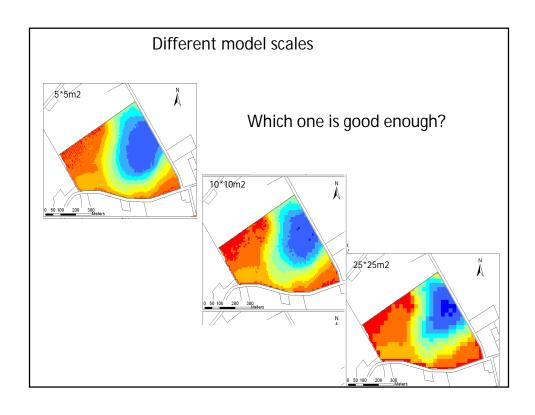
cases

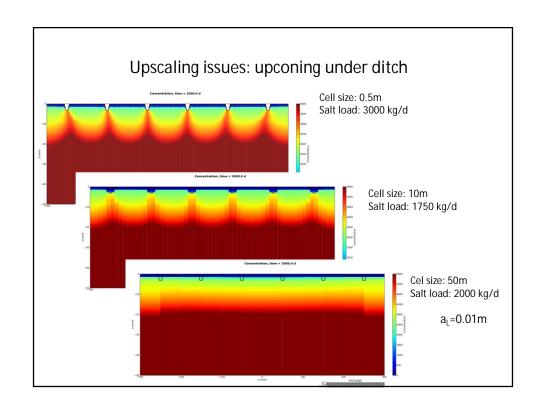
Salt water pocket in a fresh environment (VI)

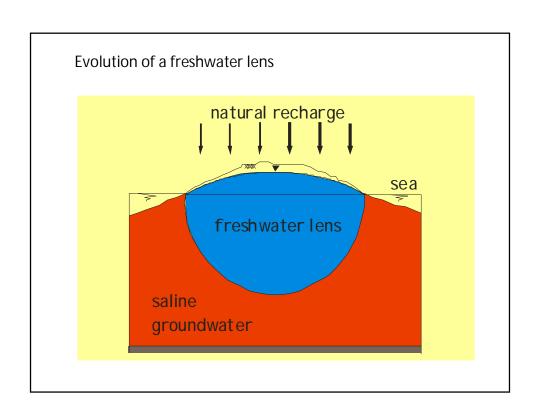
Conclusion:

- For some physical processes, a large number of cells is necessary
- Check always grid convergence!









cases

Question:

How long does it take before the volume of a freshwater lens is filled?:

a. 5 years

b. 25 years

c. 100 years

d. 500 years

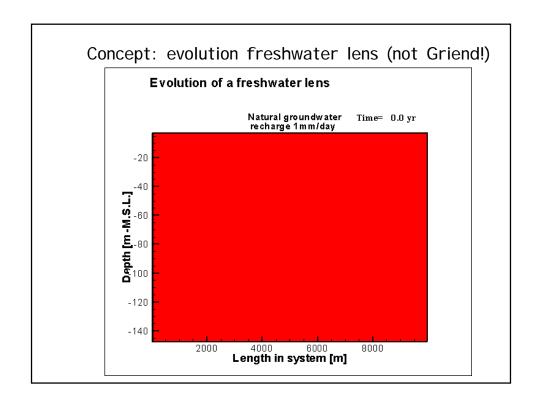
T = specific time scale

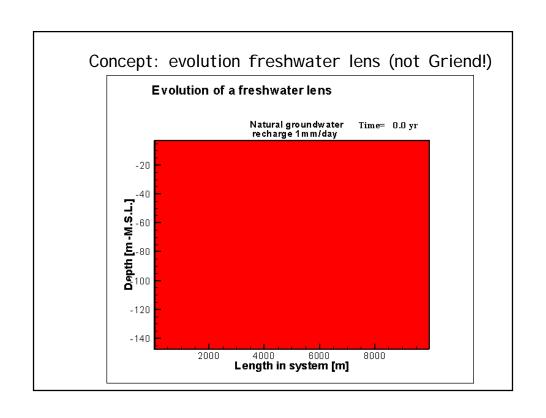
T = time period before the lens has reached 95% of its final form

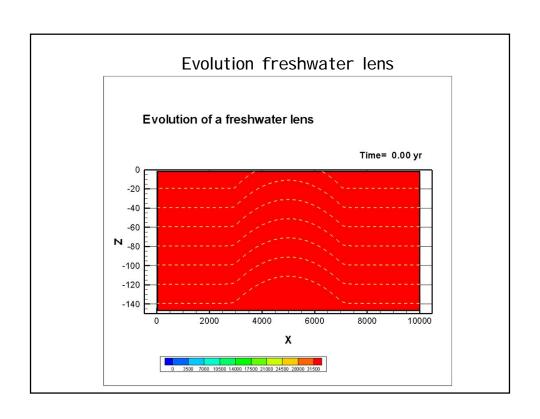
In the Netherlands: T = 75-200 jaar,

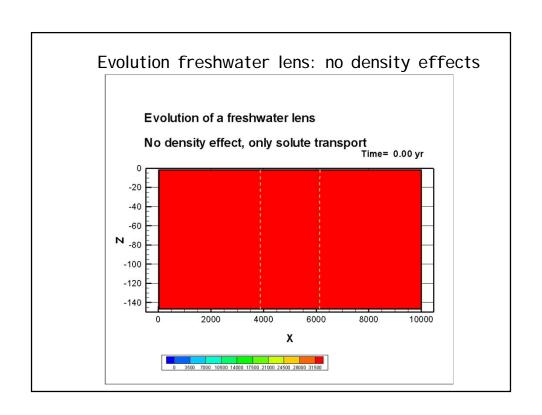
depends on:

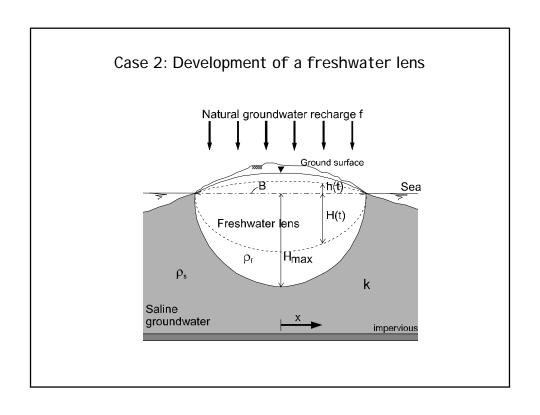
- width dune area
- natural groundwater recharge
- hydraulic conductivity soil

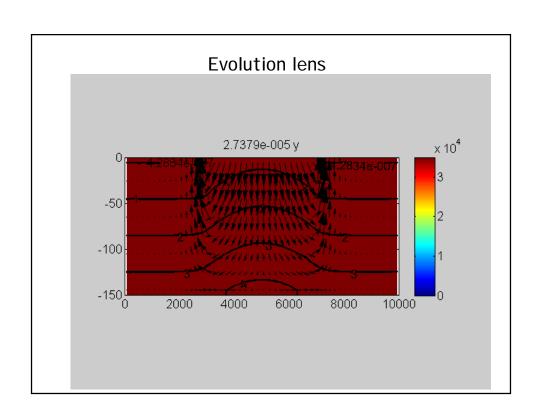


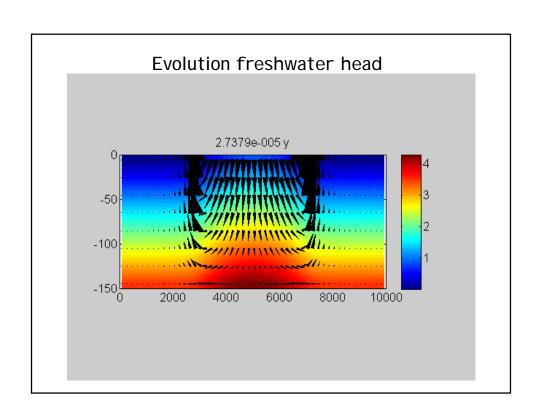








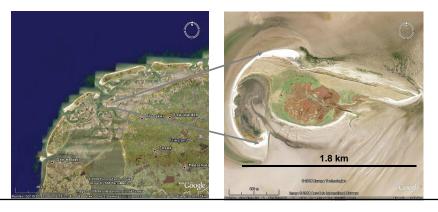


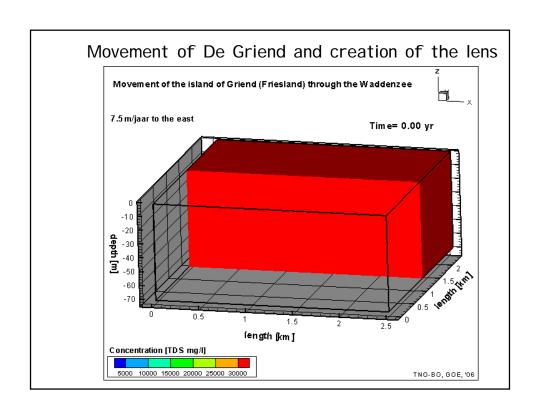


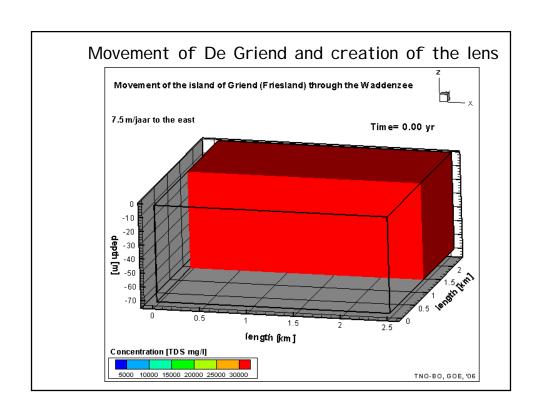
The island of Griend

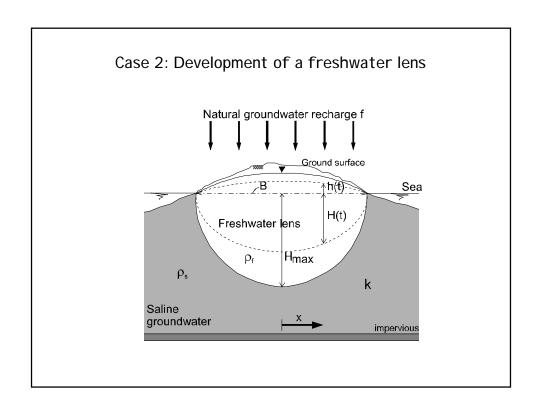
I ssues:

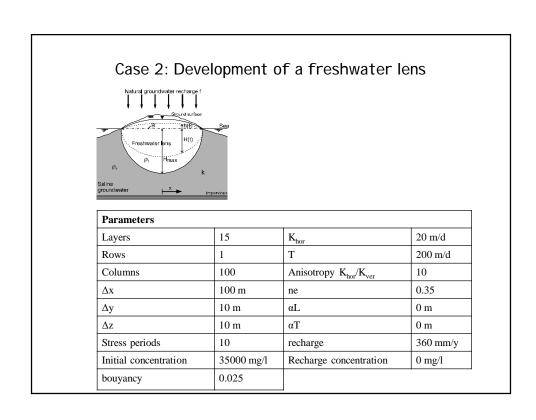
- 1. Small island moves ~7.5m per year to the east
- 2. Effect on the volume of the freshwater lens:
 - Can a lens be developed?
 - What is the thickness of the lens?

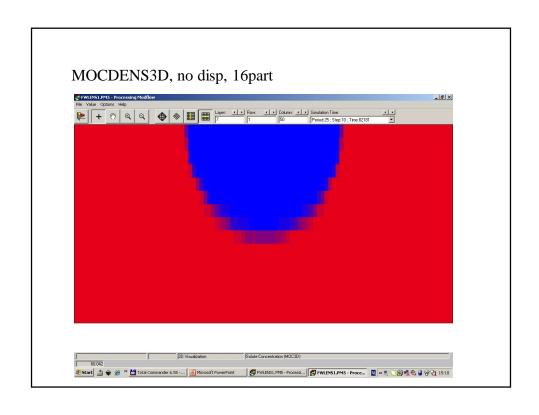


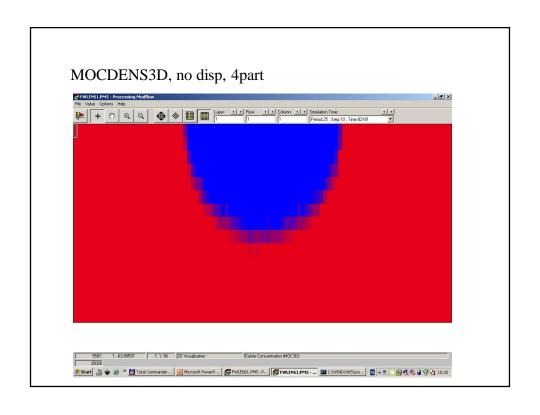


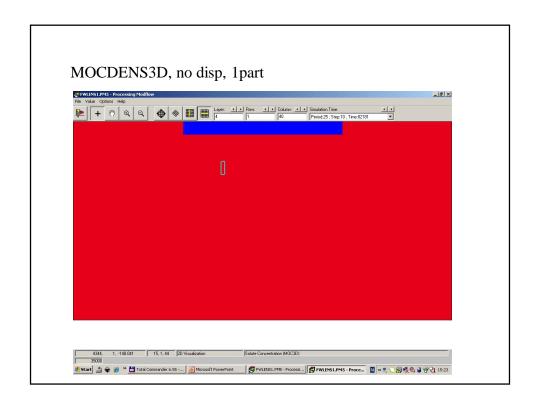


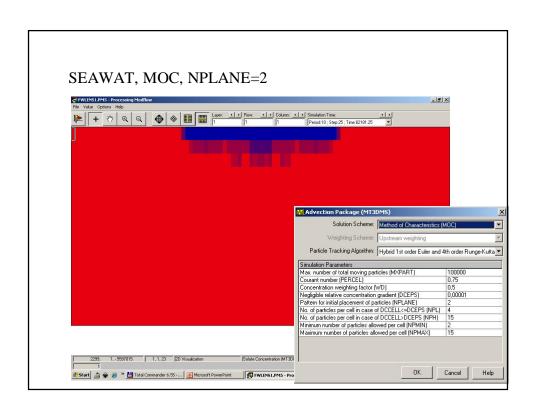


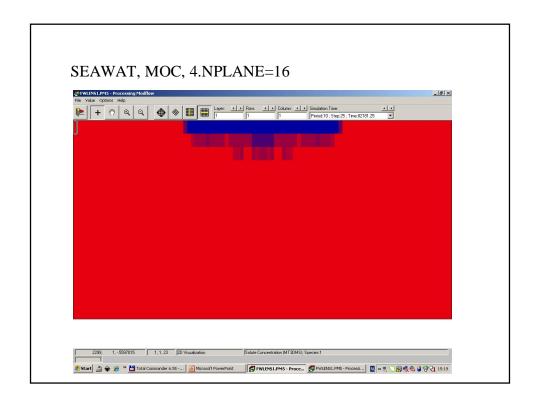


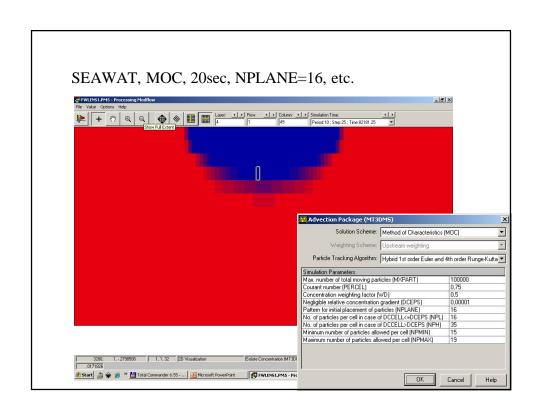


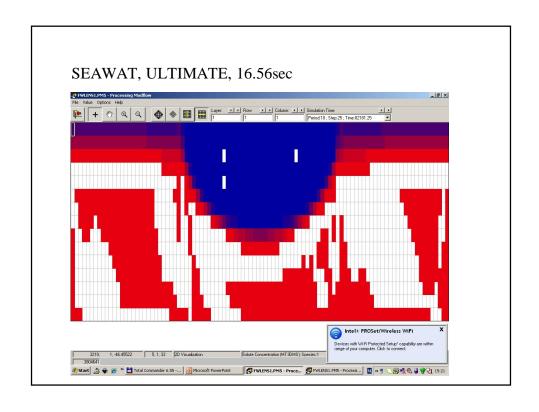


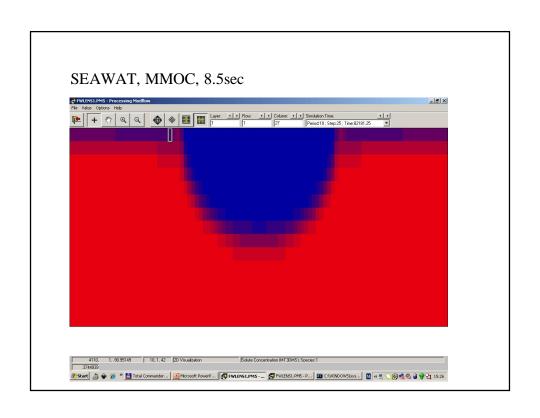


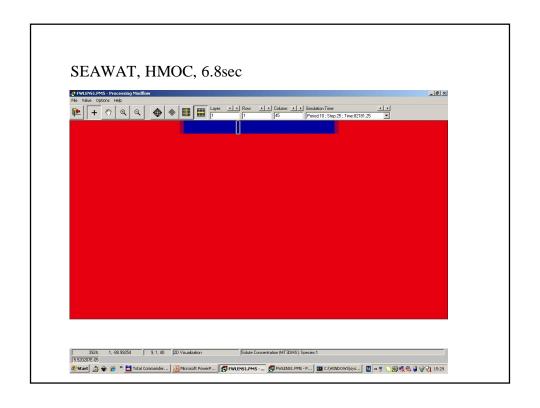


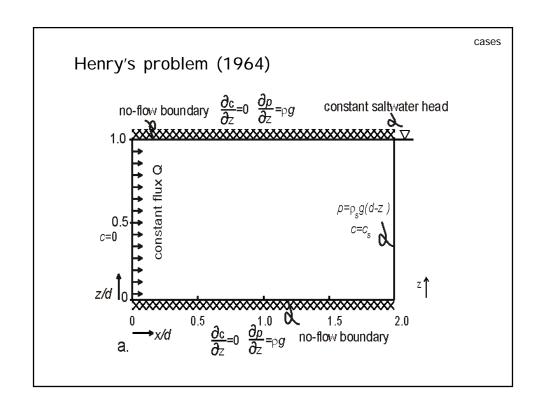


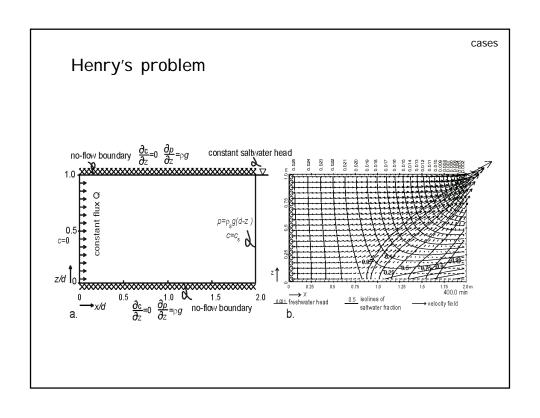


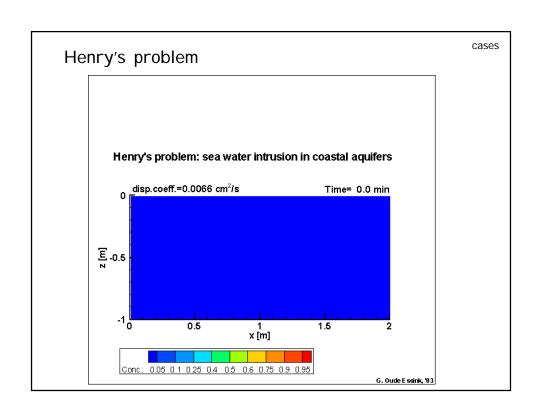








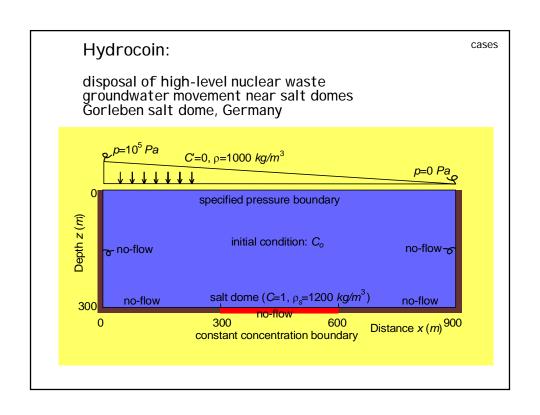


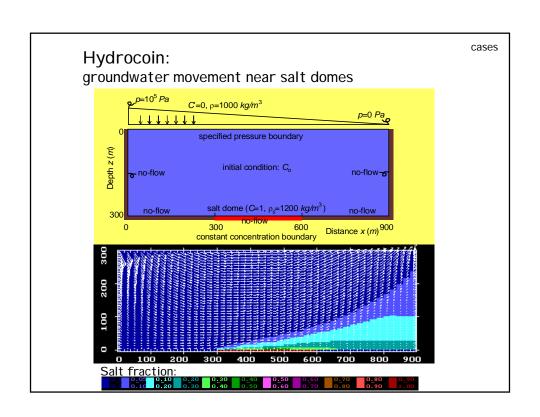


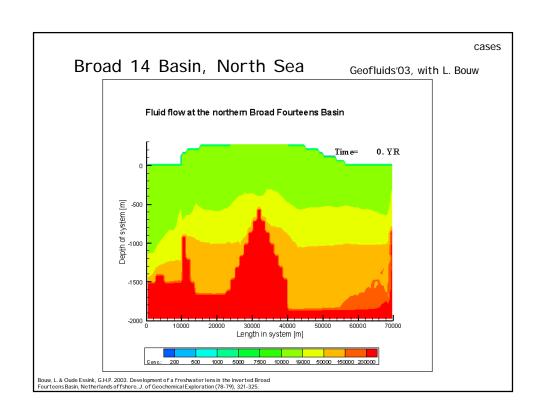
Henry's problem

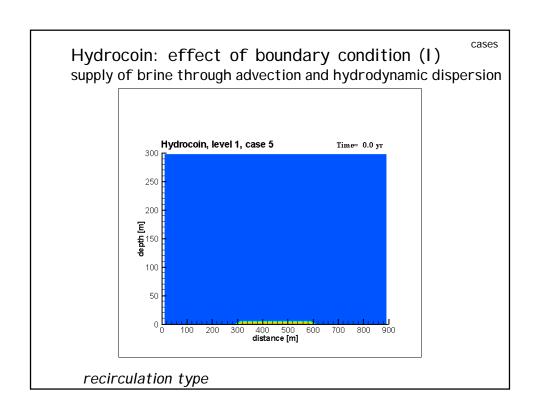
cases

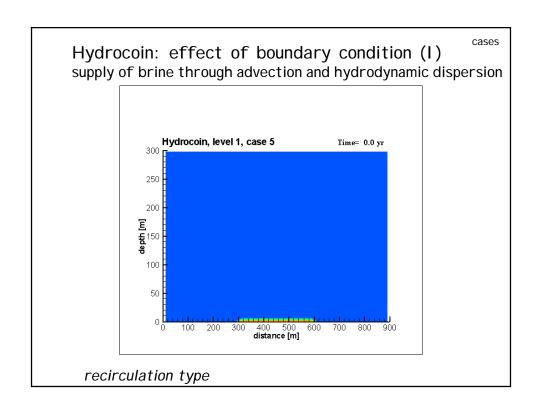
Don't use the Henry problem as a variable-density benchmark, because even with a constant density model, the results are more or less the same!

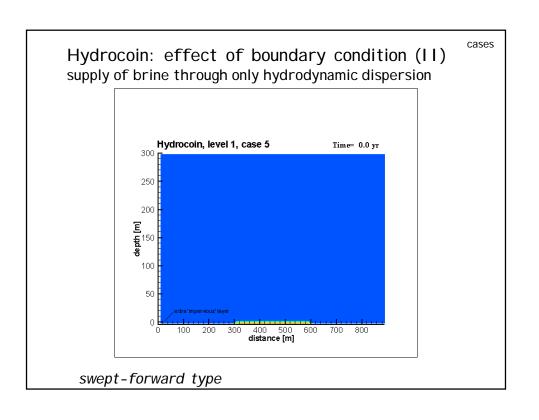


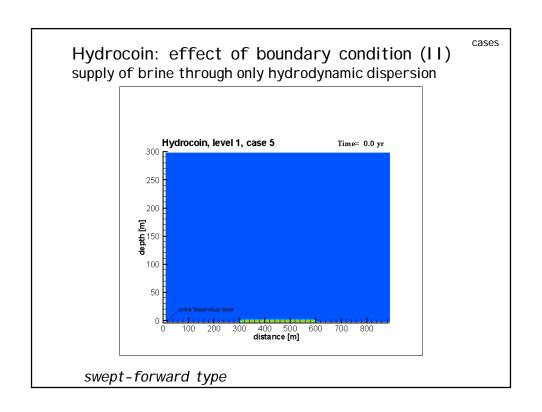


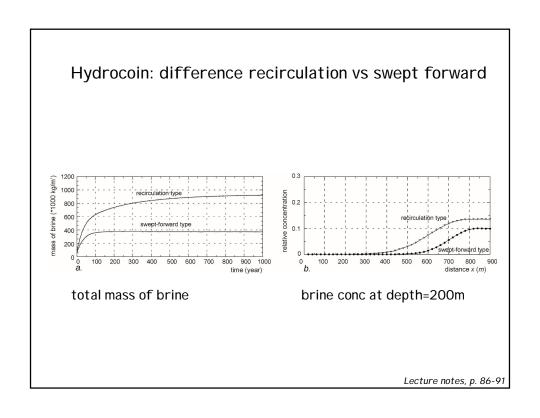


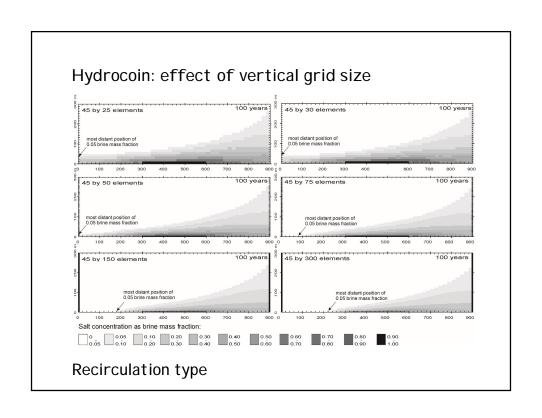


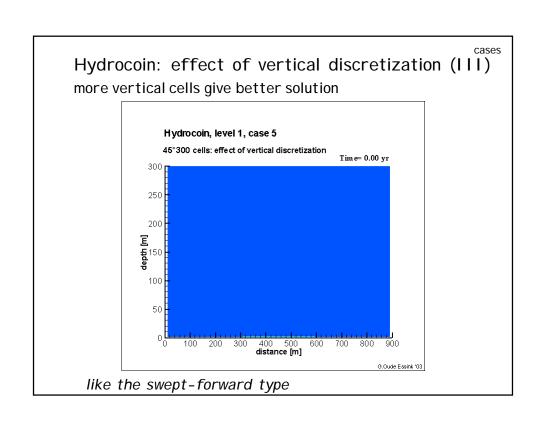


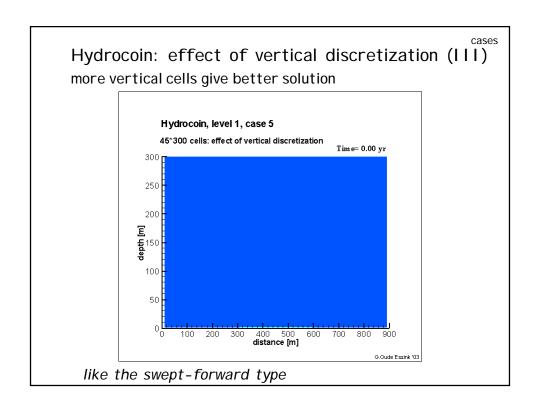


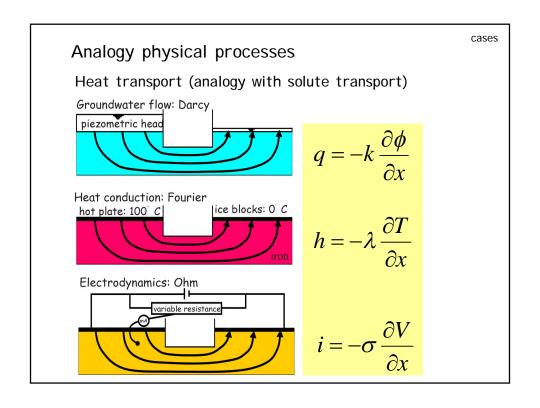












Heat transport

Conduction and convection of heat

(fluid flow)

continuity equation

(Fourier)

specific heat capacity [Joule/(kg
$$^{\circ}$$
C)]
$$-\frac{\partial h}{\partial x} = \rho' c' \frac{\partial T}{\partial t} \qquad \rho' c' = n_e \rho c_{fluid} + (1 - n_e) \rho_{solid} c_{solid}$$

Analogy solute and heat transport

cases

Solute: advection-dispersion equation

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i} \left(D_{ij} \frac{\partial C}{\partial x_j} \right) - \frac{\partial}{\partial x_i} \left(CV_i \right) + \frac{\left(C - C \right)'W}{n_e}$$

Heat: convection-conduction equation

$$\rho'c'\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} \left(\Lambda_{ij} \frac{\partial T}{\partial x_j} \right) - \rho c_f \frac{\partial T q_i}{\partial x_i} + \Gamma$$

