Flow over a backward-facing step, a convergence study comparing a z-coordinate model and a sigma-coordinate model

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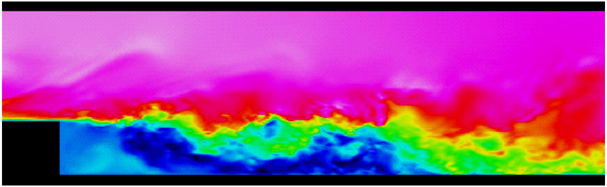
EU IP CarboOcean - Cordino

Backward-facing step

- Numerical testcase
  - Simplicity
  - Fixed separation point
  - Reattacment of the flow
  - Recovery of the boundary layer
  - High Reynolds numbers
    - bifucation of eddies

- Classical problem in applied aerodynamics
- Numerous numerical and laboratory experiments



Horizontal velocities, DNS,
Direct Numerical Simulations of Turbulent Flow over a Backward-Facing Step Hung Le and Parviz Moin

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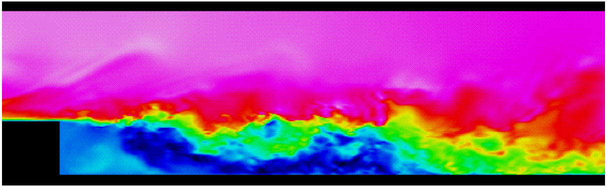

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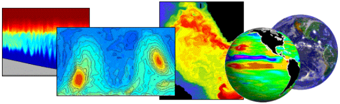
MITgcm and Bergen Ocean Model

MITgcm

-z-coordinates
-preconditioned conjugated method - non-hydrostatic
-Boussinesq
-C-grid
-finite-volume
-advection scheme super-bee limiter function
-modular fashion

Bergen Ocean Model

-σ-coordinates
- modesplit
- SOR - non-hydrostatic
- Boussinesq
- C-grid
- finite-differences
- advection TVD superbee limiter function



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Equations

The equation of continuity:

$$\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0$$

The Reynolds averaged momentum equations:

$$\begin{aligned}
\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z} &= -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + A_M\frac{\partial^2 U}{\partial x^2} + K_M\frac{\partial^2 U}{\partial z^2} \\
\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial x} + W\frac{\partial W}{\partial z} &= -\frac{1}{\rho_0}\frac{\partial P}{\partial z} - \frac{g\rho}{\rho_0} + A_M\frac{\partial^2 W}{\partial x^2} + K_M\frac{\partial^2 W}{\partial z^2}
\end{aligned}$$

Conservation of density:

$$\frac{\partial \rho}{\partial t} + U\frac{\partial \rho}{\partial x} + W\frac{\partial \rho}{\partial z} = A_H\frac{\partial^2 \rho}{\partial x^2} + K_H\frac{\partial^2 \rho}{\partial z^2}$$

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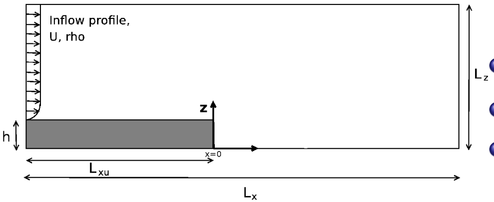
Setup



- **h: 20m**
- L_x: 45h=900 m
- L_{xu}: 18h=360 m
- L_z: 6h =120 m
- ρ: 1028 kg m^{-3}
- T: 24 h

- δ_{99}: 1.2h
- 2 open boundaries
- Initialization
  - Depth integrated velocity
- Convergence study:
  - Δx: 12-1.5 m
  - Δz: 2.4-0.3 m
- $K_{M/H}$: $10^{-2} - 10^{-6}$ m^{2}s^{-1}
- $A_{M/H}$: 10^{-2} m^{2}s^{-1}

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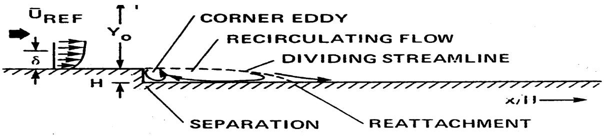
Backward-facing sharp step



