

An unstructured-grid, finite-volume
model of the Irish Sea and Celtic Sea:
Application to tides, surges, and
temperature evolution.

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Introduction

Advantages of unstructured grids;

Applications of finite element models at NOC

(e.g., TELEMAC, ADCIRC, ICOMS);

A model of Irish Sea and Celtic Sea based on
FVCOM (ICS-FVCOM).

An introduction to FVCOM

Developed at Massachusetts-Dartmouth (Chen et al, 2003);

Hydrostatic, terrain-following σ co-ordinate (similar to POM, BOM or POLCOMS);

Finite-volume method on triangle grids;

Tracer advection: MPDATA (Smolarkiewicz 2005); Mellor-Yamada turbulence closure;

Advantages including grid flexibility at complex coastal line, better conservation, etc. ;

Many modules: sediment, ecosystem, wind waves, ice, etc.,

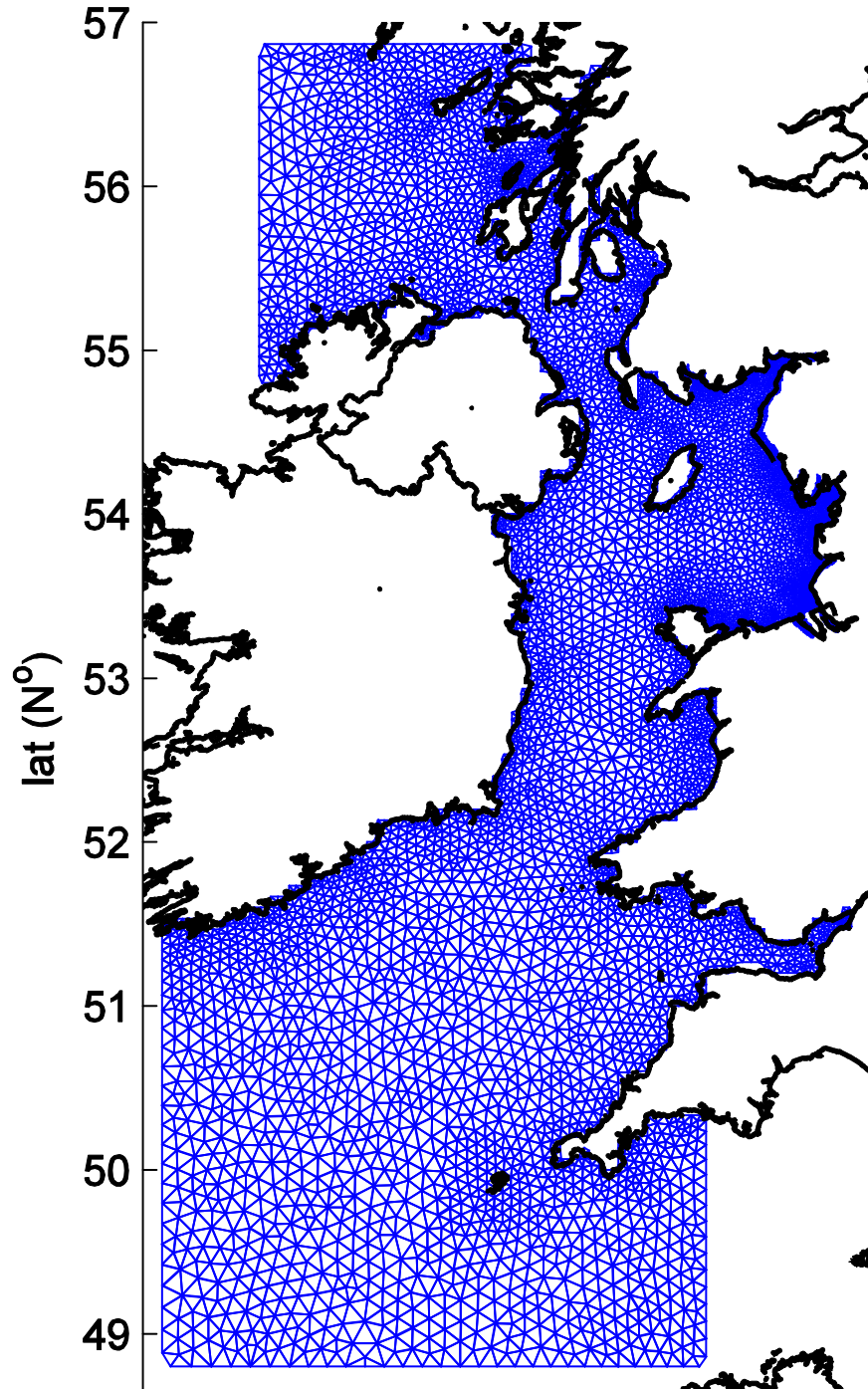
Long-term integration of the model has not been tested;

Improvements are made for the model of the Irish Sea.

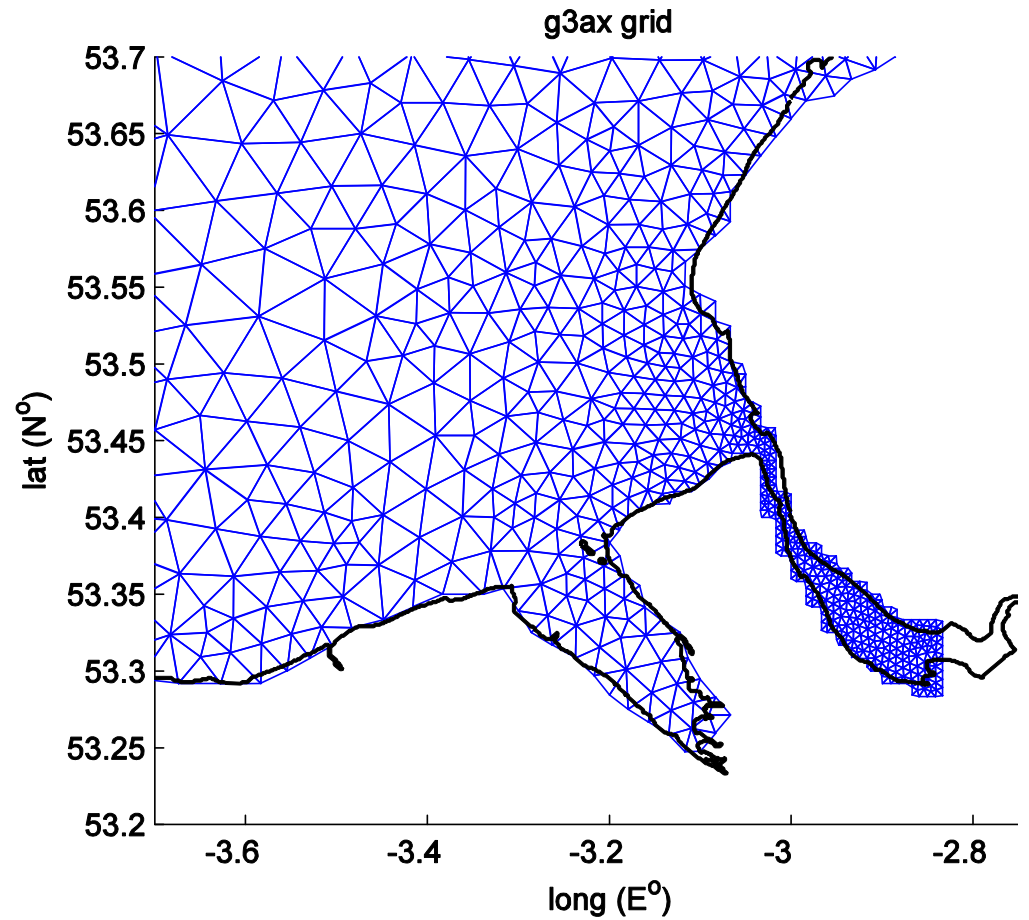
Application to tides, surges and temperature evolution in the Irish Sea and Celtic Sea

1. Barotropic tides: forced at the open boundary
Jones + Davies (Ocean Modelling, 2007) ;
2. Idealized wind force of 1 N/m^2 (effects of breaking waves)
3. Realistic meteorological forcing due to wind and heating and tides (temperature evolution);
4. ROFI in Liverpool Bay .

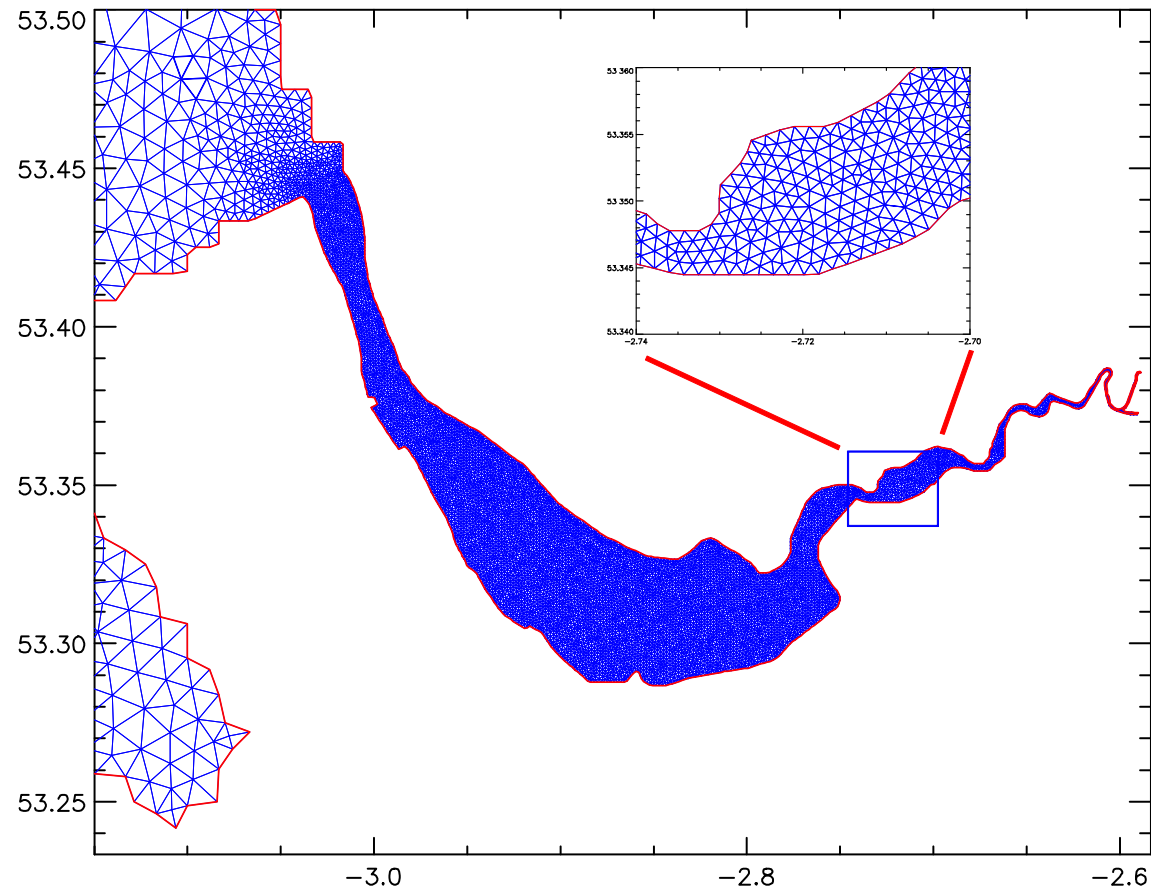
g3ax grid



Irish sea model grids (g3ax)