Heat transport

Analogy heat and solute transport

Heat transport

Convection-conduction equation

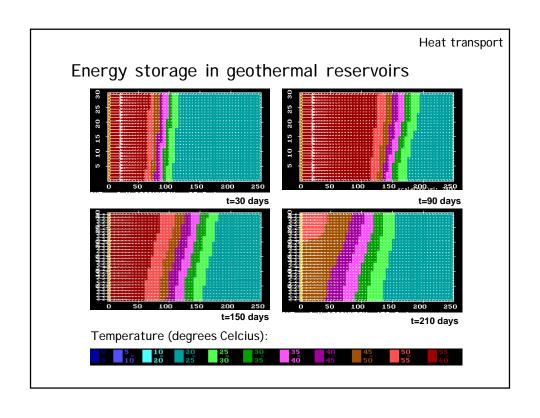
$$\rho'c'\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} \left(\Lambda_{ij} \frac{\partial T}{\partial x_j} \right) - \rho c_f \frac{\partial T q_i}{\partial x_i} + \Gamma$$

Equation of state: relation density & temperature

$$\rho_{i,j,k} = \rho_f (1 - \alpha_f T_{i,j,k})$$

Analogy between solute and heat transport

Solute	Heat
C	T
R_d	$1 + \frac{(1 - n_e)\rho_s c_s}{n_e \rho c_f}$
$D_{\!\scriptscriptstyle m}$	$\frac{n_e \lambda_e + (1 - n_e) \lambda_s}{n_e \rho c_f}$
λ	0



Elder problem (I)

It is originally a heat transport problem

Phases:

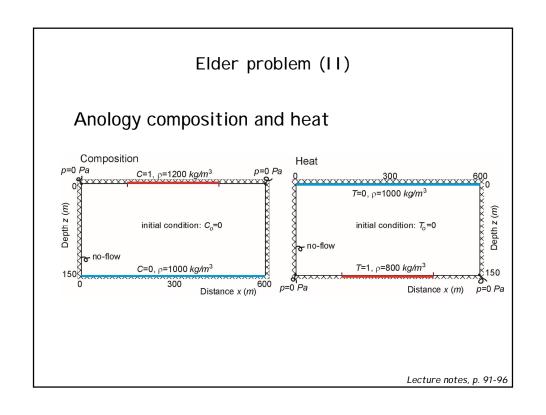
1. Stable growth diffusive boundary layer

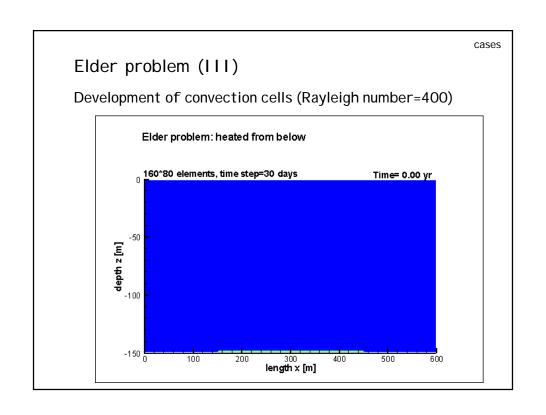
2. Development flow cells embedded in boundary layer

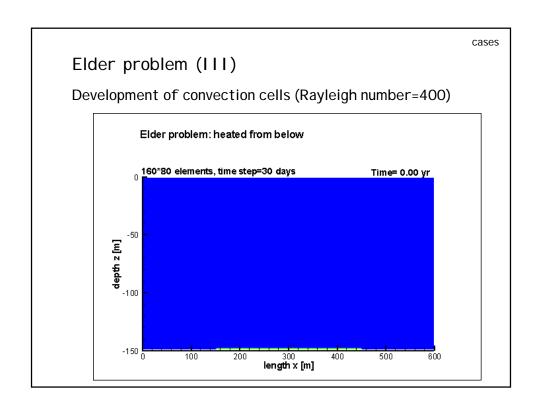
3. Emergence of disturbances that grow into fingers

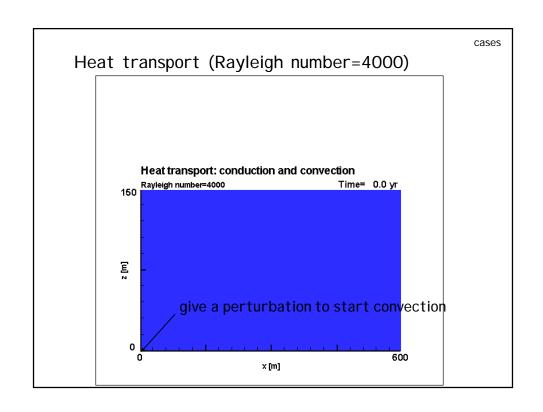
Convection of heat occurs when:

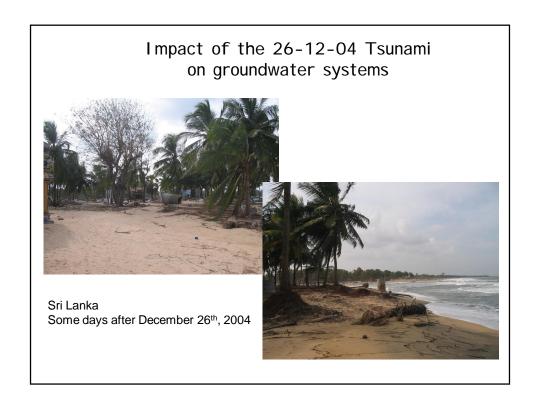
Rayleigh number > $4\pi^2$ Elder, J. Fluid Mech. 32, 69-96, 1968











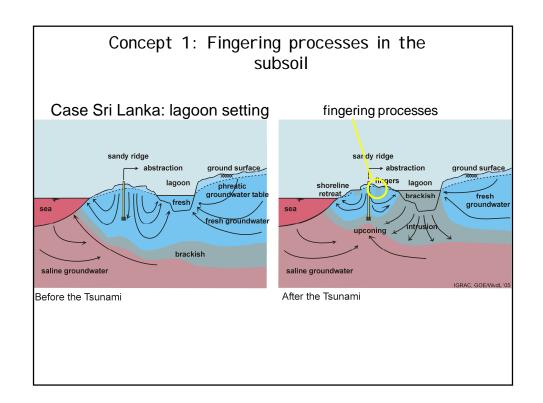
Impact of the 26-12-04 Tsunami on groundwater systems

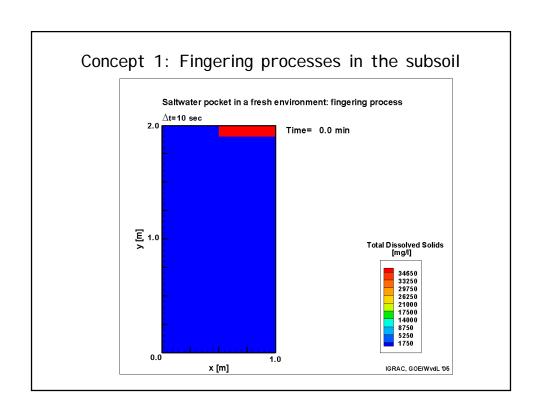
Impression of relevant salinisation processes by conceptual models of salt water intrusion in coastal aquifers:

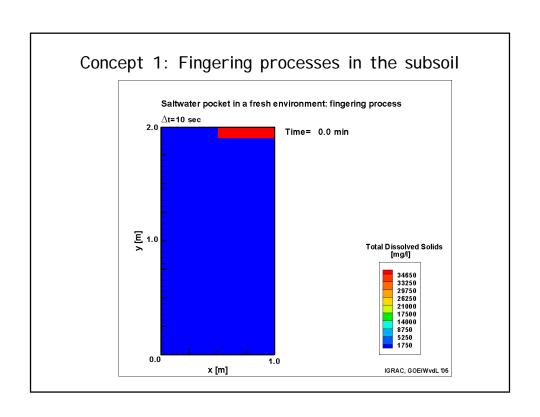
- 1. Fingering processes in the subsoil
- 2. Evolution of a freshwater lens after flooding by sea water
- 3. Freshwater lens in a coastal aquifer with a brackish lagoon

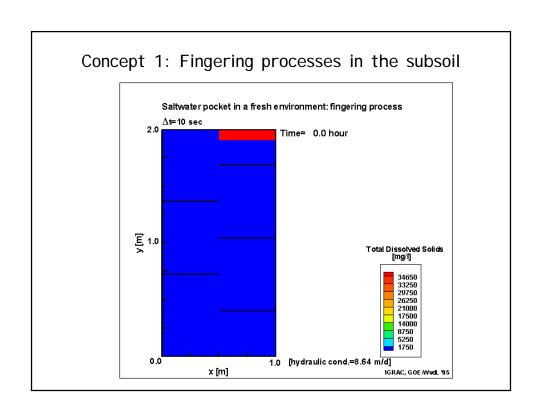
Next step:

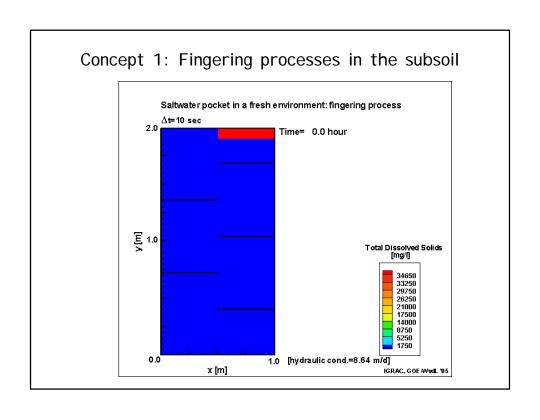
quantifying processes in real situations, using topographic and hydrogeological data, and ending up with vulnerability maps

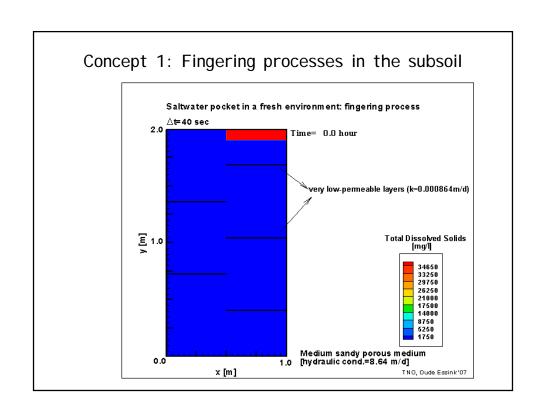


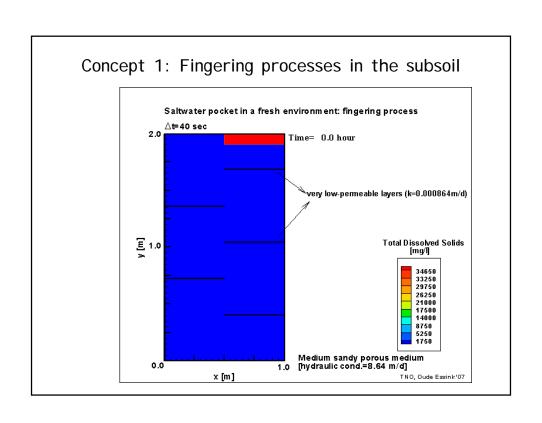


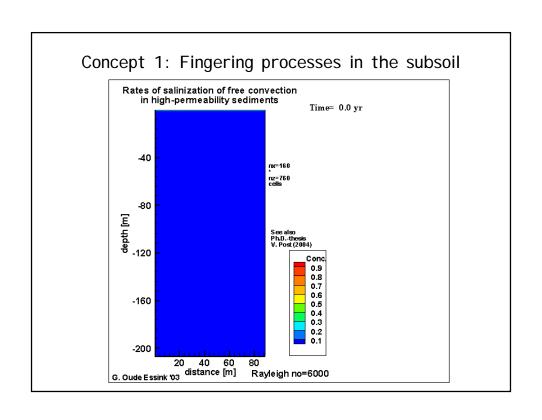


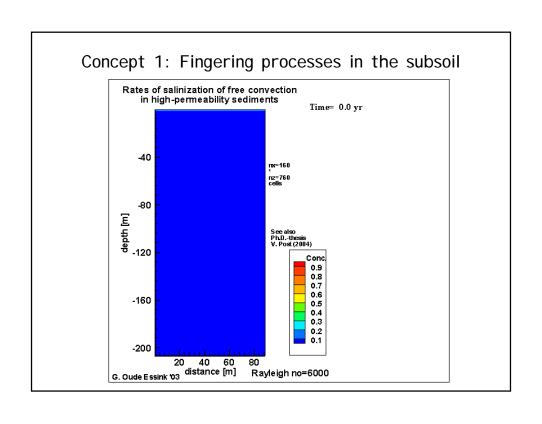


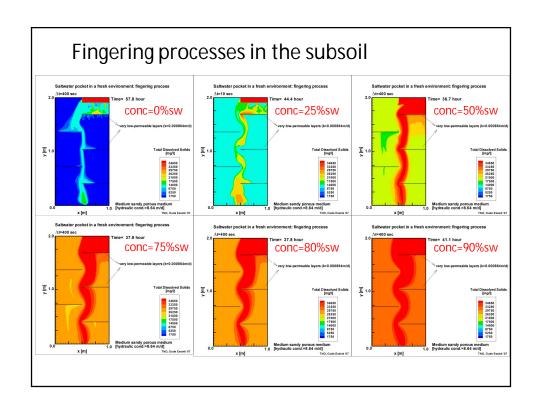


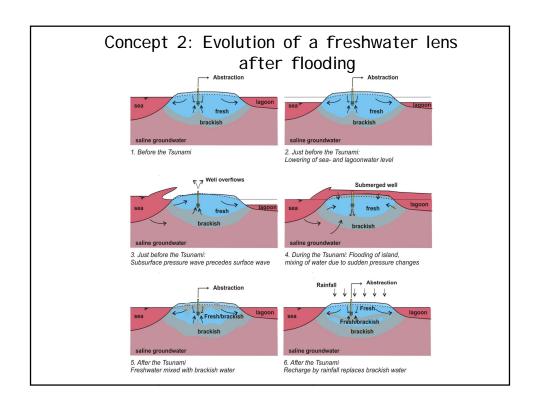


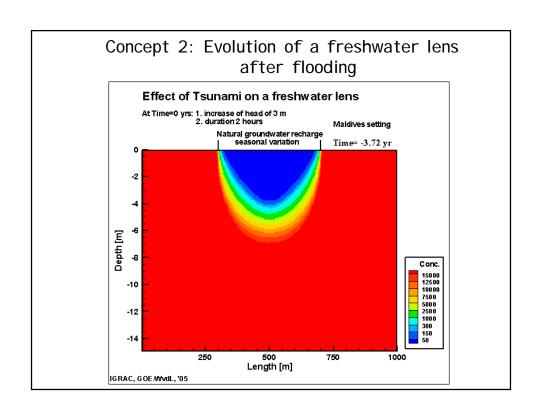


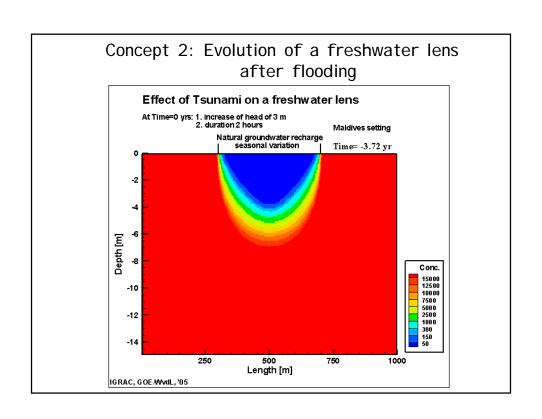


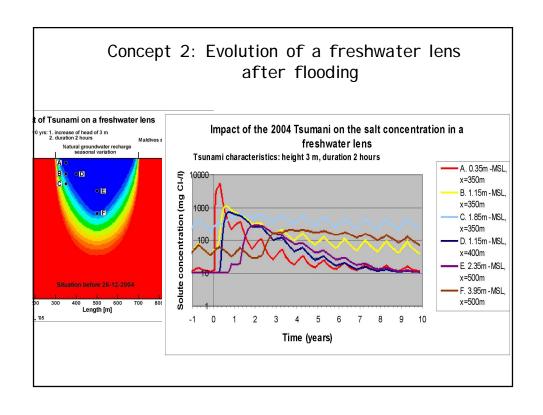


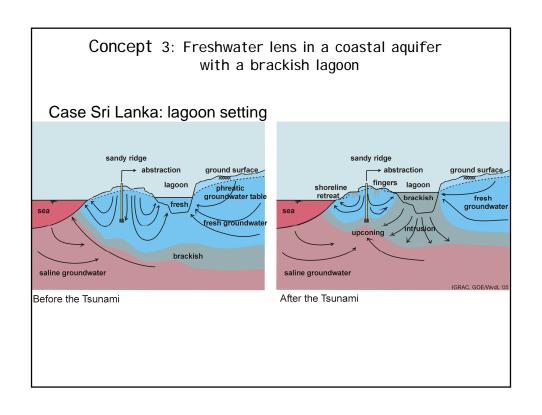


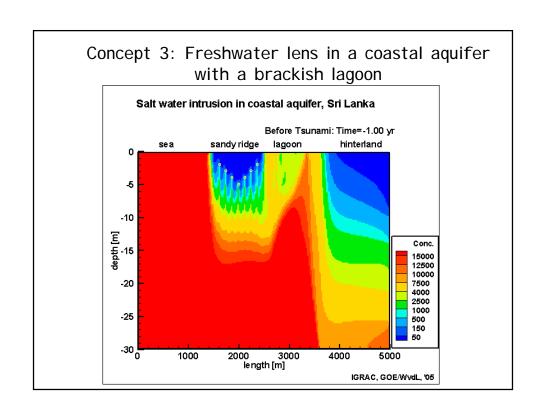


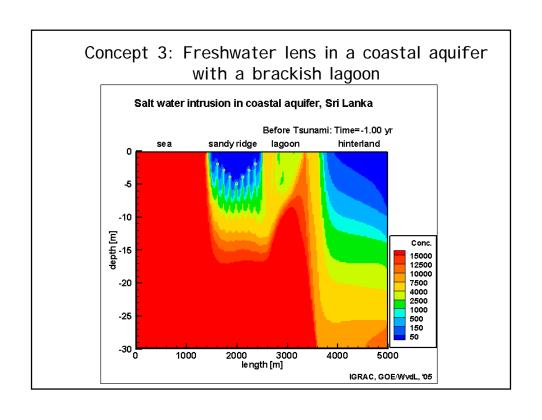


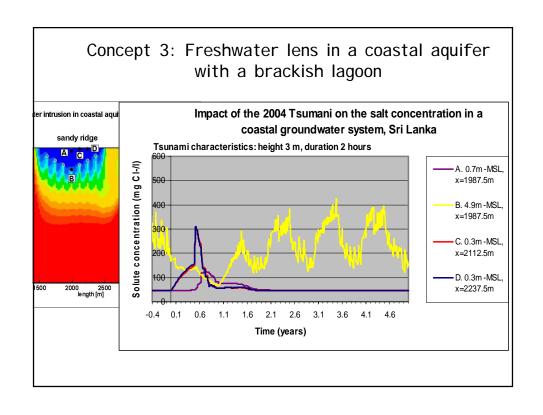








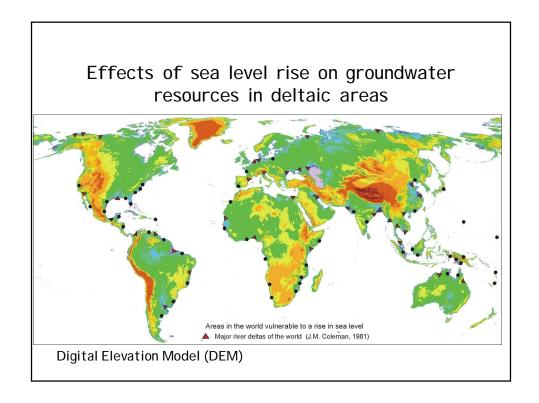


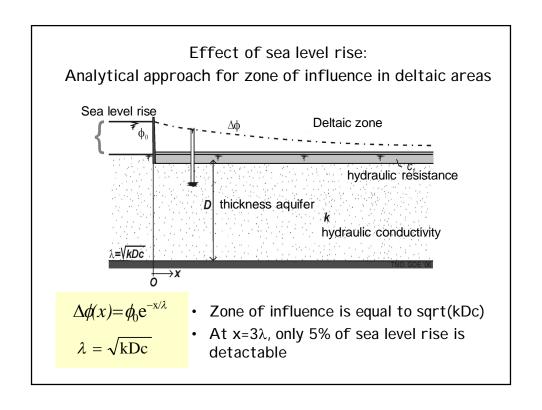


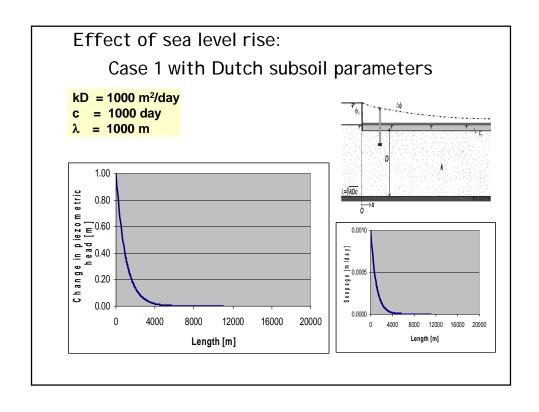
Effect sea level rise

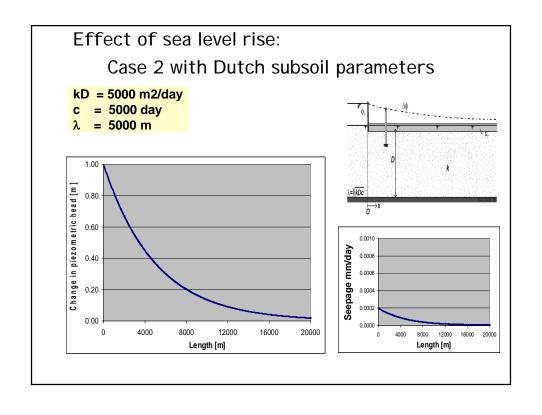
Effects of sea level rise on groundwater resources in deltaic areas

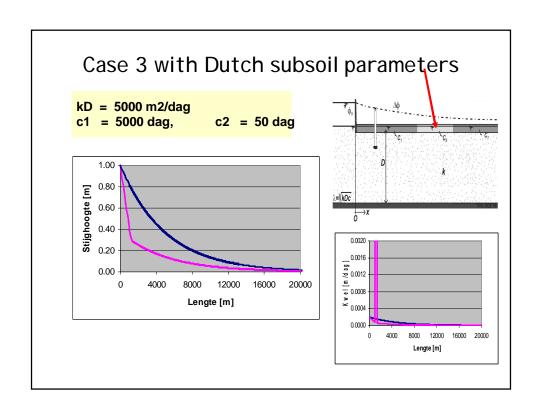
- 1. Increase of salt water intrusion
- 2. Increase of upconing under groundwater extraction wells
- 3. Increase of piezometric head
- 4. Increase of seepage and salt load to the surface water system
- 5. Risk of instable Holocene aquitards
- 6. [Decrease of fresh groundwater reservoirs due to decrease in natural groundwater recharge]



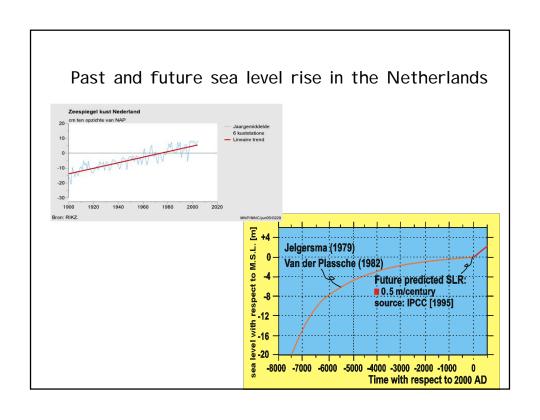






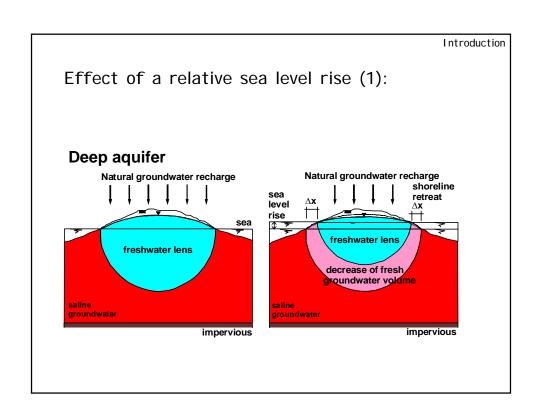


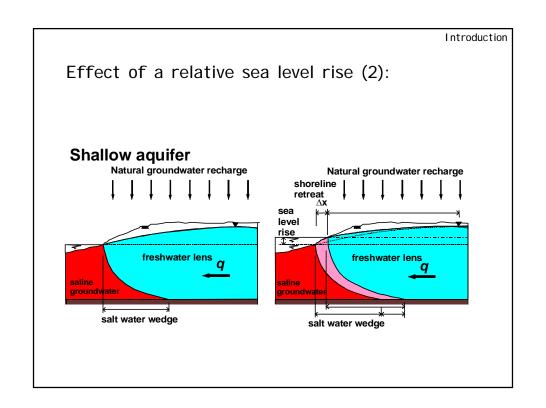


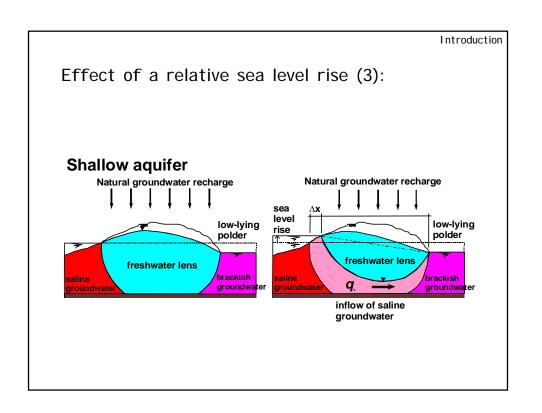


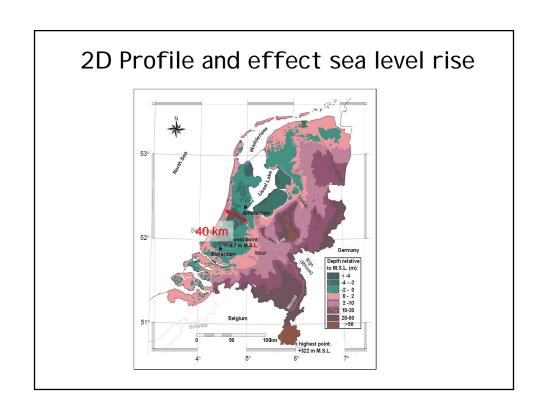
Implementing new KNMI 06 climate scenarios

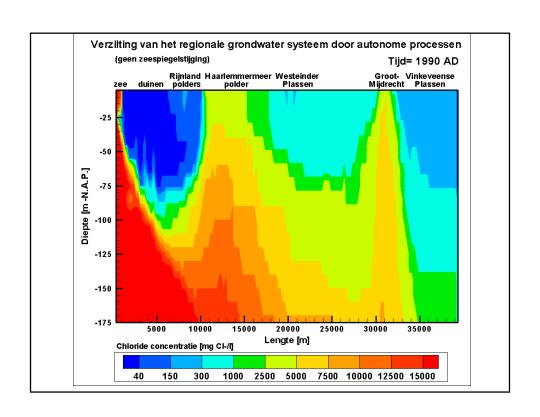
21	00	G	G+	W	W+	С	C+
Worldwide temperature rise in 2050		+1°C	+1°C	+2°C	+2°C	+3°C	+3°C
Worldwide temperature rise in 2100		+2°C	+2°C	+4°C	+4°C	+6°C	+6°C
Change in Europ	airstream pattern Western	no	yes	no	yes	no	yes
Winter	Average temperature	+1,8°C	+2,3°C	+3,6°C	+4,6°C	+5,4°C	+6,9°C
	Coldest winter day each year	+2,1°C	+2,9°C	+4,2°C	+5,8°C	+6,3°C	+7,8°C
	Average precipitation	7%	14%	14%	28%	21%	42%
Summer	Average temperature	+1,7°C	+2,8°C	+3,4°C	+5,6°C	+5,1°C	+8,4°C
	Hottest summer day each year	+2,1°C	+3,8°C	+4,2°C	+7,6°C	+6,3°C	+11,4° C
	Average precipitation	6%	-19%	12%	-38%	18%	-57%
Sea level rise	Absolute rise (cm)	35-60	35-60	40-85	40-85	45-110	45-110

