Investigation of the spreading and dilution of domestic waste water inputs into a tidal bay using the finite volume model FVCOM

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Location of study area


River Weser
Jade Bay

Input of waste water pollutants during heavy rain falls

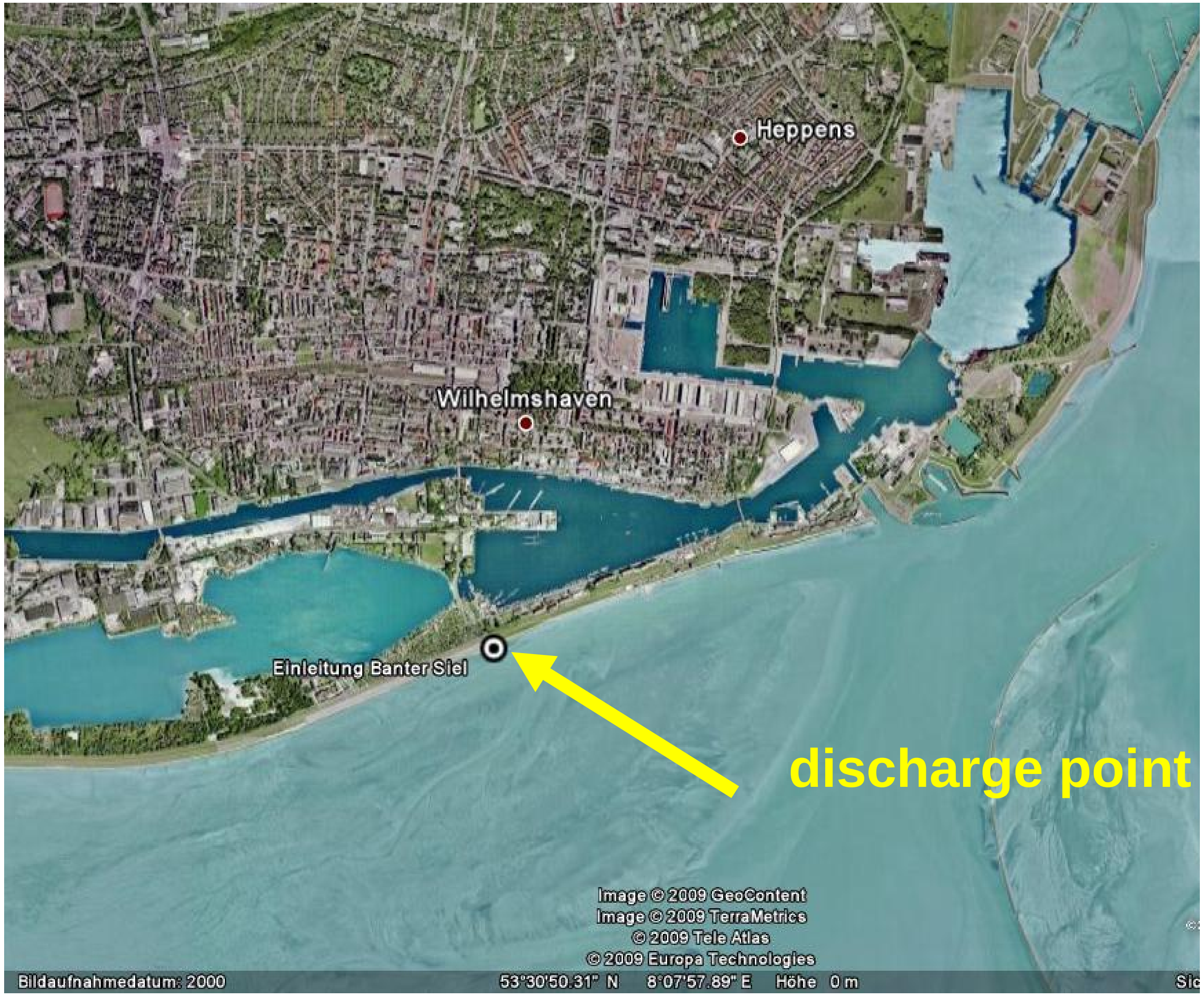

Photo of the discharge location


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The FVCOM model

Finite volume model, (version 2.7) , see e.g. Chen et al., 2006

horizontal: unstructured triangular grid
vertical :   sigma-coordinates

some **weting and drying** scheme, which is very important for our coastal application

vertical turbulence schemes: (Mellor-Yamada, k-ε-schmeme)
horizontal schemes: constant eddy diffusivities or Smagorinsky para.

we adapted the sediment module to our application, which includes:
- sinking of particles
- some decay process of the particles/bacteria
- accumulation of particles in sea floor sediments

Mesh Generation

The FVCOM group proposes the comercial software module from SMS (Surface Water Model System), with which they had good experiences.

Instead we took some free software packages:

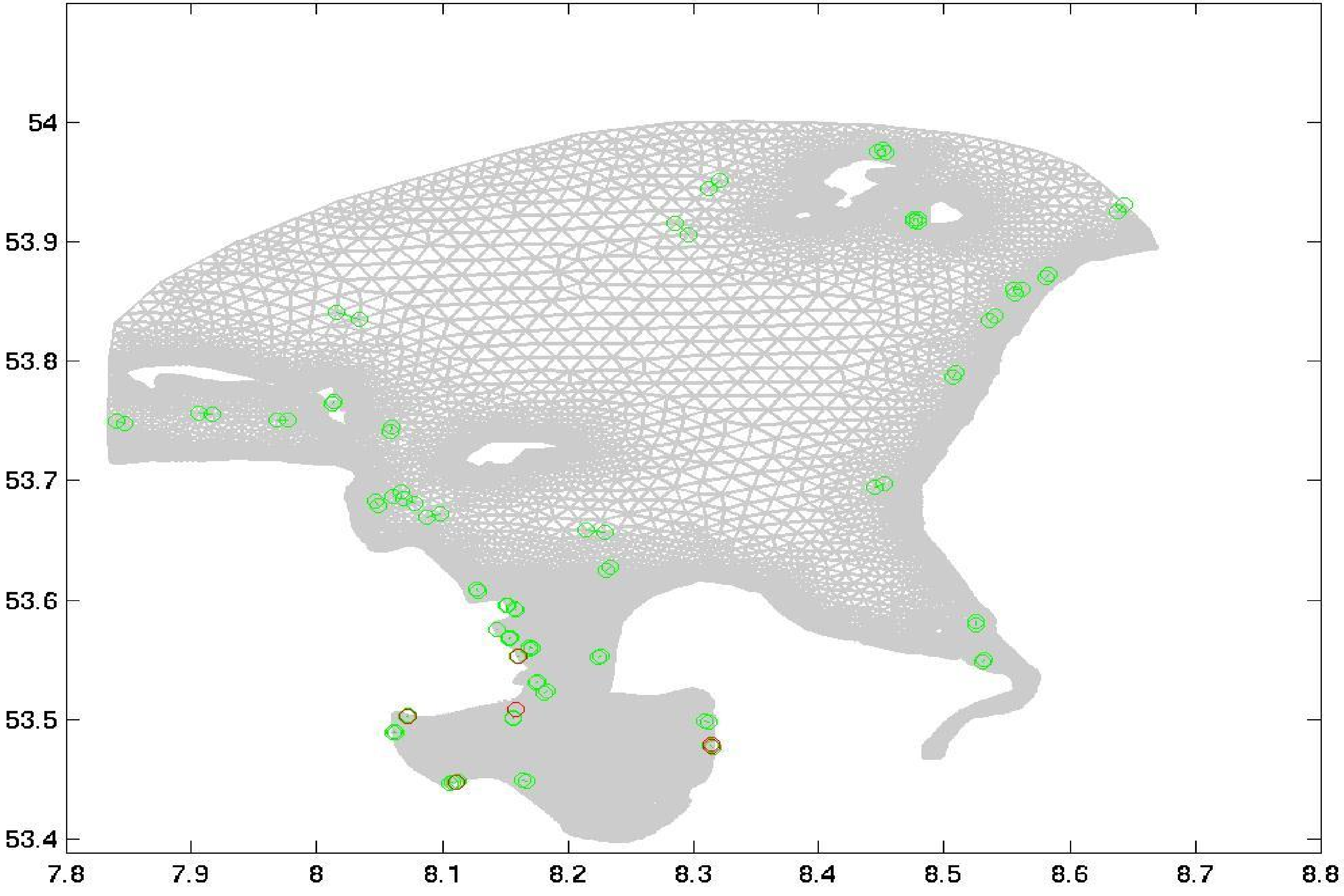
- BATTRI (Bilgili et al. 2006) which is a MATLAB frontend to TRIANGLE
- GMSH (Christophe Geuzaine and Jean-Francois Remacle)