Driver and Seegmiller (1985), AIAA Journal

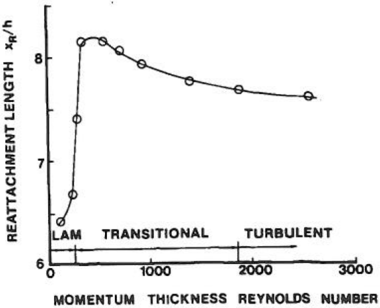
- Reynolds number, $Re = \frac{U_\infty h}{\nu}$
- Low Reynolds number: stationary flow
- Reattachment zone
  - Counter-clockwise eddy near the step
  - Clockwise eddy
- Oscillating reattachment location (Friedrich &Arnal, 1990, J Wind Eng Indust.Aerodyn)
  - Release of eddies from the reattachment zone (Le et al, 1997, J. Fluid Mech)

Backwardfacing sharp step

- Laminar: $Re < 400$
- Transient:
  $400 < Re < 3400$
- Turbulent: $Re > 3400$

Armaly, 1983, Journal of Fluid Mechanics

- Assume: Reattachment zone based on average fields
  1. $\bar{U} = 0$
  2. zero wall-shear stress $\frac{\partial U}{\partial z} = 0$
  3. Mean dividing streamline $\psi = 0$
  4. Location of 50% forward flow fraction



Eaton, 1981, AIAA

Reattachment - Sharp step

Hydrostatic consistency:

$$\left|\frac{\sigma}{\delta\sigma}\frac{\delta H}{H}\right| < 1 \; .$$

- z-coordinate model

		Laminar	Transient	Turbulent		
Re		$2 \cdot 10^2$	$2 \cdot 10^3$	$2 \cdot 10^4$	$2 \cdot 10^5$	$2 \cdot 10^6$
1st eddy	150x100	-	0.90	1.5	1.8	1.5
	300x200	-	2.1	1.8	1.7	1.5
	600x400	-	3.8	1.6	1.2	1.3
2nd eddy	150x100	6.6	14.1	6.9	9.6	8.4
	300x200	6.8	11.1	6.6	6.6	5.3
	600x400	7.0	12.5	5.7	4.5	5.3
Literature		7.0[a]	10-13.5[b]	1.76/6.28[c]		7.0[d]

Table: The end of the first and the second eddies as a function of Reynolds number and numerical resolution. [a] Beaudoin et al 2007, European Journal of Mechanics B/Fluids, [b], Rani et al, 2007, Journal of Fluid Mechanics, [c]Le et al, 1997, Journal of Fluid Mechanics (Re=5100), [d] Kim et al, 1985, Journal of Fluids Engineering.

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Streamlines

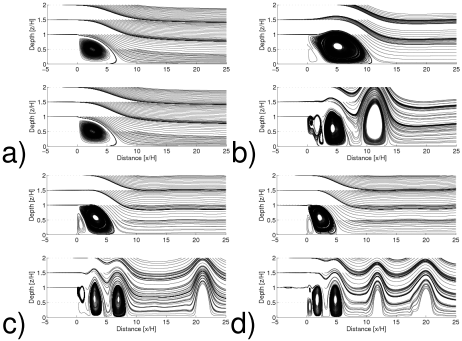


Figure: Mean streamlines 12-24 h, and a snapshot of the streamlines at 24 h: Re= a) $2{\cdot}10^2$, b) $2{\cdot}10^3$, c) $2{\cdot}10^4$, and d) $2{\cdot}10^6$, 300x200 grid cells

- Re=200: Stationary eddy
- Re=2000: Transient regime. Longer reattachment zone
- Re$\geq 2 \cdot 10^4$: Turbulent regime. More and smaller eddies.