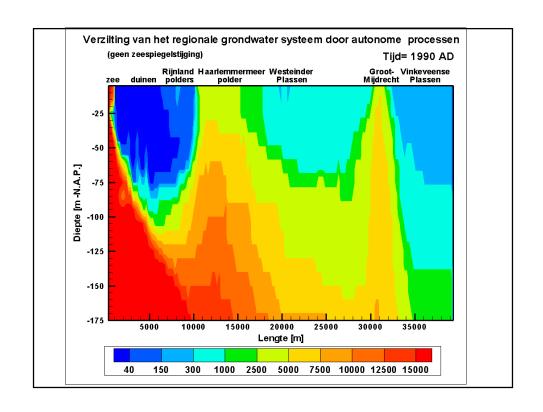


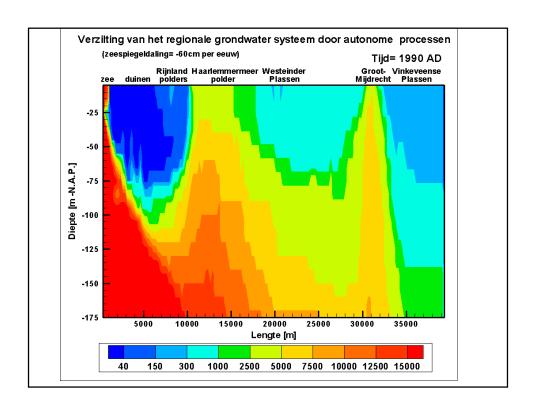


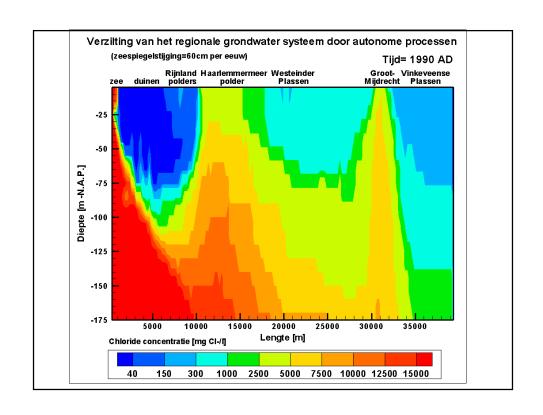


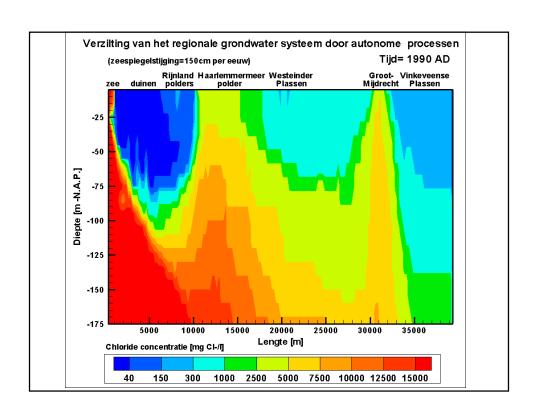




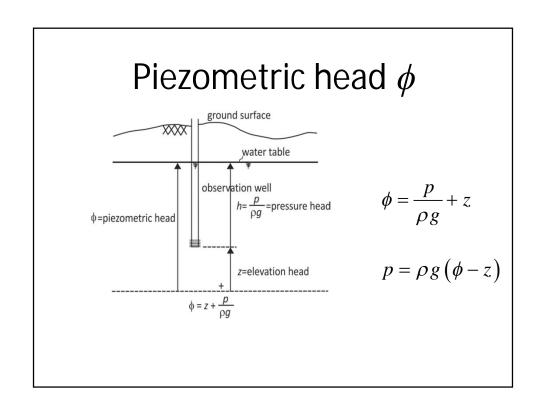








Point water head and Freshwater head ϕ_f



Freshwater head ϕ_f

$$\phi_f = \frac{p}{\rho_f g} + z$$

- 1. Groundwater with different densities can be compared
- 2. Fictive parameter
- 3. Hydrologists like to use heads instead of pressures
- 4. Pressure sometimes better
- 5. Confusing (heads not perpendicular to streamlines)

Freshwater head ϕ_f

$$h_f = \frac{\rho}{\rho_f} h$$

$$\phi_f = h_f + z$$

$$\phi_f = \frac{\rho}{\rho_f} h + z$$

e.g.: ρ_s =1025kg/m3 h=10m ϕ_i =10.25m

Special case: hydrostatic pressure:
$$q_z$$
=0

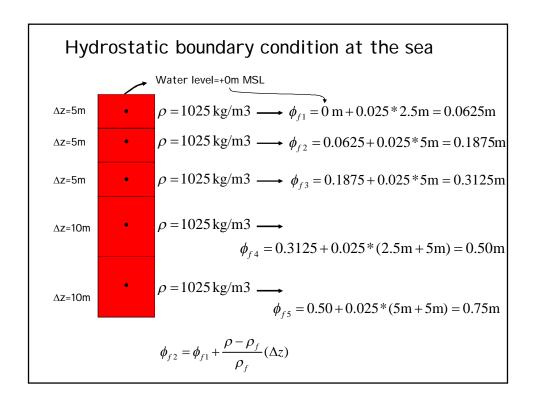
$$q_z = -\frac{\kappa_z \rho_f g}{\mu} \left(\frac{\partial \phi_f}{\partial z} + \frac{\rho - \rho_f}{\rho_f} \right)$$
 no vertical flow

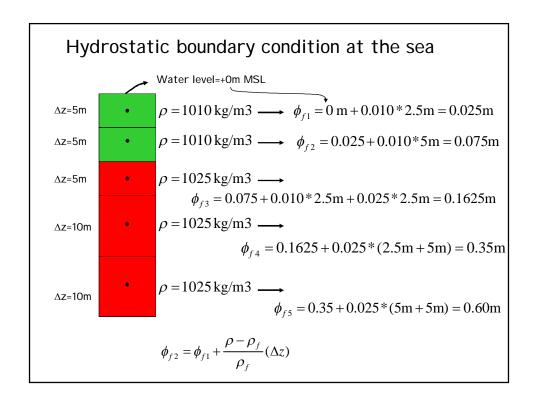
$$0 = \left(\frac{\partial \phi_f}{\partial z} + \frac{\rho - \rho_f}{\rho_f}\right)$$

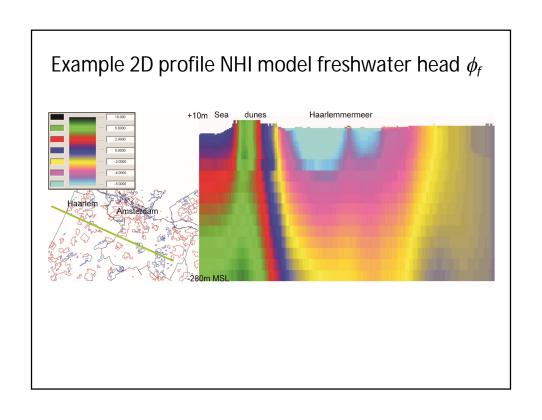
$$\partial \phi_f = -\frac{\rho - \rho_f}{\rho_f} \partial \mathbf{Z}$$

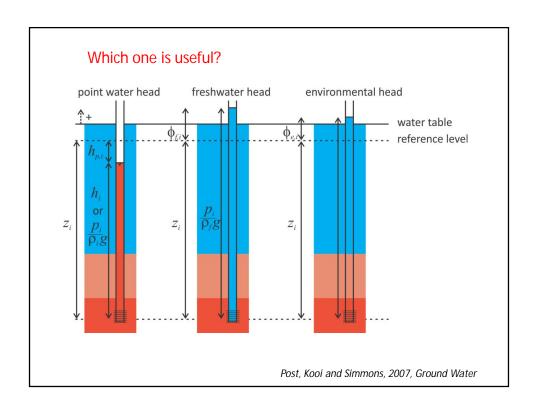
$$\phi_{f2} = \phi_{f1} - \frac{\rho - \rho_f}{\rho_f} (z2 - z1)$$

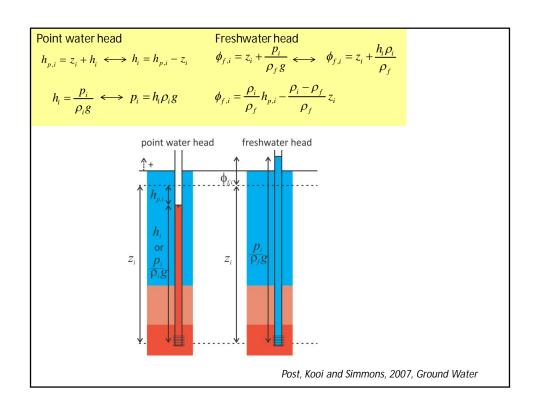
$$\downarrow + \qquad \phi_{f2} = \phi_{f1} + \frac{\rho - \rho_f}{\rho_f} (\Delta z)$$

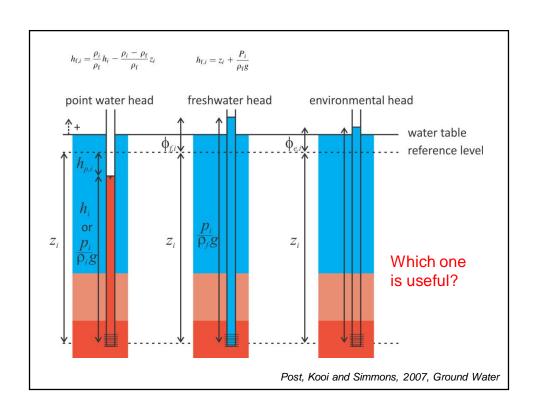


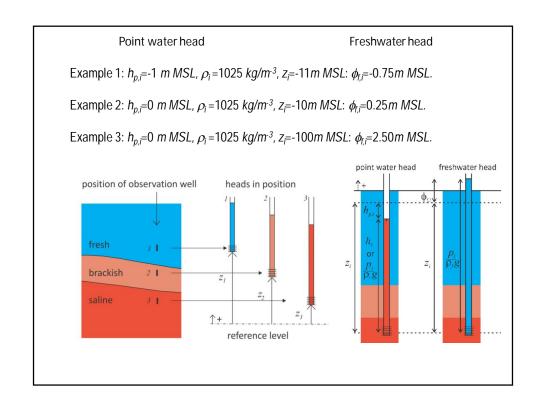


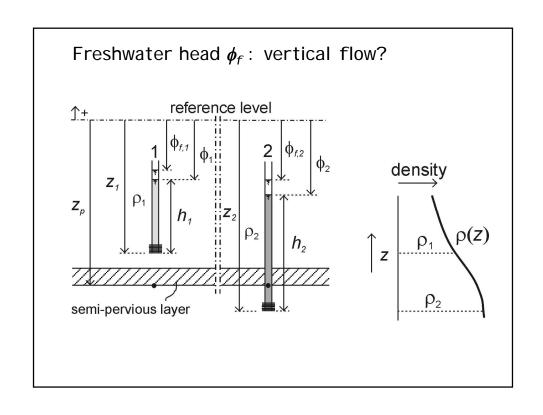


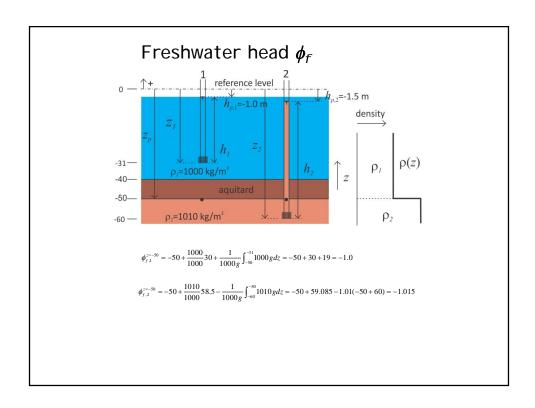


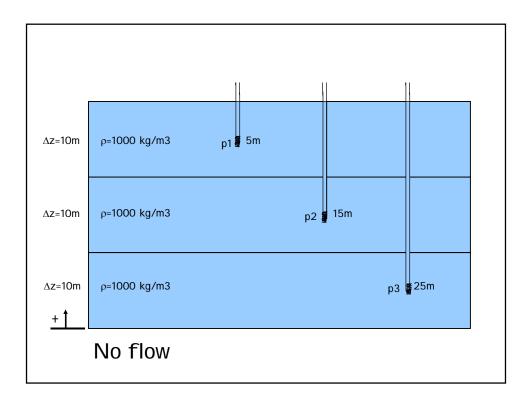


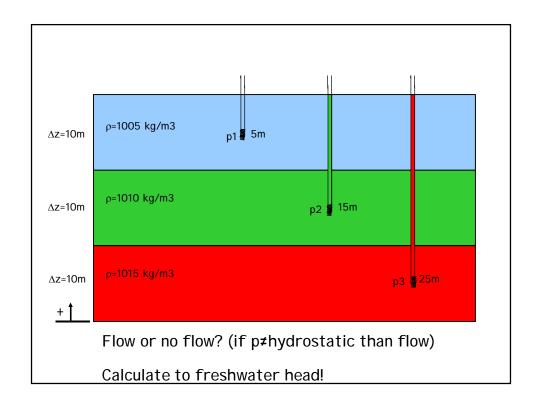


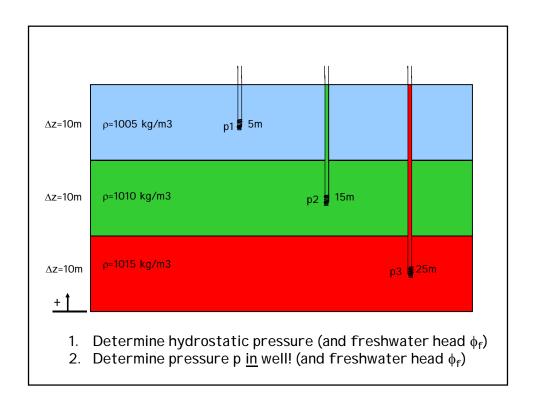


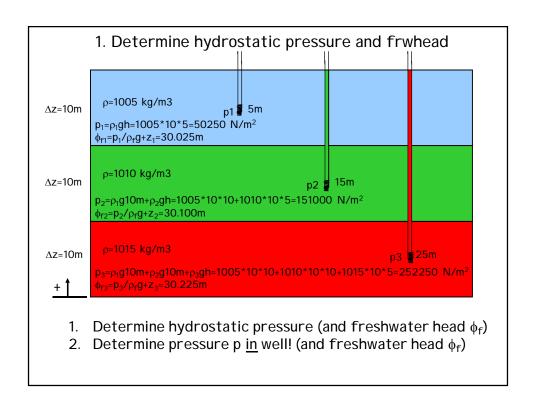


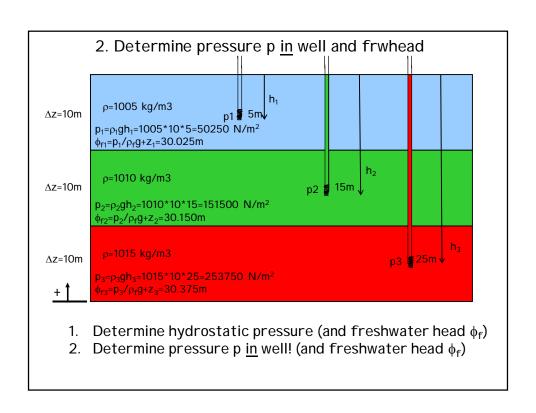


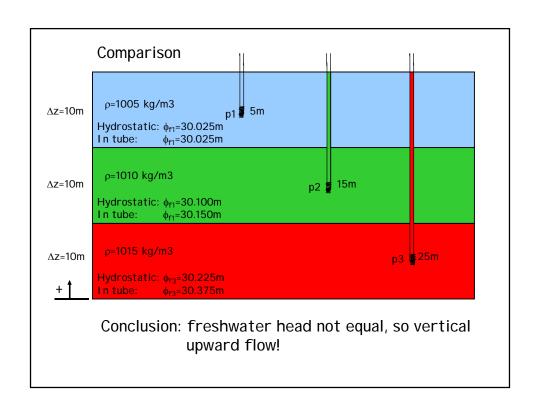


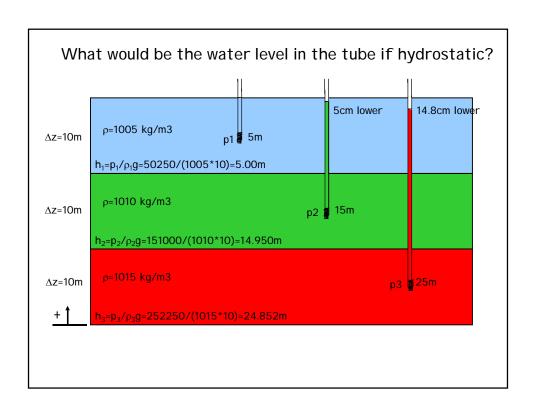


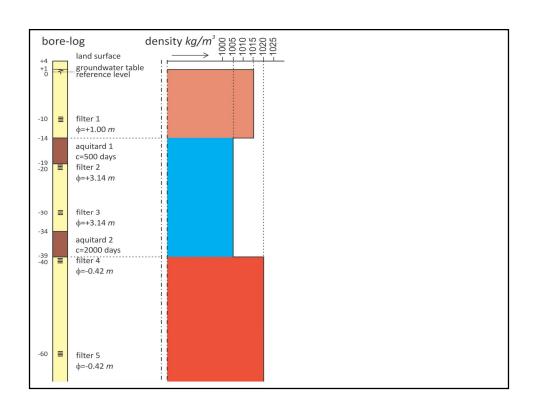








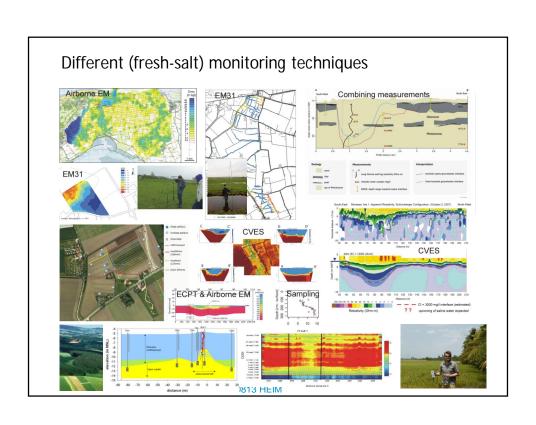




Take home message

- In coastal area (with fresh-brackish-saline groundwater), always measure head and Electrical Conductivity (EC)
- 2. Convert EC to density
- 3. Determine freshwater head with lecture notes and ppt
- 4. Determine flow

Monitoring

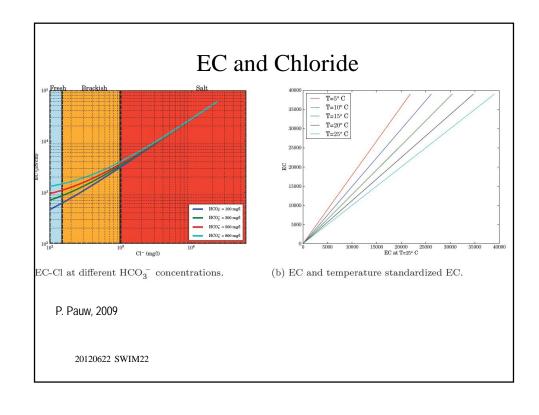


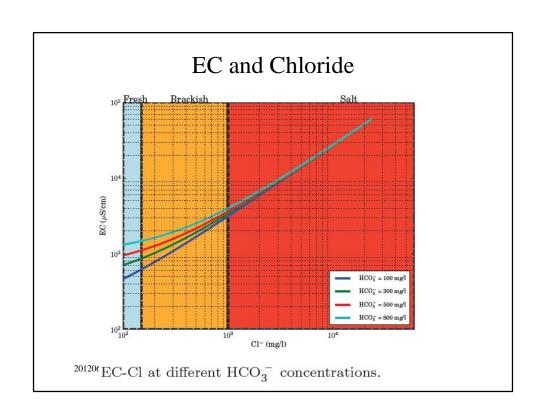
Monitoring salt in groundwater

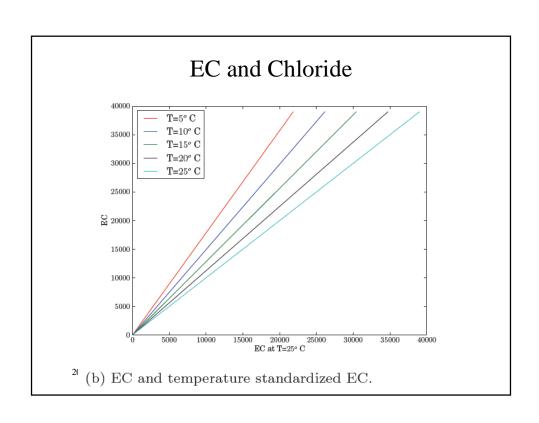
- Why monitoring?
 - Mapping salt concentrations in the groundwater
 - Detection of trends (upconing near pumping stations)
 - System and process knowledge
 - Input for a groundwater model
- Methods:
 - 1. Direct: water sample available
 - 2. Indirect: conductance of the subsoil

Pumping stations with salinisation

Pumping stations closed due to salinisation







Airborne measurements

Measuring system	Physical parameter	Geology/terrain information	
radar	EM traveltime	Terrain elevation	
Infrared photography	Infrared radiation	Surface temperature	
Time domain EM Frequency domain EM	Electr. resistivity from induced EM fields	Lithology Water salinity	
Magnetic gradiometer	Magnetic field (variations)	Lithology (magnetite) Artefacts Steel/Iron objects	
Spectral gamma	Radiation (gamma)	Soil type Surface lithology Recent disturbence	

Source: Koos Groer

Surface measurements

Measuring system	Physical parameter	Geology/terrain information	
Ground penetrating radar	EM traveltime, diaelectric constant,	Lithology Soil moisture	
ERT	Electr. resistivity	Lithology Water salinity	
Time domain EM Frequency domain EM	Electr. resistivity	Lithology Water salinity	
Magnetometer (total field, gradiometer)	Magnetic field (variations) magnetic susceptibility	Lithology (magnetite) Artefacts Steel/Iron objects (UXO)	
Spectral gamma	Radiation (gamma)	Soil type Surface lithology Recent disturbence	

99

Cone Penetration Tests

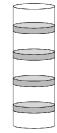
Measuring system	Physical parameter	Geology/terrain information
mechanical CPT	Cone resistance Friction resistance	Lithology Geotechnical parameters
Electrical conductivity	Electrical formation conductivity	Water salinity
Contnuous water pressure	Water pressure	Lithology Piezometric head
Water pressure dissipation in clay layers	Water pressure in time	Permeability clays
BAT sampling in CPT casing		Water chemistry
ROST, MIP		Contamination of hydrocarbons (high concentration)
Camera sonde	Visual view	Lithology, contamination, gas

__ _ ...__

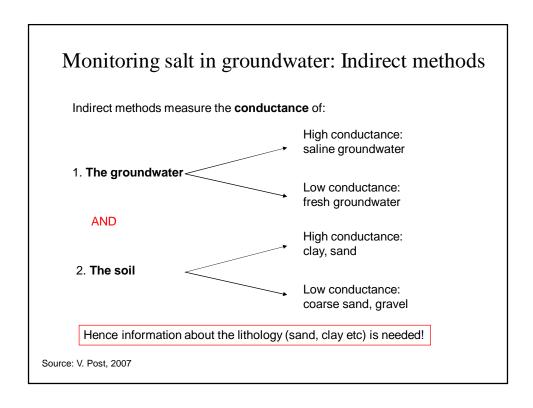
Monitoring salt in groundwater: Direct methods

Method	Advantage	Disadvantage
1. Observation well	•High accuracy •Detection trends	•Costly •Point measurement
2. Well screens in observation well	•High accuracy •Detection trends •High vertical resolution	•Costly
3. Sediment sample (extraction milliliters of water)	•High accuracy •High vertical resolution	•Very costly and time consuming





Direct methods 1 and 2



Monitoring salt in groundwater: Indirect methods

Method	Advantages	Disadvantages
1. Electrical conductance measurements	•High resolution (3D) •Depth ~200 m	•Time consuming
2. Electromagnetic measurements	•Fast	•Limited vertical resolution
		•Sensitive for underground conductors (pipes)
3. Satellites	•Suitable for large areas	•Small vertical resolution •Low accuracy

Method used at Deltares Number of measurements bottom Holocene top layer: direct methods and Vertical Electric Soundings (VES) Direct measurements Electrical conductance measurements Surface (VES) Borehole Number of measurements bottom Holocene top layer: direct methods and Vertical Electric Soundings (VES) Source: Oute Essink et al (2005)

Electrical conductance measurements

- 1. Measuring:
 - Inside a borehole
 - From surface level
 - From the air



Source: TNO

Electrical conductance measurements

1. Measuring:

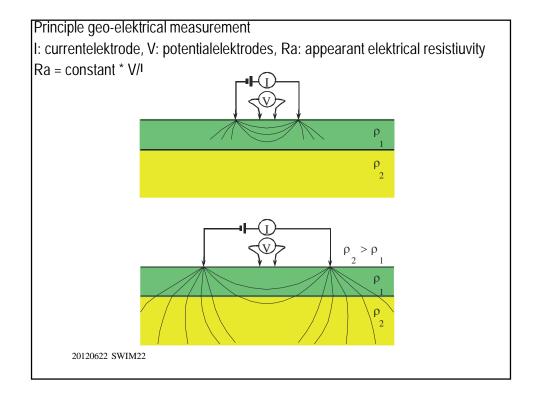
- Inside a borehole
- From surface level (depth ~ 200 m)
- From the air

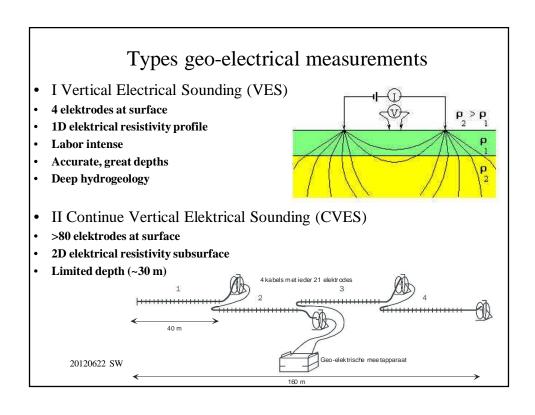


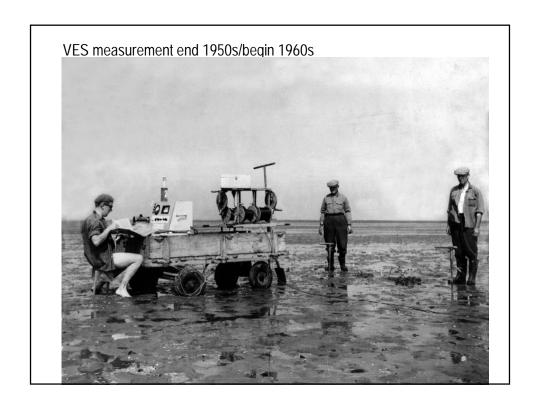


Source: Vitens

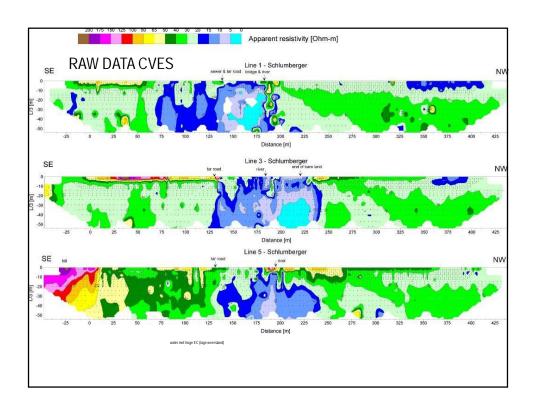
Source: V. Post, 2007

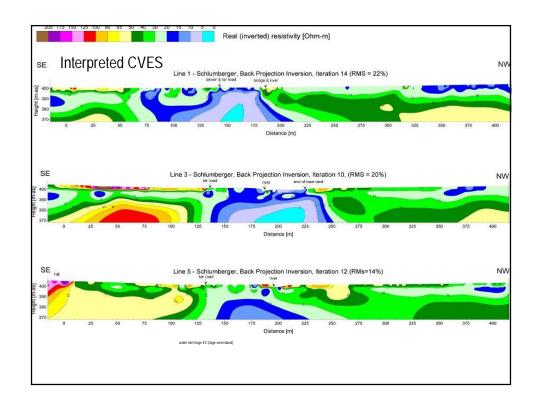


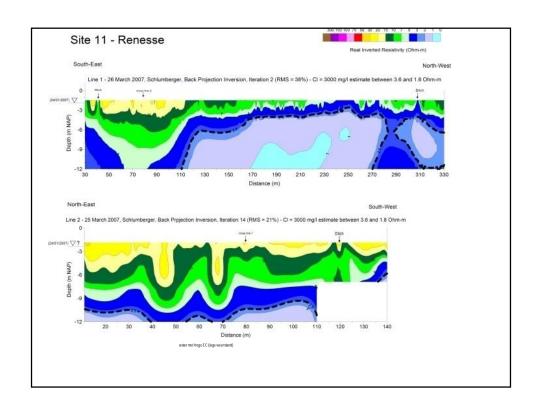


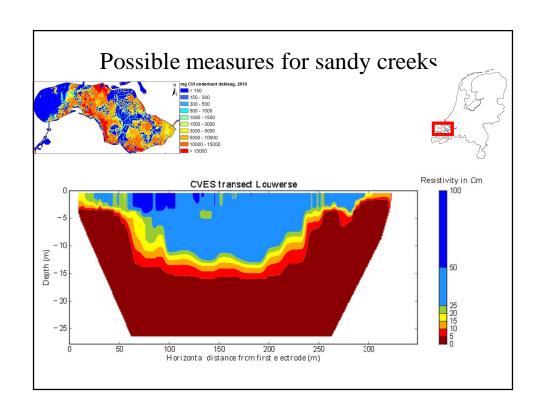


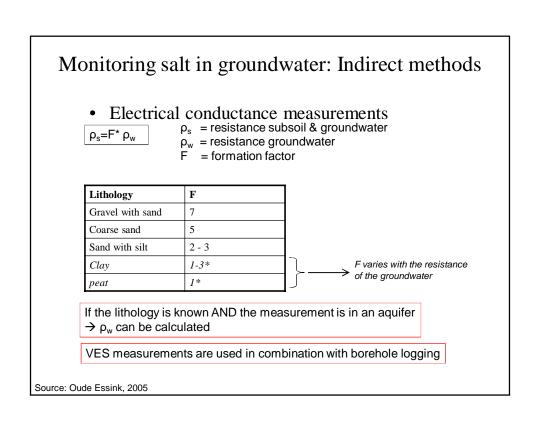




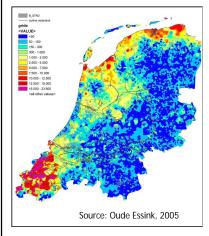








Result: chloride concentration bottom Holocene toplayer



- Software Geological Survey of the Netherlands (TNO) is used to determine the salt concentration of the groundwater in the measurements
- Inter- and extrapolation is used to make a continuous field
- 2D Result is an combination of:

 - Direct measurements (3500)
 Electrical conductance in boreholes (2000)
 - 3. Vertical Electric Sounding (VES) measurements (10.000)

T-EC probe





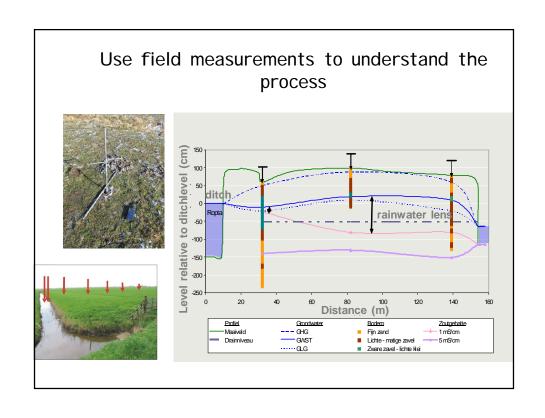


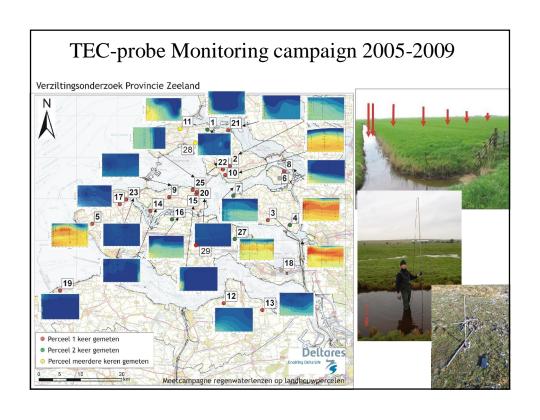








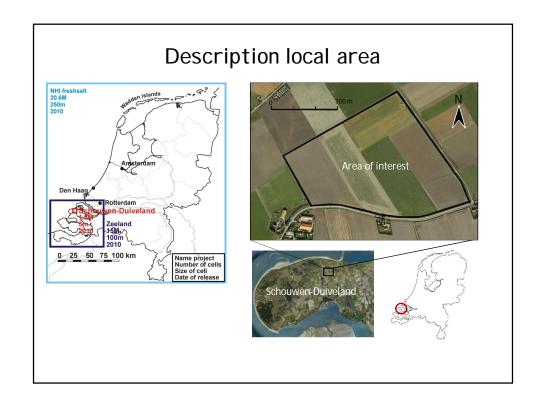


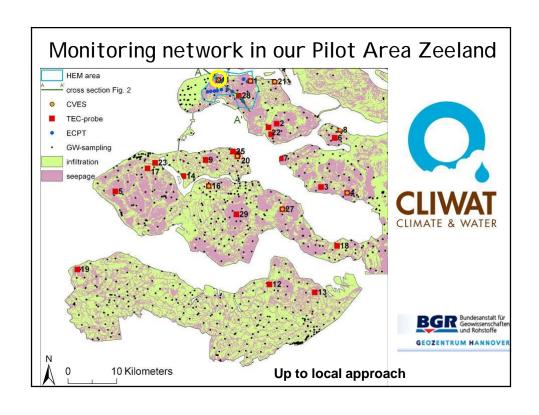


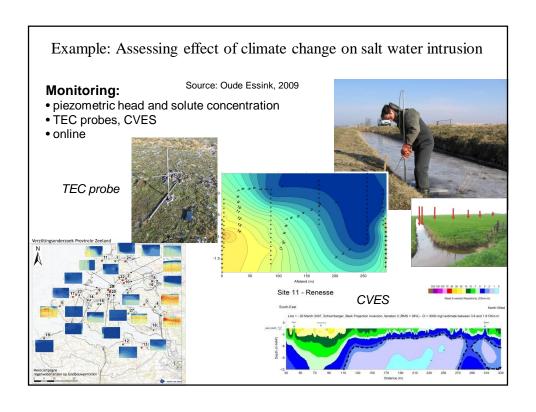
CliWat www.cliwat.eu

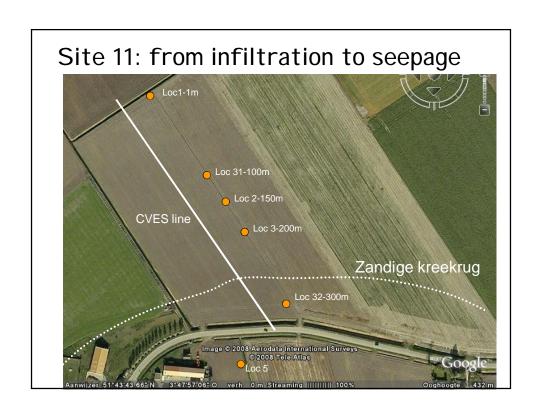
- Transnational project in the North Sea Region
- Main objectives:
 - to evaluate the physical and chemical impacts of climate change on groundwater and surface water systems
 - to provide data for adaptive and sustainable water management and infrastructure.
- Different innovative monitoring techniques (Helicopter EM, CVES, CPT, TEC-probe) are used to map the salinization status of the coastal groundwater system.