  This tool provides good quality meshes

We checked mesh quality with the following criteria:

- Minimum interior angle is larger than 30°
- Maximum interior angle less than 130°
- Area change of adjacent triangles is less than 2

Checking mesh quality



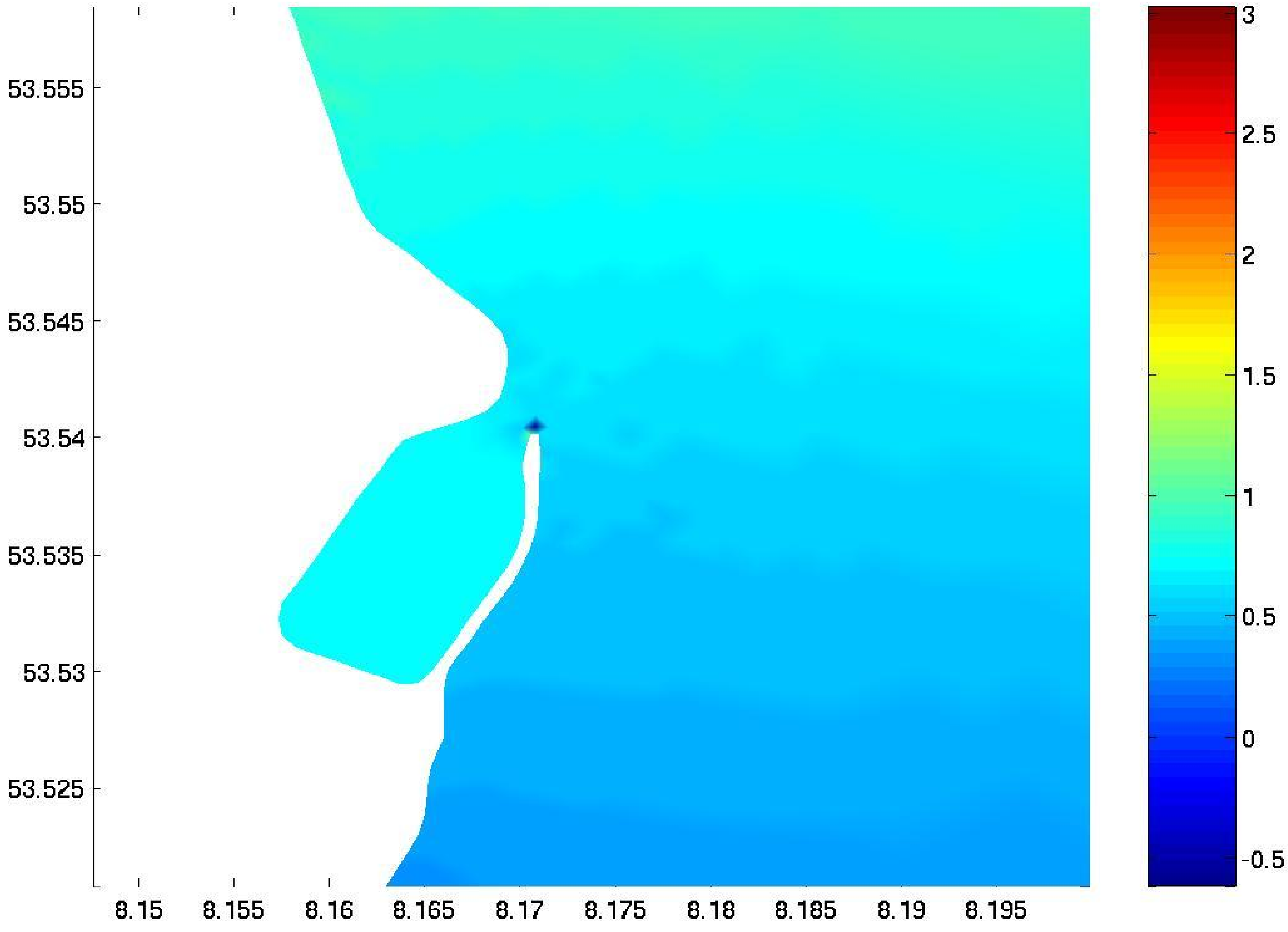
Problems with simulation of sea surface elevation under tidal forcing