Refined grids in river Mersey

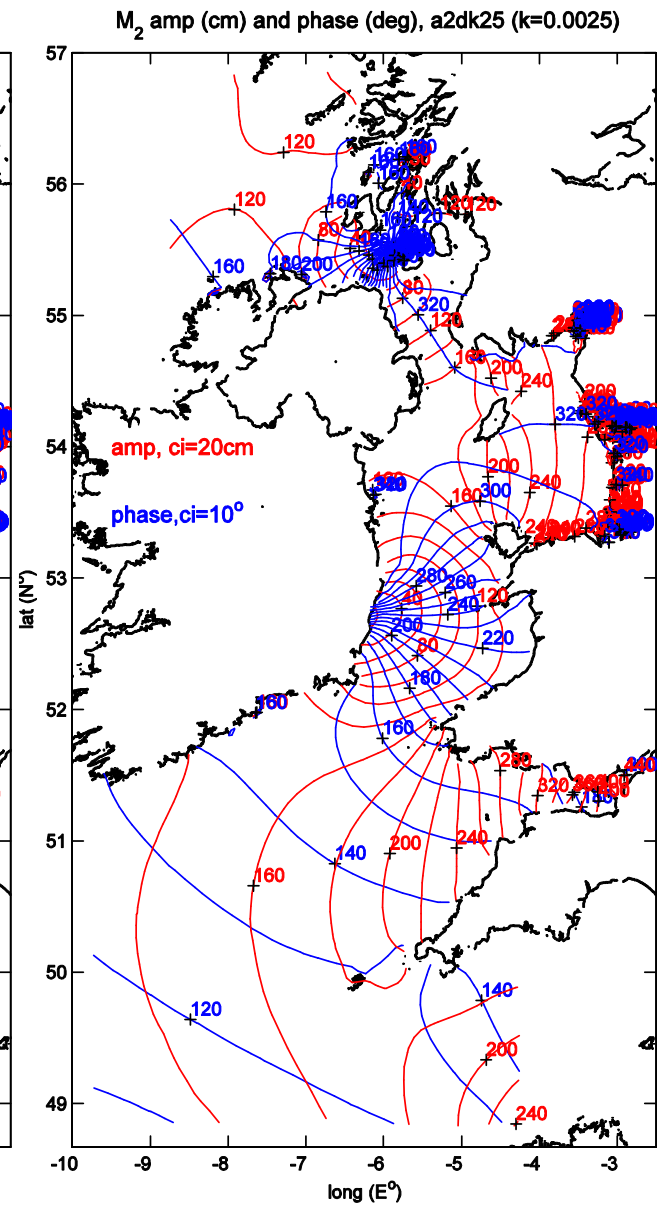
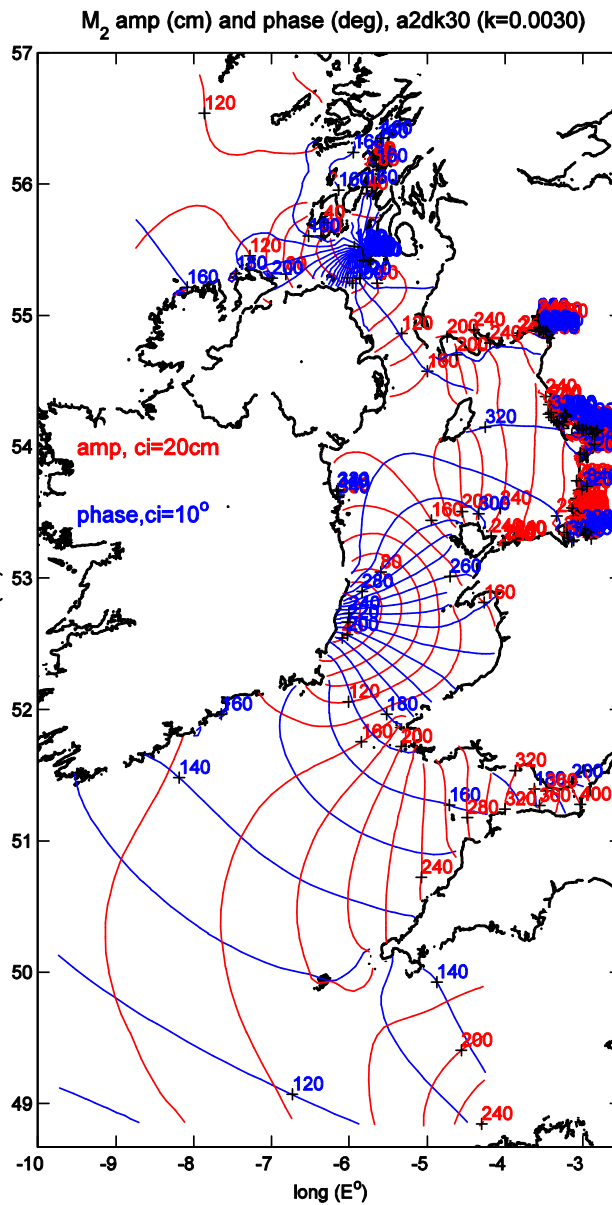
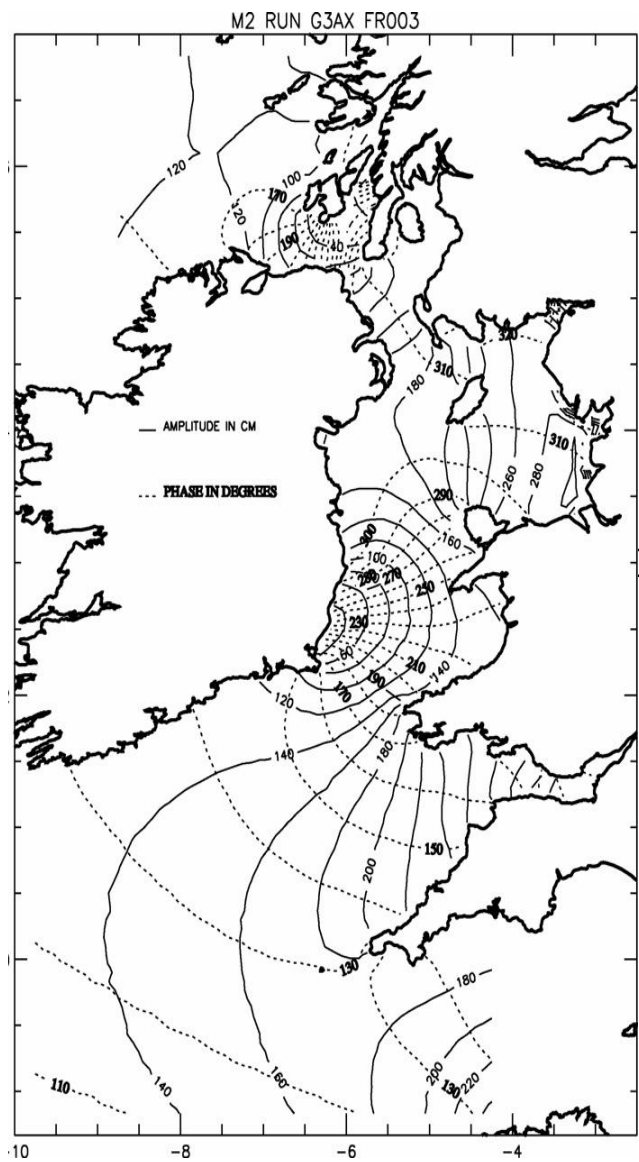


M2 tide:

TELMAC 2D (k=0.003)

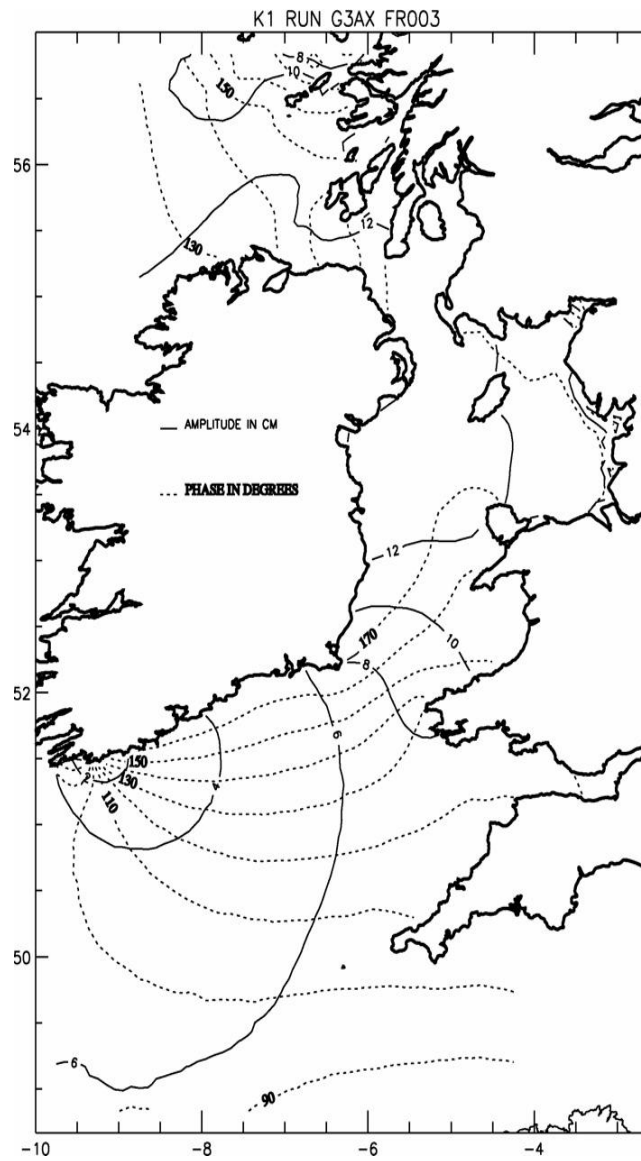
ICS-FVCOM (k=0.003)

ICS-FVCOM(k=0.0025)

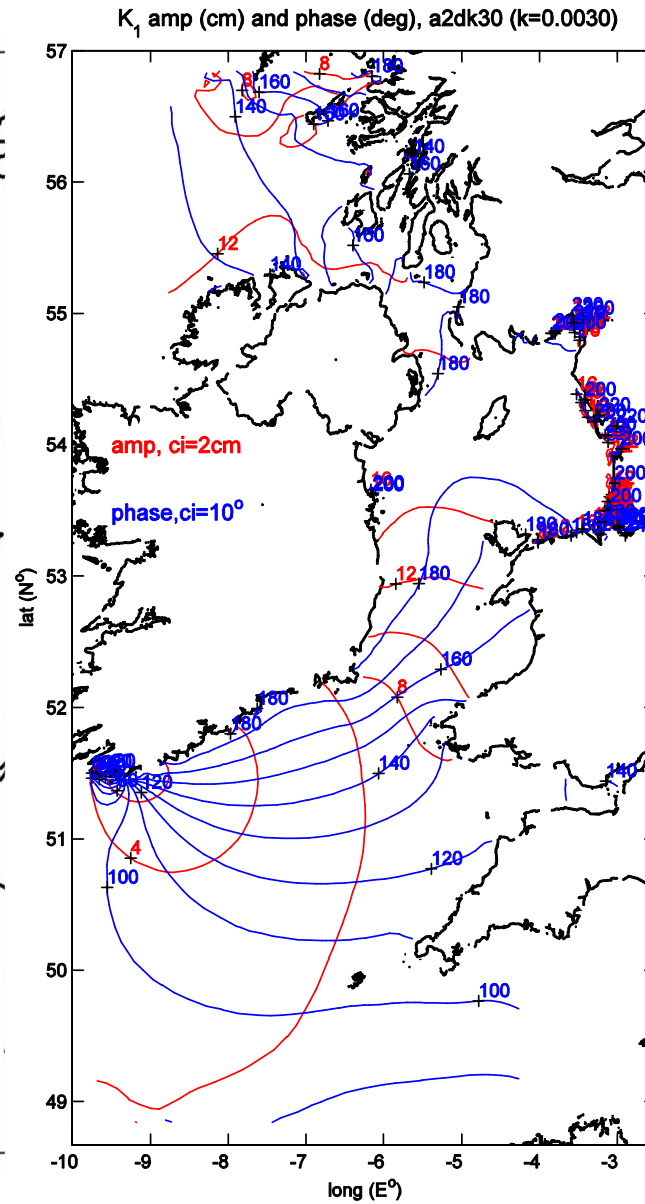


K1

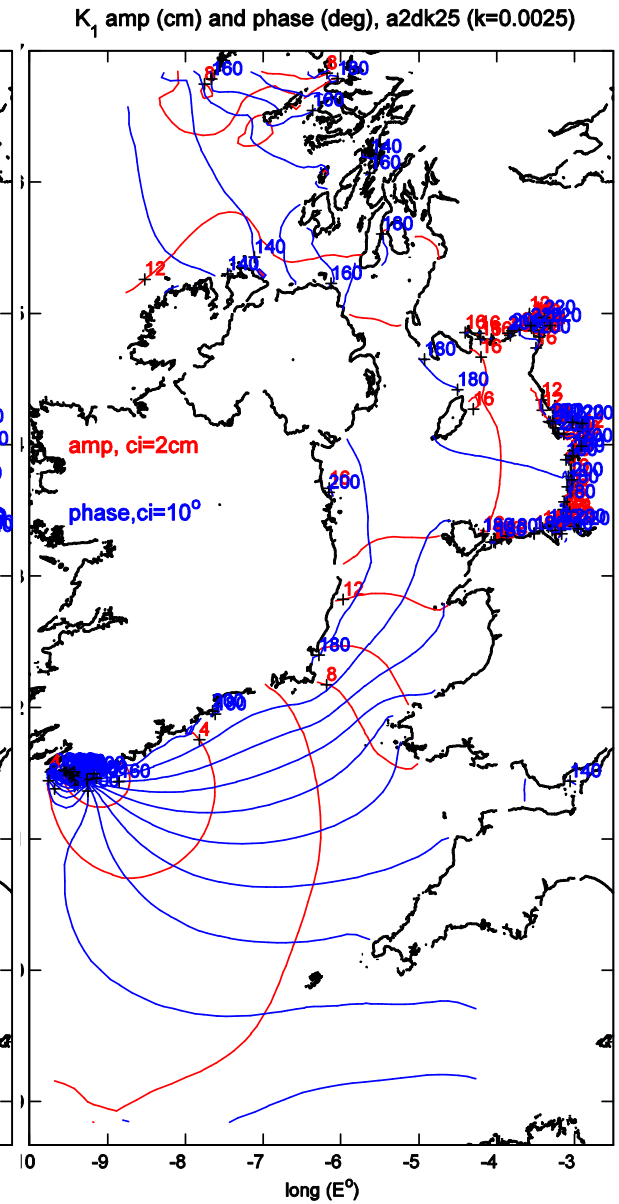
TELMAC 2D (k=0.003)