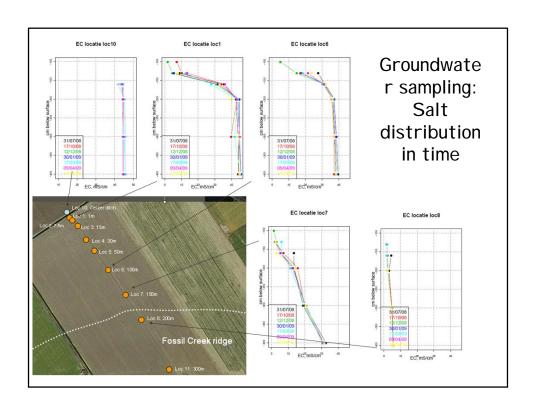


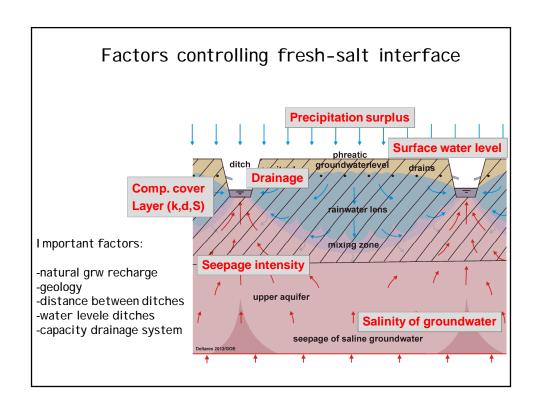


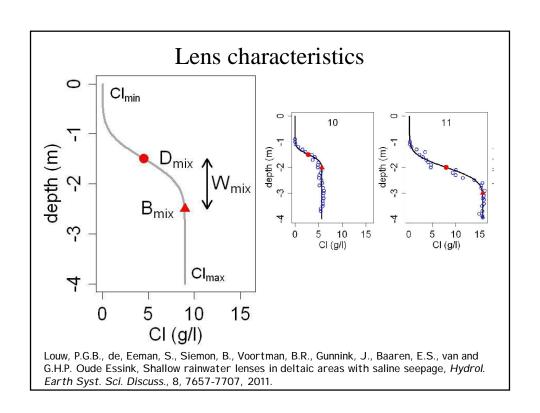


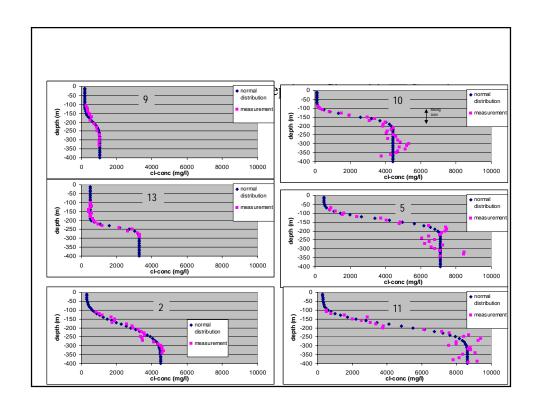


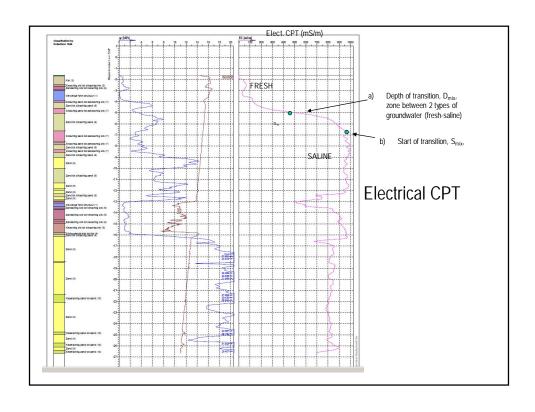


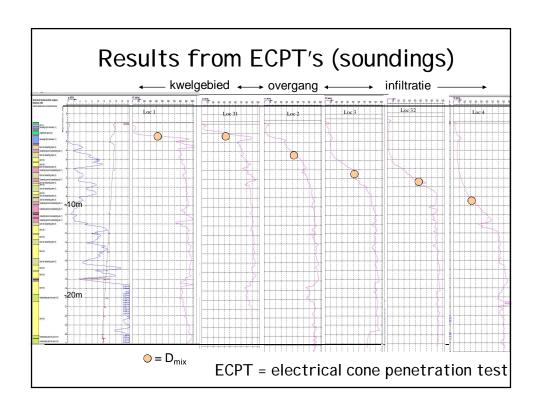


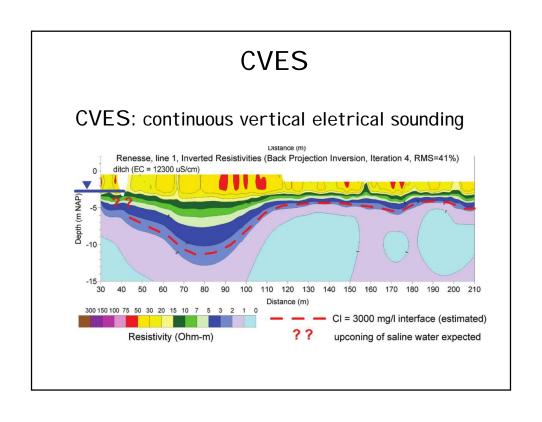


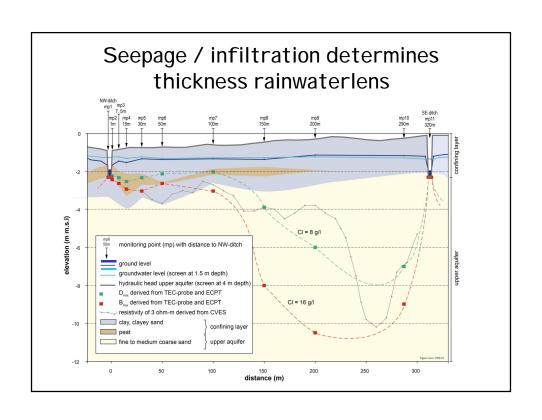


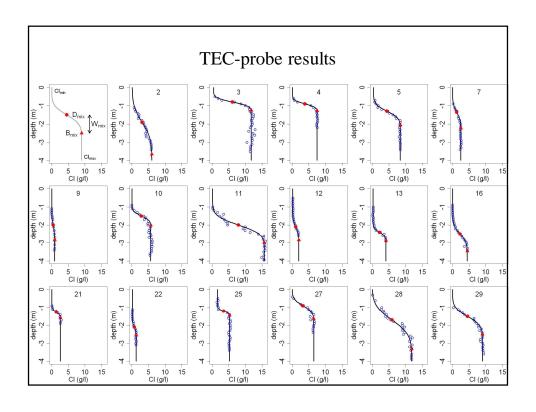


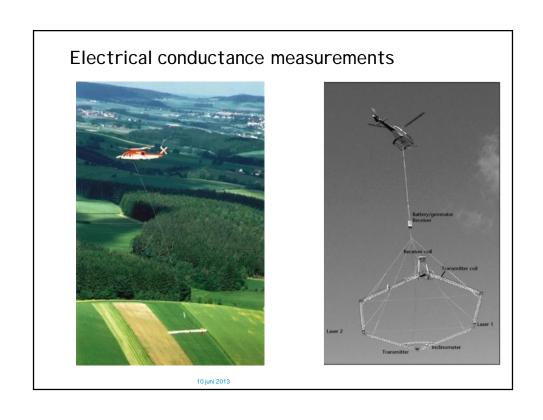


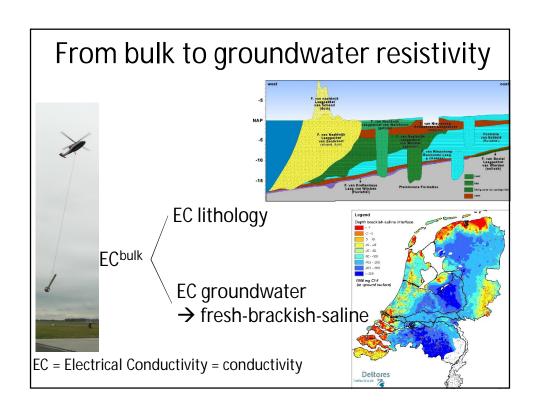


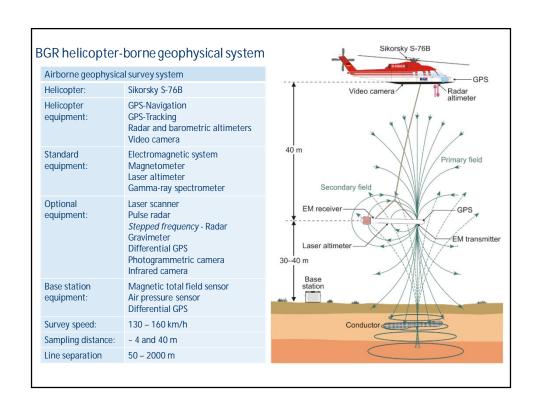


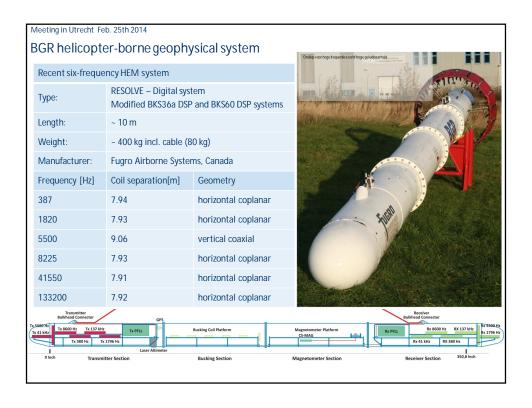


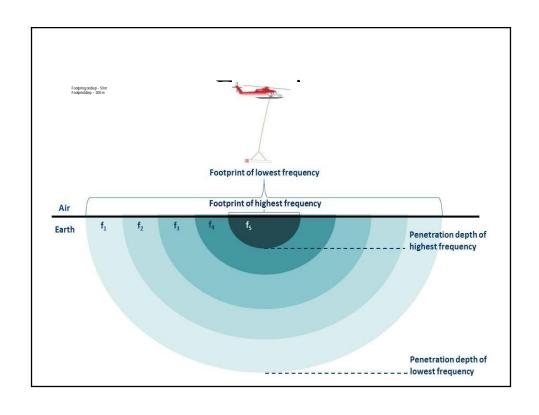


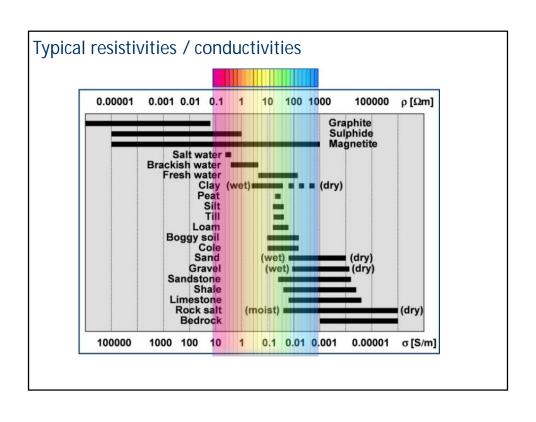


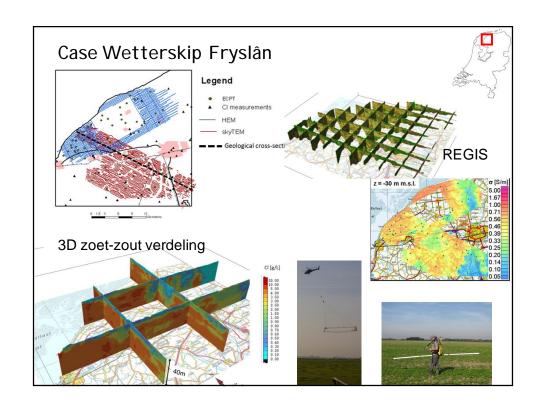


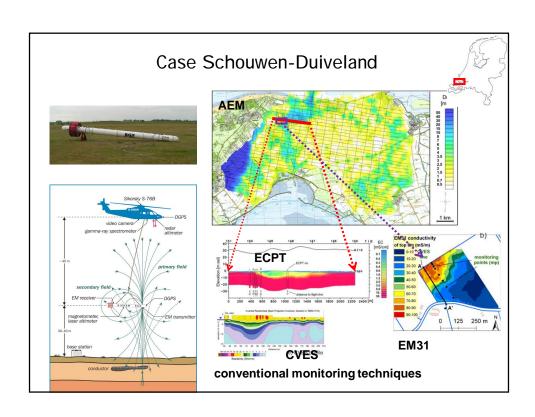


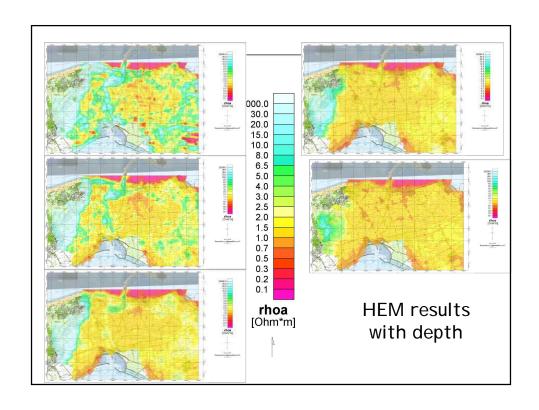


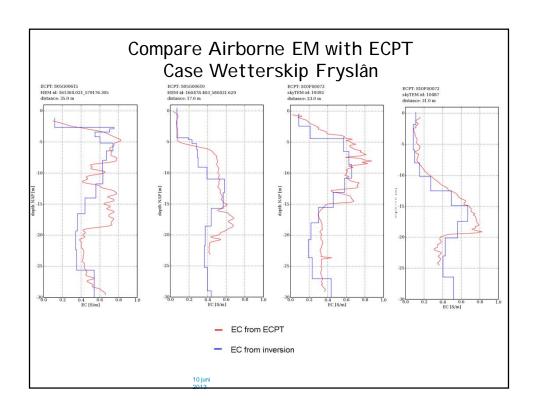


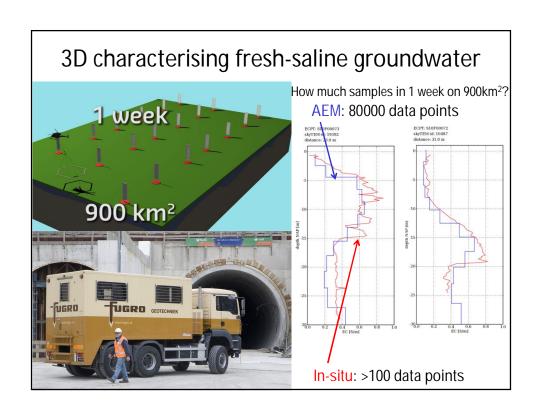


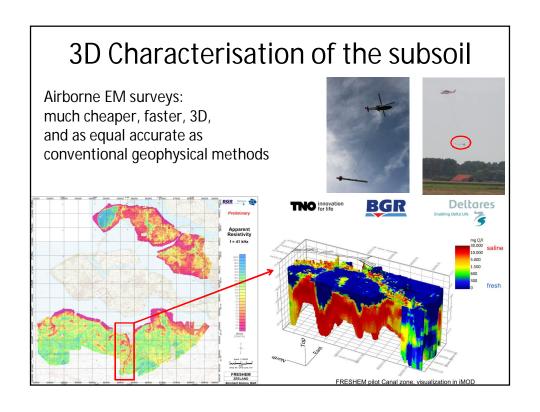


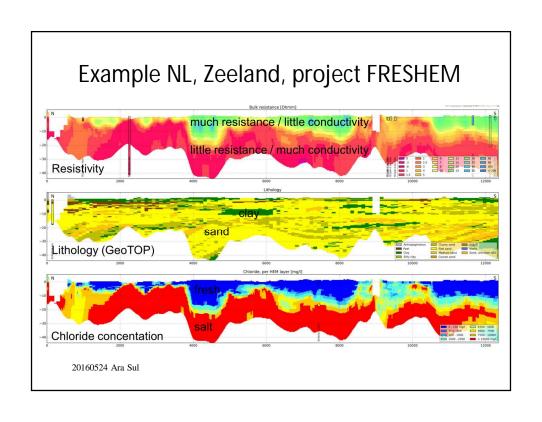


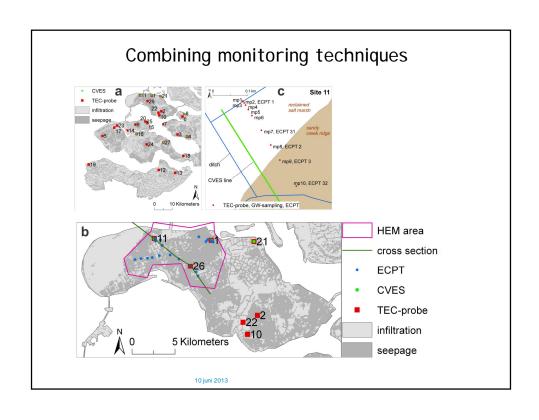


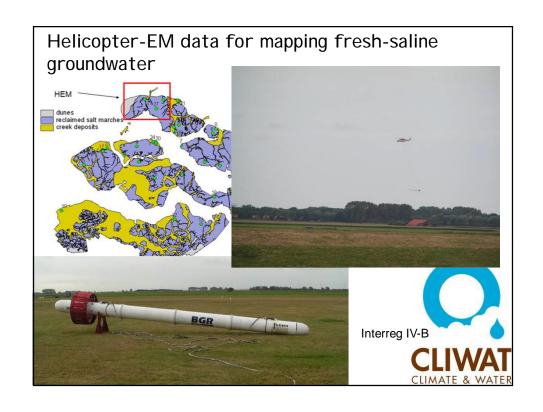


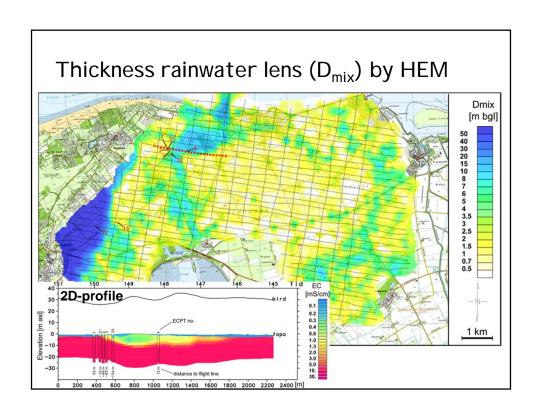


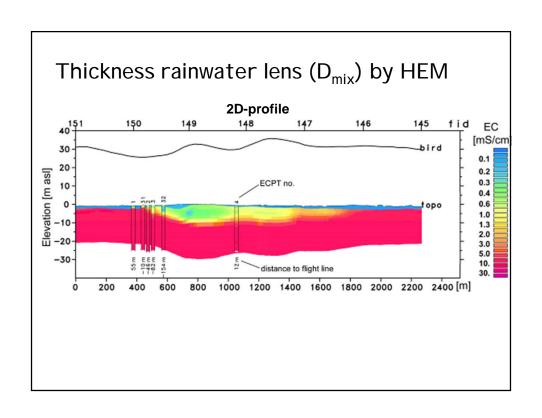


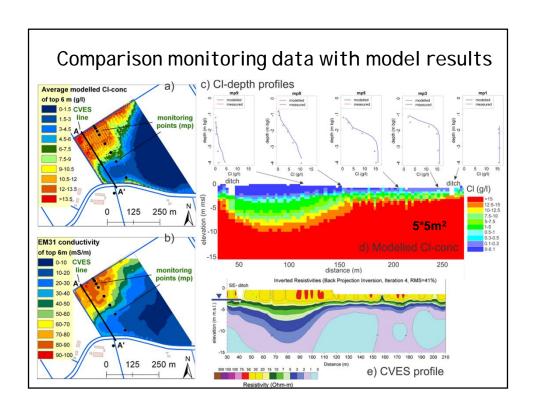


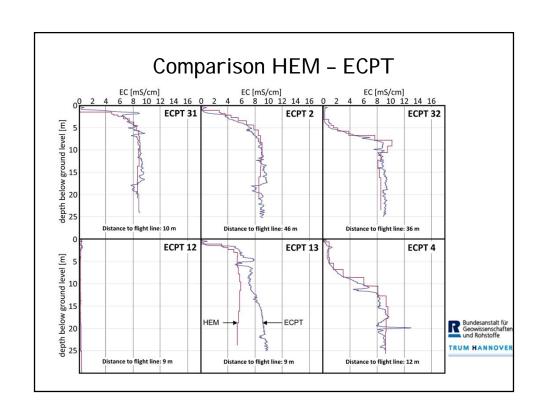


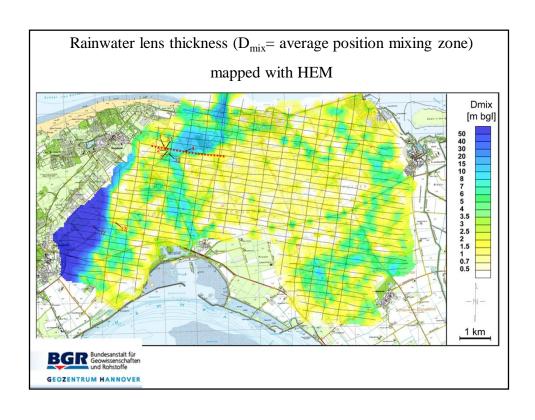


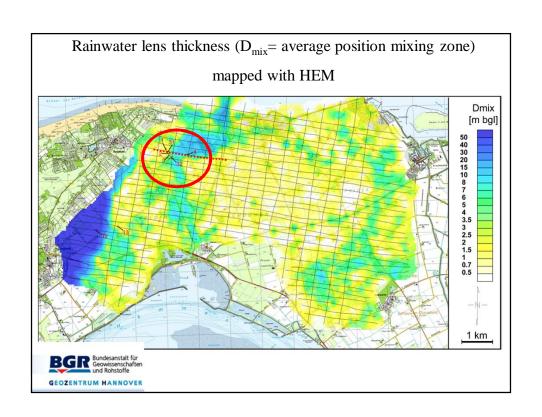






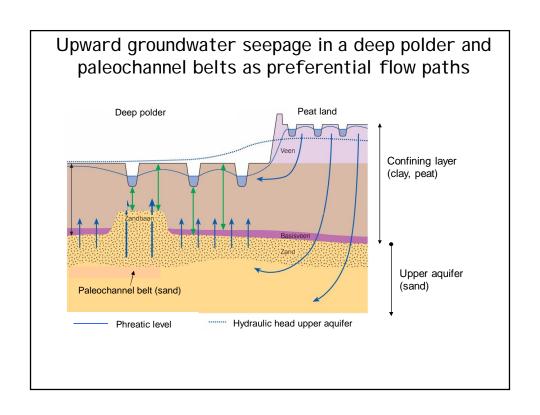


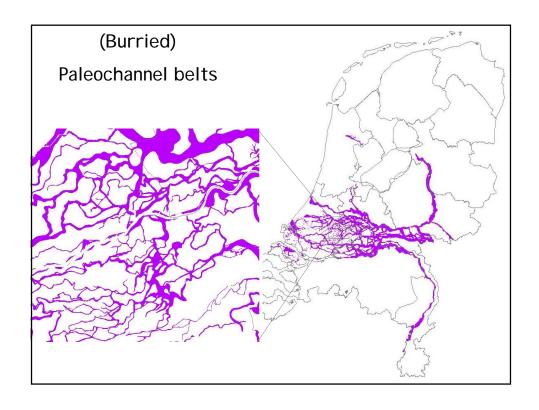


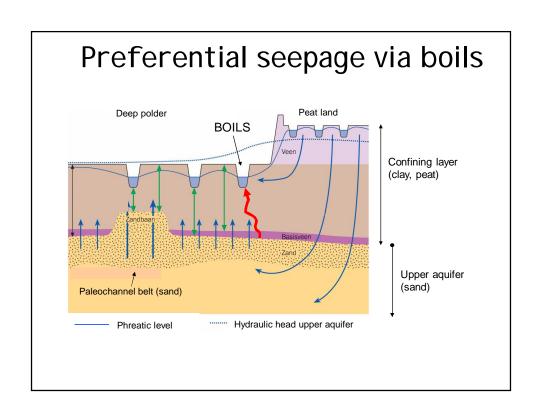


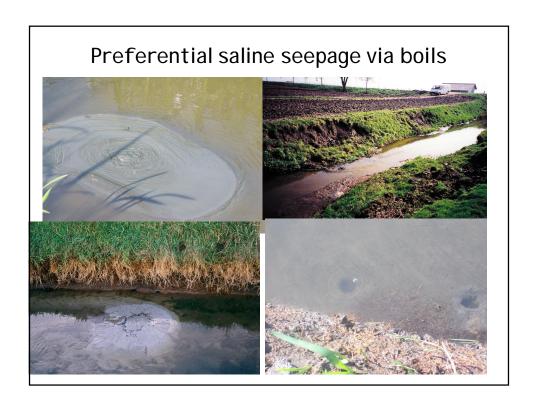
Salty boils

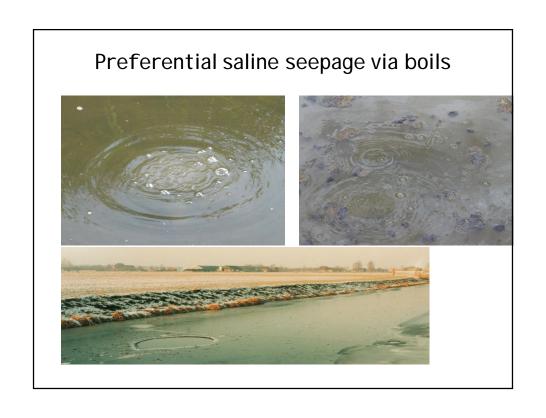
Louw, P.G.B., de, Oude Essink, G.H.P., Stuyfzand, P.J., Zee, van der, S.E.A.T.M., 2010, Upward groundwater flow in boils as the dominant mechanism of salinization in deep polders, The Netherlands, J. Hydrol. 394, 494-506.

