Numerical Studies of Flow Over a Sill: Sensitivity of the Non-Hydrostatic Effects to the Grid Size

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Non-hydrostatic ocean models

- Status on non-hydrostatic ocean models
- The non-hydrostatic pressure estimation in σ-coordinate models
- When do we need to include non-hydrostatic pressure effects?
- Tidal flow over a sill
- Hydrostatic versus non-hydrostatic results as functions of grid size and SGS closure

Non-hydrostatic ocean models

- In z-coordinates: The MITgcm
- In σ-coordinates: Kanarska and Maderich (2003) - POM
- In ROMS: Kanarska, Shchepetkin, and McWilliams (2007)
- Also available in other models
- May be expensive to compute non-hydrostatic pressure

Non-hydrostatic pressure in σ-models

After transformation from z-coordinates (KM03) :

$$
\frac{\partial}{\partial x}\left(D\frac{\partial P}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{\partial P}{\partial \sigma}\left(\sigma\frac{\partial D}{\partial x} + \frac{\partial \eta}{\partial x}\right)\right) + \frac{1}{D}\frac{\partial^2 P}{\partial^2 \sigma} -
$$

$$
\frac{\partial}{\partial \sigma}\left(\frac{\partial P}{\partial x}\left(\sigma\frac{\partial D}{\partial X} + \frac{\partial \eta}{\partial x}\right)\right) + \frac{1}{D}\frac{\partial}{\partial \sigma}\left(\frac{\partial P}{\partial \sigma}\left(\sigma\frac{\partial D}{\partial x} + \frac{\partial \eta}{\partial x}\right)^2\right) =
$$

$$
\frac{\rho_0}{\Delta t}\left(\frac{\partial \tilde{U} D}{\partial x} + \frac{\partial \tilde{\omega}}{\partial \sigma} + \frac{\partial \eta}{\partial t}\right).
$$

Non-hydrostatic pressure in σ-models

Simplified version in Berntsen and Furnes (2005) :

$$\frac{\partial}{\partial x}\left(D\frac{\partial P}{\partial x}\right)+\frac{1}{D}\frac{\partial^2 P}{\partial^2\sigma}=\frac{\rho_0}{\Delta t}\left(\frac{\partial \tilde{U}D}{\partial x}+\frac{\partial\tilde{\omega}}{\partial\sigma}+\frac{\partial\eta}{\partial t}\right).$$

5 versus 9 points involved in 2D studies
7 versus 19 points involved in 3D studies
Total errors = Model errors + Numerical errors

Surface boundary condition for P?

- $P = 0$ in KM03
- $\frac{\partial P}{\partial n} = 0$ in MITgcm
- Also mixed BCs suggested

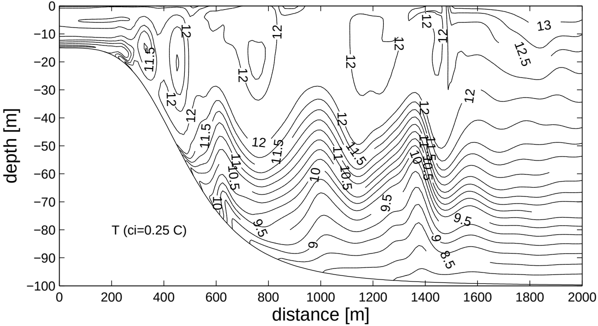
When do we need non-hydrostatic pressure?

- Scale analysis- non-hydrostatic parameter
- 1 to 10 km suggested in Marshall et al. 1997
- Run time decisions suggested in Wadzuk and Hodges 2004
- Process oriented studies
- Sensitive to SGS parameterisation
- If we allow P_{NH} effects without resolving the real non-hydrostatic processes, we may get strong aliasing effects

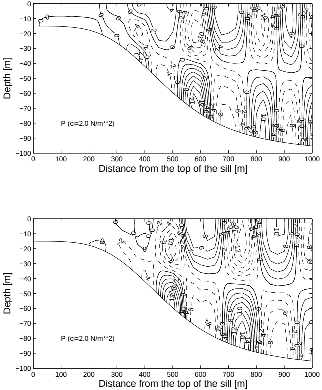
Super-critical flow over a sill (Loch Etive)

See Inall et al. 2004,2005, Stashchuk et al. 2007, and several papers by Xing and Davies.
XD demonstrated strong non-hydrostatic effects using the MITgcm and Δx = 12.5m.