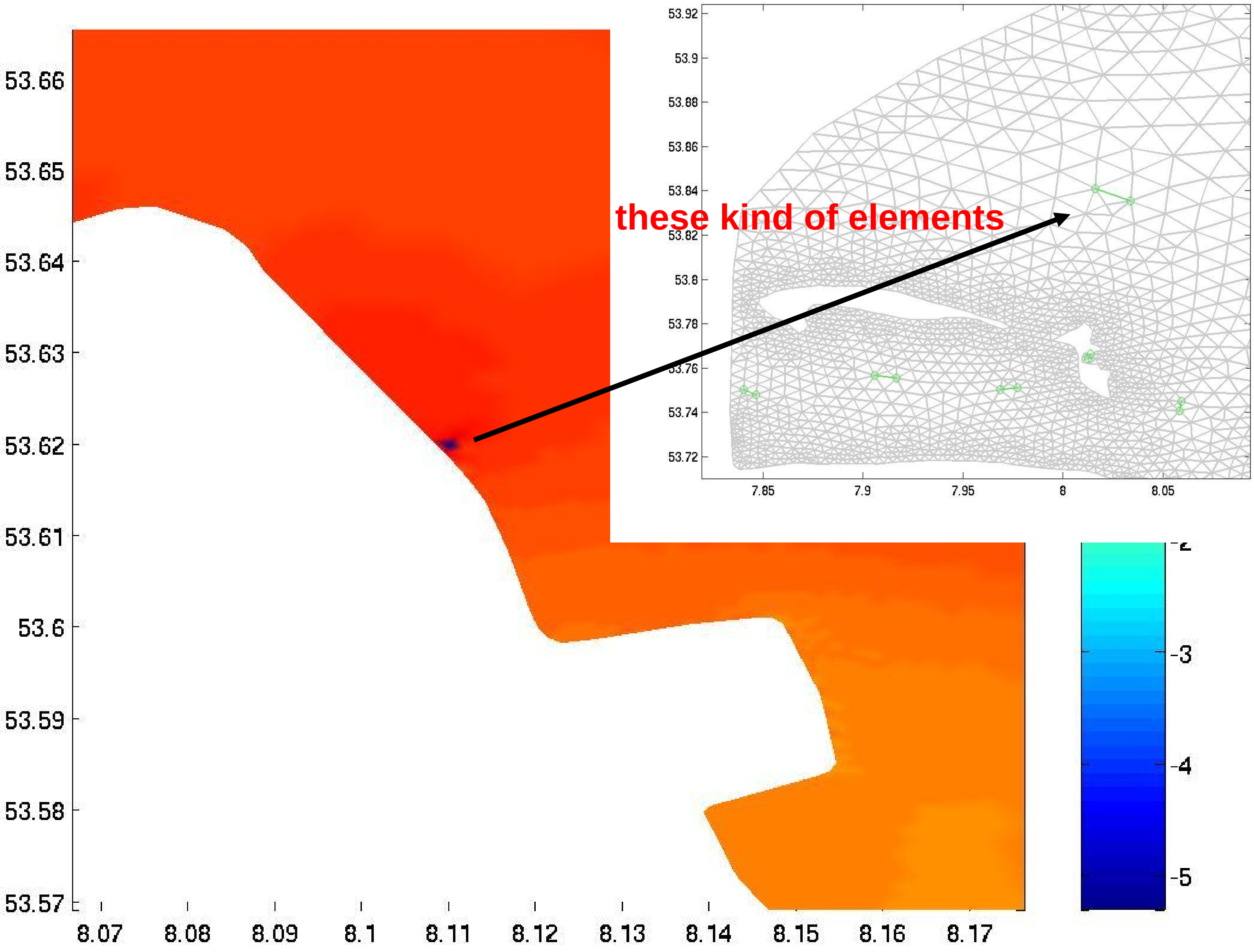
only M2 tidal forcing at open boundary

flat bottom **to avoid problems due to topography**

we tried to find critical regions based on the finite element mesh itself

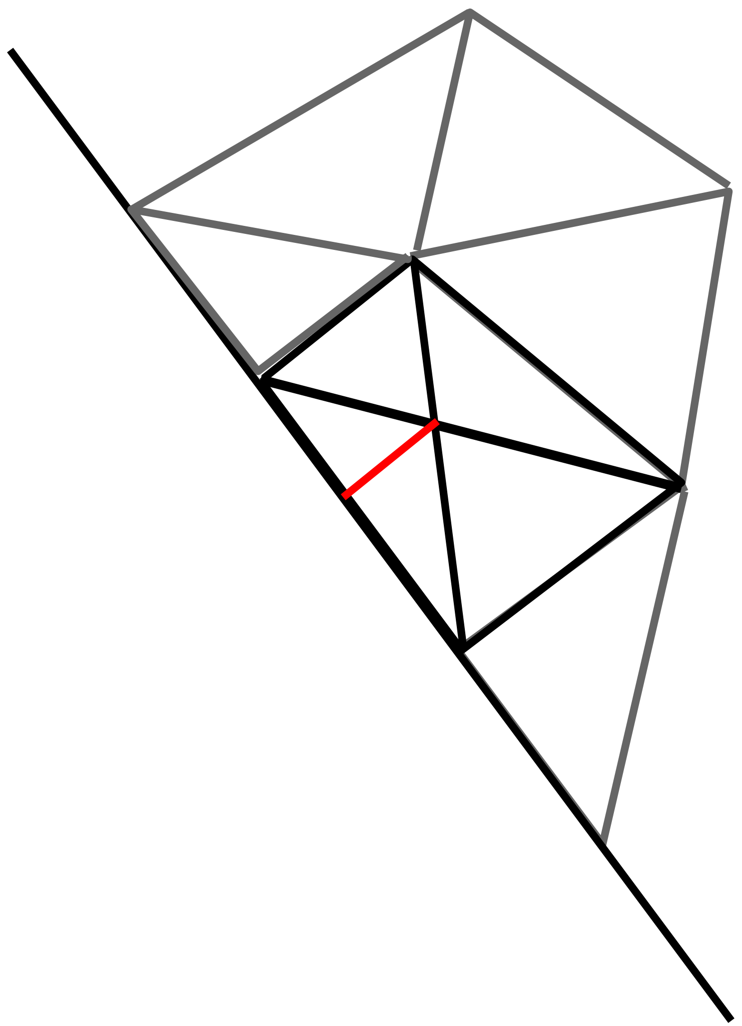
The beginning of an instability





these kind of elements
53.66
53.65
53.64
53.63
53.62
53.61
53.6
53.59
53.58
53.57
8.07
8.08
8.09
8.1
8.11
8.12
8.13
8.14
8.15
8.16
8.17
-2
-3
-4
-5
53.92
53.9
53.88
53.86
53.84
53.82
53.8
53.78
53.76
53.74
53.72
7.85
7.9
7.95
8
8.05

„Solution“ to the problem

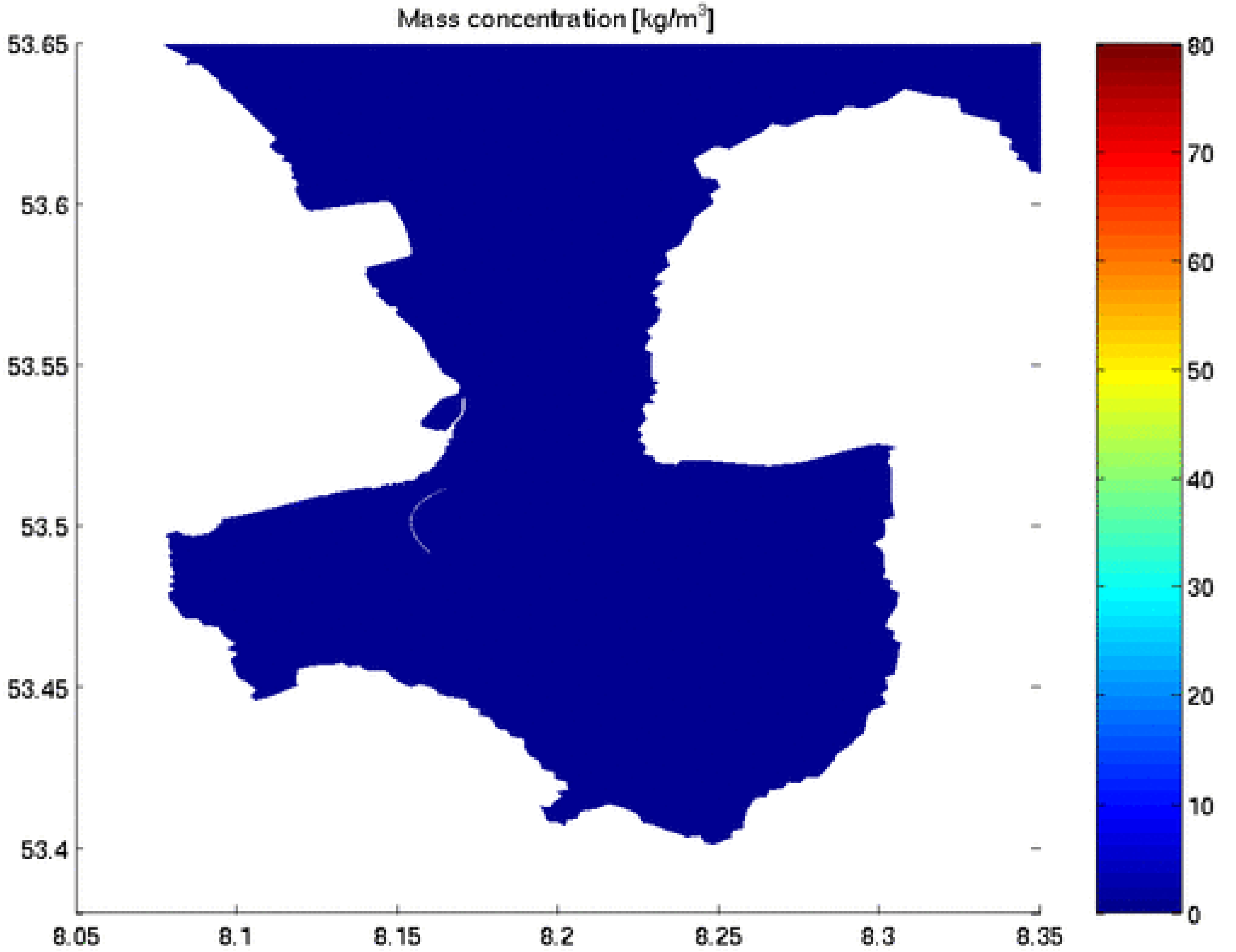


Why cause some of those „4-triangle“ configurations instabilities ?