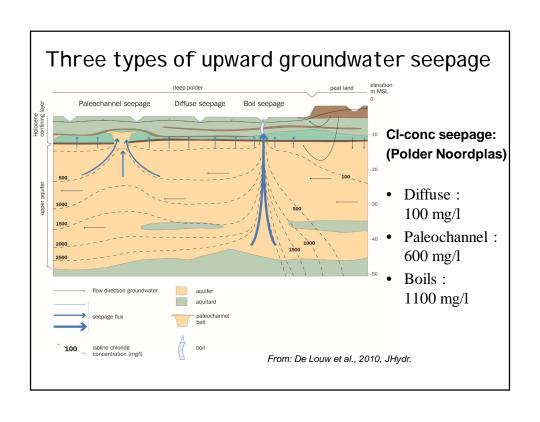








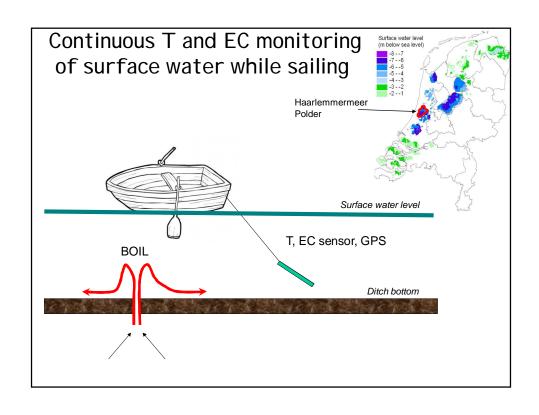


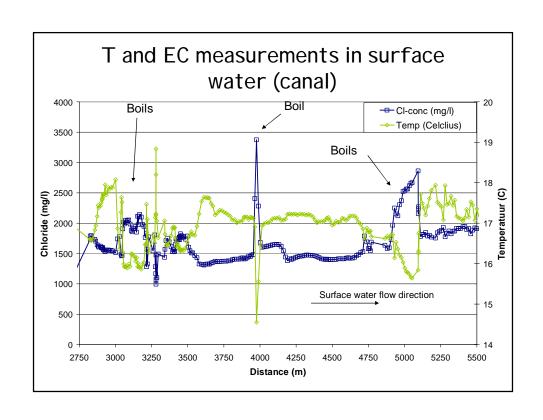


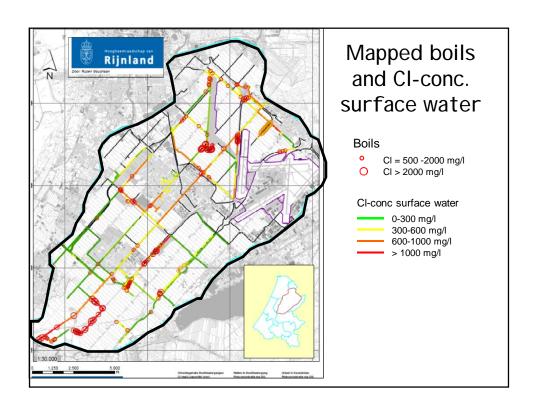












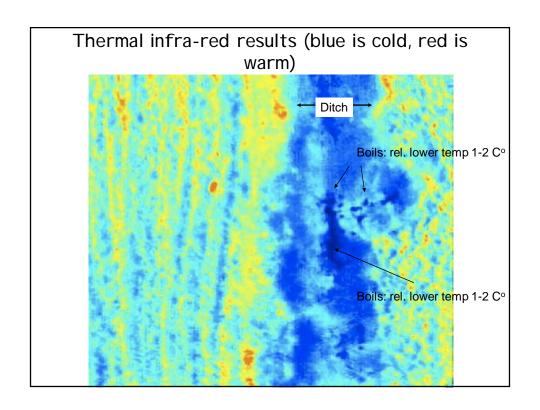
LARS technology (TNO Industry): <u>Thermal Infra-red</u>

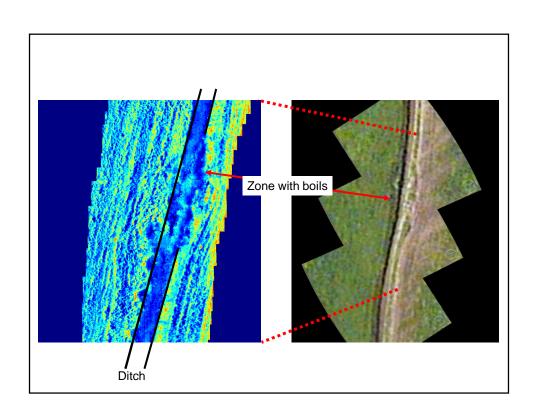
• Altitude: 0-150 m

 Temp-detection using Thermal Infra Red sensors (only surface!)

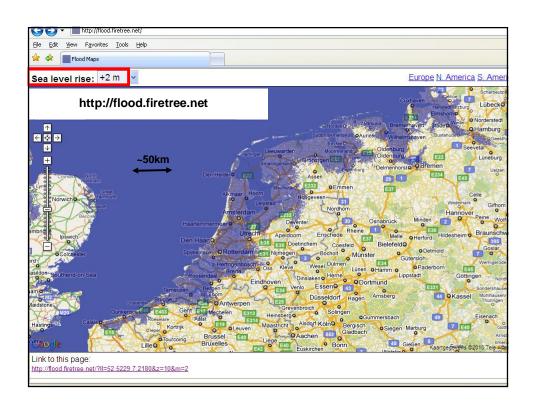


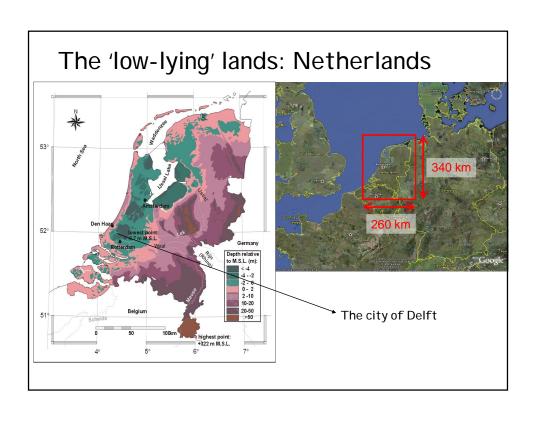






To get an idea about the possible future effects of sea level rise and climate change in your delta
evaluate of the past water management in the Dutch delta
Salt water intrusion in the Netherlands





Case study: The Netherlands

The Dutch coastal zone is already theathened by sea level rise and land subsidence for many centuries

Intensive water management system

Coping with salt water intrusion problems since 1950's

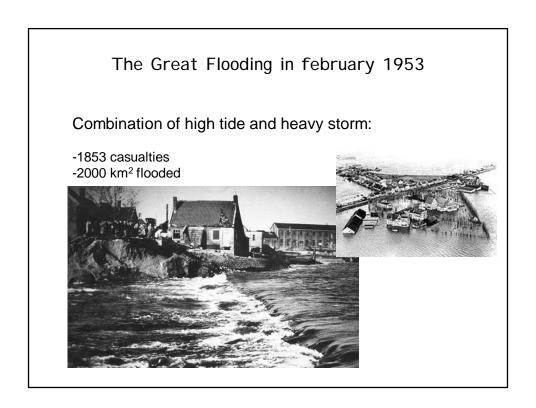


The 'low-lying' lands: Netherlands

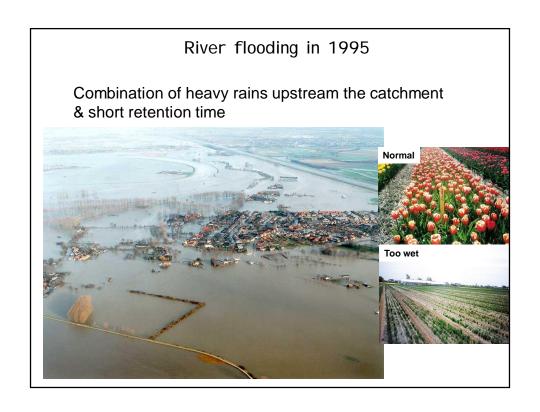
The facts:

- a deltaic area with 3 rivers: Meuse, Scheldt & Rhine
- 25% of land surface is lying below mean sea level
- 65 % would be flooded regularly if there were no dunes and dikes
- 8 million people would be endangered









Dike collapse 2003

Combination of peat dike instability and very dry summer

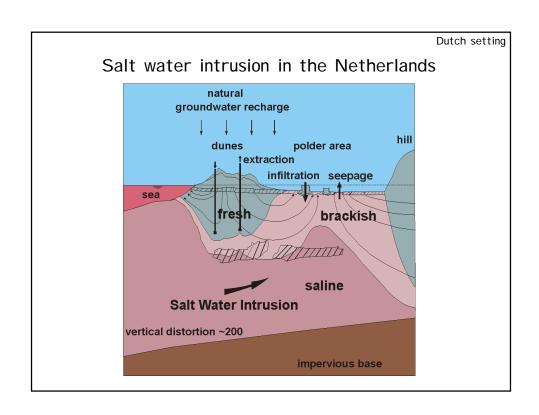
Estimated water management costs 'to keep our feet dry'

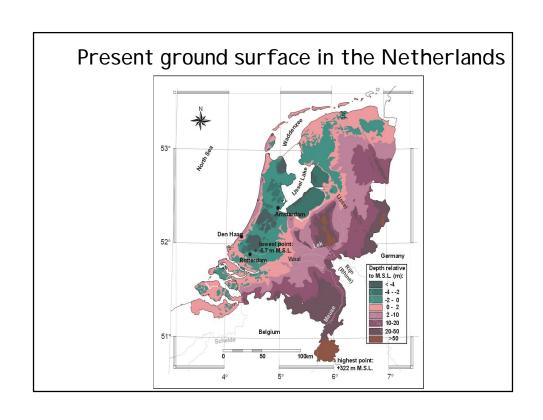
Costs up till 2050 in billion euros:

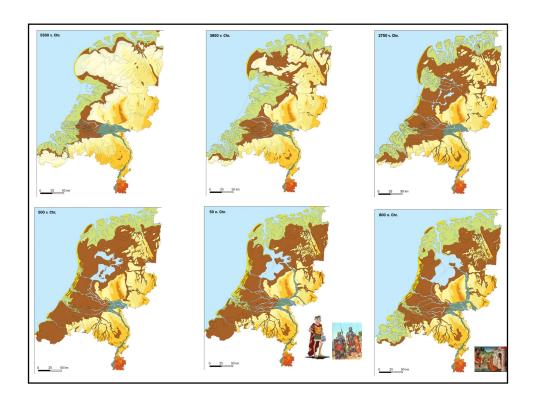
rivers: upper part	5.7
rivers: lower part	5.6
low-lands	1.7
coastal zone	8.0

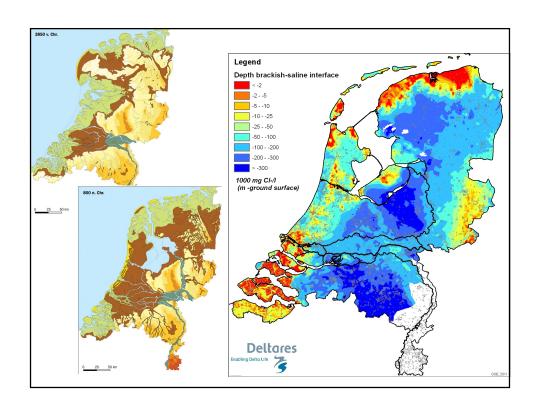
infrastructure 3.5 purchase of ground 2.0

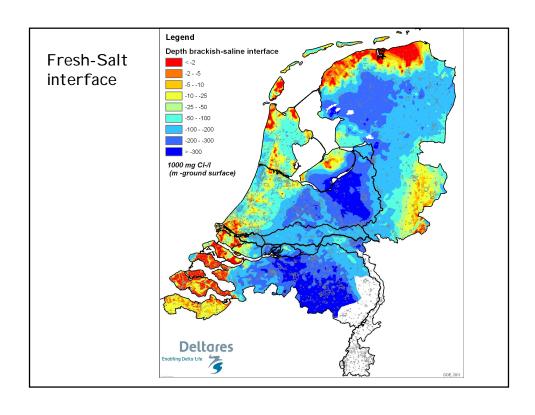
26.5 billion euros

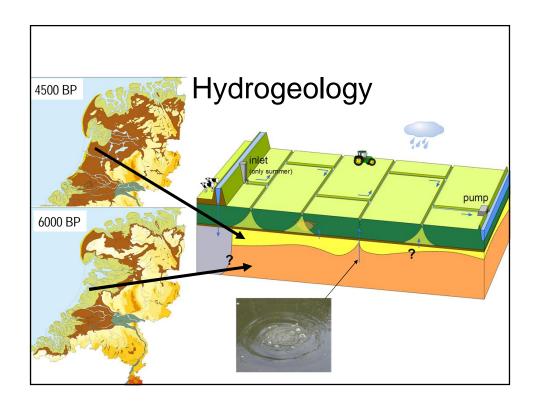












Dutch setting

Salinisation of the Dutch subsurface

Physical transport processes:

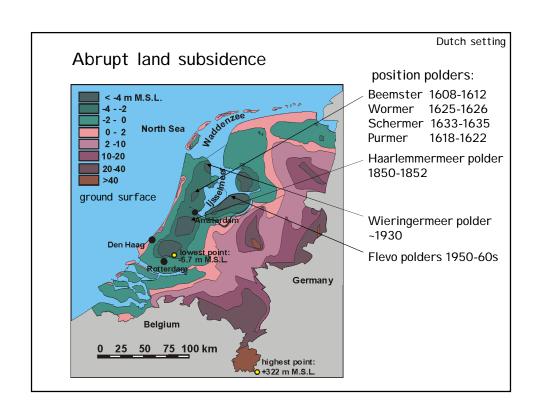
- advective: e.g. trans- and regressions
- dispersive: mixing with marine deposits
- diffusive: e.g. I Jsselmeer lake
- chemical: solution, precipitation, ion-exchange

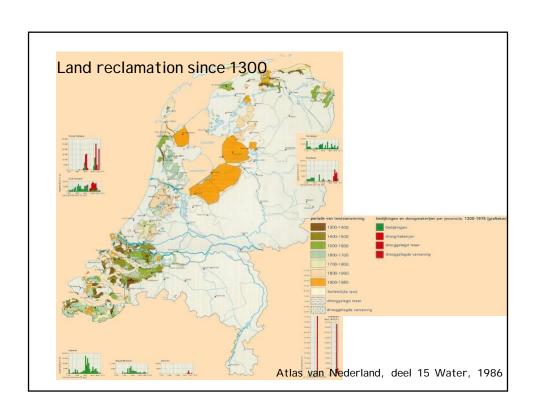
Anthropogenic causes:

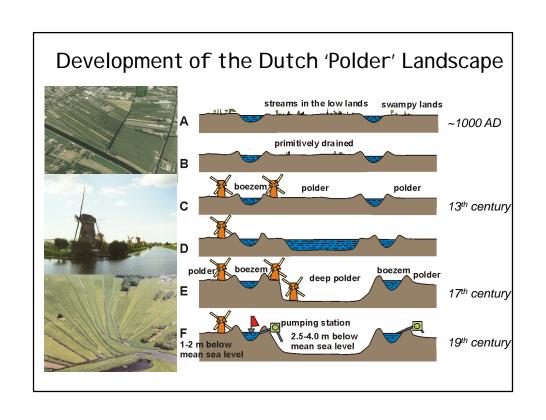
- land subsidence
- polder level lowering
- groundwater extractions

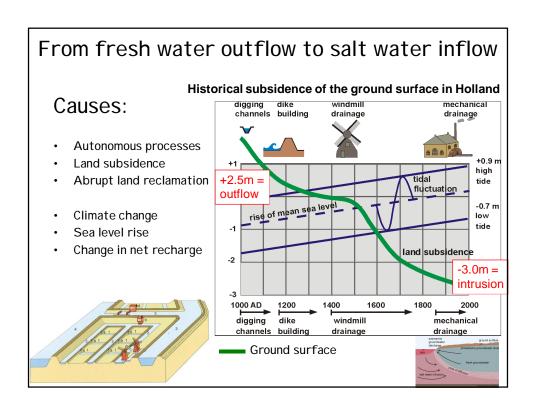
Future developments (climate change):

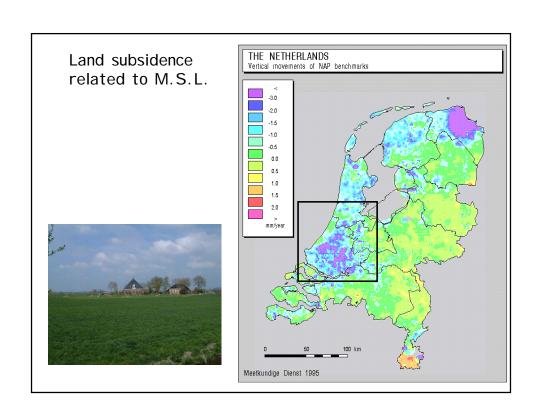
- sea level rise
- changes in recharge

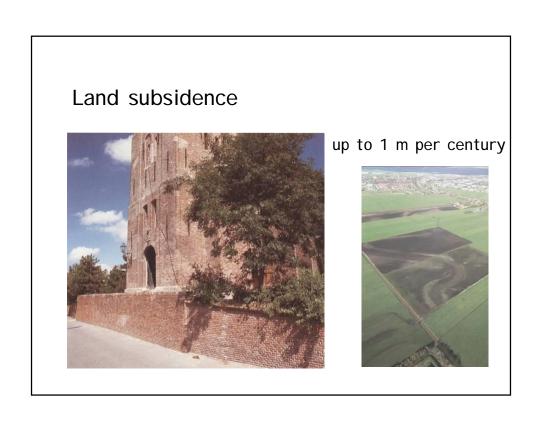


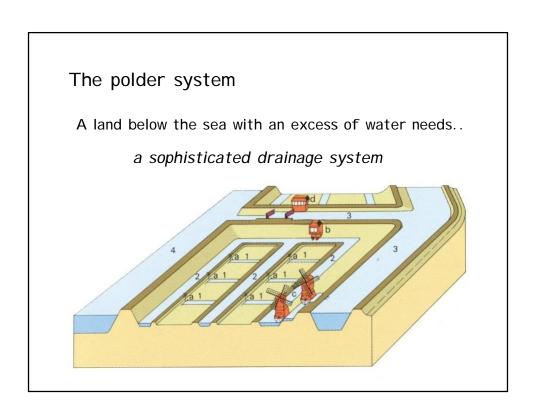


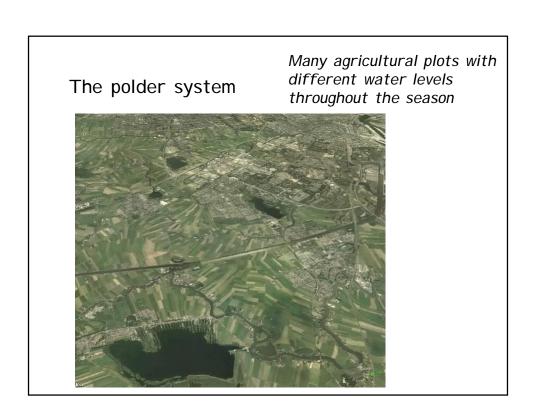


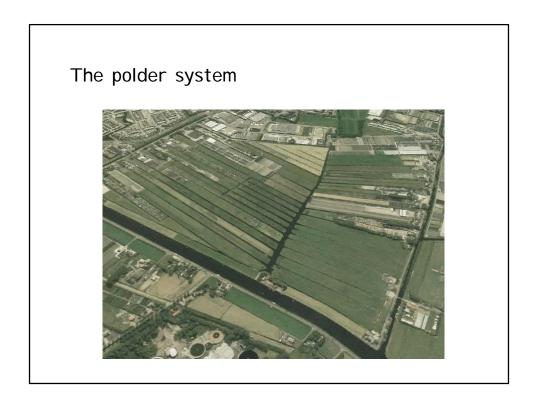


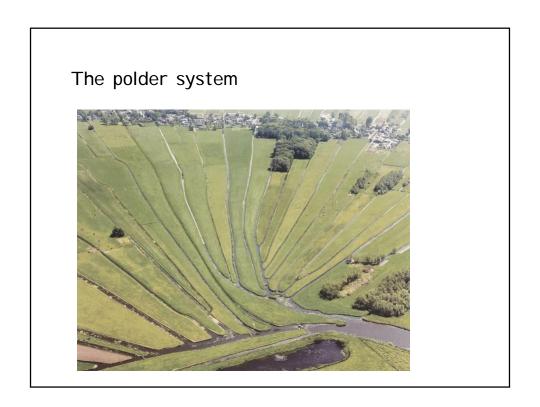


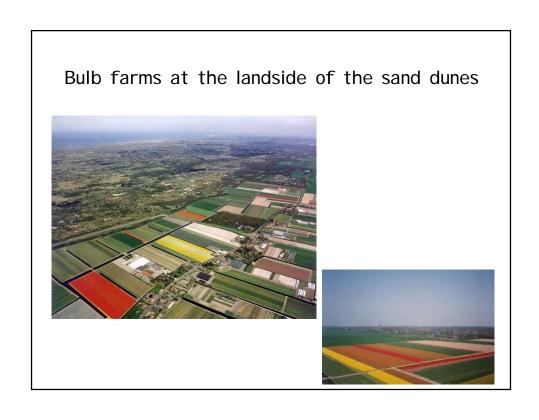


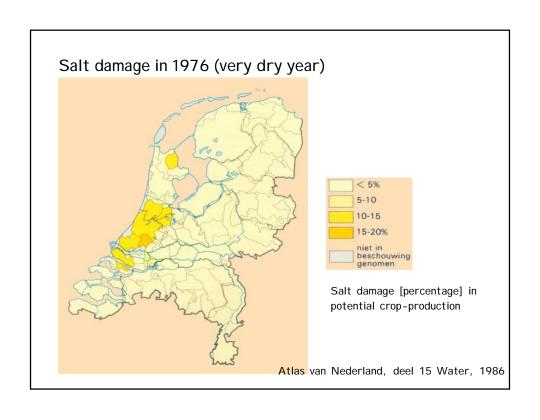


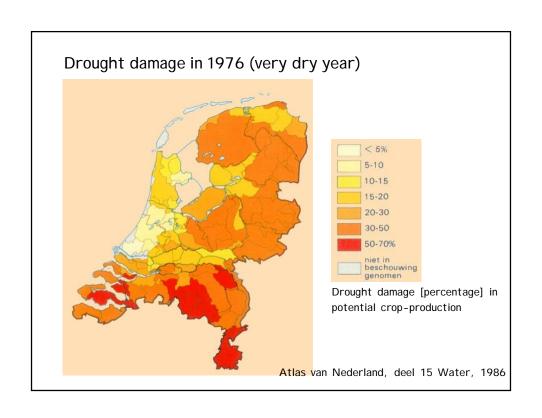


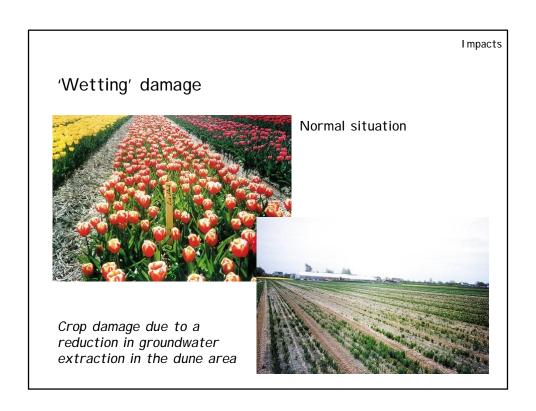


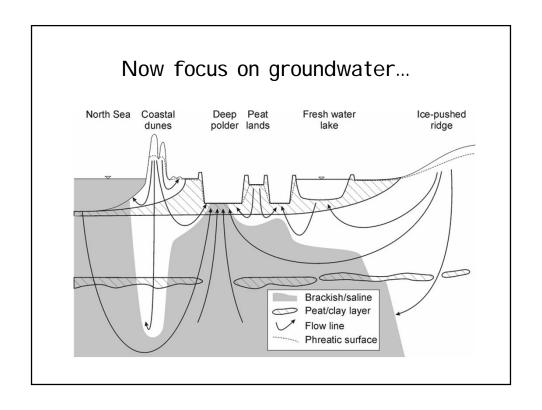












Threats to water management due to climate change:

Short term threats:

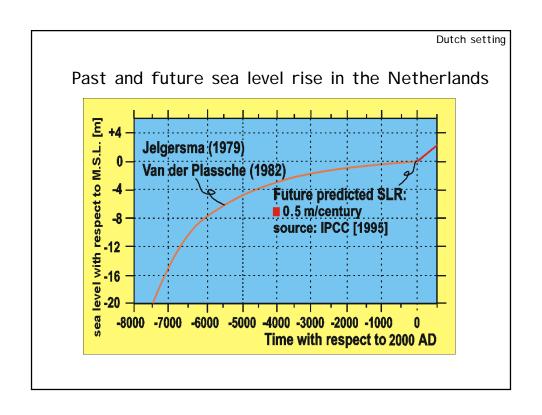
- -flooding
- -dike collapse
- -drought

asks for operational water management

Long term threats:

- -salt water intrusion
- -land subsidence
- -smaller fresh groundwater resources

asks for strategic water management



Numerical variable density models at Deltares

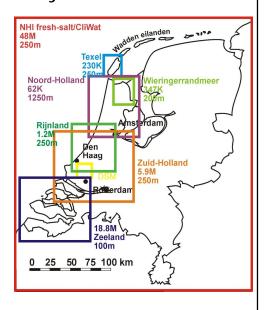
Characteristics:

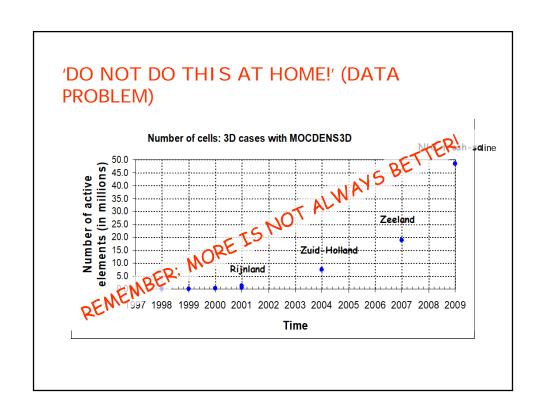
- variable-density groundwater
- · fresh, brackish and saline
- · 3D, non-steady
- coupled solute transport

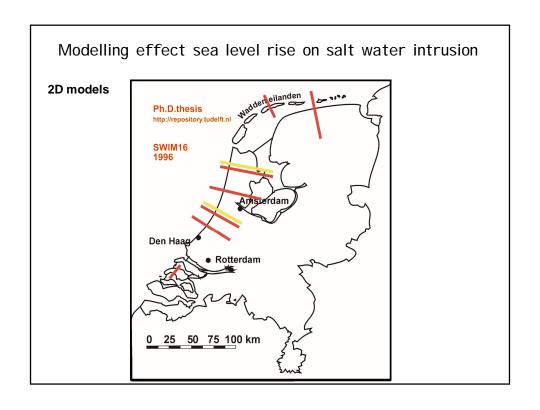
Code (MODFLOW family): MOCDENS3D SEAWAT

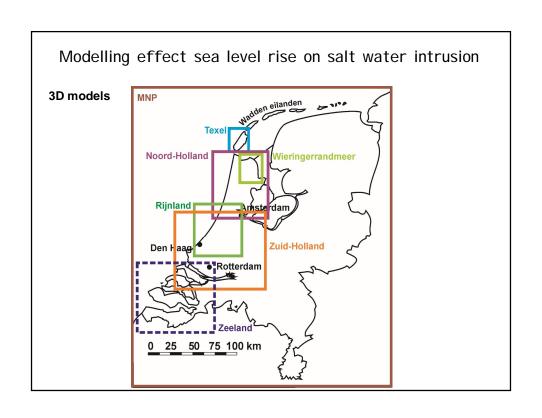
Assessing effects:

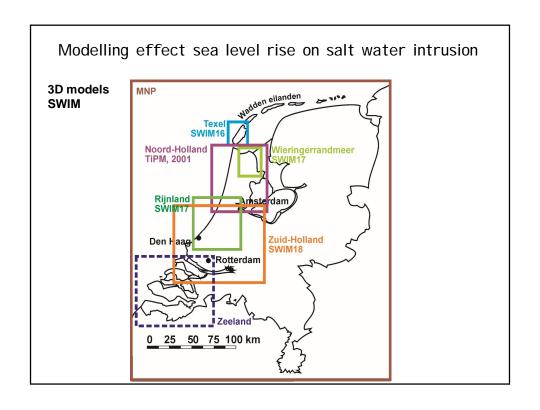
- autonomous salinisation
- · sea level rise
- · changing recharge pattern
- land subsidence
- changing extraction rates
- adaption measures

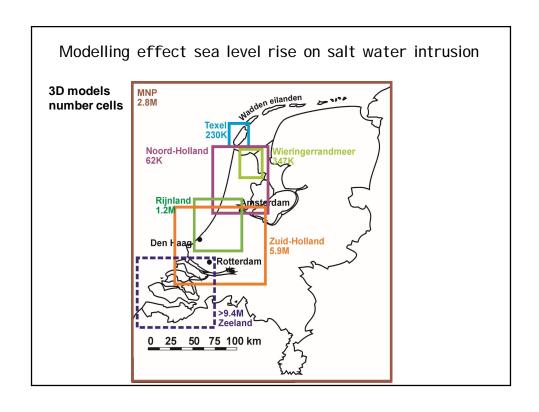








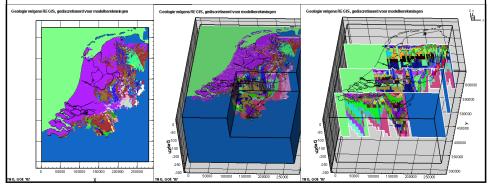




Recent model study for the whole Netherlands on the effect of sea level rise of water management (1)

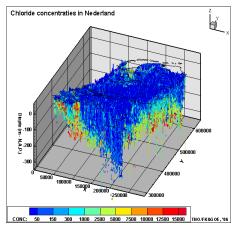
Using the national subsoil parametrisation

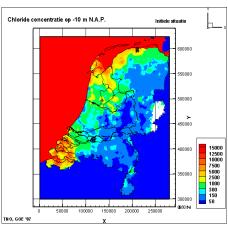
- REGIS V2
- Top geological system from +10m up to -280m M.S.L.
- 31 modellayers with thicknesses: 2*5m; 10*2m; 8*5m en 11*20m
- cellsize 1000x1000m (coarse)