Bottom velocities

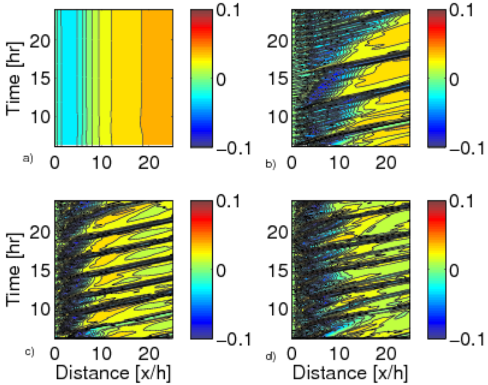
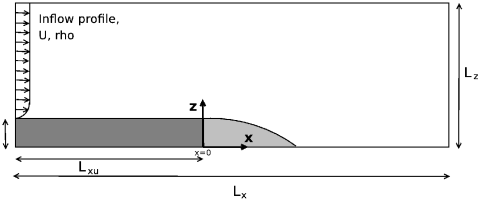


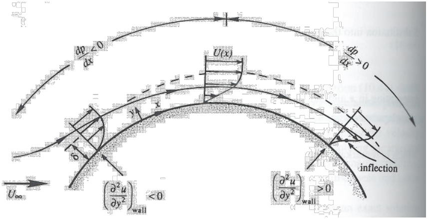
Figure: Horizontal bottom velocities. Re= a) $2{\cdot}10^2$, b) $2{\cdot}10^3$, c) $2{\cdot}10^4$, and d) $2{\cdot}10^5$, 300x200 grid cells

- Stationary eddy: Re=200
- Bifurcation of eddies Re$\geq$ 2000
- Increasing number of eddies are released from the reattachment point for increasing Re

Backward-facing rounded step



$$H(x) = \begin{cases} -100\ , & x < 0\ , \\ -120 + 20 \cdot \exp^{-\frac{x^2}{L^2}}\ , & x \leq 0\ . \end{cases}$$



Kundu and Cohen, Fluid Mechanics, 2004

- Adverse pressure force
- $P > P_c$ separation - recirculation region
- Thicker boundary layer
- Flow downstream separation point dependent on Re

Backward-facing rounded step

Observations from a water-tunnel experiment (Bao& Dallmann, 2004, Aerospace Science and Technology)

- Re$=$2700: steady laminar separation bubble
- Re$>$ 2700: bursting/vortex shedding
- The separated flow is filled with downwardsmoving vortexes
- New vortexes generated just downstream the separation line
- The angle of the separation line is constant in time
- The separation point is constant in time
- Mean-flow separation bubble - defined only by appropriate time-averaging and spanwise-averaging

Separation point

- $\bar{U} = 0$: Separation point, reattachment point

	Model	$2{\cdot}10^2$	$2{\cdot}10^3$	$2 \cdot 10^4$	$2 \cdot 10^5$	$2 \cdot 10^6$
150x100	BOM	-	-	**5.9**	4.1	3.8
300x200	BOM	-	-	**5.3**	1.4	0.80
600x400	BOM	-	-	**5.8**	2.1	5.8[a]
150x100	MITgcm	-	-	**6.2**	6.2[b]	6.8
300x200	MITgcm	-	-	**5.2**	9.7	3.5[b]
600x400	MITgcm	-	-	**5.5**	4.6	3.4

Table: Separation point L=8. [a] Two clock-wise eddies occur. [b] Negative velocities is observed at $x = -7.0$.

Reattachment point

	Model	$2{\cdot}10^2$	$2{\cdot}10^3$	$2 \cdot 10^4$	$2 \cdot 10^5$	$2 \cdot 10^6$
150x100	BOM	-	-	**21.2**	17.6	17.0
300x200	BOM	-	-	**15.4**	8.15	13.0
600x400	BOM	-	-	**18.0**	11.0	10.3/18.4[a]
150x100	MITgcm	-	-	**16.1**	16.7	17.3
300x200	MITgcm	-	-	**16.4**	14.6	13.7
600x400	MITgcm	-	-	**16.5**	13.3	12.2

Table: Reattachment point for L=8. [a] Two clock-wise eddies occur.