ICS-FVCOM (k=0.003)

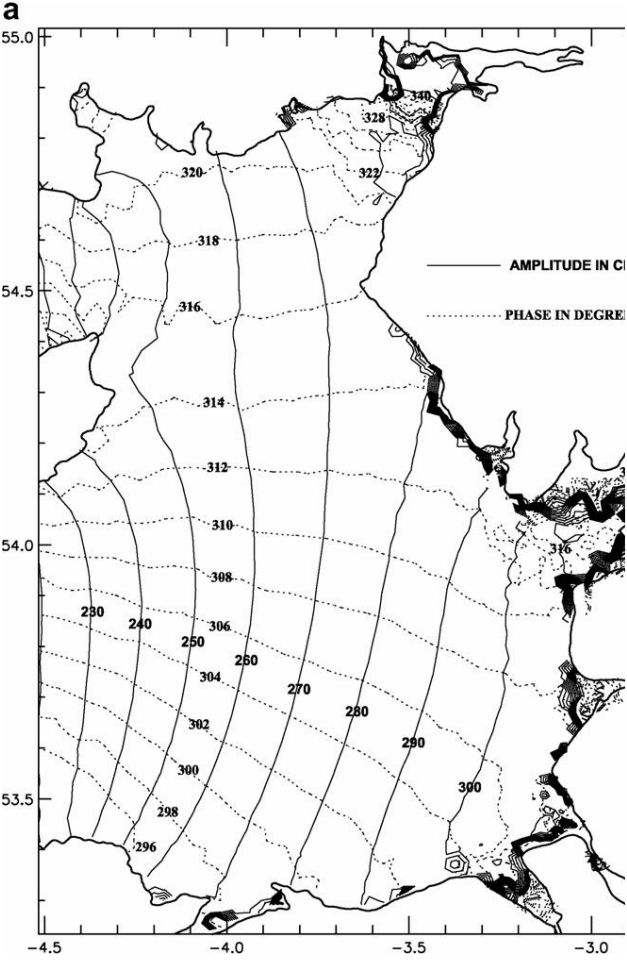


ICS-FVCOM(k=0.0025)



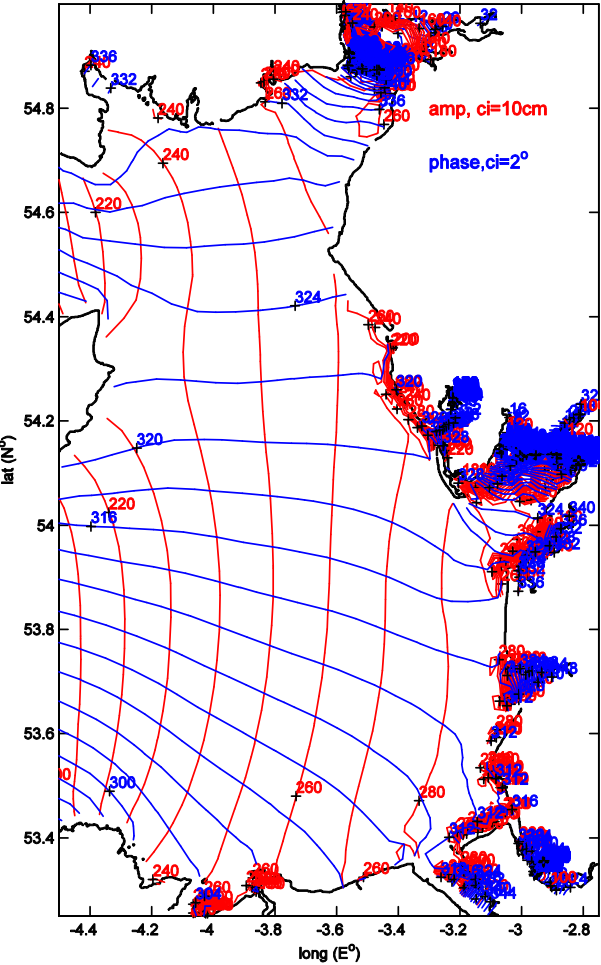
Computed M2 tide in the Eastern Irish Sea

TELEMAC 2D



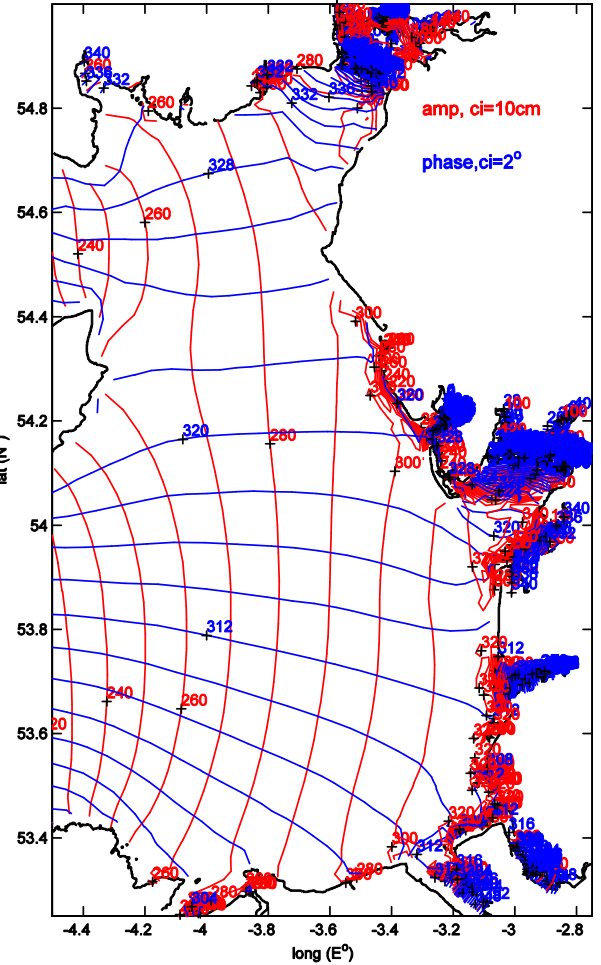
ICS-FVCOM 2D

M₂ amp (cm) and phase (deg), a2dk30 (k=0.0030)



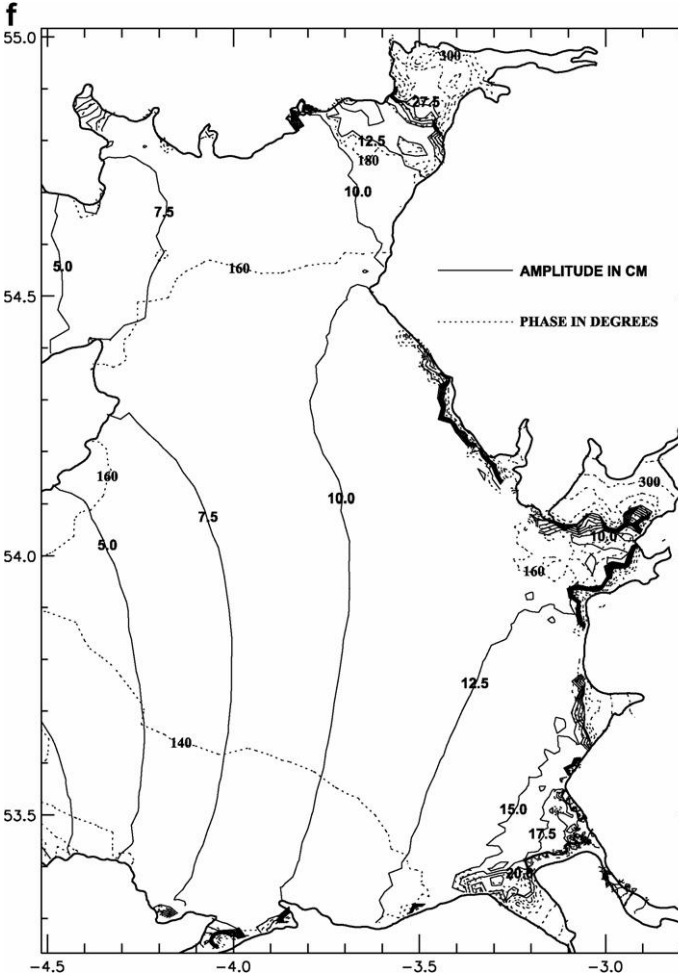
ICS-FVCOM 3D

M₂ amp (cm) and phase (deg), a3d



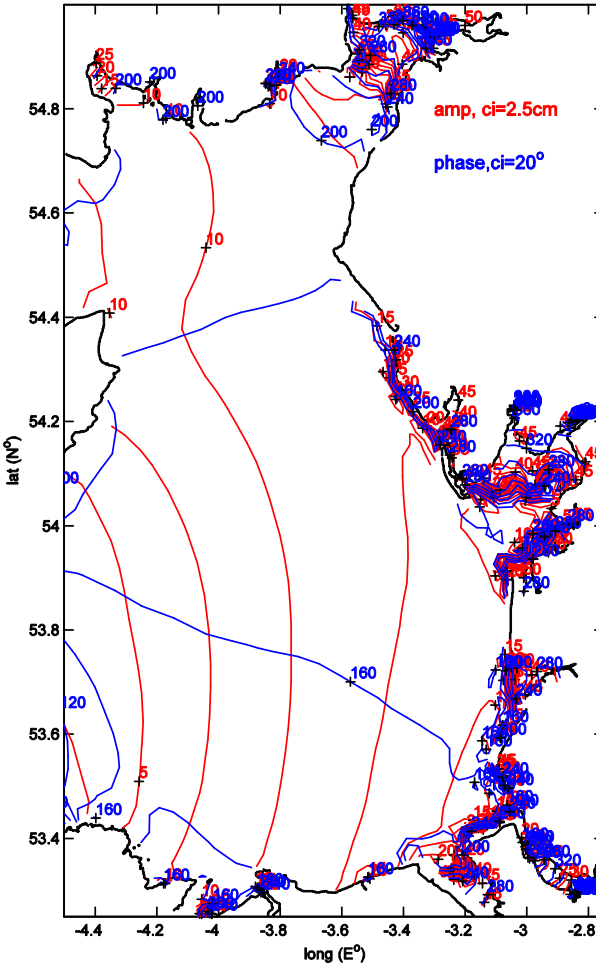
Computed M4 tide in the Eastern Irish Sea

TELEMAC 2D



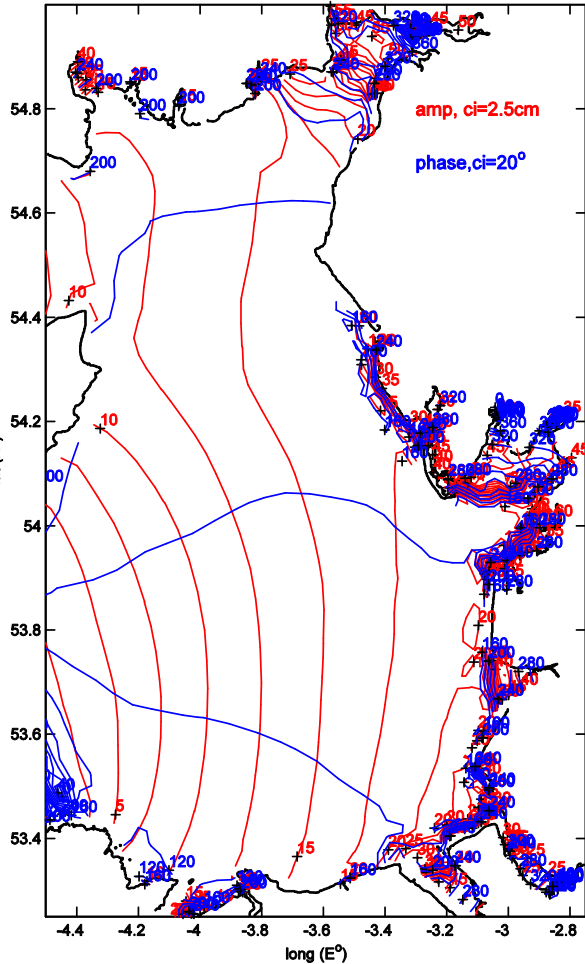
ICS-FVCOM 2D

m_4 amp (cm) and phase (deg), a2dk30 ($k=0.0030$)