Recent model study for the whole Netherlands on the effect of sea level rise of water management (2)

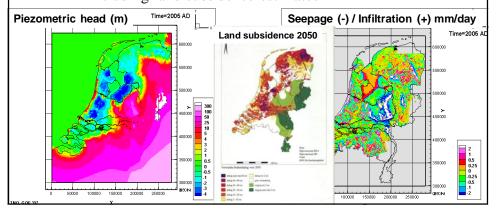
Using the national 3D salt concentration in groundwater Zoet-Zout REGIS: ~65000 measuring points (analyses, VES, Borehole)

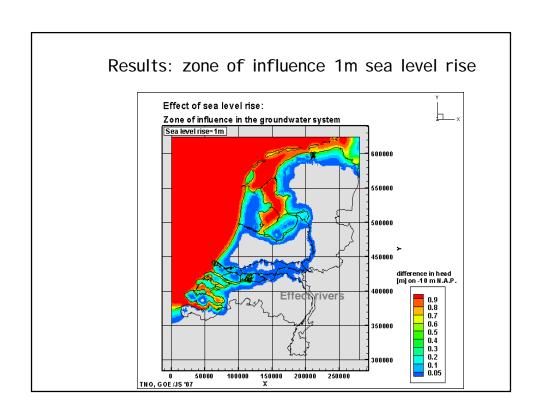


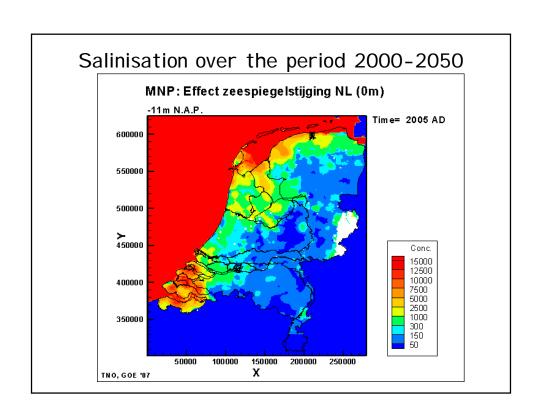


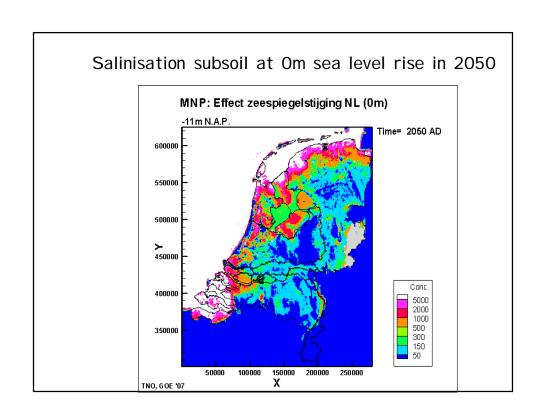
Recent model study for the whole Netherlands on the effect of sea level rise of water management (3)

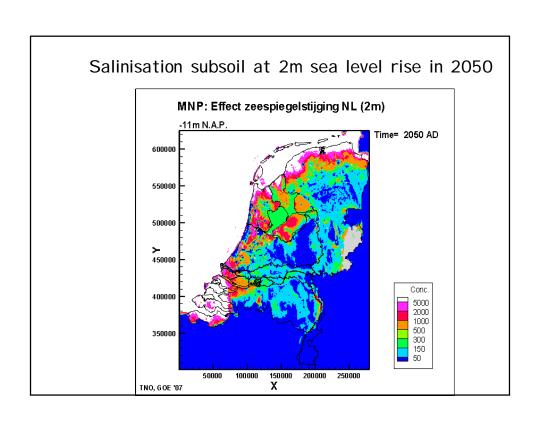
- Variable-density 3D groundwater flow model and coupled solute transport
 - 10 scenario's, including extreme sea level rise
 - including land subsidence estimates

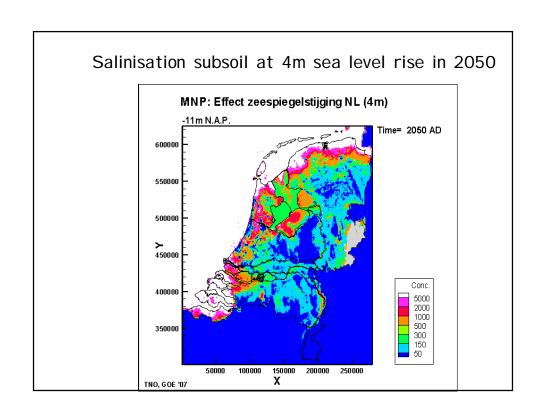




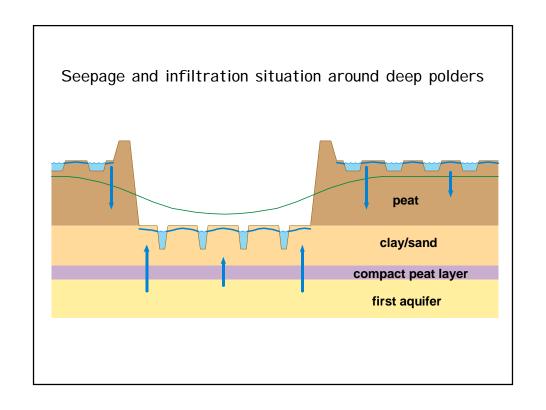


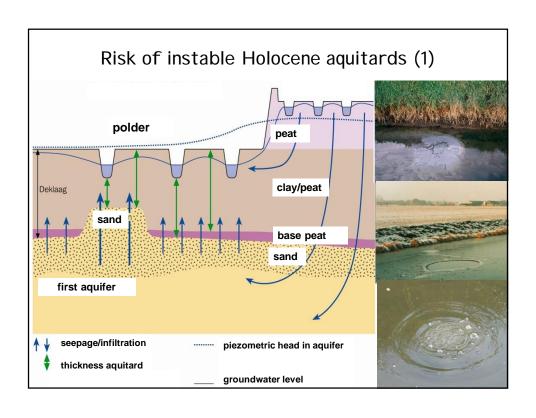


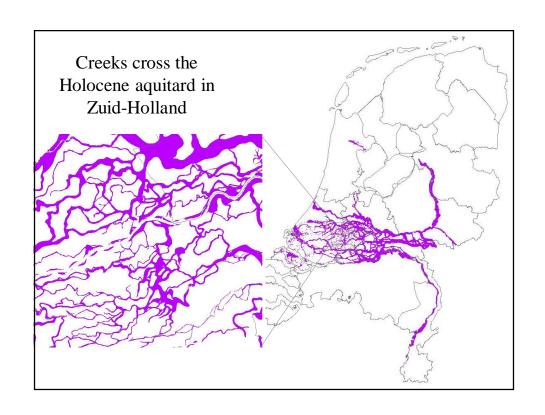




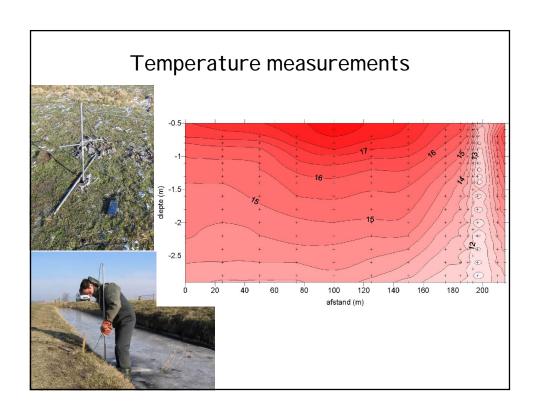
Salty wells

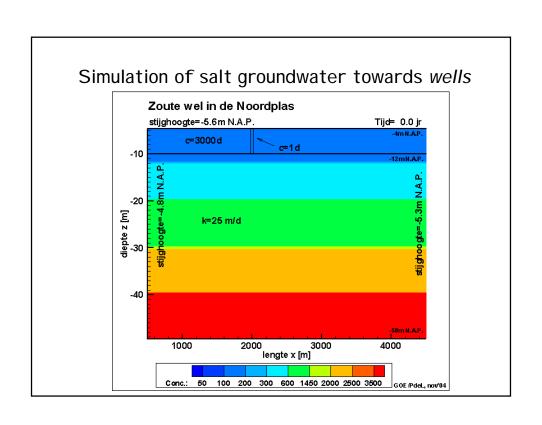


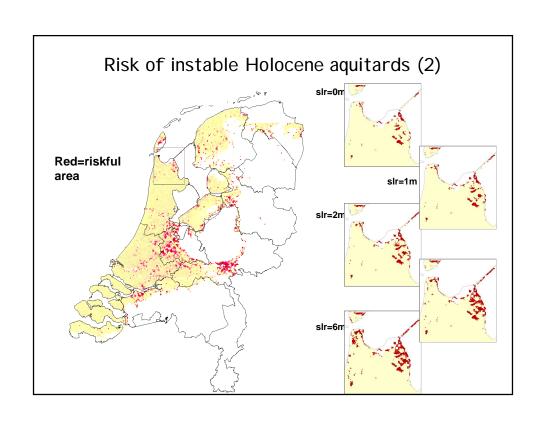


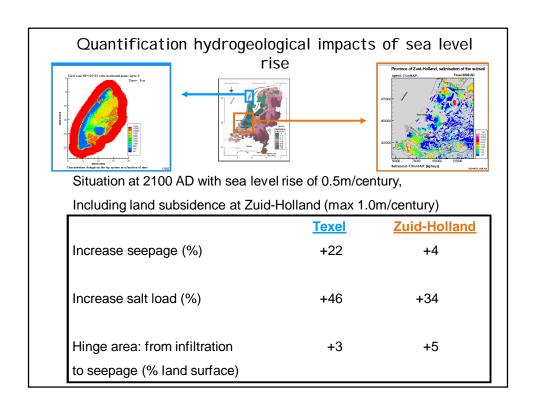


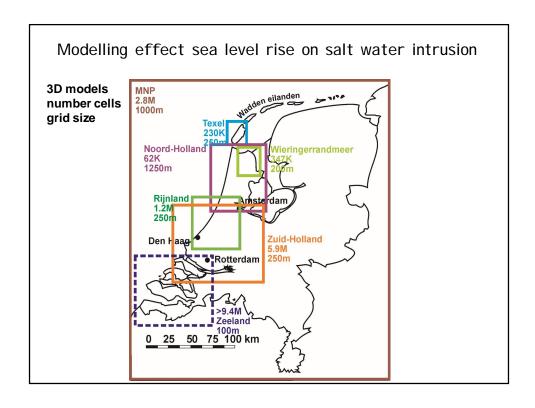












3D modelling

Characteristics 3D Cases (I): geometry & subsoil

Case	Kop van Noord-Holland	Texel	Wieringer- meerpolder	Rijnland
total land surface [km²]	2150	130	200	1100
L _x *L _y modelled area [km]	65*51	20*29	23*27	52*60
depth system [m -N.A.P]	290	302	385	190
aquifer hydr.cond. [m/d]	5-70	5-30	15-40	12-70
aquitard hydr.cond. [m/d]	0.12-0.001	0.01-1	0.012-0.056	2.5E-4-0.8
porosity	0.35	0.3	0.25	0.25
anisotropy [k _z /k _x]	0.4	0.4	0.25	0.1
long. dispersivity α_L [m]**	2	2	2	1
# head&conc. observations	not applicable*	111	95	1632
characteristics	not applicable*	Δφ =0.24 m	Δφ =0.34 m	Δφ =0.60 m
head calibration		σ=0.77 m	σ=0.21 m	σ=0.77 m

^{*} calibration with seepage & salt load in polders

^{**}molecular diffusion=10⁻⁹ m²/s; trans. disp.=1/10 long. disp.

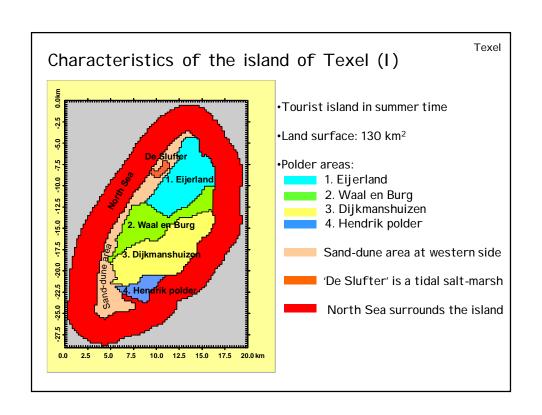
3D modelling

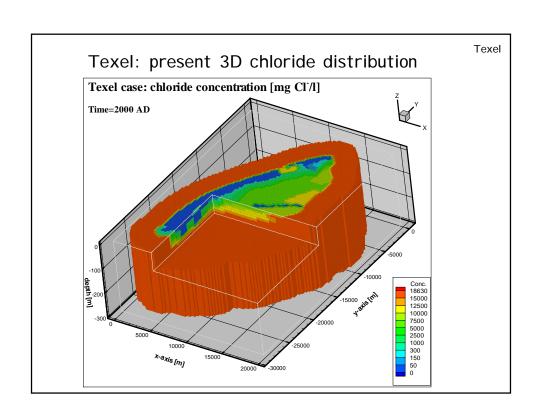
Characteristics 3D Cases (II): model parameters

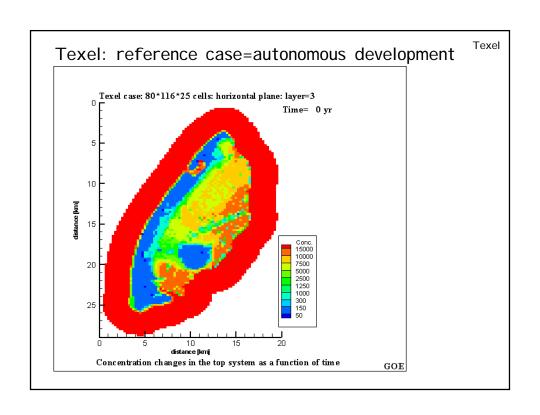
Case	Kop van Noord-Holland	Texel	Wieringer- meerpolder	Rijnland (=391 EM RAM)
horizontal cell size [m]	1250*1250	250*250	200*200	250*250
vertical cell size [m]	10	1.5 to 20	2 to 70	5 to 10
total # active cells	~40.000	~126.000	~312.000	~1.200.000
# cells	41*52*29	80*116*23	116*136*22	209*241*24
# particles per cell	27	8	8	8
total time [yr]	1000	500	50	500

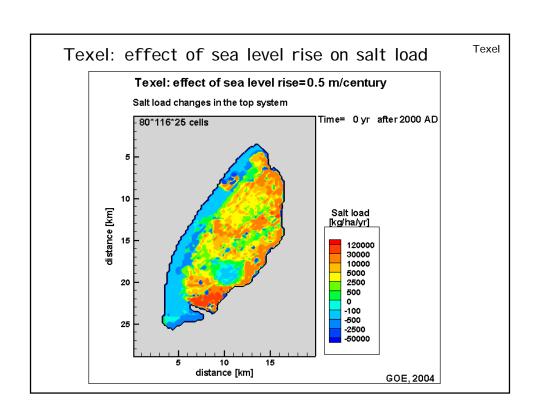
convergence head criterion= $10^{\text{-}5}/10^{\text{-}4}$ m flow time step $\Delta t\text{=}1$ year

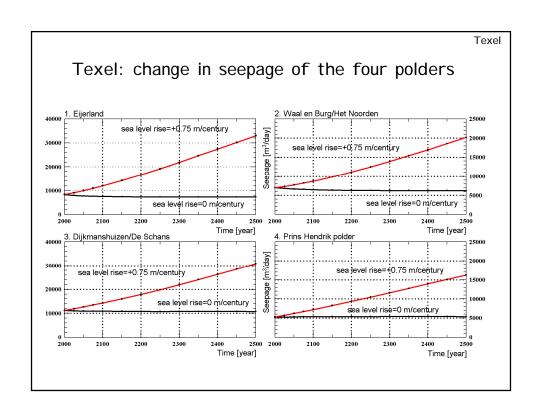
Model of the island of Texel

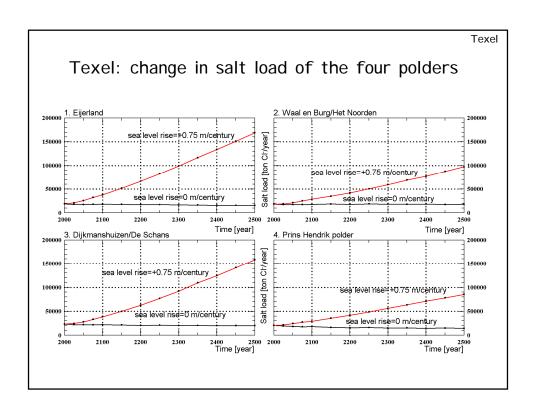




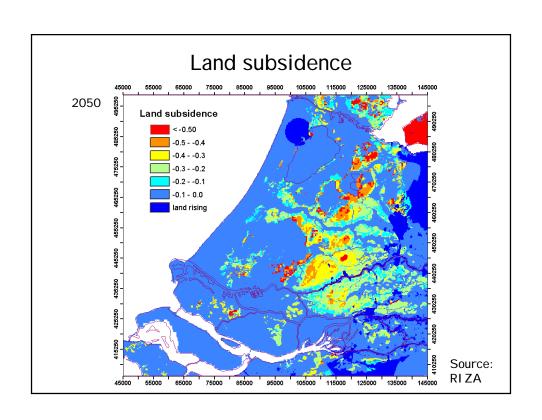


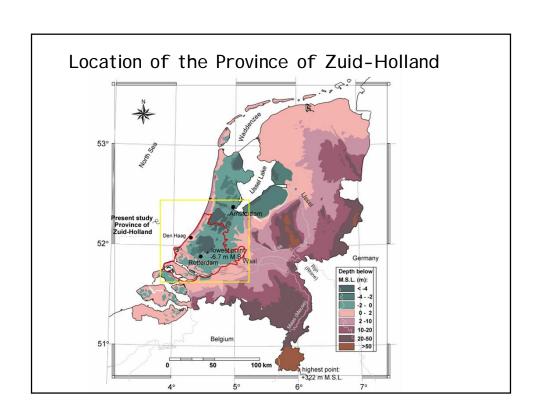






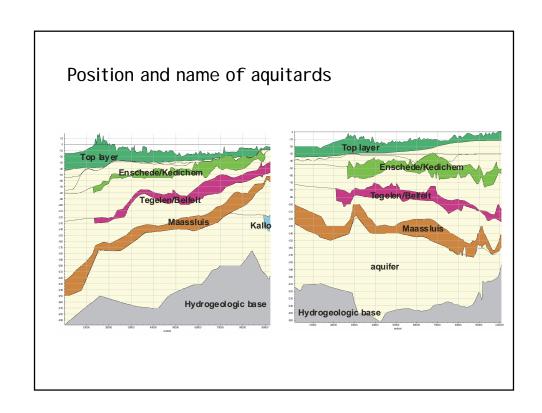
Model of the Province of Zuid-Holland			
Case study: Province of Zuid-Holland			
European water framework directive "in 2015, state of all groundwaters and surface waters must be good"			
I dentification of all fresh groundwater bodies in the province			
How fast is the salinisation process?			
More seepage, more salt load?			

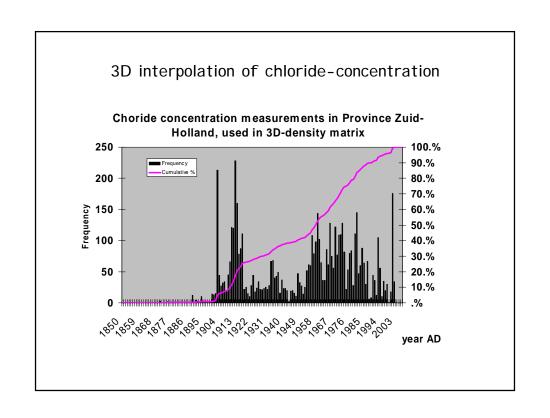


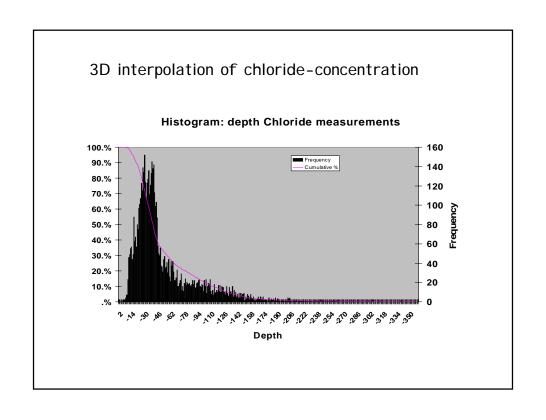


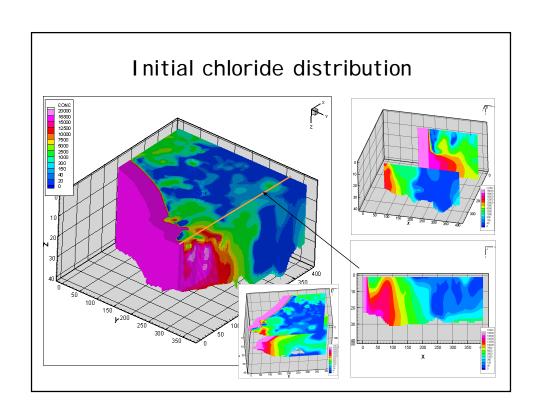
Numerical model description

- variable-density groundwater flow
- coupled solute transport
- MOCDENS3D
- area: 100km * 92.5km * 300m depth
- 400 *370 cells, 40 layers
- ~4 million active cells
- uses most accurate Dutch 3D subsurface schematization available
- 9 aquifers and aquitards
- uses 5772 chloride concentration measurements





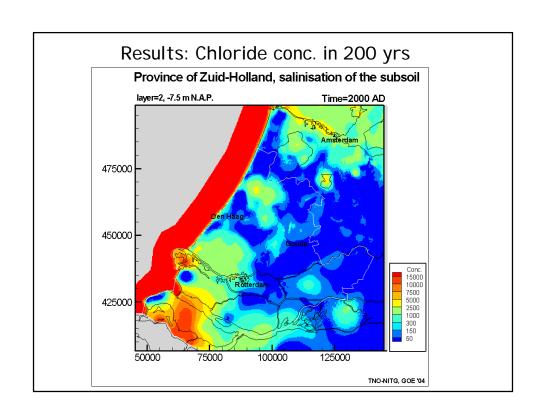


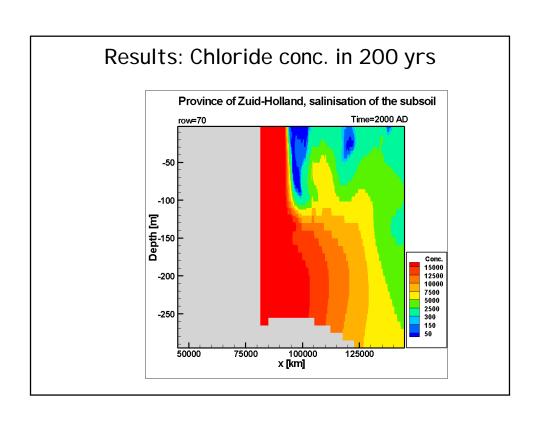


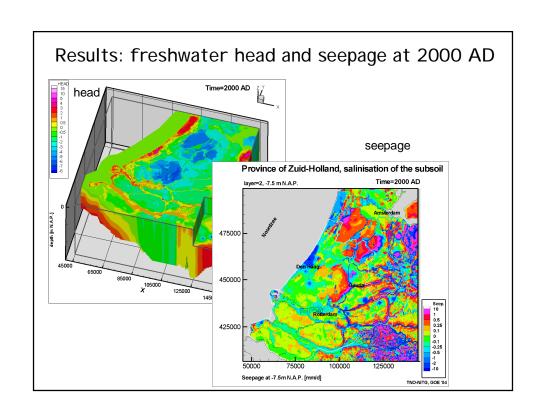
Present freshwater volume

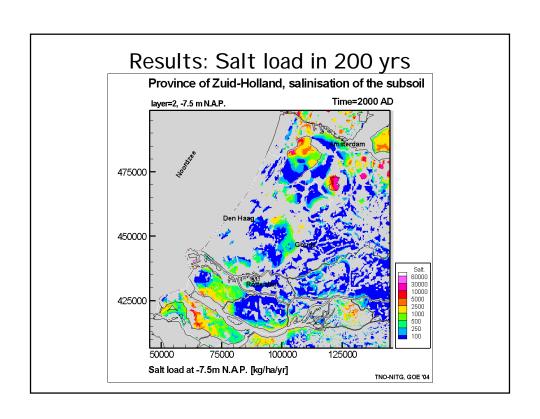
27 billion m³

36% fresh, 14% brackish, 50% saline



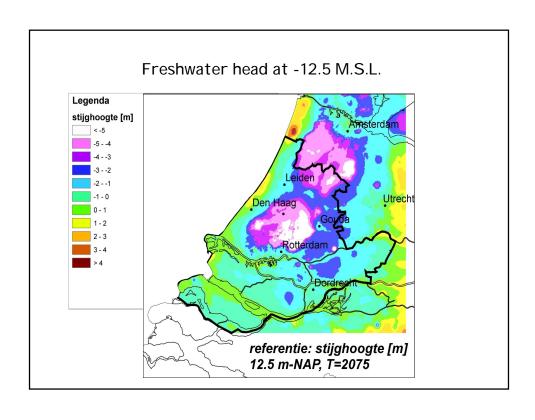


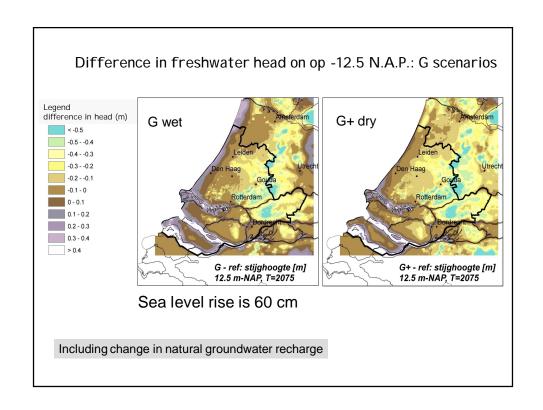


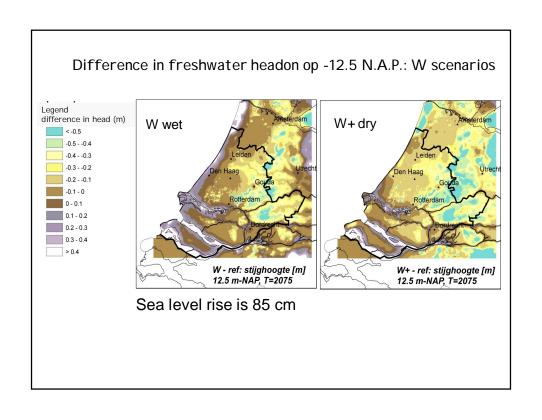


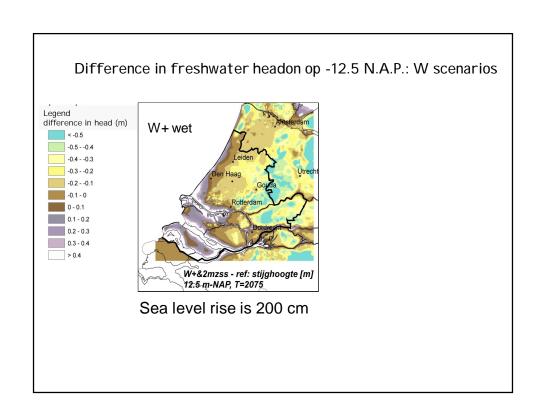
Effect sea level rise, change in natural groundwater recharge and land subsidence on freshwater head in aquifer