Bottom velocities

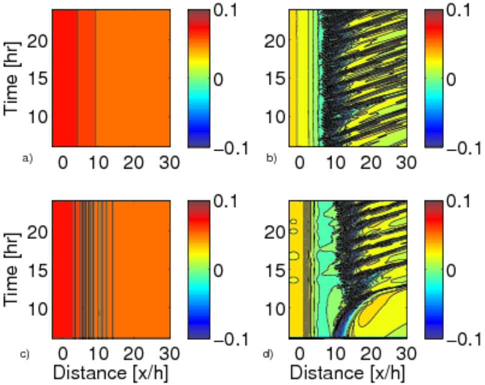


Figure: Horizontal bottom velocities. L=8 a) BOM, Re=$2 \cdot 10^2$, b) BOM, Re=$2 \cdot 10^4$, c) MITgcm, Re=$2 \cdot 10^2$, and d) MITgcm, Re=$2 \cdot 10^4$.

- Stationary flow Re=200
- Stationary separation point
- Fluctuating reattachment point

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Vorticity

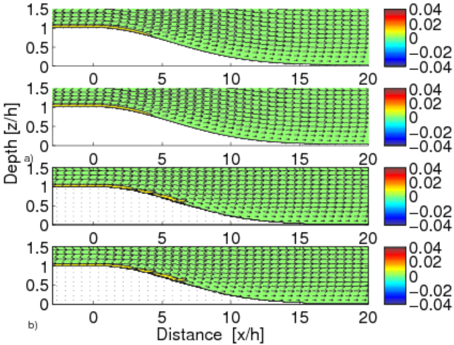

Figure: Mean vorticity (12-24 h) and a snapshot of the vorticity (24 h) for a) BOM and b) MITgcm, Re=$2 \cdot 10^3$, 300x200 grid cells.

Vorticity

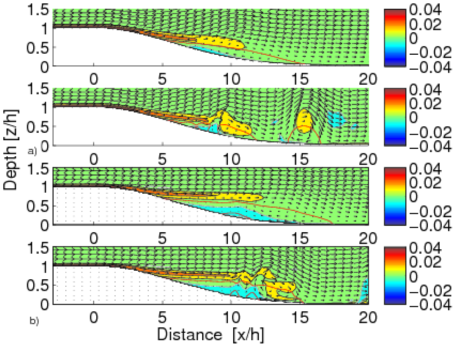

Figure: Mean vorticity (12-24 h) and a snapshot of the vorticity (24 h) for a) BOM and b) MITgcm, Re=$2 \cdot 10^4$, 300x200 grid cells.

Vorticity

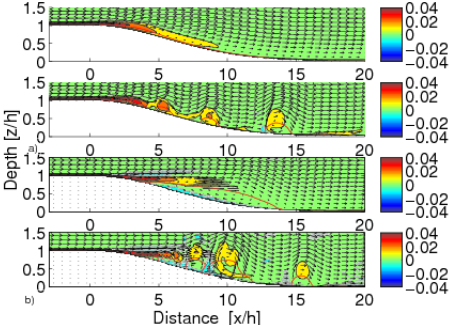

Figure: Mean vorticity (12-24 h) and a snapshot of the vorticity (24 h) for a) BOM and b) MITgcm, Re=$2 \cdot 10^6$, 300x200 grid cells.

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Summary

- The simulations represent both the laminar, transient, and turbulent regime
- Within the turbulent regime $K_{M/H} = 10^{-4}$ m^{2}s^{-1} gives the best result
- Rounded step:
  - Stagnant separation point
  - Fluctuation reattachment point
  - σ-coordinates: reduced height of reattachment zone
  - z-coordinates: more noise on the slope

Future work

- 3D simulations
- Include particles - food access for cold water corals
- Extend to also study flow over a trench and a circular trench - pockmark shape
  - Suitable locations for cold water corals

Thank you for your attention!