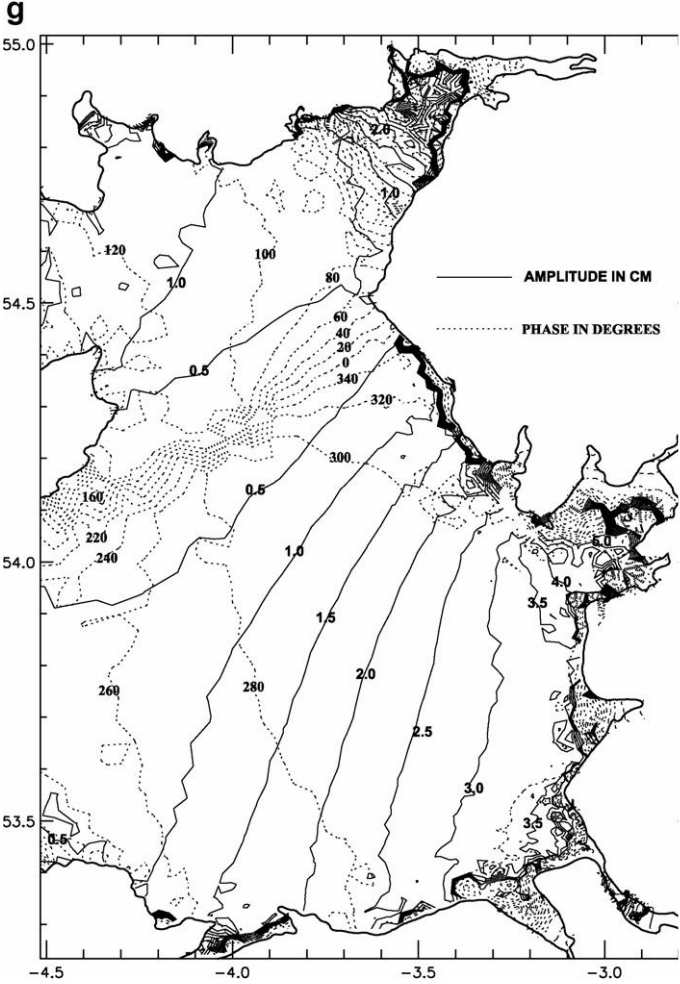
ICS-FVCOM 3D

m_4 amp (cm) and phase (deg), a3d



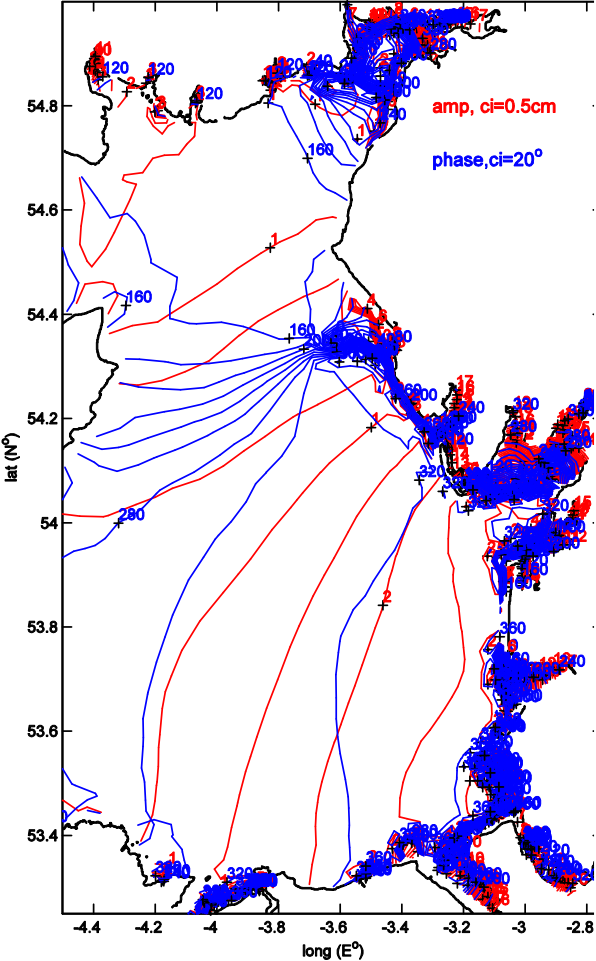
Computed M6 tide in the Eastern Irish Sea

TELEMAC 2D



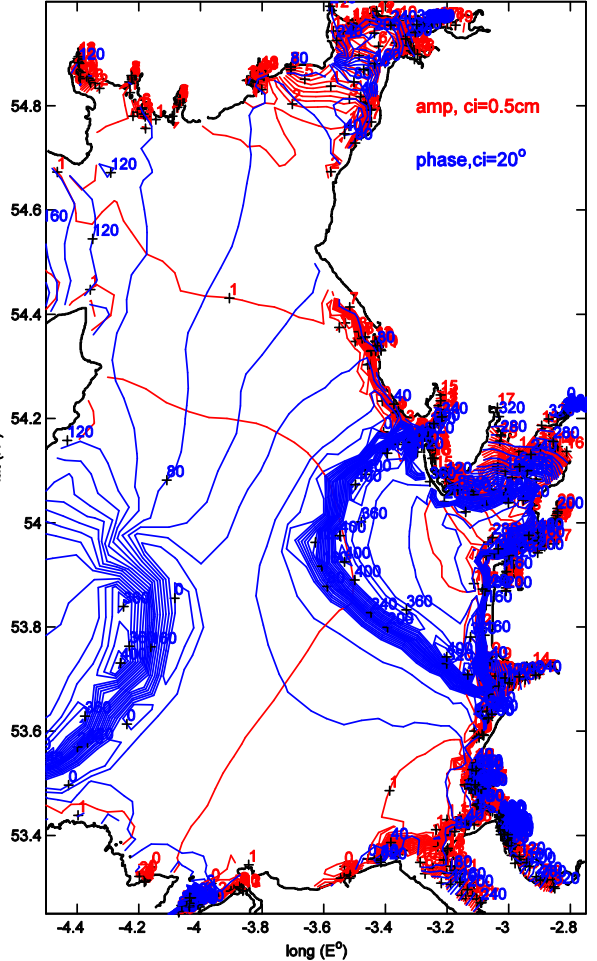
ICS- FVCOM 2D

M_6 amp (cm) and phase (deg), a2dk30 ($k=0.0030$)

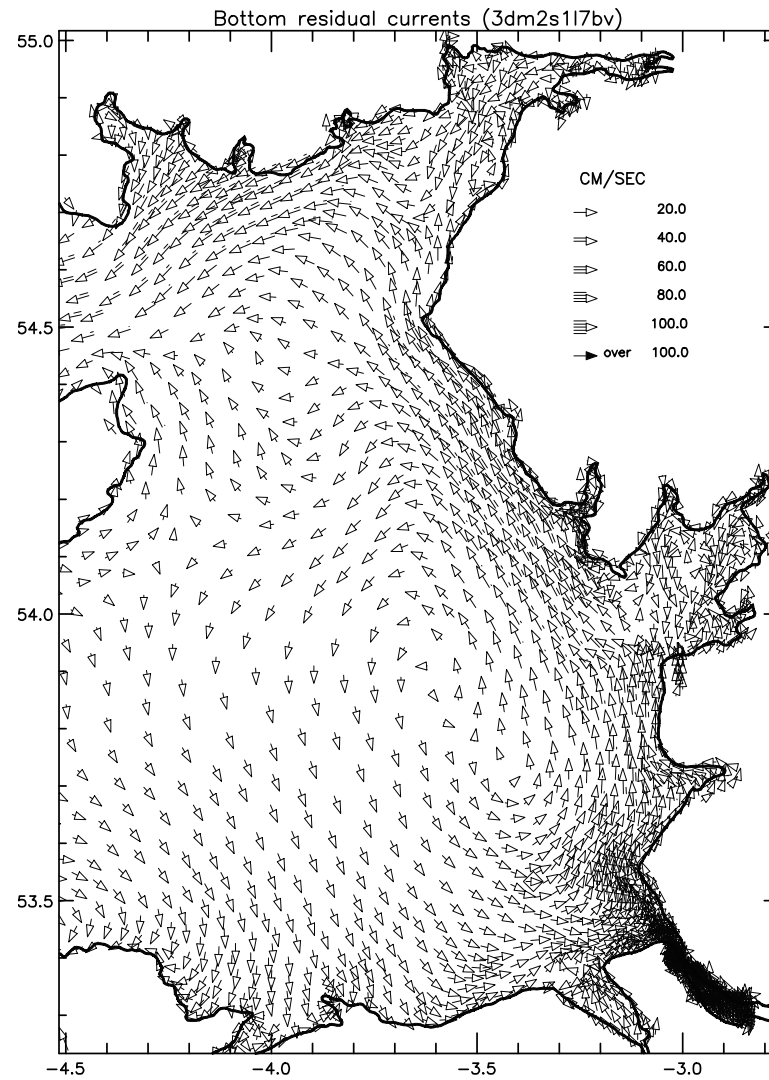
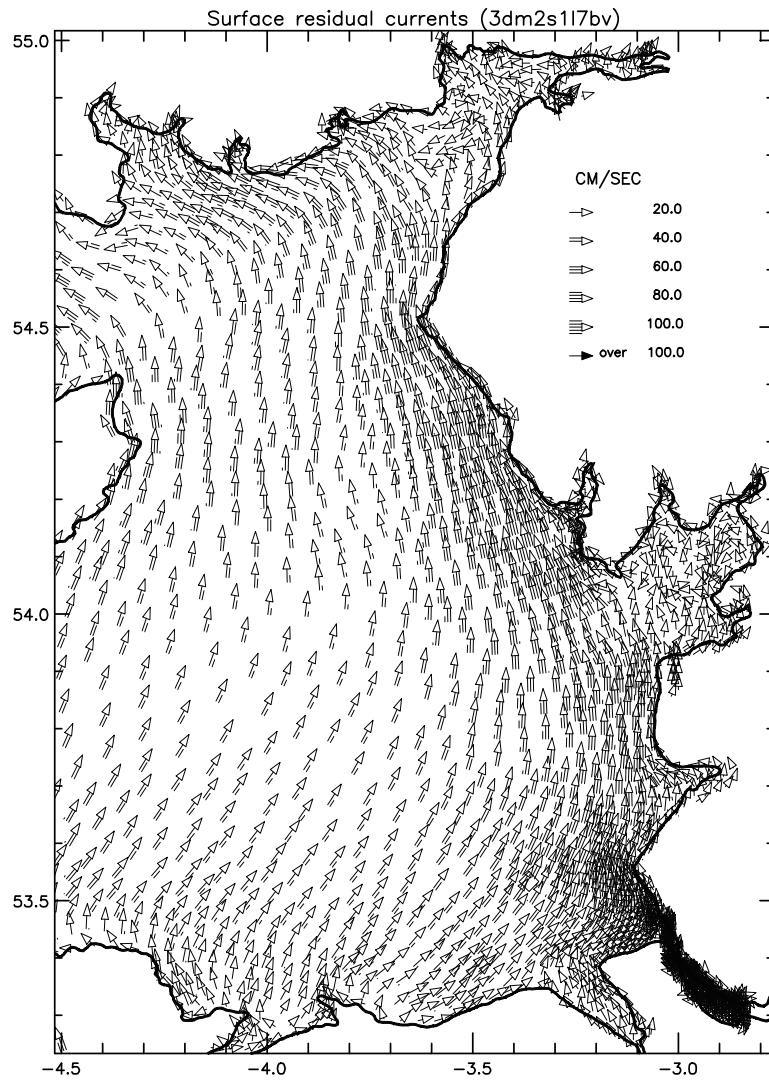