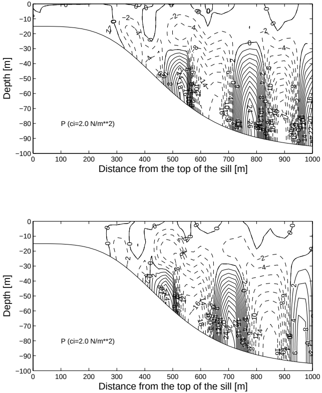
Temperature field after 1/4 T with Δx = 12.5 m.



Pressures at maximum inflow



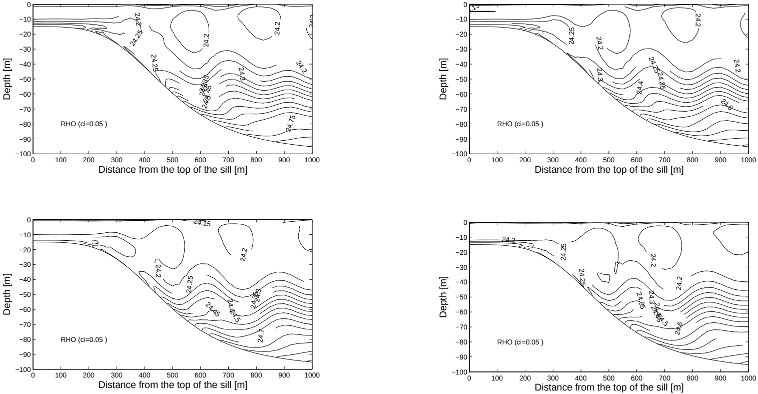
$\partial P / \partial n$ **= 0 at the surface**



P **= 0 at the surface**

Upper panel KM-method and lower panel BF-method

Densities at maximum inflow

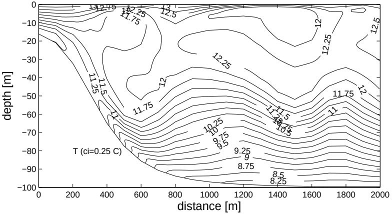


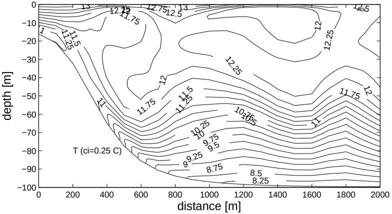
$\partial P / \partial n$ **= 0 at the surface**

P **= 0 at the surface**

Upper panel KM-method and lower panel BF-method
(From paper submitted with Keilegavlen)

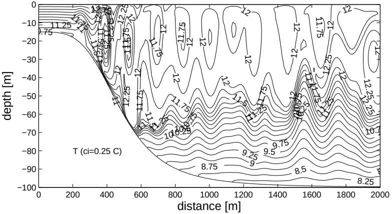
Temperatures at maximum inflow for $\Delta x = 100$ m

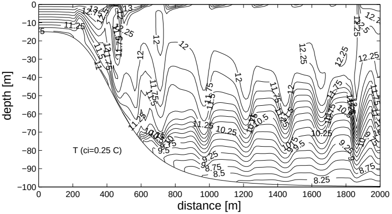




Non-hydrostatic results on top and hydrostatic results below

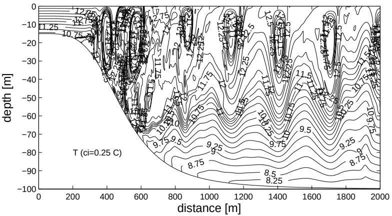
Temperatures at maximum inflow for Δx = 12.5 m

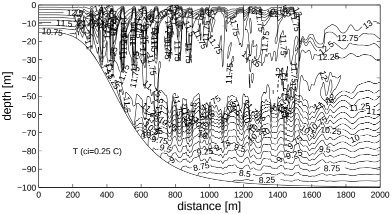




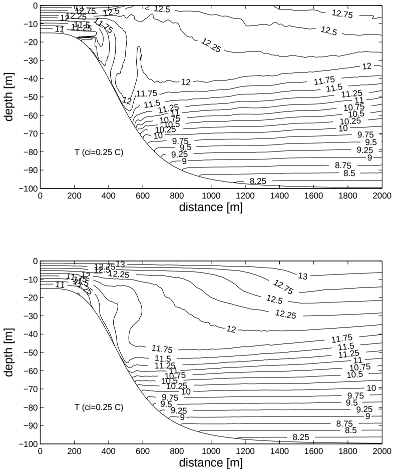
Non-hydrostatic results on top and hydrostatic results below