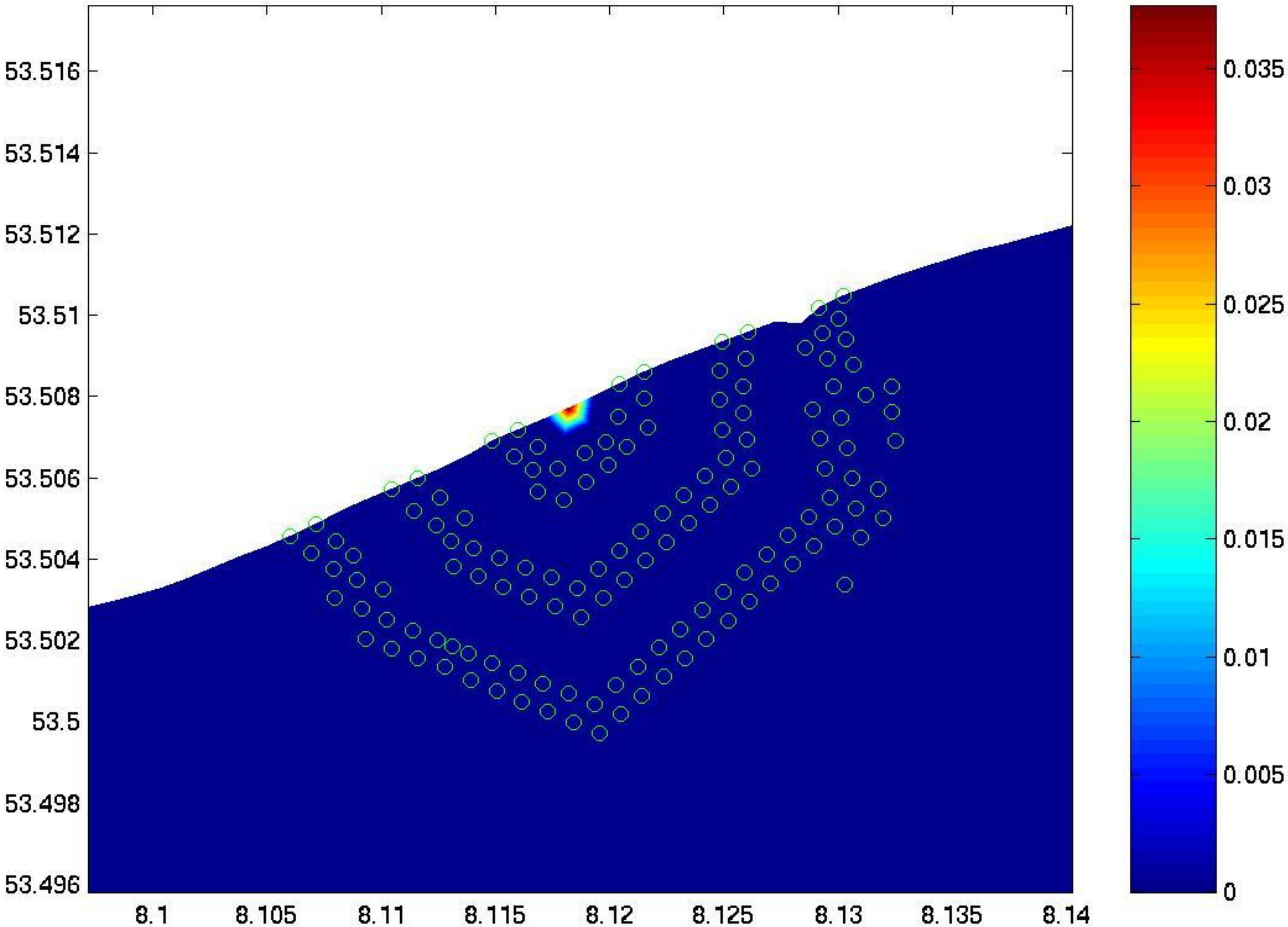
Meanwhile there is an option in GMSH to avoid those configurations

Is it really necessary to avoid them?
Still open question

Input of pollutants



Problem with negative concentrations



Tried to solve problem by:

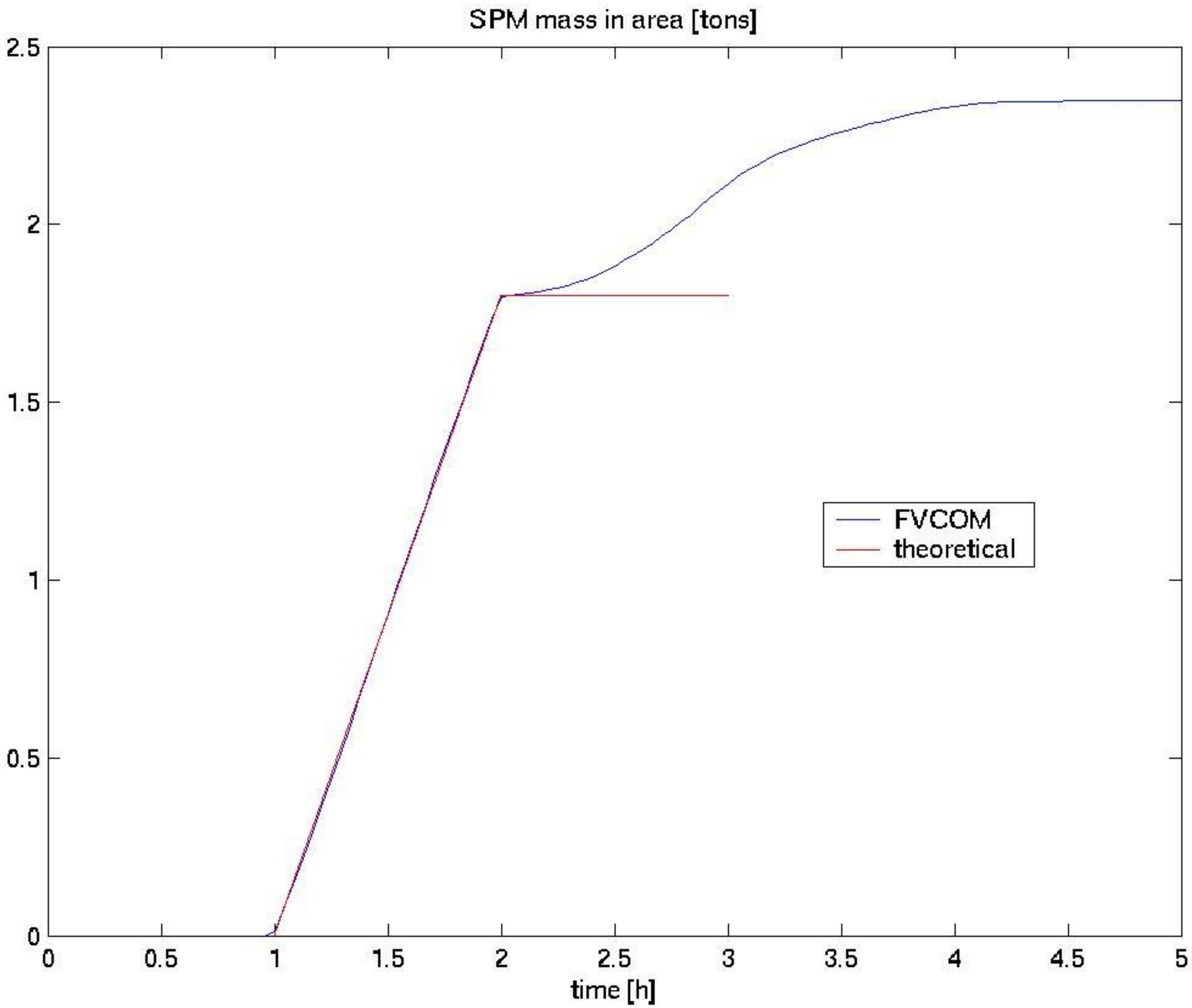
1) reduction of time step

2) monotone advection scheme (MPDATA)

3) simply cut-off negative concentrations

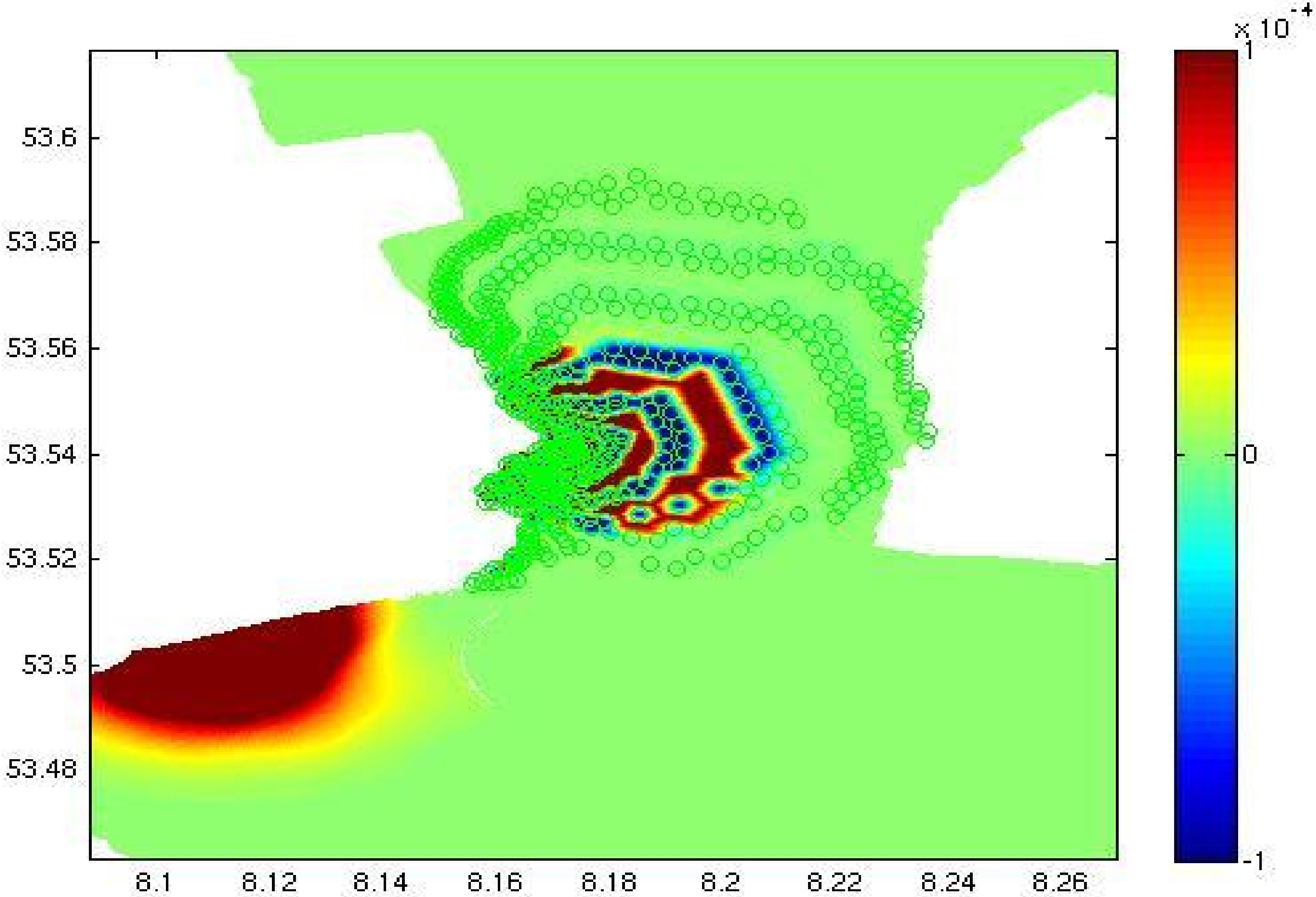
4) large horizontal background diffusivity to smooth concentration oscillations

Cuting-off negative values



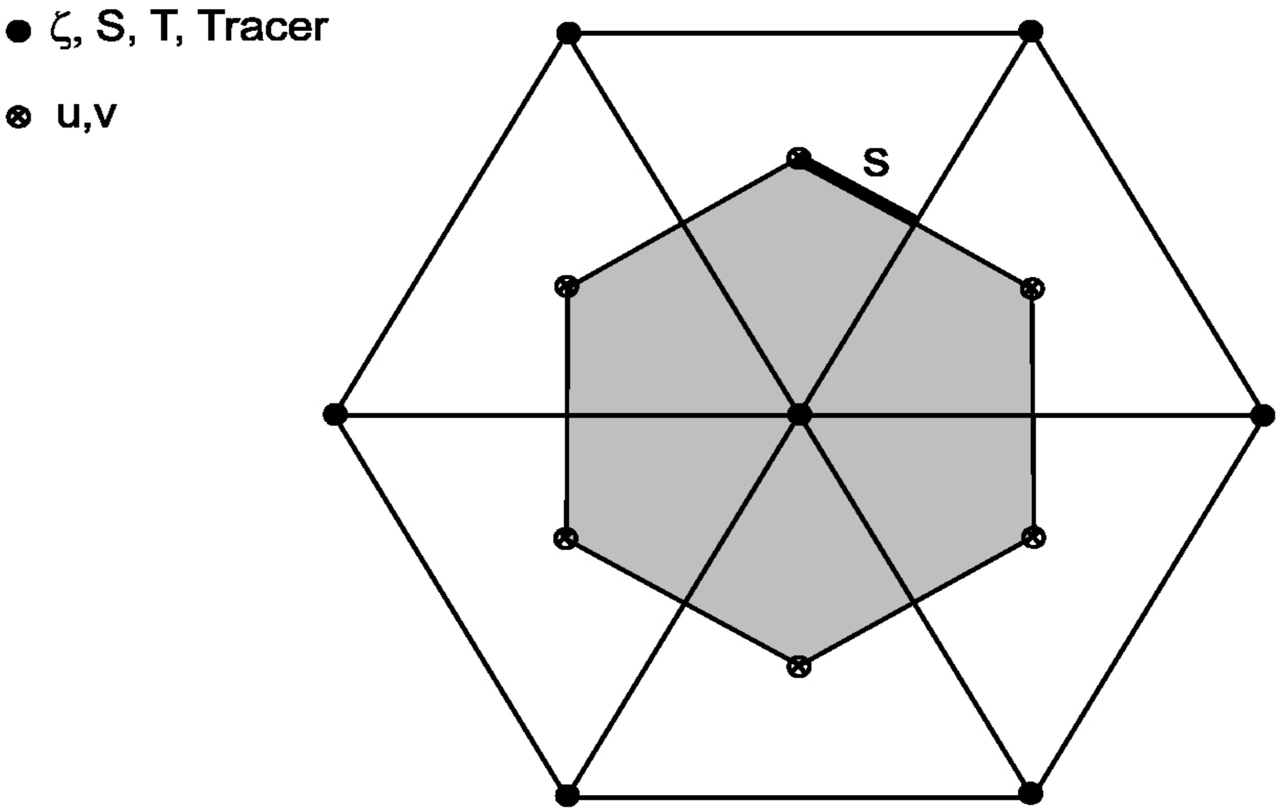
Experiment with constant mass input over 1 hour starting after 1 hour.