ICS- FVCOM 3D

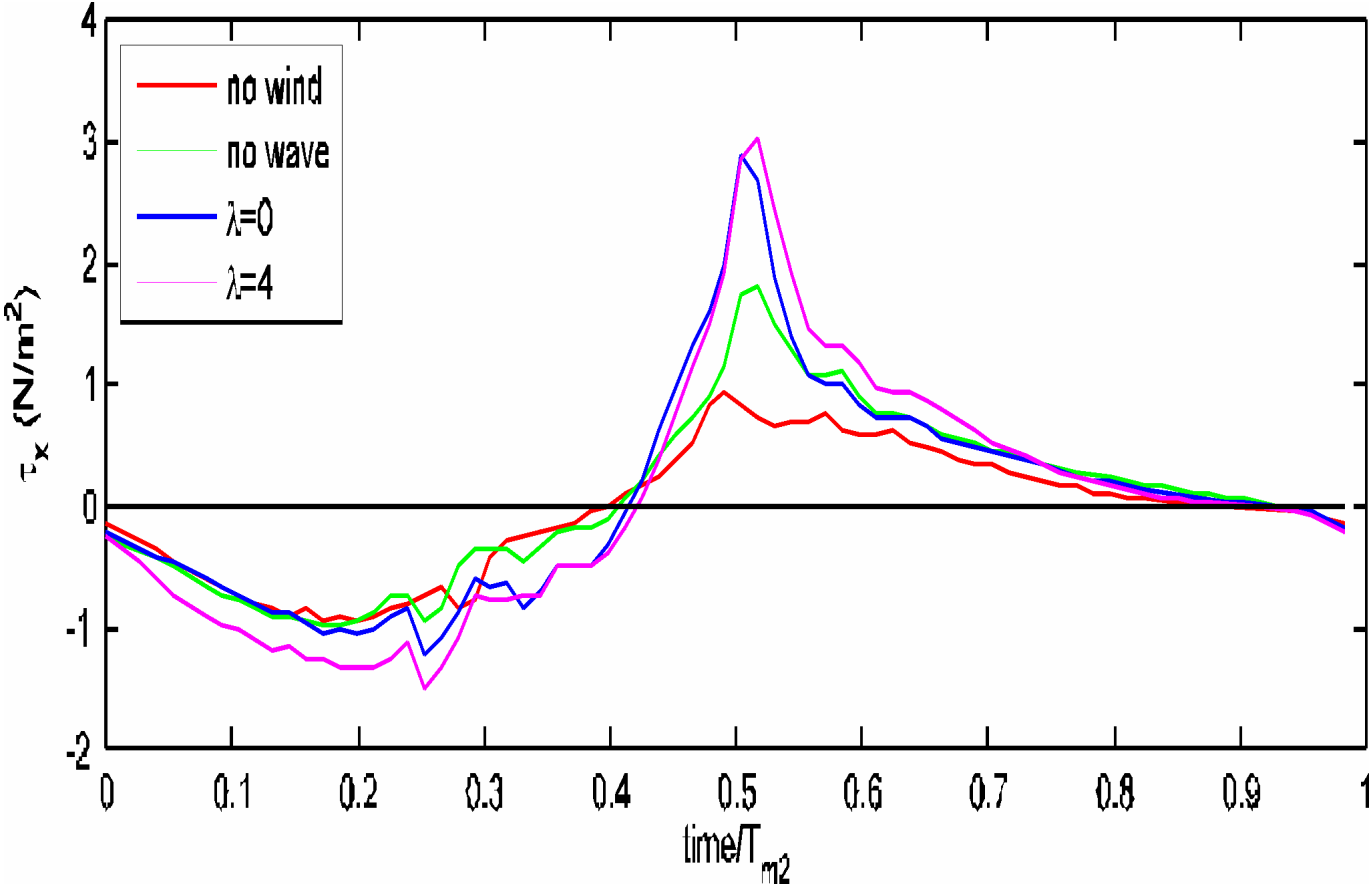
M_6 amp (cm) and phase (deg), a3d



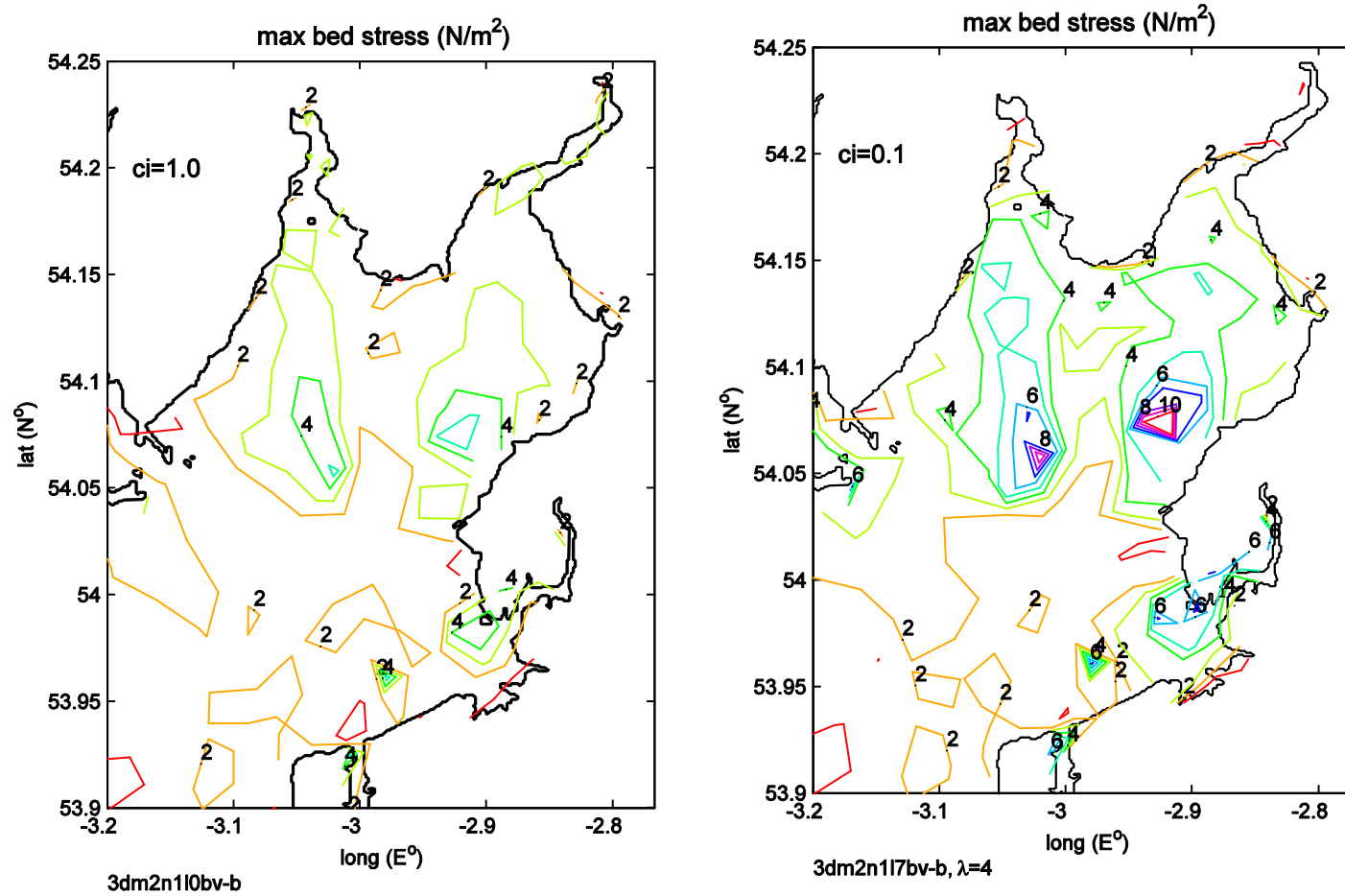
Surface and bottom currents (south wind stress 1pa)



Bed stress at a location in the Liverpool Bay

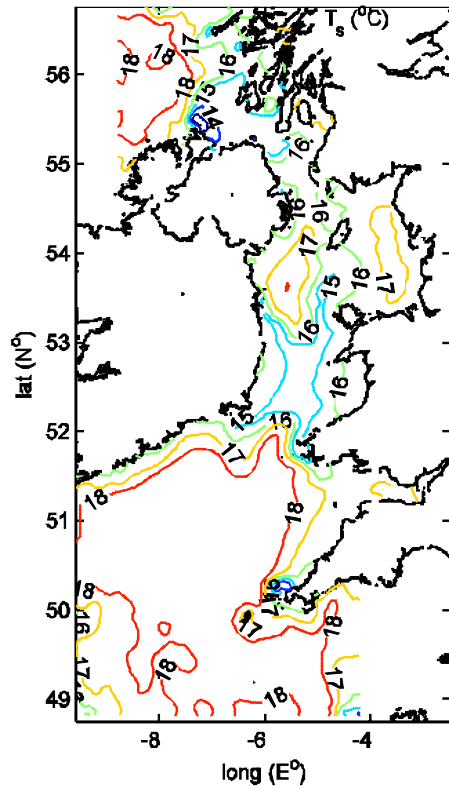


Effects of wave breaking parameterization on maximum bed stress in the shallow water region

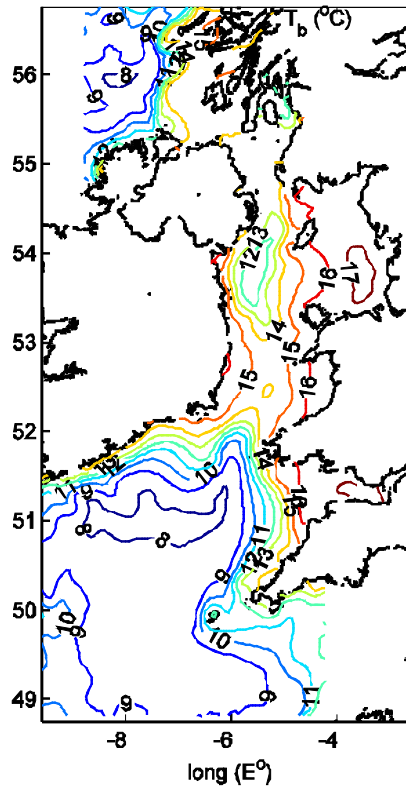


Temperature on 16 Aug. 1995

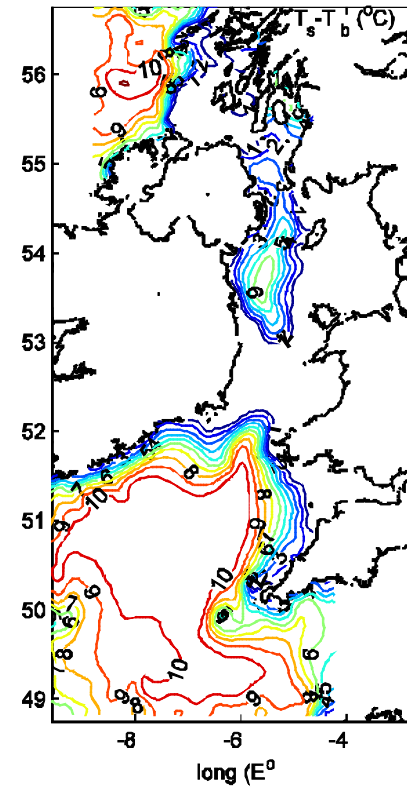
Surface temp (T_s)



Nearbed temp (T_b)