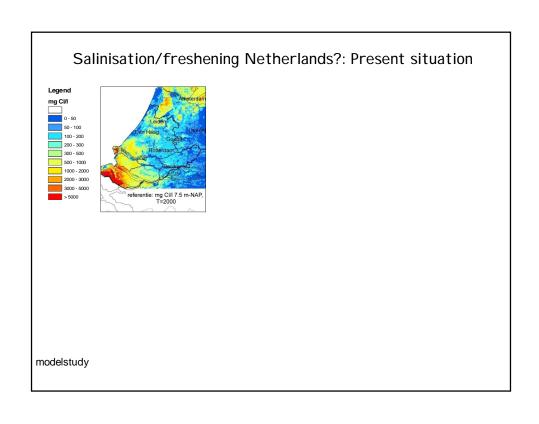
Some regional modelling results

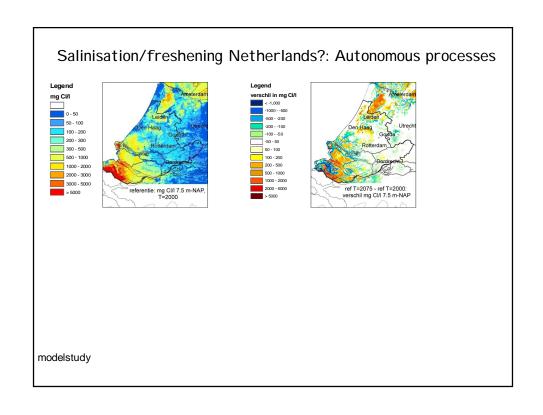


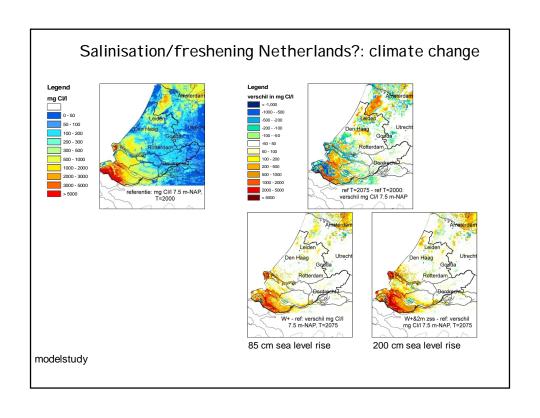


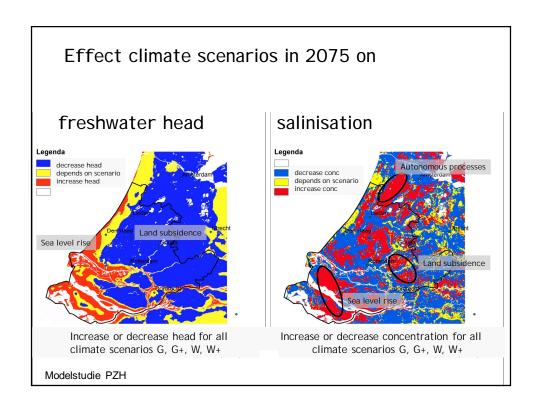










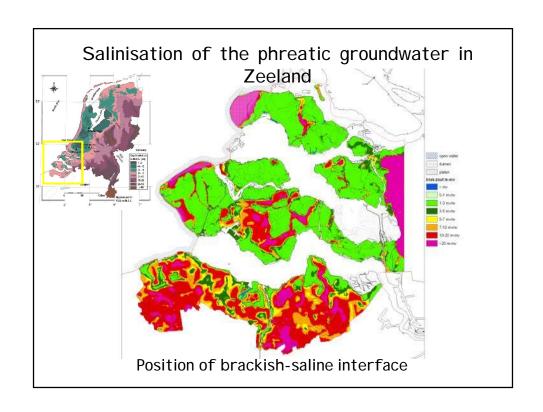


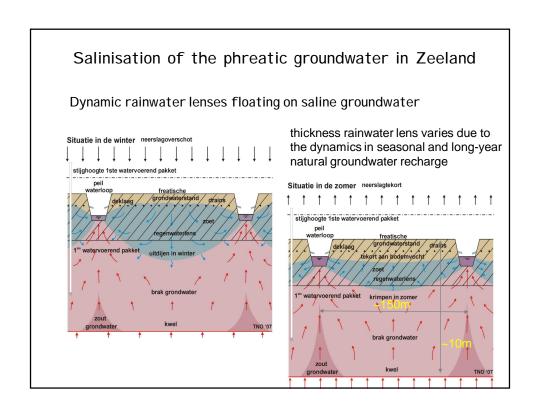
Rainwater lens

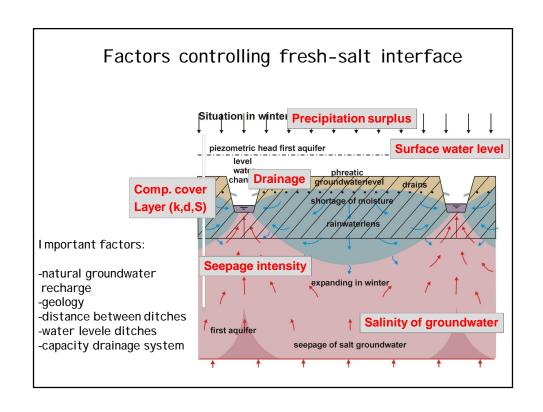
Rainwater lenses in an agricultural setting

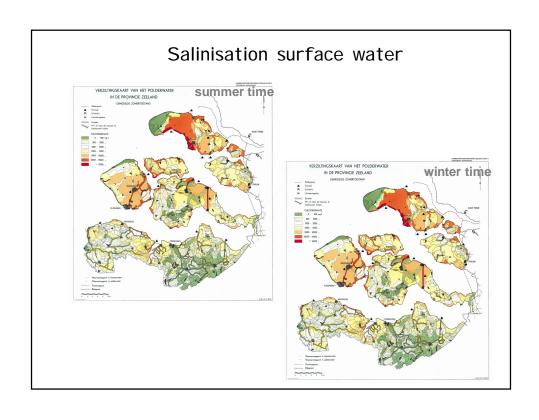
Shallow dynamic freshwater bodies flowing upon brackishsaline groundwater

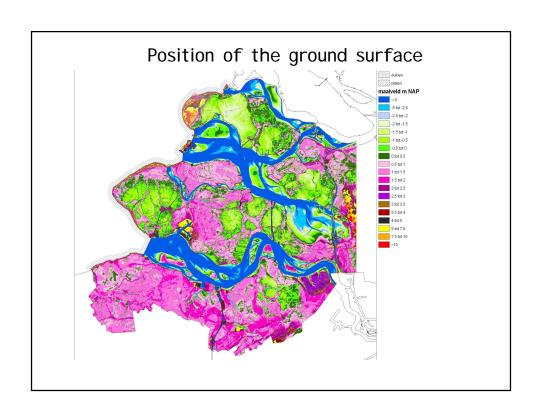
-density dependent-dynamics: seasonal & long-year

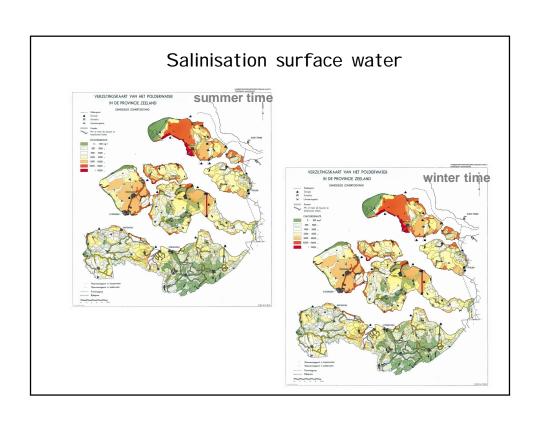












Problem definition dynamic freshwater lenses



Salt in the agricultural plots originates from:

- surface water system (irrigation water)
- groundwater system (salt load to the root zone)

The salinisation will increase due to:

- · sea level rise
- · climate change
- · water level management



How to tackle the problem?

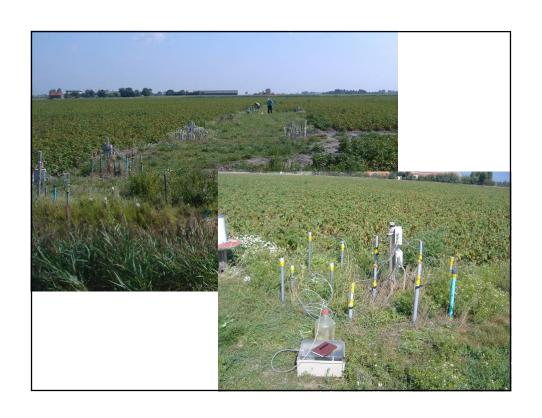
Field measurements at parcels

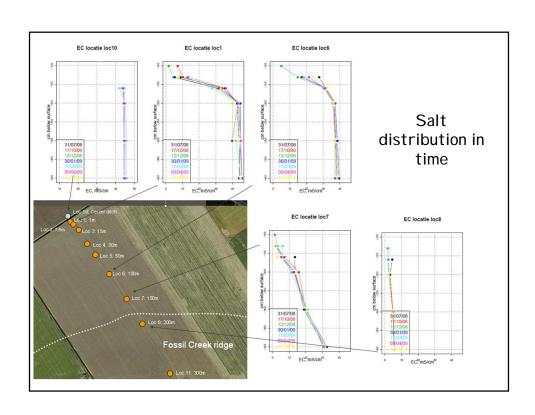
- fresh-brackish-salt interface at local scale using T-EC-probe and later CVES and ERT
- groundwater level and quality
- surface water level and quality

Modelling

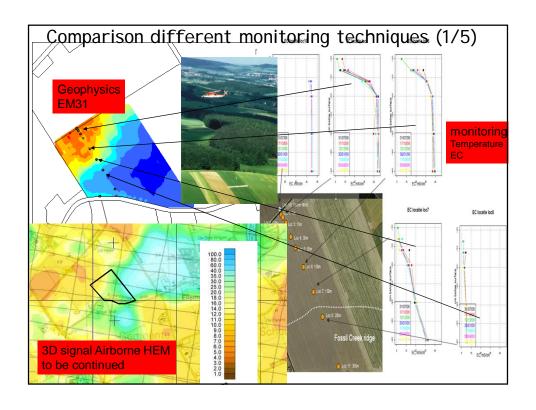
- density dependent groundwater flow
- two different scales:
 - regional scale: transect perpendicular at coast
 - local scale: parcel between two ditches

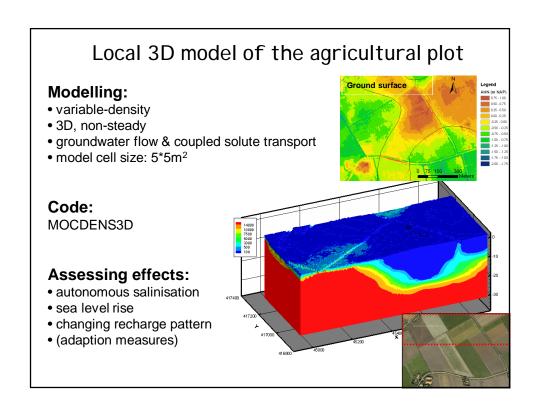


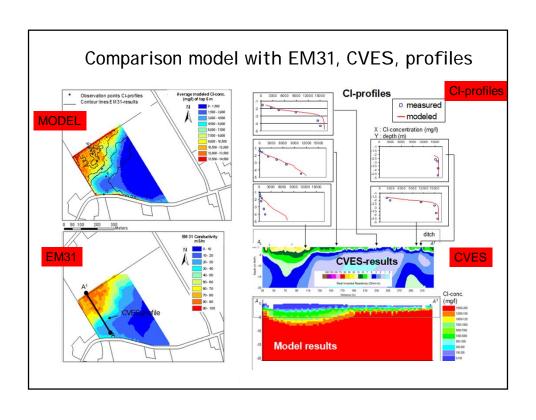


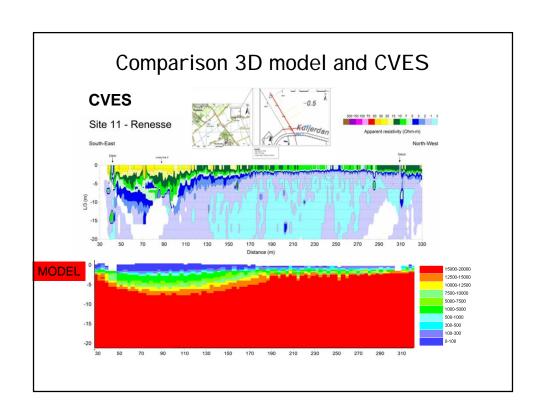


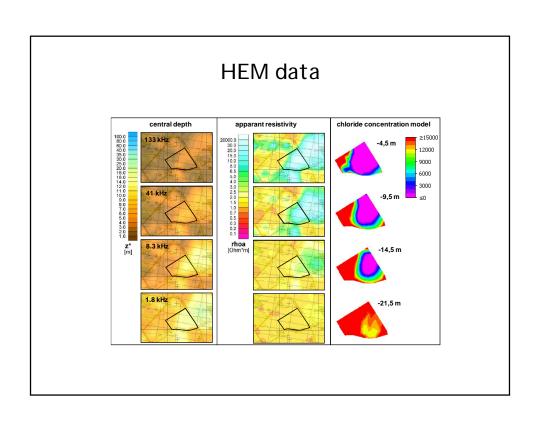
- TEC
- sampling
- EM31
- CVES
- HEM
- ECPT
- Numerical models (2D and 3D)

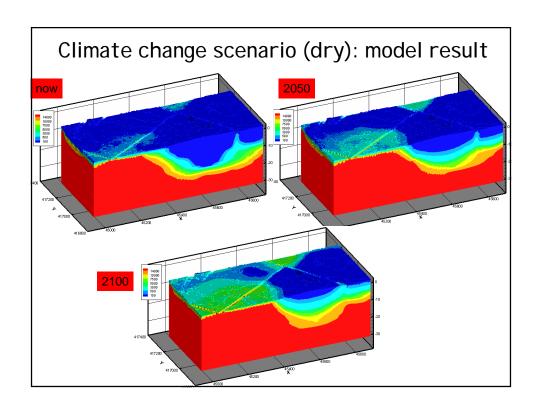






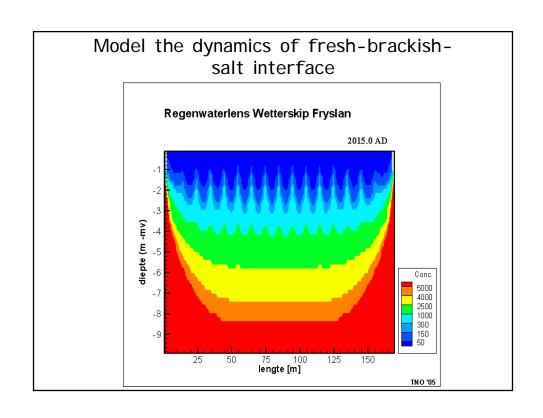


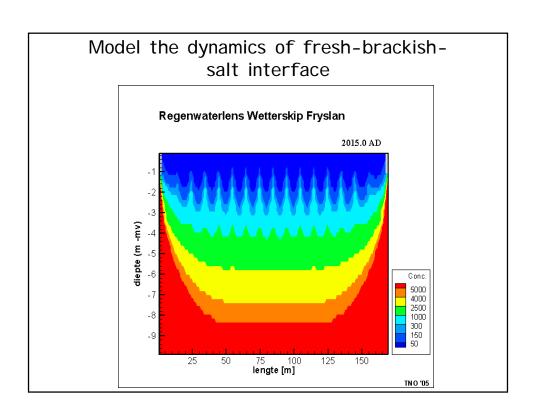


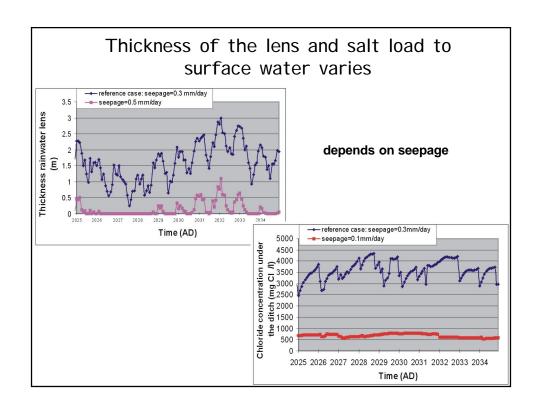


To be continued...

- Implementing more realisations of 3D geology and initial 3D fresh-saline
 - Analyse the differences
- Running climate change scenarios (on national and regional level)
 - Effect on surface water (salt load)
 - Effect on root zone (rainwater lenses)
 - Effect on freshwater volumes (drinking water)
- Compare model results of different scales and give recommendations





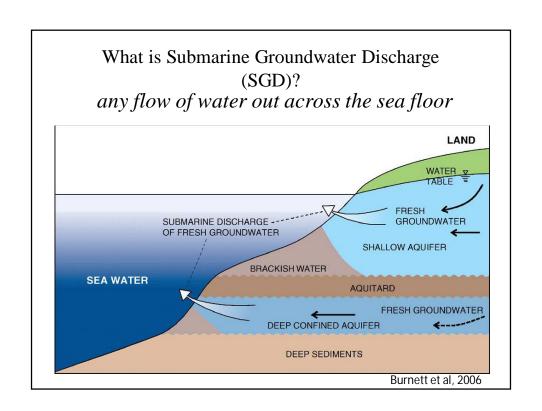


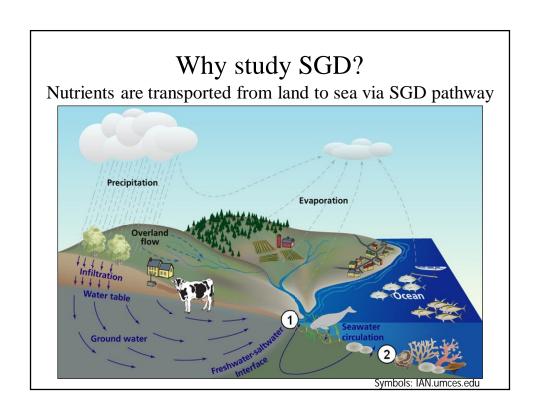
Conclusions (salinisation Dutch aquifers):

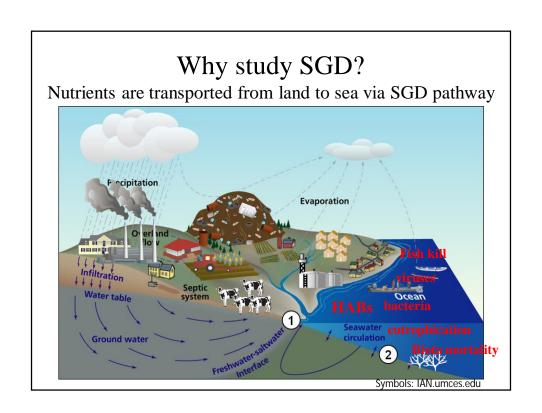
- •Salinisation in the Netherlands is a non-stationary process
- •Three physical processes threaten the Dutch aquifers:
 - -autonomous development
 - -land subsidence
 - -sea level rise
- •Increase in seepage and salt load can be severe during the coming 50/100 years
- •Modelling techniques are available to assess possible effects

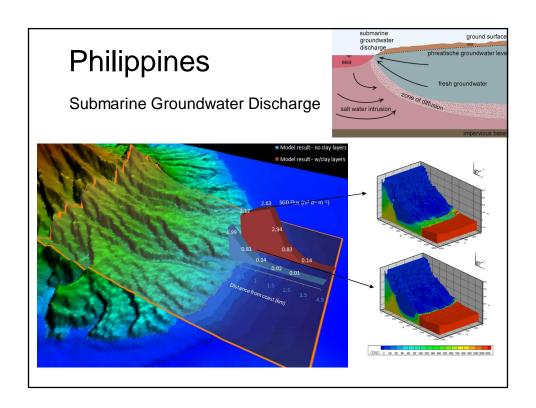
Recommendations (salinisation Dutch aquifers):

- •Number of quality measurements should be increased
- •Feasibility study is necessary to implement potential technical measures to compensate salt water intrusion









Conclusions (modelling of variable-density flow)

- Don't use the Henry problem to test your variable-density code
- Use enough cells to model the Hydrocoin and Elder problem

For modelling 3D systems:

- Remember the Peclet discretisation limitation for cell sizes (unless you're using the method of characteristics!)
- Longitudinal dispersivity should not be too large (e.g. <10m)
- It's important to derive a very accurate density distribution (as that significantly effects the velocity field!)
- Watch out for numerical problems at the outflow face to the sea

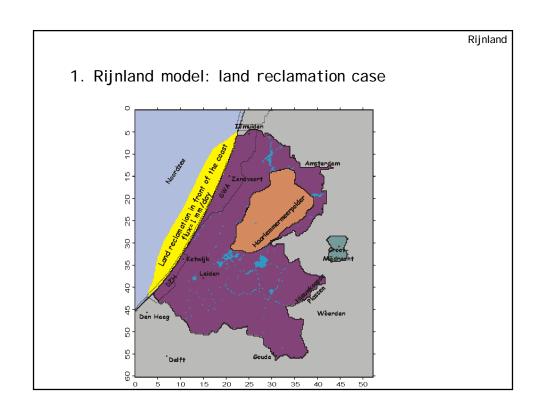
Challenges for the future

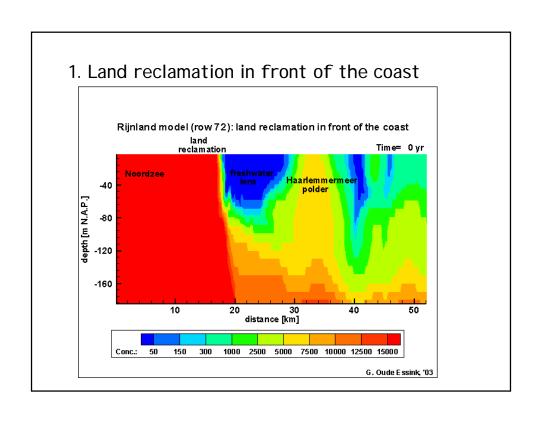
- Improve the 3D density matrix, e.g. by more types of measurements
- I mplement effect of climate change and sea level rise on coastal aquifers
- Optimalisation of (ground)water management in coastal aquifers by using 3D variable-density flow models
- I mprove calibration of 3D models by using transient data of solute concentrations
- $\bullet \ \ Incorporate \ reactive \ multicomponent \ solute \ transport$

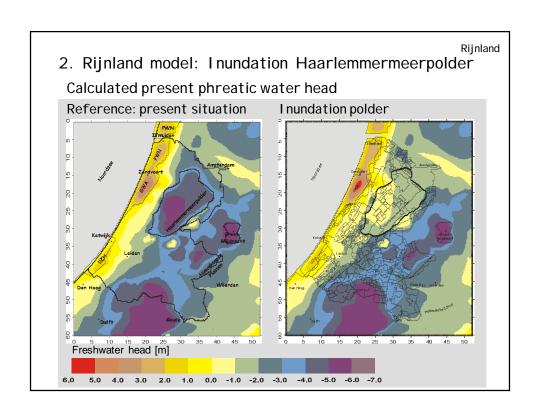
Solutions

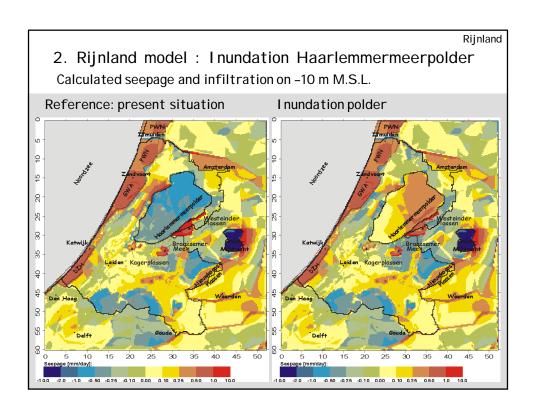
Possible measures to compensate salt water intrusion

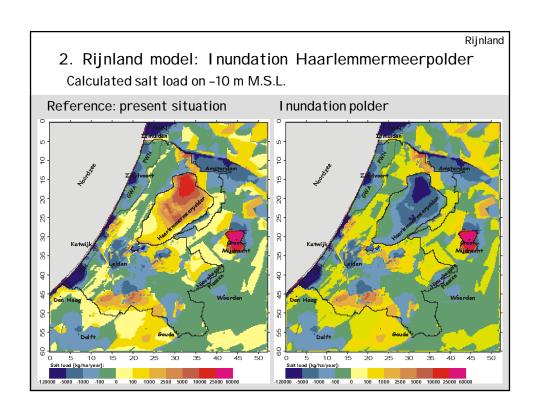
- 1. Land reclamation in front of the coast
- 2. I nundation of low-lying polders
- 3. Extraction of saline/brackish groundwater
- 4. Infiltration of fresh surface water
- 5. Creating physical barriers

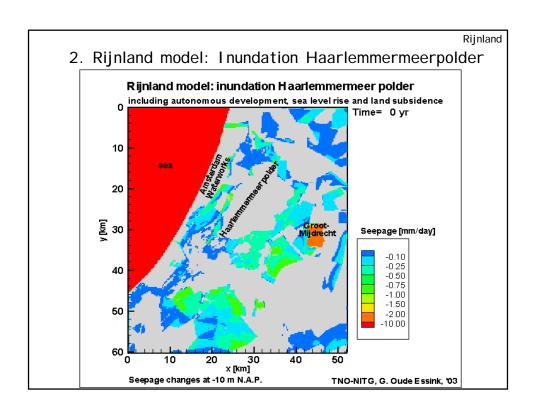


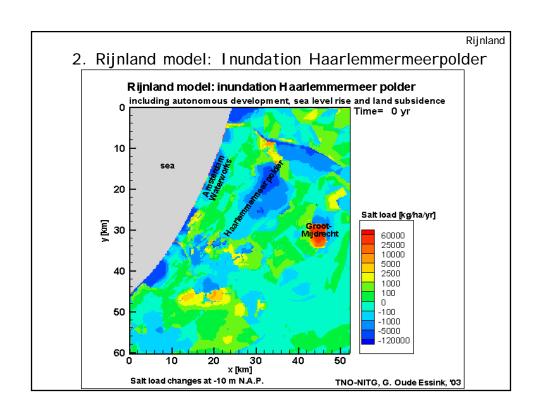


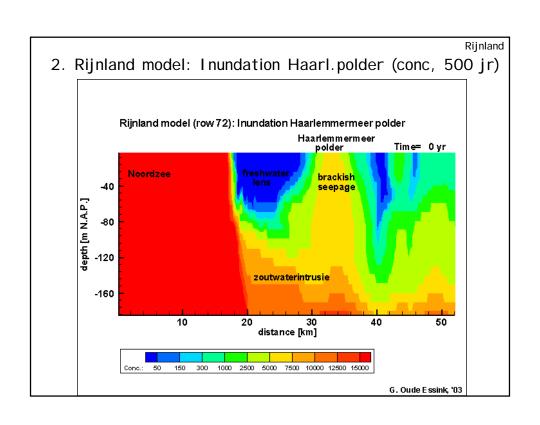


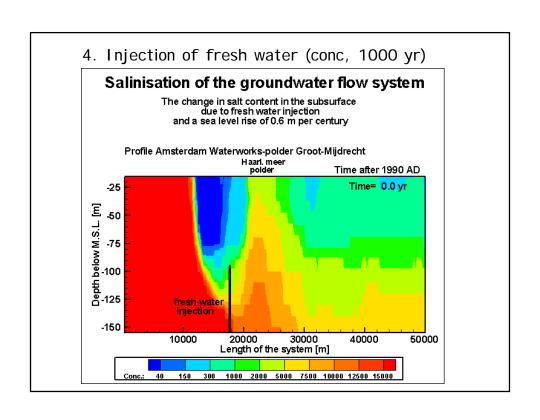


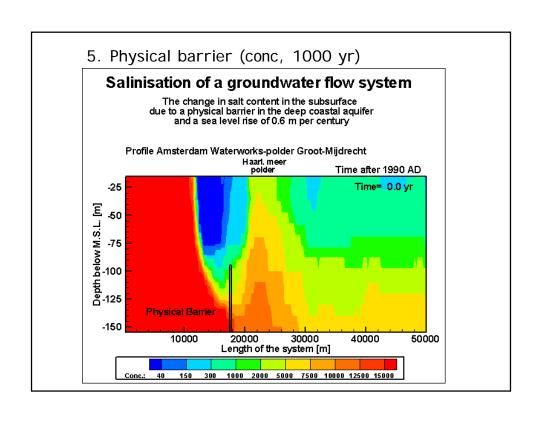


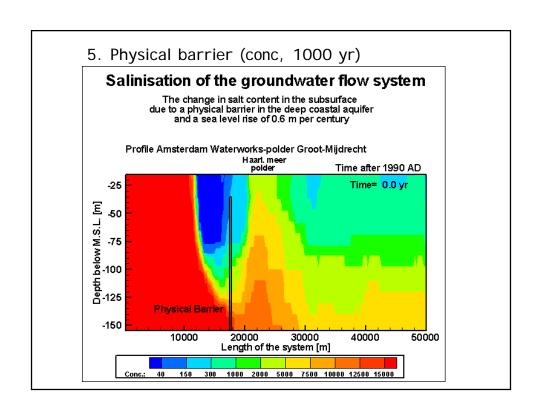


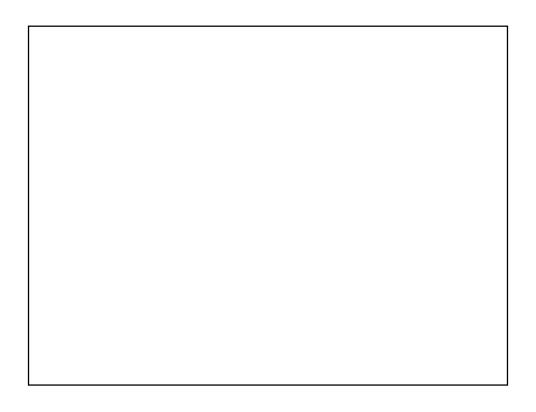












modelling

Solute transport models

Combine
the groundwater flow equation
and
the advection-dispersion equation
by means of
an equation of state

modelling

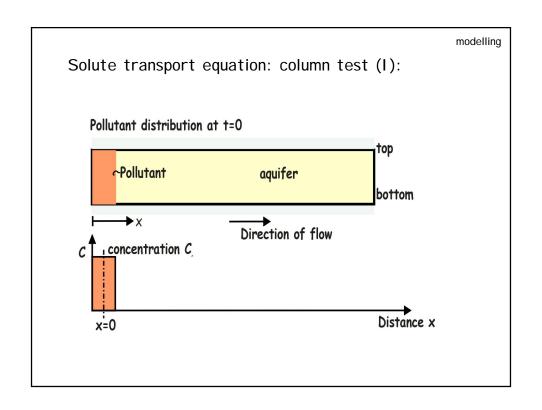
Solute transport equation

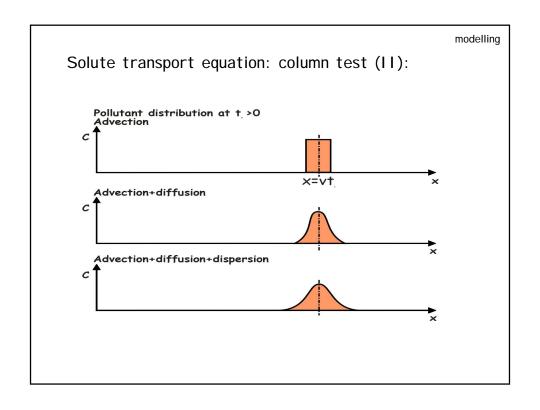
Partial differential equation (PDE):

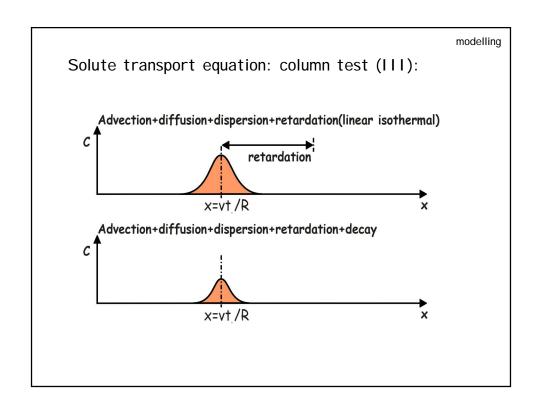
$$R_{d} \frac{\partial C}{\partial t} = \frac{\partial}{\partial x_{i}} \left(D_{ij} \frac{\partial C}{\partial x_{j}} \right) - \frac{\partial}{\partial x_{i}} \left(CV_{i} \right) + \frac{(C - C)'W}{n_{e}} - R_{d} \lambda C$$

change dispersion advection source/sink decay in concentration diffusion

 D_{ij} =hydrodynamic dispersion [L^2T^{-1}] R_d =retardation factor [-] λ =decay-term [T^{-1}]