Adding high horizontal background diffusion



Problem with the diffusion mechanism?

Tracer control element



The change of a tracer at the central node of these control elements is calulated from the fluxes across the bounderies of the tracer control elements (TCE)

Diffusion on a triangle: an example

$$\frac{\partial T}{\partial t} = \nabla \cdot (A_h \nabla T)$$

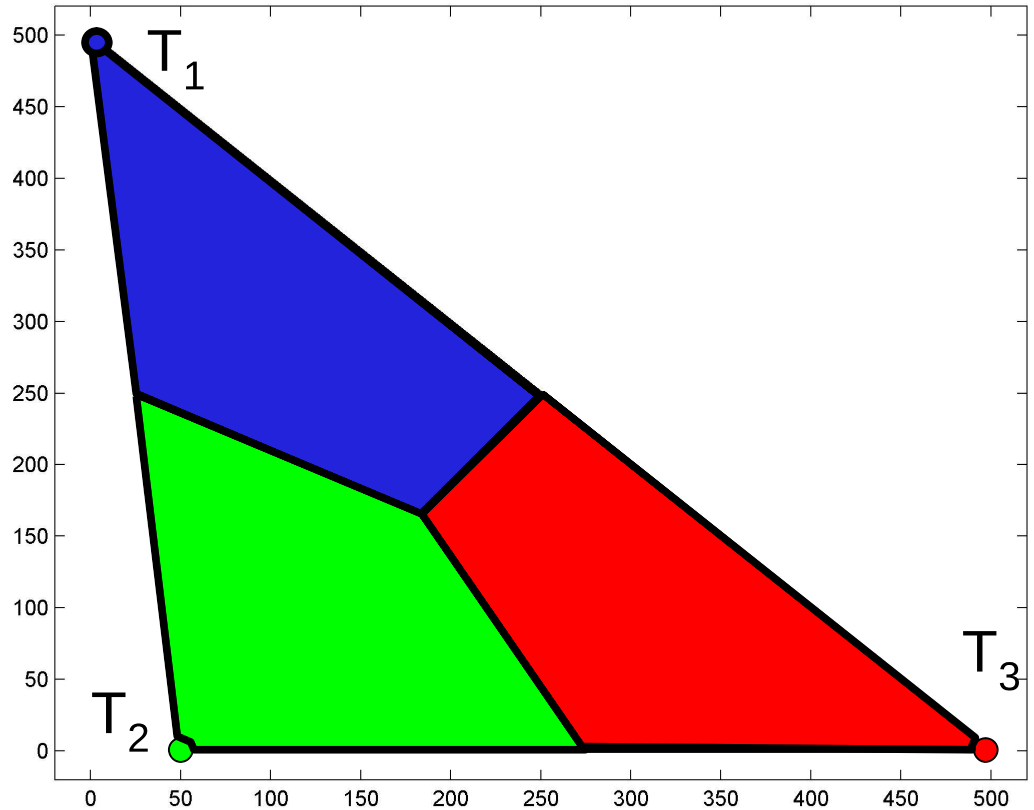
$A_h = 200\ m^2/s$

initial conditions:

$T_1=1.0$

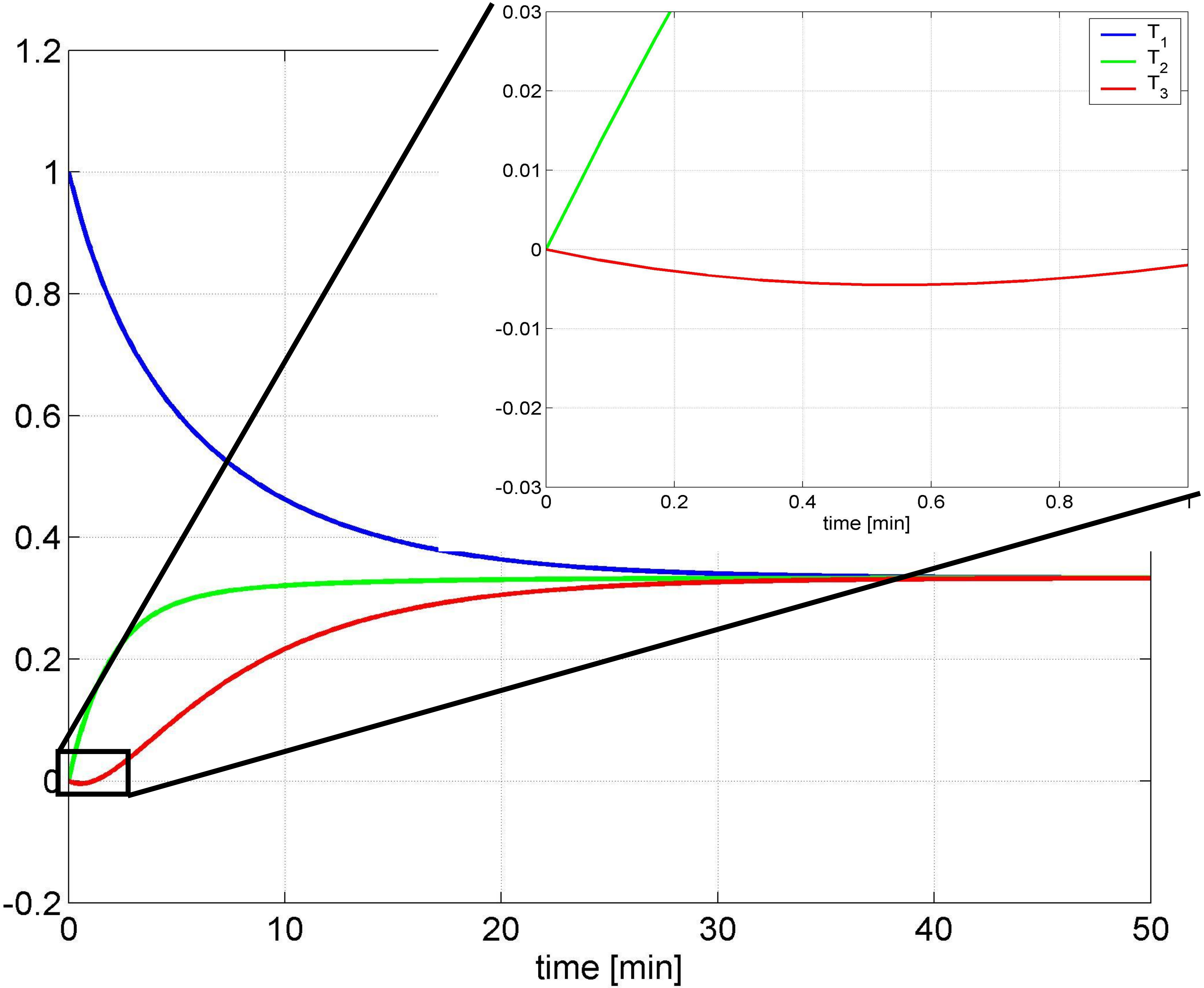
$T_2=0.0$

$T_3=0.0$



no diffusive fluxes across the triangle sides

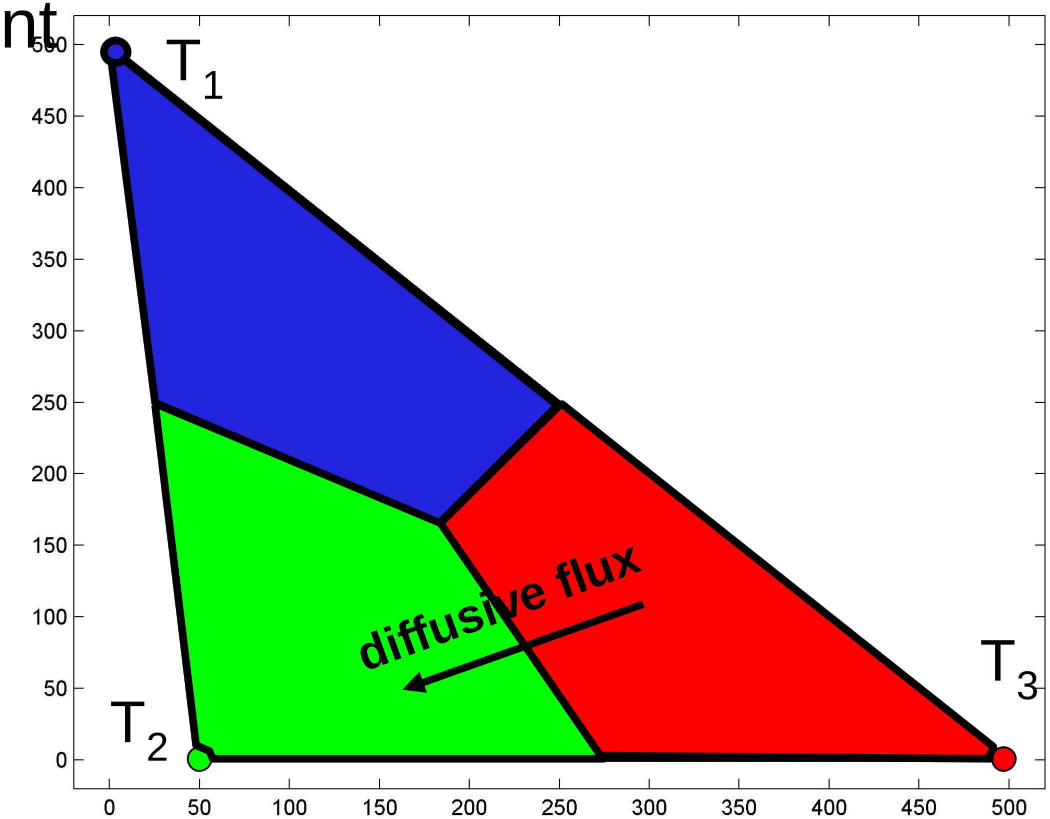
Implementation of the diffusive scheme of FVCOM in MATLAB



1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
10
20
30
40
50
time [min]
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0
0.2
0.4
0.6
0.8
time [min]
T_1
T_2
T_3

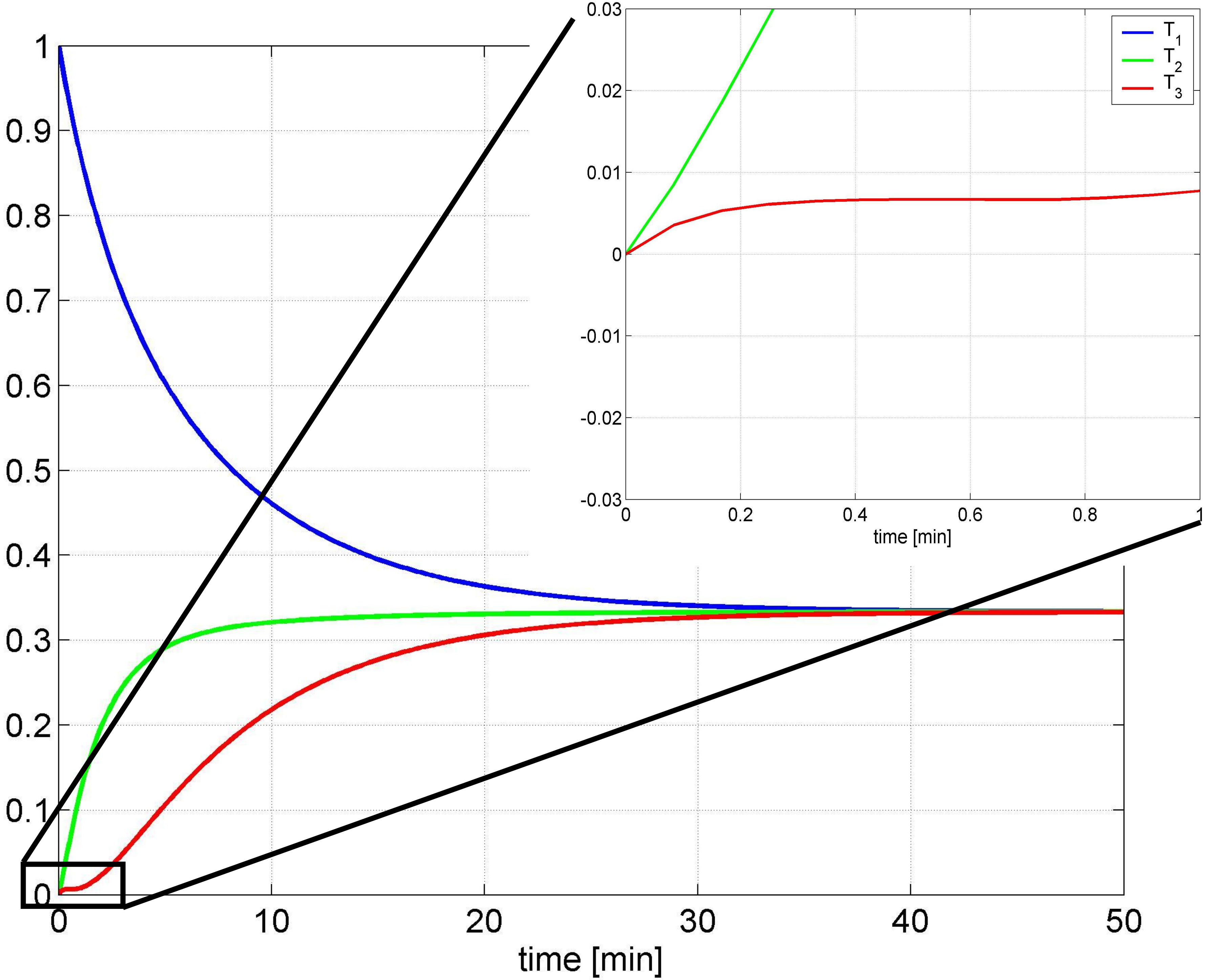
Explanation for the negative values

Due to the constant gradient over the whole triangle, there is a computated diffusive outflux from TCE3 to TCE2, although there is no „mass“ inside TCE3 in the begining.