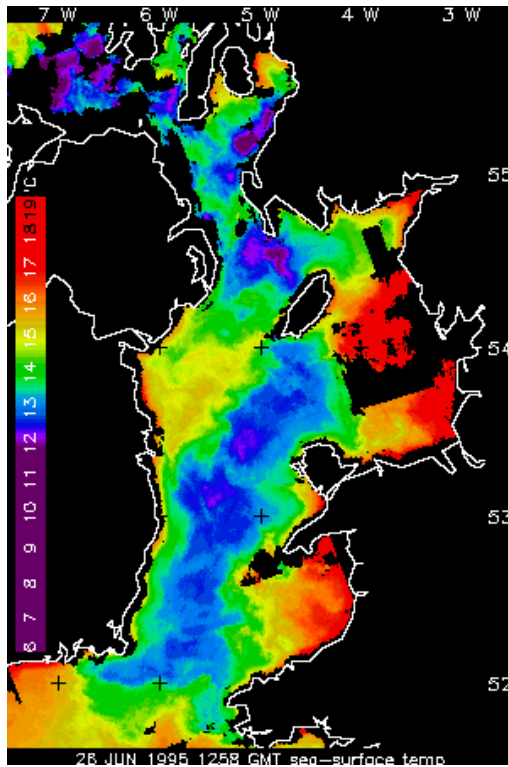


$T_s - T_b$

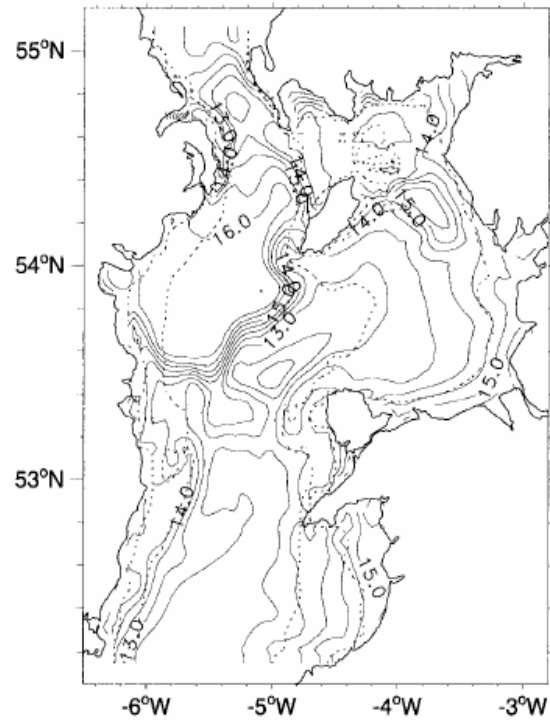


Surface temperature on 26 June 1995

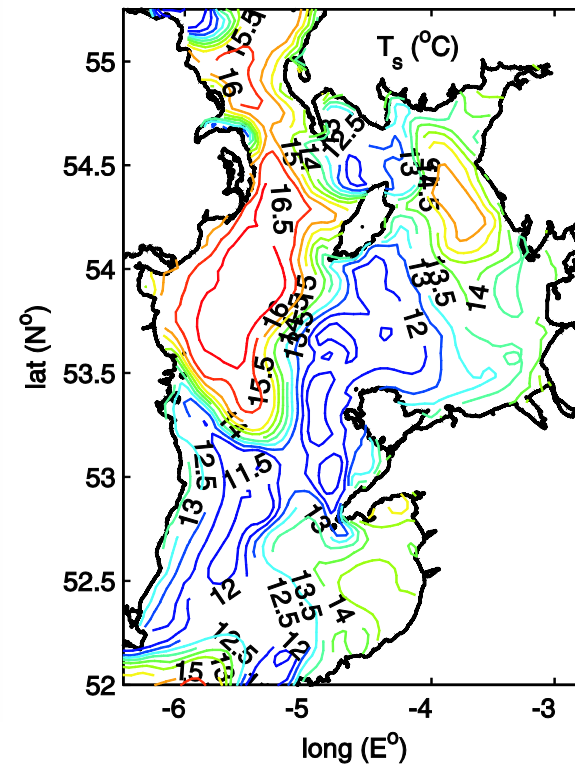
OBS



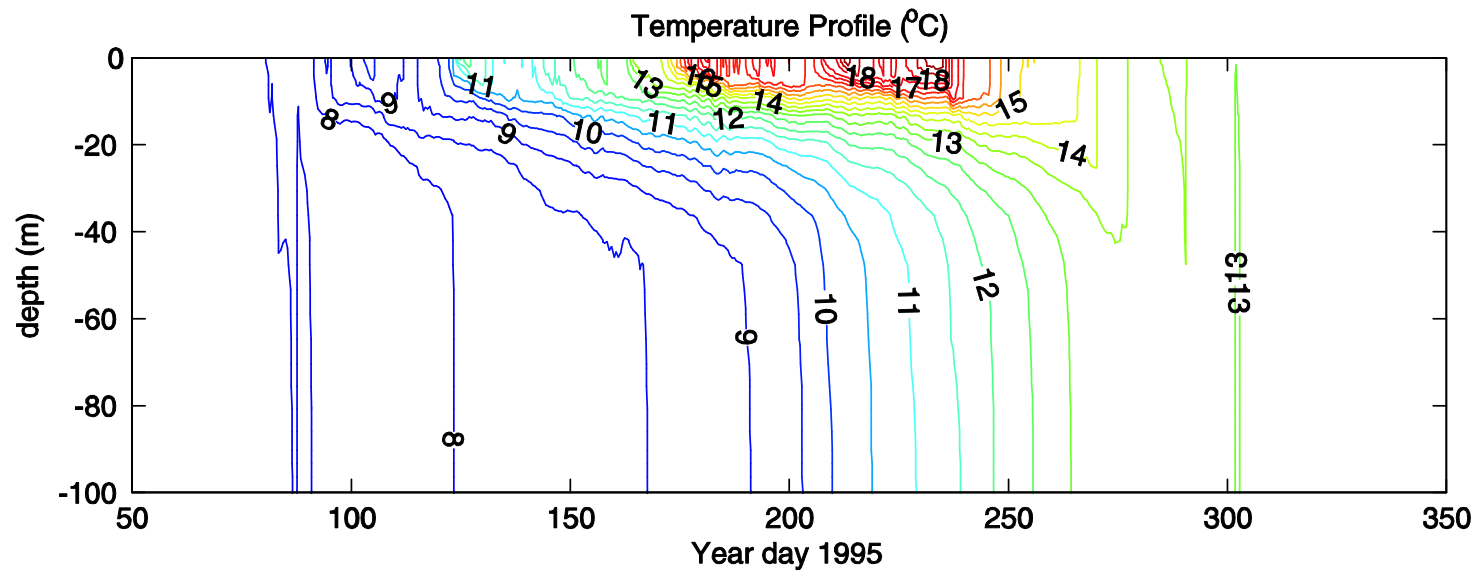
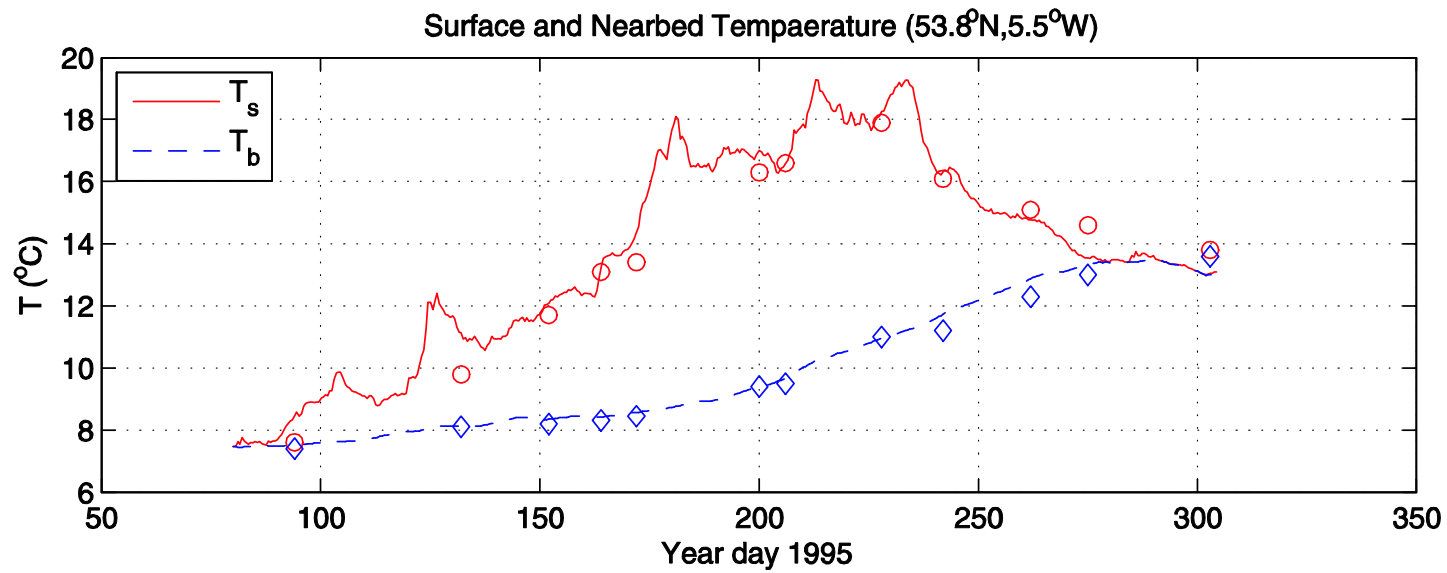
Xing+Davies JPO 2000



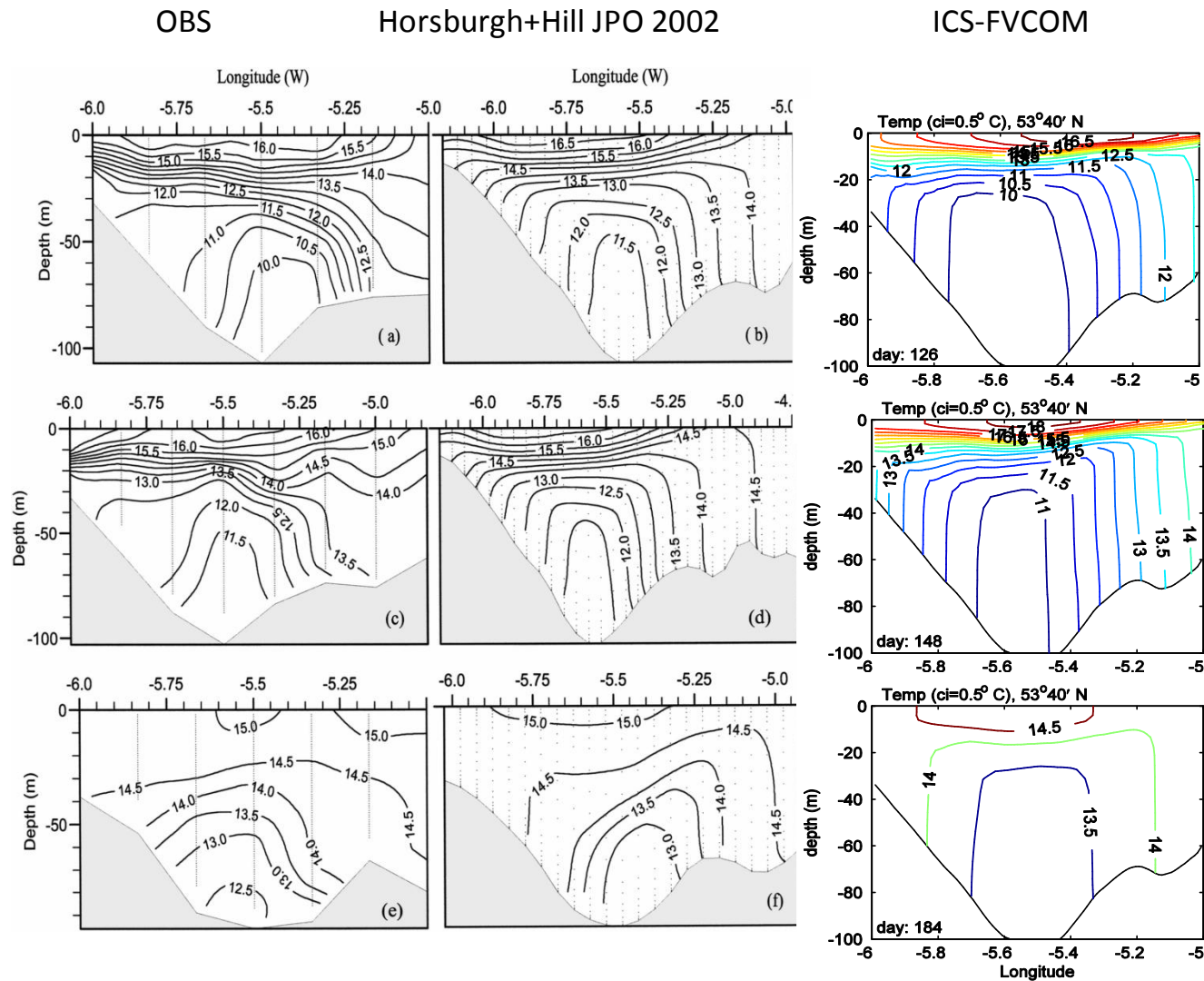
ICS-FVCOM



Time series of temperature at the West Irish sea

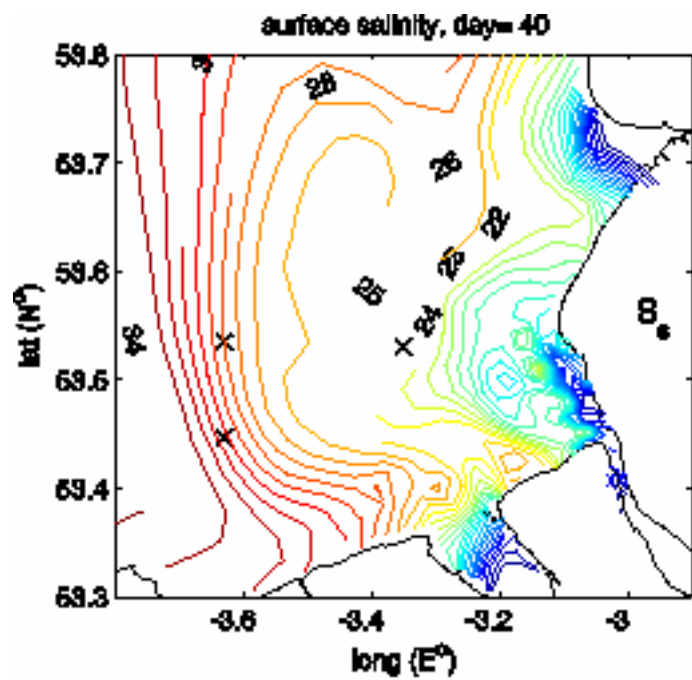


(x-z) section of temperature along 53° 40' N from observations, models of POM and ICS-FVCOM