Temperatures at maximum inflow for Δx = 1.5625 m



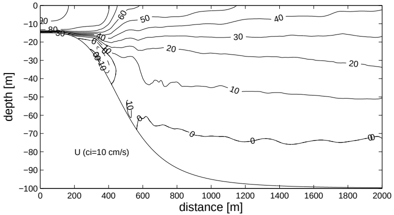


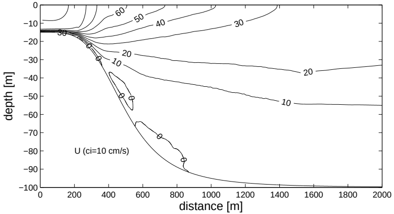
Time mean temperatures for Δx = 1.5625 m



Non-hydrostatic results on top and hydrostatic results below

Time mean U-velocities for Δx = 1.5625 m





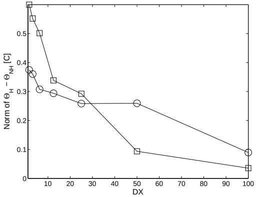
Non-hydrostatic results on top and hydrostatic results below

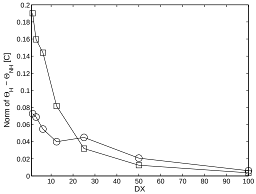
Measuring the effects in the 2-norm

$$\| \phi_H - \phi_{NH} \| = \sqrt{\frac{1}{Area} \int_{0m}^{2000m} \int_{H(x)}^{\eta(x,t)} (\phi_H - \phi_{NH})^2 dxdz}$$

2-norms also computed for the time mean fields

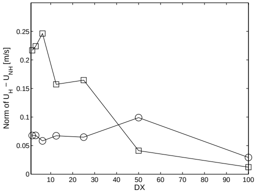
2-norms of differences in temperatures

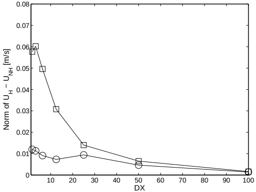




Results at max. inflow on top and for time mean fields below
Squares used for low visc./diff. exp.
Circles used for large visc./diff. exp.

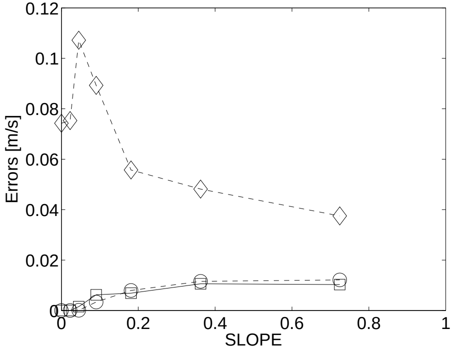
2-norms of differences in U-velocities





Results at max. inflow on top and for time mean fields below
Squares used for low visc./diff. exp.
Circles used for large visc./diff. exp.

Sensitivity to the slope



Errors in time average *U* as function of slope.
Squares used for errors from non-hydrostatic *P* errors
Circles used for errors from internal pressure gradient errors
Diamonds used for 'errors' due to uncertainties in the SGS closure

Discussion-I

- The primary model fields in non-hydrostatic σ-coordinate models robust to the choice of method for the pressure, and to the surface boundary condition
- If viscosity/diffusivity small, stronger non-hydrostatic effects with smaller Δx
- Also clear non-hydrostatic effects on the time mean fields
- For the sill case, the turbulent rotor in the lee is missing in the hydrostatic results
- For the sill case, small non-hydrostatic effects with Δx larger than 25 m
- Possible aliasing effects with too large Δx
- Consequences for 3D studies

Discussion-II

- For full non-hydrostatic effects: DNS
- Internal waves + Kelvin-Helmholtz instabilities represented
- Parameterizations of non-hydrostatic effects into larger grid size hydrostatic models
- There is a cost involved when computing the non-hydrostatic pressure
- We need an elliptic pressure solver
- 10 times SOR per time step