Hydrodynamic dispersion

hydrodynamic dispersion

mechanical dispersion+ diffusion

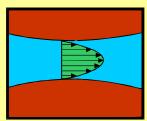
mechanical dispersion:
tensor
velocity dependant

diffusion:
molecular process
solutes spread due to concentration differences

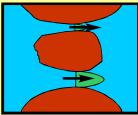
modelling

modelling

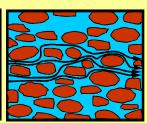
Mechanical dispersion



Differences in velocity in the pore



Differences in velocity due to variation in pore-dimension



Differences in velocity due to variation in velocity direction

Solute transport equation: diffusion (I) diffusion is a slow process: diffusion equation

only 1D-diffusion means: R_d =1, V_i =0, λ =0 and W=0

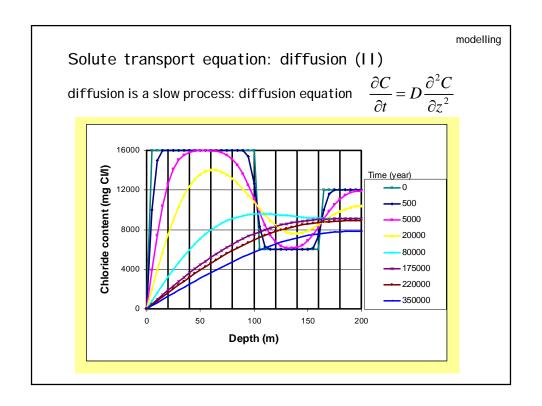
$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}$$

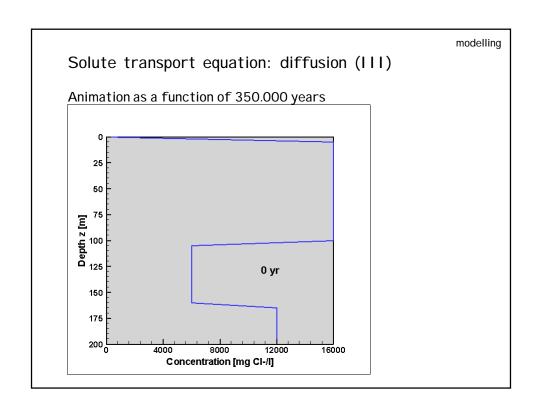
similarity with non-steady state groundwater flow equation

$$S\frac{\partial \phi}{\partial t} = T\frac{\partial^2 \phi}{\partial x^2} + N \qquad \frac{T\Delta t}{S\Delta x^2} < 0.5$$

$$\phi_i^{t+\Delta t} = \phi_i^t + \frac{N\Delta t}{S} + \frac{T\Delta t}{S\Delta x^2} \left(\phi_{i+1}^t - 2\phi_i^t + \phi_{i-1}^t \right)$$

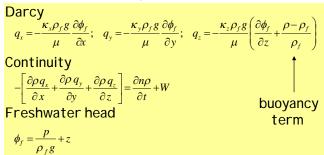
$$C_i^{t+\Delta t} = C_i^t + \frac{D\Delta t}{\Delta z^2} \left(C_{i+1}^t - 2C_i^t + C_{i-1}^t \right) \qquad \frac{D\Delta t}{\Delta z^2} < 0.5$$





MOCDENS3D

Groundwater flow equation (MODFLOW, 1988)



Advection-dispersion equation (MOC3D, 1996)

$$\frac{\partial C}{\partial t} = \frac{1}{nR_f} \frac{\partial}{\partial x_i} \left(nD_{ij} \frac{\partial C}{\partial x_j} \right) - \frac{V_i}{R_f} \frac{\partial C}{\partial x_i} + \frac{\sum [W(C'-C)]}{nR_f} - \lambda C$$

Equation of state: relation density & concentration

$$\rho_{i,j,k} = \rho_f (1 + \beta C_{i,j,k})$$

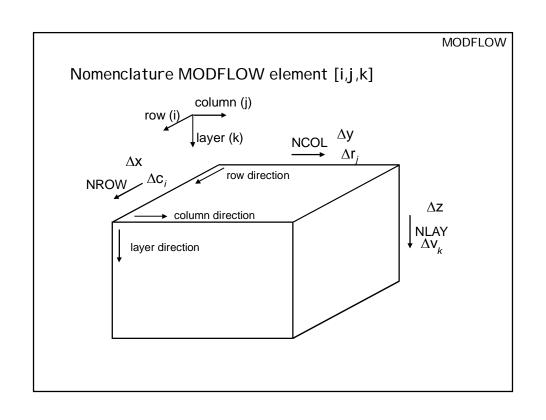
MOCDENS3D

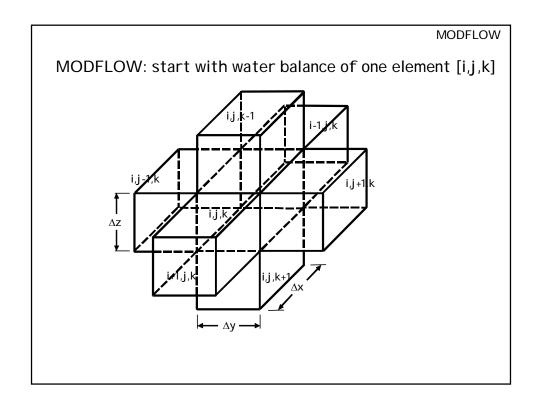
MOCDENS3D is based on MODFLOW

a modular 3D finite-difference ground-water flow model

(M.G. McDonald & A.W. Harbaugh, from 1983 on)

- USGS, 'public domain'
- non steady state
- heterogeneous porous medium
- anisotropy
- coupled to reactive solute transport MOC3 (Konikow et al, 1996) MT3D, MT3DMS (Zheng, 1990) RT3D PHT3D (Prommer, 2004)
- easy to use due to numerous Graphical User Interfaces (GUI's)
 PMWI N, GMS, Visual Modflow, Argus One, Groundwater Vistas, etc.





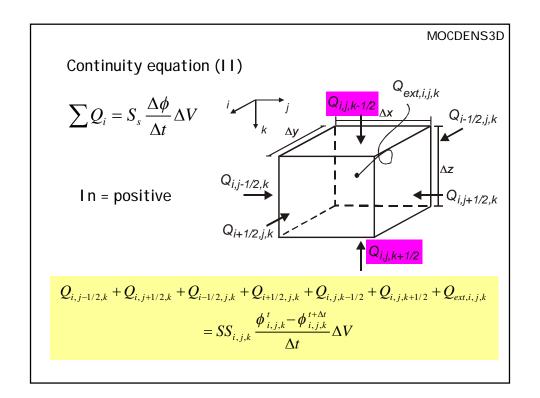
MODFLOW

Continuity equation (I)

In - Out = Storage

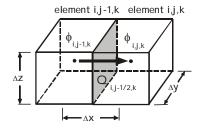
$$\frac{\partial}{\partial x} \left(k_{xx} \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_{yy} \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_{zz} \frac{\partial \phi}{\partial z} \right) - W = S_s \frac{\partial \phi}{\partial t}$$

$$\sum Q_i = S_s \frac{\Delta \phi}{\Delta t} \Delta V$$



MOCDENS3D

Flow equation (Darcy's Law)



$$Q = surface * q = surface * k \frac{\partial \phi}{\partial x}$$

$$Q_{i,j-1/2,k} = k_{i,j-1/2,k} \Delta y \Delta z \frac{\phi_{i,j-1,k} - \phi_{i,j,k}}{\Delta x}$$

$$Q_{i,j-1/2,k} = CR_{i,j-1/2,k} \left(\phi_{i,j-1,k} - \phi_{i,j,k} \right)$$

where $CR_{i,j-1/2,k} = \frac{k_{i,j-1/2,k} \Delta y \Delta z}{\Delta x}$ is the conductance [L²/T]

MOCDENS3D

Density dependent vertical flow equation

$$q_z = -\frac{\kappa_z \rho_f g}{\mu} \left(\frac{\partial \phi_f}{\partial z} + \frac{\rho - \rho_f}{\rho_f} \right)$$
$$q_z = -k \left(\frac{\partial \phi_f}{\partial z} + \frac{\rho - \rho_f}{\rho_f} \right)$$

$$q_{z} = -\frac{\kappa_{z}\rho_{f}g}{\mu} \left(\frac{\partial \phi_{f}}{\partial z} + \frac{\rho - \rho_{f}}{\rho_{f}} \right)$$

$$q_{z} = -k_{z} \left(\frac{\partial \phi_{f}}{\partial z} + \frac{\rho - \rho_{f}}{\rho_{f}} \right)$$

$$= surface * k_{z} \left(\frac{\partial \phi_{f}}{\partial z} + \frac{\rho - \rho_{f}}{\rho_{f}} \right)$$

$$Q_{i,j,k-1/2} = k_{i,j,k-1/2} \Delta x \Delta y \left(\frac{\phi_{f,i,j,k-1} - \phi_{f,i,j,k}}{\Delta z} + BUOY_{i,j,k-1/2} \right)$$

$$Q_{i,j,k-1/2} = CV_{i,j,k-1/2} \left(\phi_{f,i,j,k-1} - \phi_{f,i,j,k} + BUOY_{i,j,k-1/2} \Delta z \right)$$

where
$$BUOY_{i,j,k-1/2} = \left(\frac{(\rho_{i,j,k-1/2} + \rho_{i,j,k})/2 - \rho_f}{\rho_f}\right)$$
 =buoyancy term [-]

where
$$CV_{i,j,k-1/2} = \frac{k_{i,j,k-1/2} \Delta x \Delta y}{\Delta z}$$
 =conductance [L²/T]



Density dependent groundwater flow equation

$$\begin{aligned} Q_{i,j-1/2,k} &= CR_{i,j-1/2,k} \left(\phi_{f,i,j-1,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j+1/2,k} &= CR_{i,j+1/2,k} \left(\phi_{f,i,j+1,k} - \phi_{f,i,j,k} \right) \\ Q_{i-1/2,j,k} &= CC_{i-1/2,j,k} \left(\phi_{f,i-1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i+1/2,j,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1/2,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1/2,j,k} - \phi_{f,i,j,k} \right) \\ Q_{i,j-1/2,j,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1/2,j,k} - \phi_{f,i+1/2,j,k} \right) \\ Q_{i,j-1/2,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1/2,j,k} - \phi_{f,i+1/2,j,k} \right) \\ Q_{i,j-1/2,j,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1/2,j,k} - \phi_{f,i+1/2,j,k} \right) \\ Q_{i,j-1/2,j,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1/2,j,k} - \phi_{f,i+1/2,j,k} \right) \\ Q_{i,j-1/2,j,k} &= CC_{i+1/2,j,k} \left(\phi_{f,i+1/2,j,k} - \phi_{f,i+1/2,j,k} \right) \\ Q_{i,j-1/2,j,k} &= CC_{i+1/2$$

$$\begin{aligned} Q_{i,j,k-1/2} &= CV_{i,j,k-1/2} \Big(\phi_{f,i,j,k-1} - \phi_{f,i,j,k} \\ Q_{i,j,k+1/2} &= CV_{i,j,k+1/2} \Big(\phi_{f,i,j,k+1} - \phi_{f,i,j,k} \\ - \beta UOY_{i,j,k-1/2} \Delta v_{k+1/2} \Big) \end{aligned}$$

$$\begin{aligned} Q_{i,j-1/2,k} + Q_{i,j+1/2,k} + Q_{i-1/2,j,k} + Q_{i+1/2,j,k} + Q_{i,j,k-1/2} + Q_{i,j,k+1/2} + Q_{ext,i,j,k} \\ &= SS_{i,j,k} \frac{\phi_{f,i,j,k}^t - \phi_{f,i,j,k}^{t+\Delta t}}{\Delta t} \Delta V \end{aligned}$$

MOCDENS3D

The term $Q_{ext,i,j,k}$

Takes into account all external sources

Rewriting the term:

$$Q_{ext,i,j,k} = P_{i,j,k} \phi_{i,j,k}^{t+\Delta t} + Q'_{i,j,k}$$

MOCDENS3D

Thé variable density groundwater flow equation

$$\begin{split} Q_{i,j-1/2,k} + Q_{i,j+1/2,k} + Q_{i-1/2,j,k} + Q_{i+1/2,j,k} + \frac{Q_{i,j,k-1/2}}{\Delta t} + \frac{Q_{i,j,k+1/2}}{\Delta V} + Q_{ext,i,j,k} \\ &= SS_{i,j,k} \frac{\phi_{f,i,j,k}^t - \phi_{f,i,j,k}^{t+\Delta t}}{\Delta t} \Delta V \\ \text{and:} \\ Q_{ext,i,j,k} = P_{i,j,k} \phi_{f,i,j,k}^{t+\Delta t} + Q_{i,j,k}^t \\ \text{gives:} \\ & CV_{i,j,k-1/2} \phi_{f,i,j,k-1}^{t+\Delta t} + CC_{i-1/2,j,k} \phi_{f,i-1,j,k}^{t+\Delta t} + CR_{i,j-1/2,k} \phi_{f,i,j-1,k}^{t+\Delta t} \\ + \left(-CV_{i,j,k-1/2} - CC_{i-1/2,j,k} - CR_{i,j-1/2,k} - CR_{i,j+1/2,k} - CC_{i+1/2,j,k} - CV_{i,j,k+1/2} + HCOF_{i,j,k} \right) \phi_{f,i,j,k}^{t+\Delta t} \\ + CR_{i,j+1/2,k} \phi_{f,i,j+1,k}^{t+\Delta t} + CC_{i+1/2,j,k} \phi_{f,i+1,j,k}^{t+\Delta t} + CV_{i,j,k+1/2} \phi_{f,i,j,k+1}^{t+\Delta t} = RHS_{i,j,k} \end{split}$$
 with:
$$HCOF_{i,j,k} = P_{i,j,k} - SC1_{i,j,k} / (\Delta t) \\ RHS_{i,j,k} = -Q_{i,j,k}^t - SC1_{i,j,k} \phi_{f,i,j,k}^t / (\Delta t) \\ - CV_{i,j,k-1/2} BUOY_{i,j,k-1/2} \Delta v_{k-1/2} + CV_{i,j,k+1/2} BUOY_{i,j,k+1/2} \Delta v_{k+1/2} \\ SC1_{i,j,k} = SS_{i,j,k} \Delta V$$

MOCDENS3D

Equation of state

$$BUOY_{i,j,k-1/2} = \left(\frac{(\rho_{i,j,k-1/2} + \rho_{i,j,k})/2 - \rho_f}{\rho_f}\right)$$

$$\rho_{i,j,k} = \rho_f \left(1 + \frac{\rho_s - \rho_f}{\rho_f} \frac{C_{i,j,k}}{C_s}\right)$$
or
$$\rho_{i,j,k} = \rho_f \left(1 + \beta C_{i,j,k}\right)$$

MODFLOW

Method of Characteristics (MOC)

Solve the advection-dispersion equation (ADE) with the Method of Characteristics

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x_i} \left(D_{ij} \frac{\partial C}{\partial x_j} \right) - \frac{\partial}{\partial x_i} (CV_i) + \frac{(C - C)'W}{n_e}$$

Lagrangian approach:

Splitting up the advection part and the dispersion/source part:

- advection by means of a particle tracking technique
- dispersion/source by means of the finite difference method

MOC3D

Advantage of the MOC approach by splitting up the advection-dispersion equation

It is difficult to solve the whole advection-dispersion equation in one step, because the so-called Peclet-number is high in most groundwater flow/solute transport problems.

The Peclet number stands for the ratio between advection and dispersion

MOC3D

Procedure of MOC: advective transport by particle tracking

- •Place a number of particles in each element
- •Determine the effective velocity of each particle by (bi)linear interpolation of the velocity field which is derived from MODFLOW
- •Move particles during one solute time step Δt_{solute}
- •Average values of all particles in an element to one node value
- •Calculate the change in concentration in all nodes due to advective transport
- •Add this result to dispersive/source changes of solute transport

MOC3D

Steps in MOC-procedure

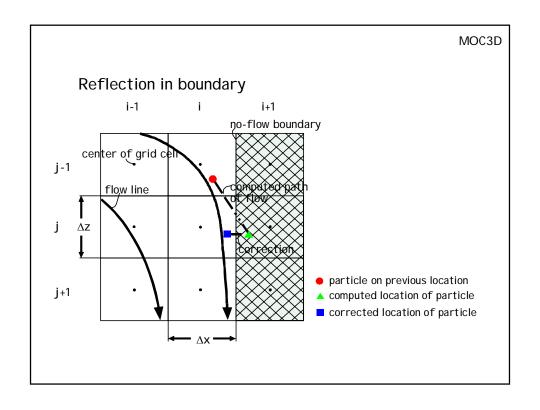
- 1. Determine concentration gradients at old timestep k-1
- 2. Move particles to model advective transport
- 3. Concentration of particles to concentration in element node
- 4. Determine concentration gradients on new timestep k*
- 5. Determine concentration in element node after advective, dispersive/source transport on timestep k

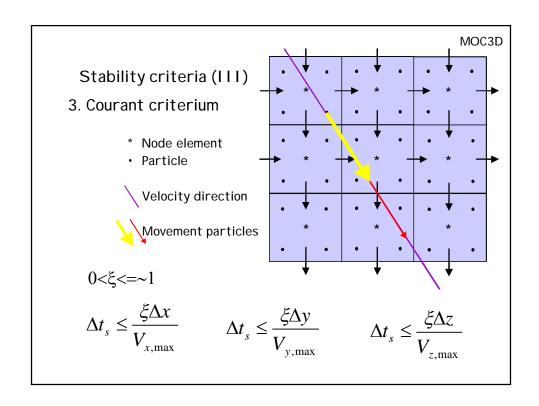
Konikow and Bredehoeft, 1978

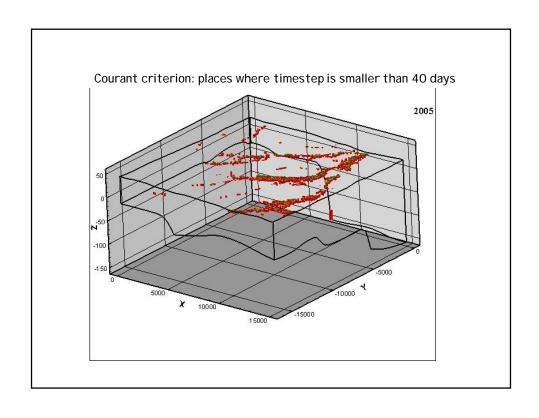
MOC3D

Causes of errors in MOC-procedure

- 1. Concentration gradients
- 2. Average from particles to node element, and visa versa
- 3. Concentration of sources/sinks to entire element
- 4. Empty elements
- 5. No-flow boundary: reflection in boundary







Numerical dispersion and oscillation Concentration Concentration Concentration Concentration Concentration Concentration Concentration Concentration Overshooting Exact solution Numerical dispersion And Concentration Overshooting Concentration Overshooting Exact solution Oscillation Undershooting $\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2} \quad C_i^{t+\Delta t} = C_i^t + \frac{D\Delta t}{\Delta z^2} \left(C_{i+1}^t - 2C_i^t + C_{i-1}^t \right) \quad \frac{D\Delta t}{\Delta z^2} < 0.5$

3D problems

Numerical dispersion problem (I)

To solve the advection-dispersion equation, standard finite difference and element techniques should consider the following spatial discretisation criterion:

Peclet number $Pe \le 2$ to 4

where:
$$Pe = \left| \frac{V\Delta x}{D_h} \right|$$

V = effective velocity [L/T] Δx = dimension grid cell [L]

 D_h = hydrodynamic dispersion [L²/T]

3D problems

Numerical dispersion problem (II)

For advection dominant grondwater flow, the Peclet number can be rewritten as:

$$\Delta x \le 2\alpha_L$$
 to $4\alpha_L$

where α_{L} = longitudinal dispersivity [L]

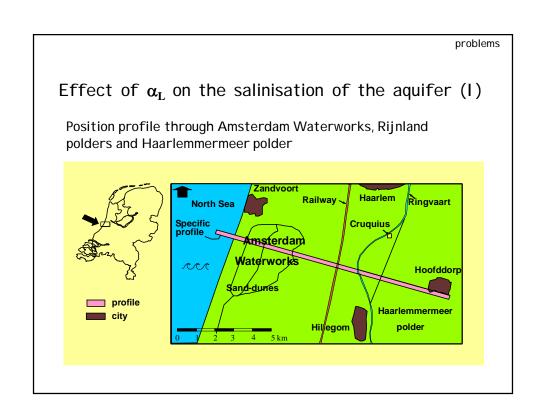
What does that mean?

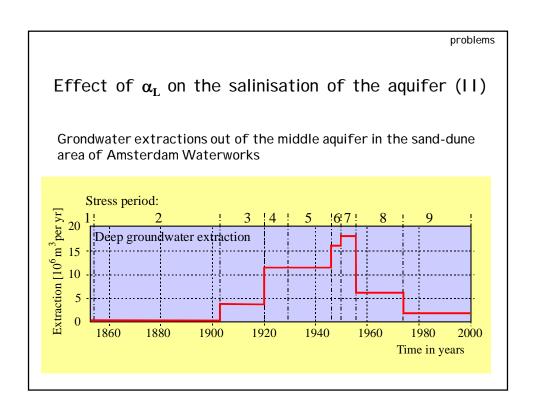
If α_L is small, then Δx should be small too!!

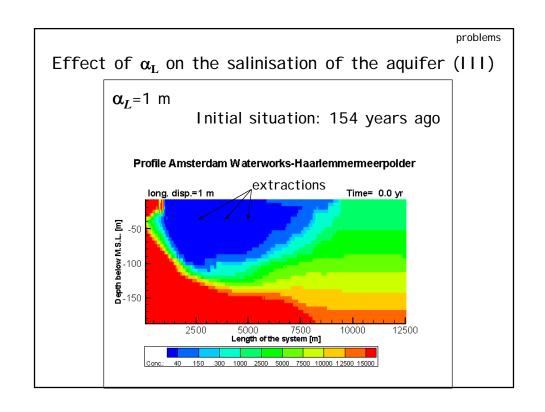
3D problems

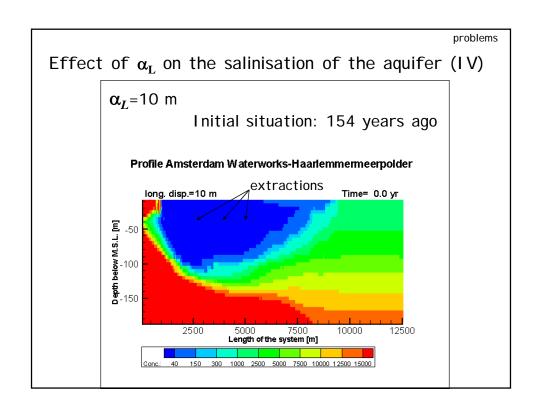
Numerical dispersion problem (III)

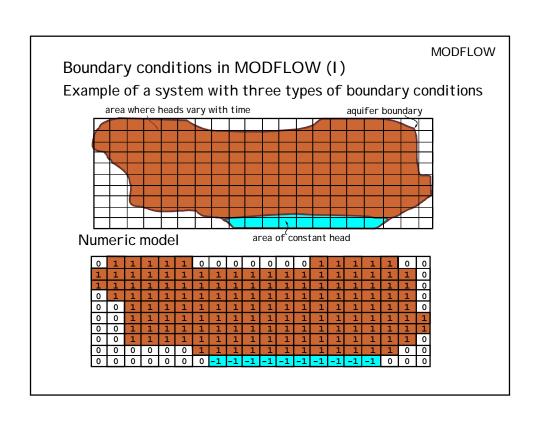
Now follows an transient salt water intrusion case to demonstrate why in many coastal aquifers the longitudinal dispersivity α_{L} [L] should be small











Boundary conditions in MODFLOW (II)

For a constant head condition: I BOUND<0
For a no flow condition: I BOUND=0
For a variable head: I BOUND>0

MODFLOW

Packages in MODFLOW

- 1. Well package
- 2. River package
- 3. Recharge package
- 4. Drain package
- 5. Evaporation package
- 6. General head package

MODFLOW

1. Well package

$$Q_{well} = Q_{i,j,k}$$

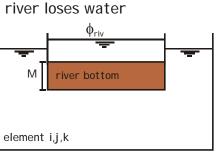
Example: an extraction of 10 m³ per day should be inserted in an element as:

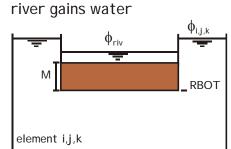
$$Q_{ext,i,j,k} = P_{i,j,k} \phi_{i,j,k}^{t+\Delta t} + Q'_{i,j,k}$$

$$Q_{ext,i,j,k} = -10$$
 (in = positive)

$$Q'_{i,j,k} = -10$$

2. River package (I)





$$Q_{riv} = KLW \left(\frac{\phi_{riv} - \phi_{i,j,k}}{M} \right)$$

$$Q_{riv} = \frac{KLW}{M} \left(\phi_{riv} - \phi_{i,j,k} \right) \iff Q_{riv} = C_{riv} \left(\phi_{riv} - \phi_{i,j,k} \right)$$

2. River package (II)

$$Q_{riv} = C_{riv} \left(\phi_{riv} - \phi_{i,j,k} \right)$$

Example: the river conductance C_{riv} is 20 m²/day and the rivel level=3 m, than this package should be inserted in an element as:

$$Q_{ext,i,j,k} = P_{i,j,k} \phi_{i,j,k}^{t+\Delta t} + Q'_{i,j,k}$$

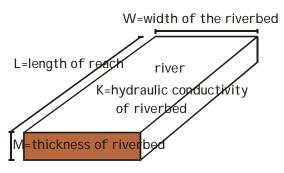
$$Q_{ext,i,j,k} = 20(3 - \phi_{i,j,k})$$

$$Q'_{i,j,k} = 60$$
 and $P_{i,j,k} = -20$

MODFLOW

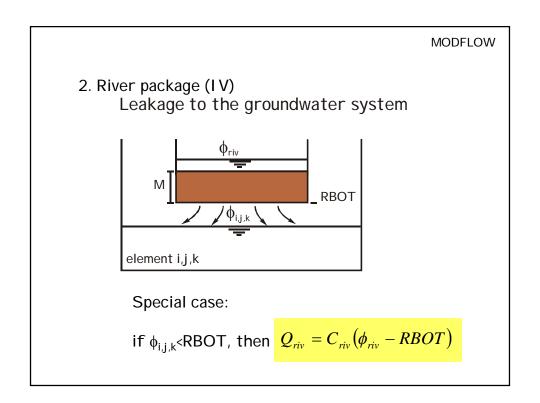
2. River package (III)

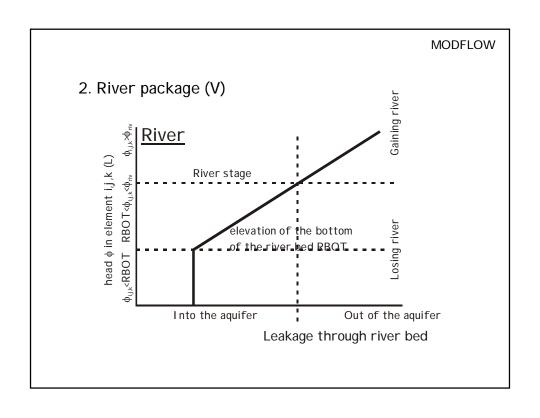
Determine the conductance of the river in one element:



where
$$C_{riv} = \frac{KLW}{M}$$
 is the

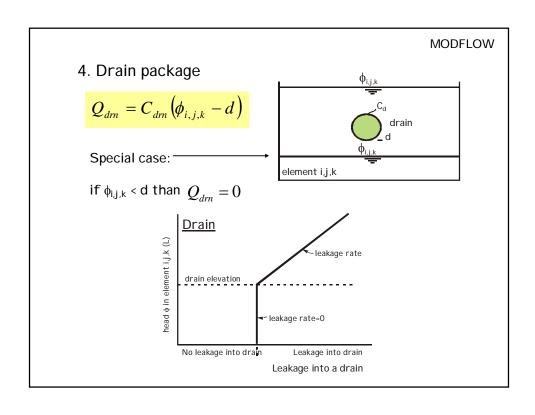
conductance $[L^2/T]$ of the river

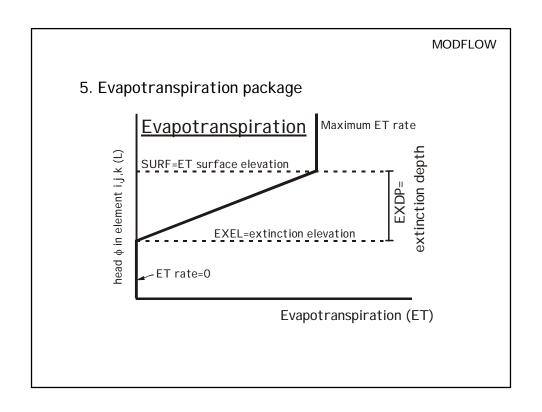


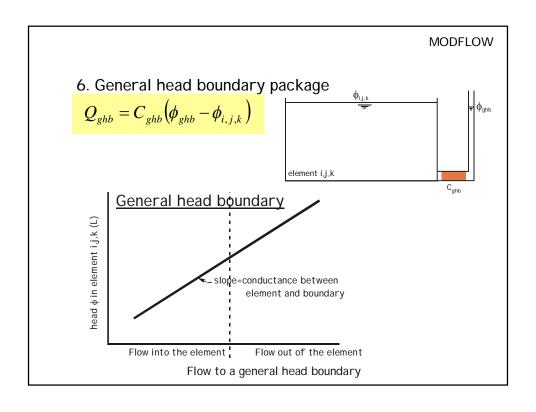


3. Recharge package

$$Q_{rec} = I \Delta x \Delta y$$







```
1 (name).bas-file
2 (name).bcf-file
3 (name).moc-file
4 (name).wel-file
5 (name).riv-file
6 (name).drn-file
7 (name).ghb-file
8 (name).sip-file
9 densin.dat-file
10 (name).nam-files
```

Time indication MODFLOW

ITMUNI=1: seconde ITMUNI=2: minute ITMUNI=3: hour ITMUNI=4: day ITMUNI=5: year