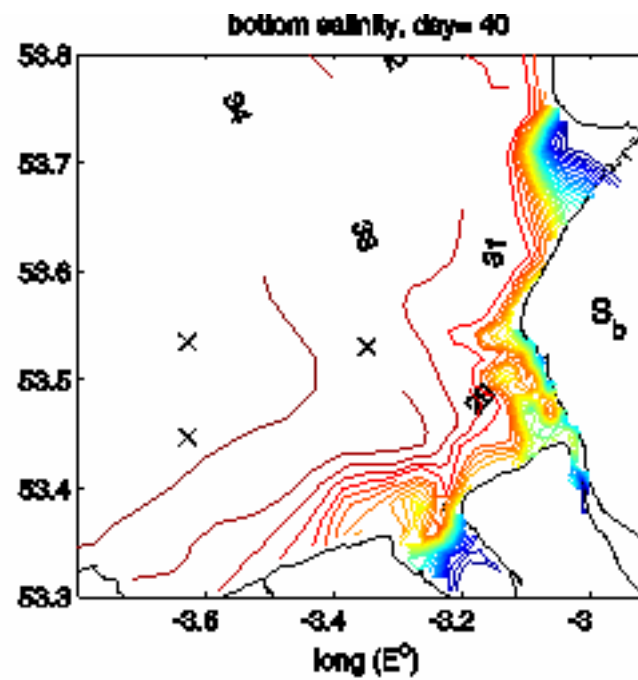


ROFI: fresh water discharge in Liverpool Bay

Surface salinity



bottom salinity



3-riv-mult, day= 40

Conclusions

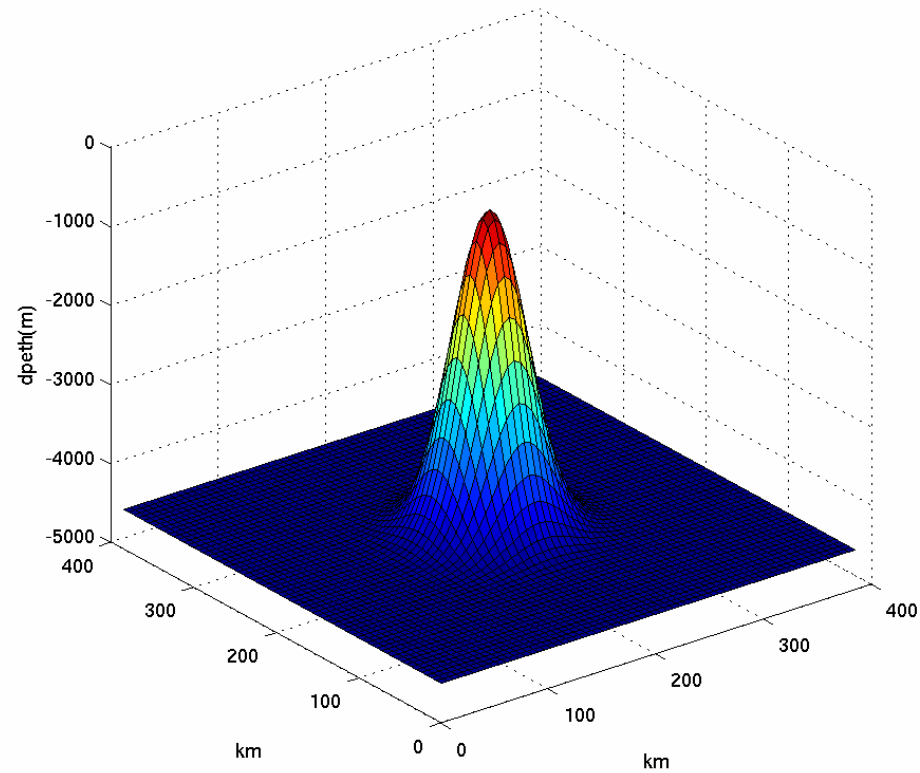
A robust finite-volume, unstructured-grid model of the Irish sea based on FVCOM is established;

Validation work is in progress (bartropic tides are well reproduced);

A shelf wide finite-volume unstructured-grid model for the purpose of climate study is achievable in the near future;

Future improvements: turbulence, HPG, efficiency ...

The seamount test case (Haidvogel+Beckman, Berntsen+Furnes 2005, etc.)



Model setup and parameters

FVCOM v2.7 (full non-linear hydrostatic model),
with uniform resolution 3, 6 km

No vertical diffusion and bottom friction

No horizontal diffusivity

Horizontal viscosity (10^3 , 10^2 m²/s)

A Gaussian form of topog. (450-4500m)

An initial exponential density profile ($\Delta\rho=1.5$
kg/m³)

Internal pressure gradient in σ -coordinate system, $[\sigma=(z-\zeta)/D]$ and finite volume method (Chen et al)

u equation and internal pressure gradient term:

$$\frac{\partial Du}{\partial t} + \nabla \cdot (\bar{V}Du) + \dots = -\frac{g}{\rho_0} \left[D \frac{\partial}{\partial x} \left(D \int_{\sigma}^0 \sigma \frac{\partial \rho}{\partial \sigma} d\sigma \right) + D^2 \frac{\partial \rho \sigma}{\partial x} \right] + \dots$$

$$\begin{aligned} IPG &= -\frac{g}{\rho_0} \left\{ \iint D \frac{\partial}{\partial x} \left[D \left(\int_{\sigma}^0 \sigma \frac{\partial \rho}{\partial \sigma} d\sigma \right) \right] dx dy + \iint D^2 \frac{\partial \rho \sigma}{\partial x} dx dy \right\} \\ &= -\frac{g}{\rho_0} \left\{ \bar{D} \oint \left(D \int_{\sigma}^0 \sigma \frac{\partial \rho}{\partial \sigma} d\sigma \right) dy + \bar{D}^2 \oint \rho \sigma dy \right\} \end{aligned}$$

Green's Theorem:

$$\oint -Ndx + Mdy = \iint \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy$$

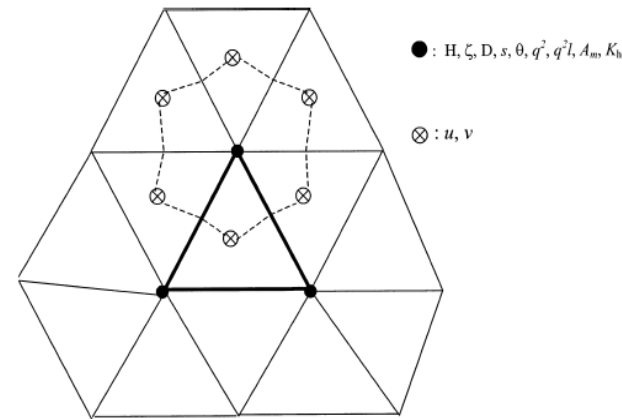
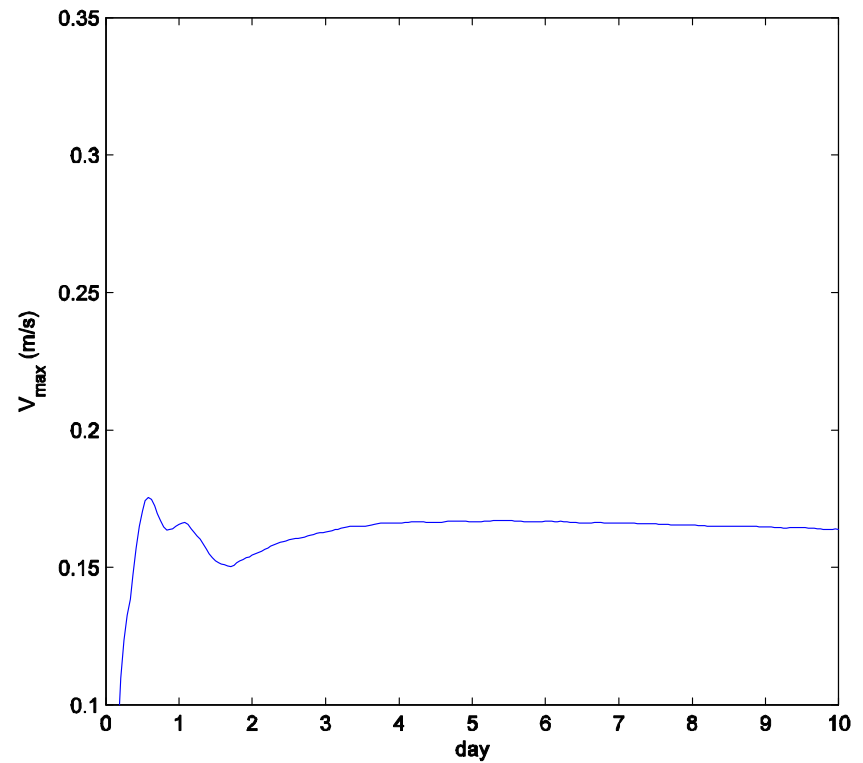
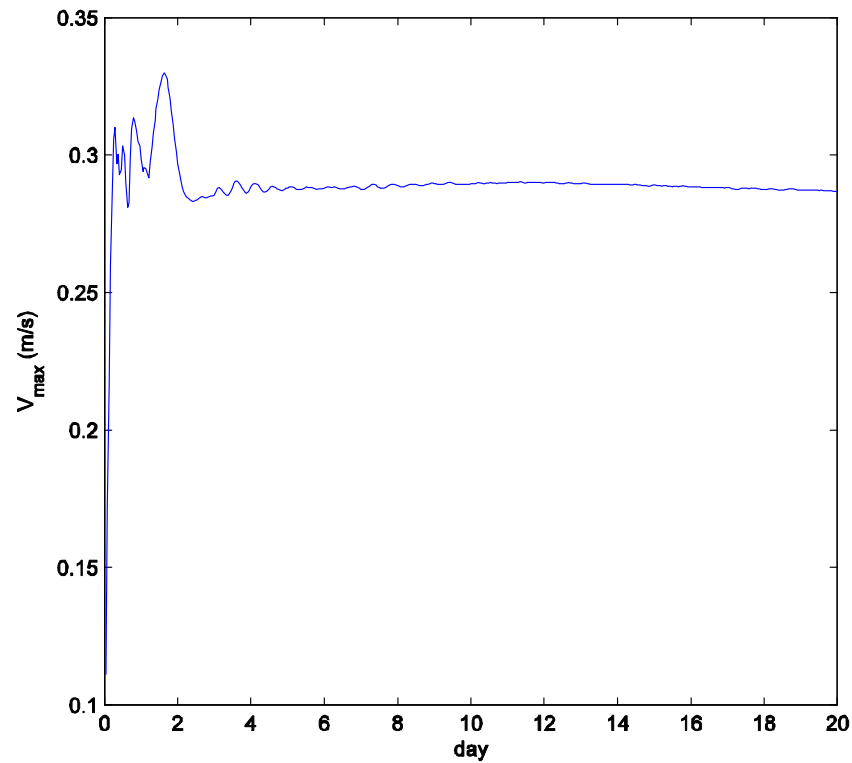
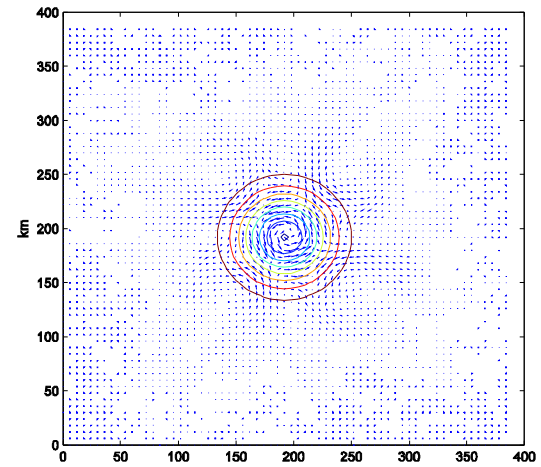
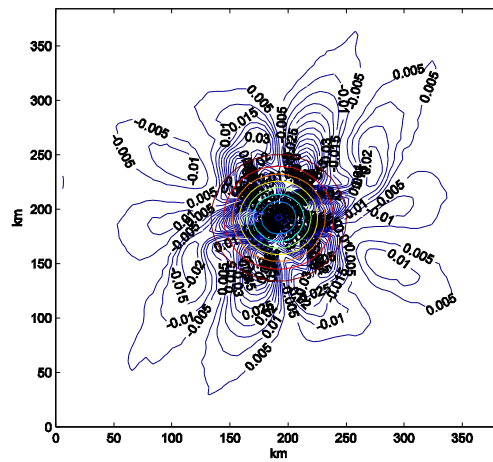
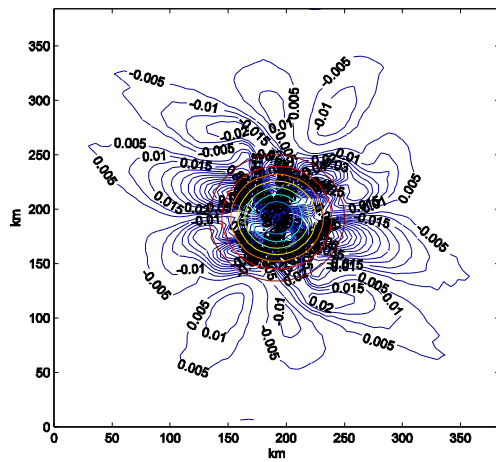
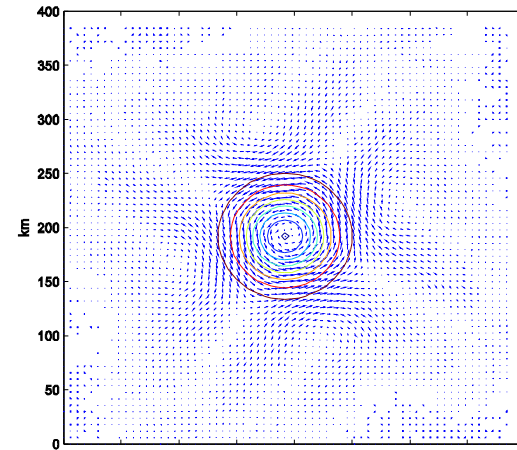
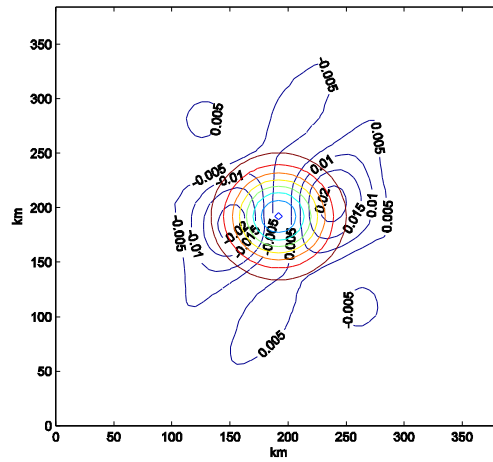
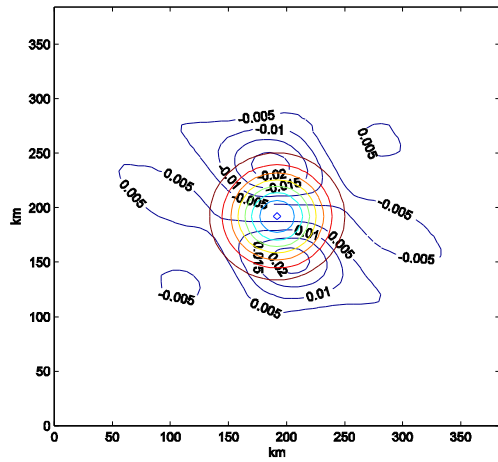


FIG. 1. The unstructured grid for the finite-volume model.

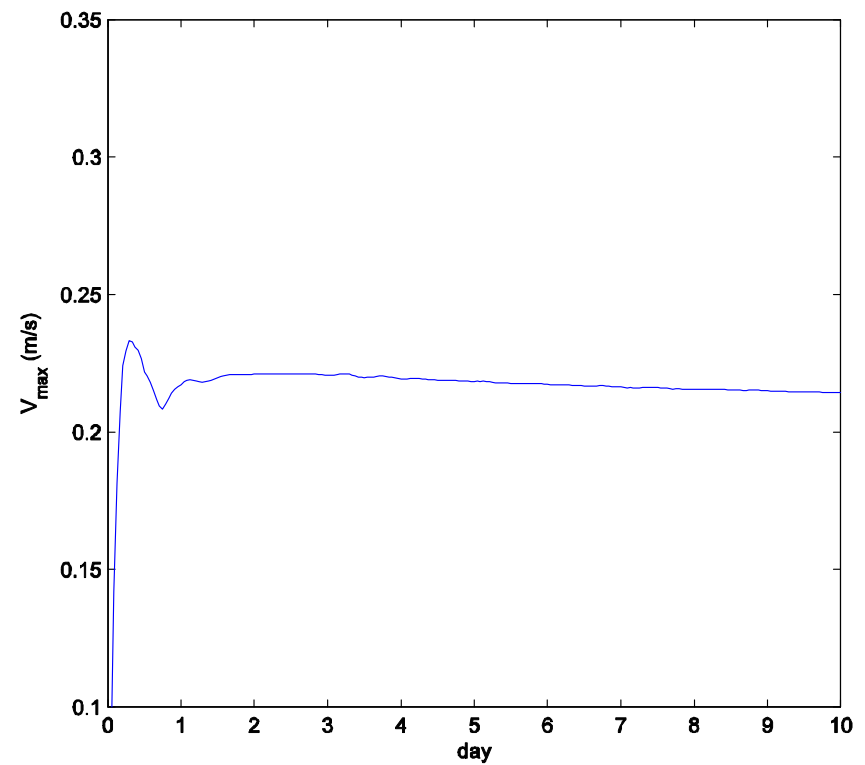
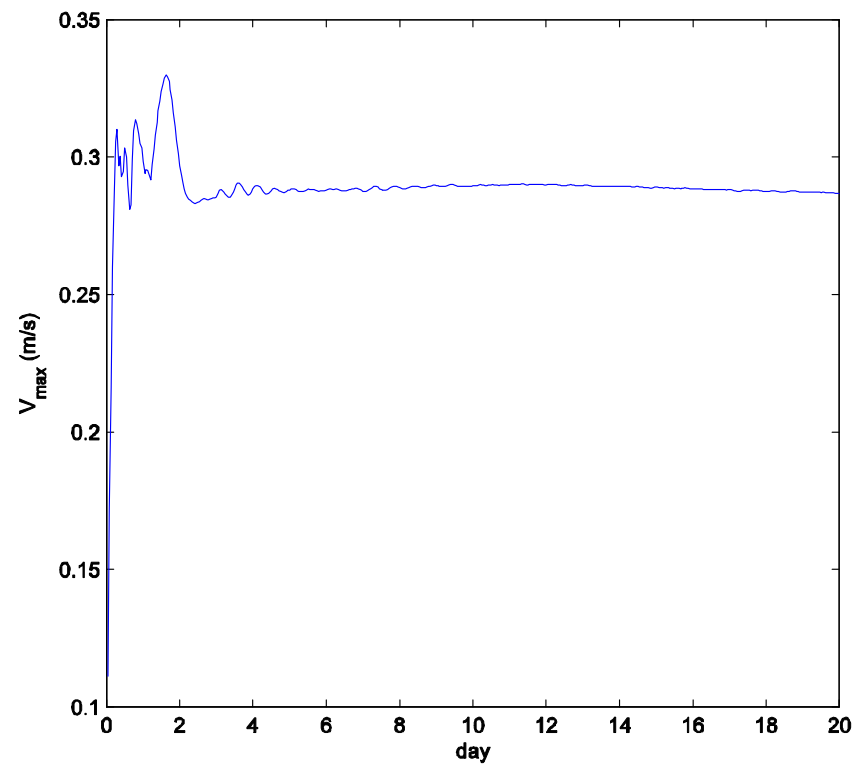
Maximum velocities ($Ah=1000\text{m}^2/\text{s}$) (isosceles, resolution, left: 6km, right: 3km)



Top and bottom layer (u,v) at day 10 (ci=0.5 cm/s, isosceles, res=3km)



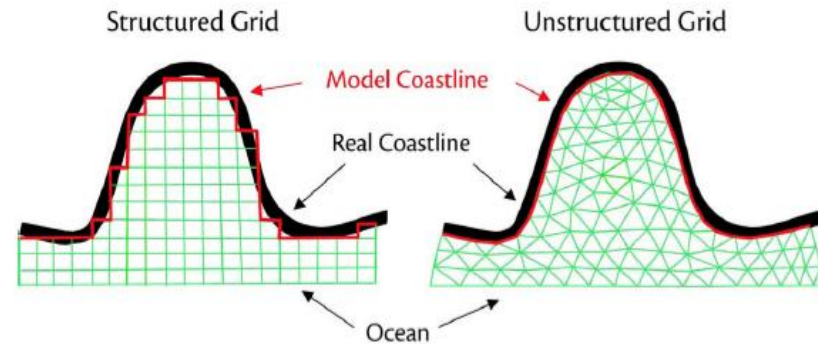
Maximum velocities ($Ah=1000\text{m}^2/\text{s}$) (left: isosceles, right: equilaterals)



Thank you for your attention !

An introduction to FVCOM

- Developed at Massachusetts-Dartmouth (Chen et al, 2003);
- Hydrostatic ; terrain-following σ coordinate (similar to POM);
- Second-order accurate discrete flux calculation in the integral form of continuity equations;
- Trace advection: MPDATA (Smolarkiewicz 2005); horizontal diffusion: Smagorinsky; vertical: Mellor Yamada closure scheme;
- Advantages include grid flexibility at complex coastal line, better conservation, etc. ;
- **Long-term integration of the model has not been tested yet;**
- **Improvements are made for the model Irish Sea.**



$$\iint \frac{\partial \zeta}{\partial t} dx dy = - \iint \left(\frac{\partial u D}{\partial x} + \frac{\partial v D}{\partial y} \right) dx dy$$

$$= - \oint V_n D ds$$

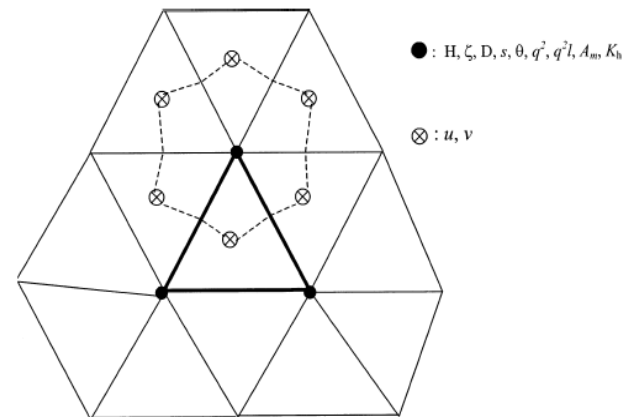


FIG. 1. The unstructured grid for the finite-volume model.

Internal pressure gradient in σ -coordinate system, $[\sigma=(z-\zeta)/D]$ and finite volume method (in FVCOM)

u equation and internal pressure gradient term:

$$\begin{aligned} \frac{\partial Du}{\partial t} + \nabla \cdot (\vec{V}Du) + \dots &= -\frac{gD^2}{\rho_0} \int_{\sigma} \left[\frac{\partial \rho}{\partial x} - \frac{\sigma}{D} \frac{\partial D}{\partial x} \frac{\partial \rho}{\partial \sigma} \right] d\sigma + \dots = -\frac{g}{\rho_0} \left[D \frac{\partial}{\partial x} \left(D \int_{\sigma} \sigma \frac{\partial \rho}{\partial \sigma} d\sigma \right) + D^2 \frac{\partial \rho \sigma}{\partial x} \right] + \dots \\ &= -\frac{gD}{\rho_0} \left[-\frac{\partial D}{\partial x} \int_{\sigma} \sigma \frac{\partial \rho}{\partial \sigma} d\sigma + D \frac{\partial}{\partial x} \int_{\sigma} \rho d\sigma \right] + \dots \end{aligned}$$

$$\int_{\sigma} \sigma \frac{\partial \rho}{\partial x} d\sigma = -\sigma \rho - \int_{\sigma} \rho d\sigma$$

$$\begin{aligned} \iint (\dots) dx dy &= -\frac{gD_m}{\rho_0} \left\{ \iint -\frac{\partial D}{\partial x} \int_{\sigma} \sigma \frac{\partial \rho}{\partial x} d\sigma dx dy + \iint D \frac{\partial}{\partial x} \int_{\sigma} \rho d\sigma dx dy \right\} \\ &= -\frac{gD_m}{\rho_0} \left\{ -\left(\int_{\sigma} \sigma \frac{\partial \rho}{\partial x} d\sigma \right)_m \iint \frac{\partial D}{\partial x} dx dy + D_m \iint \frac{\partial}{\partial x} \int_{\sigma} \rho d\sigma dx dy \right\} \\ &= -\frac{gD_m}{\rho_0} \left\{ -\left(\int_{\sigma} \sigma \frac{\partial \rho}{\partial x} d\sigma \right)_m \oint D dy + D_m \oint \int_{\sigma} \rho d\sigma dy \right\} \end{aligned}$$

Green's Theorem:

$$\oint -N dx + M dy = \iint \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} \right) dx dy$$

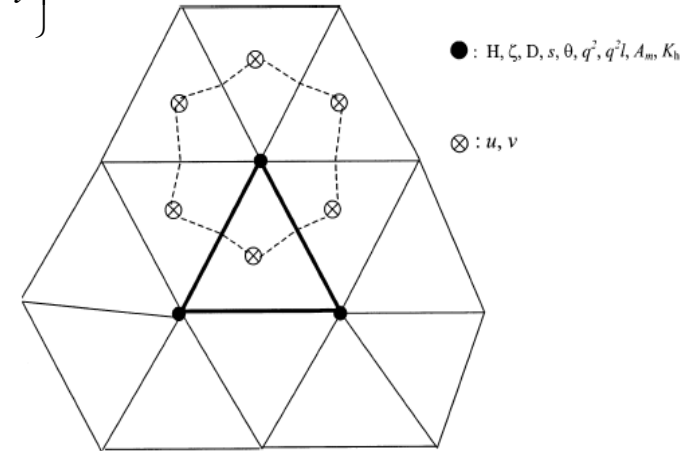


FIG. 1. The unstructured grid for the finite-volume model.