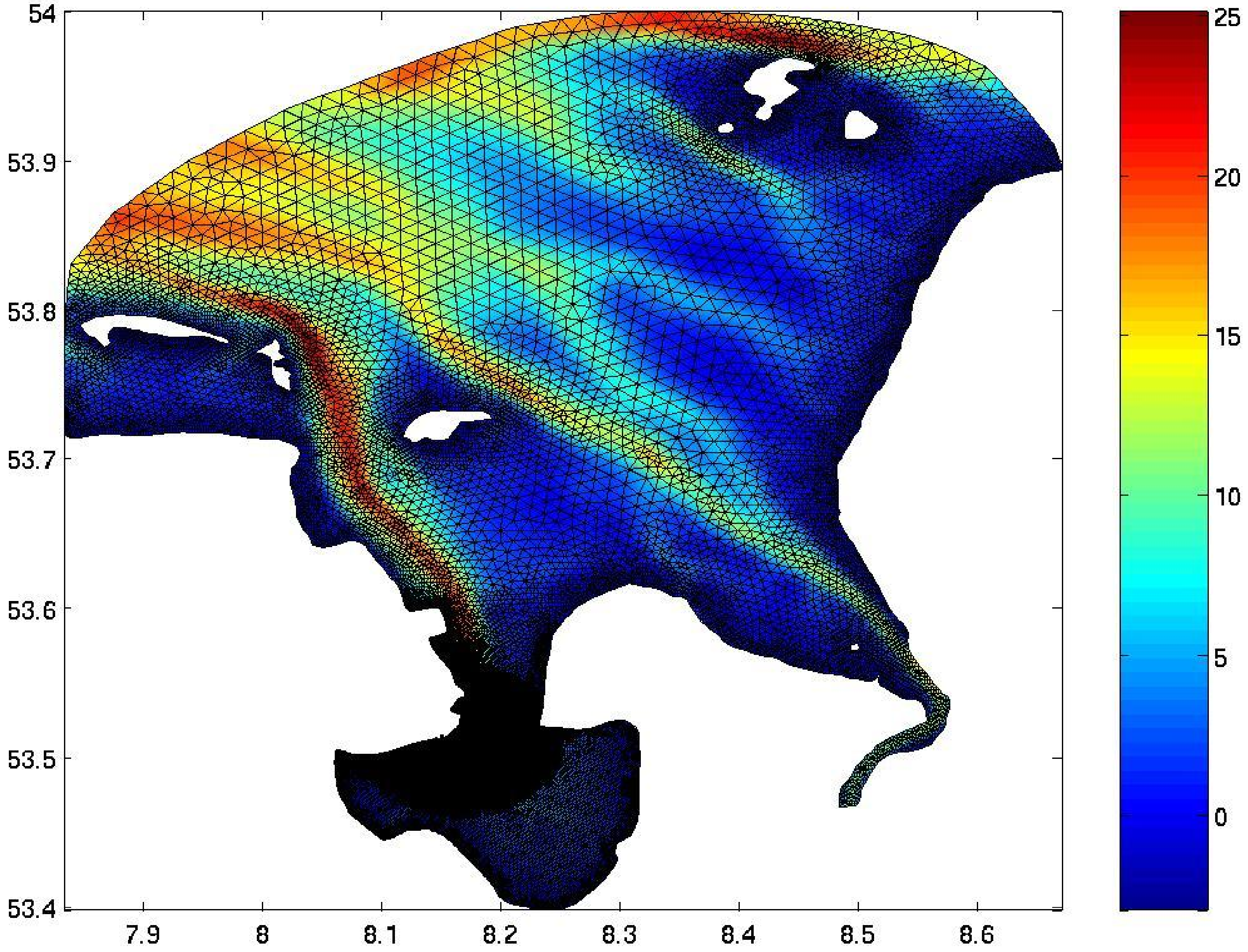
Solution to the problem:

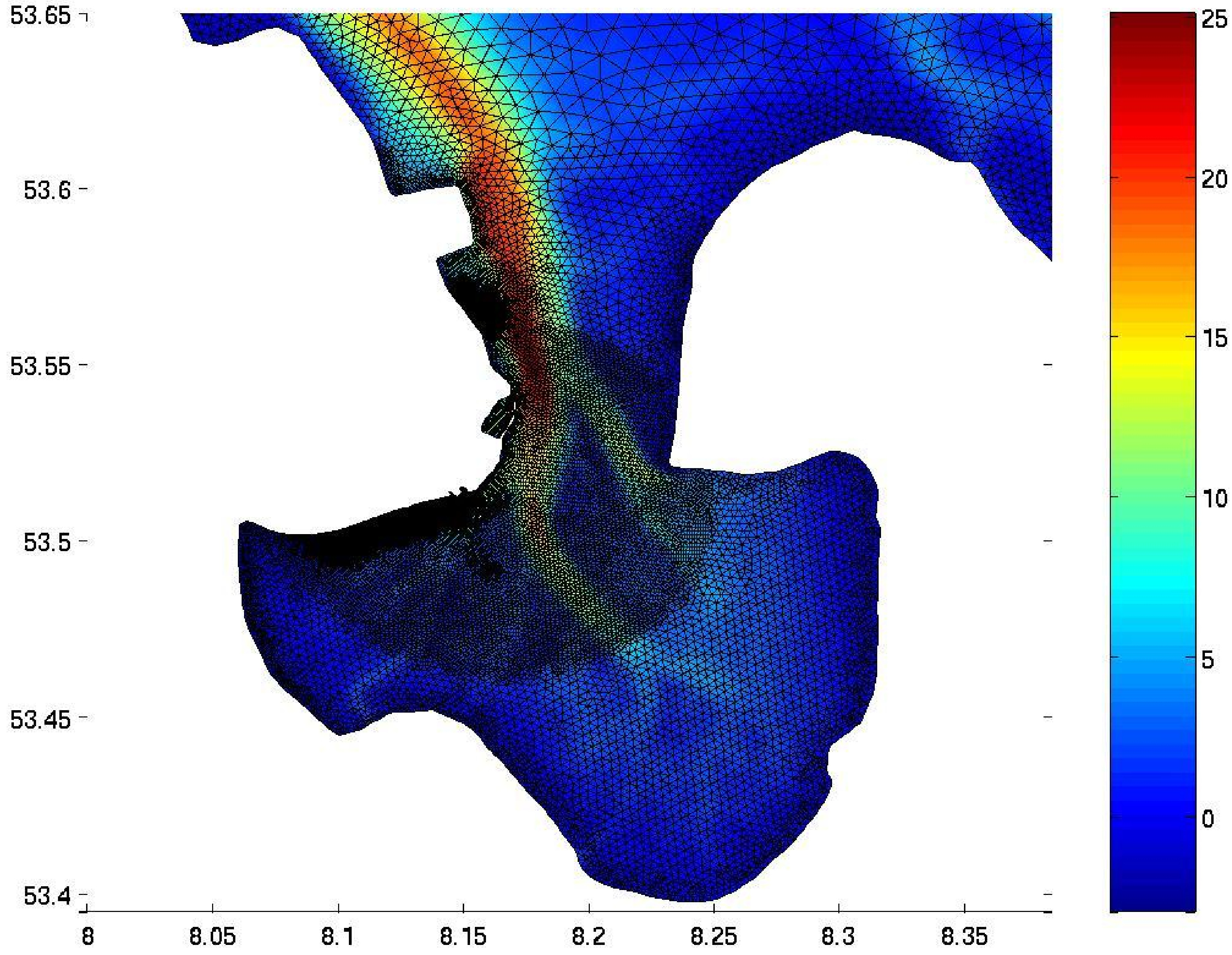
In FVCOM the outflux must be limited by the available total mass in a control element



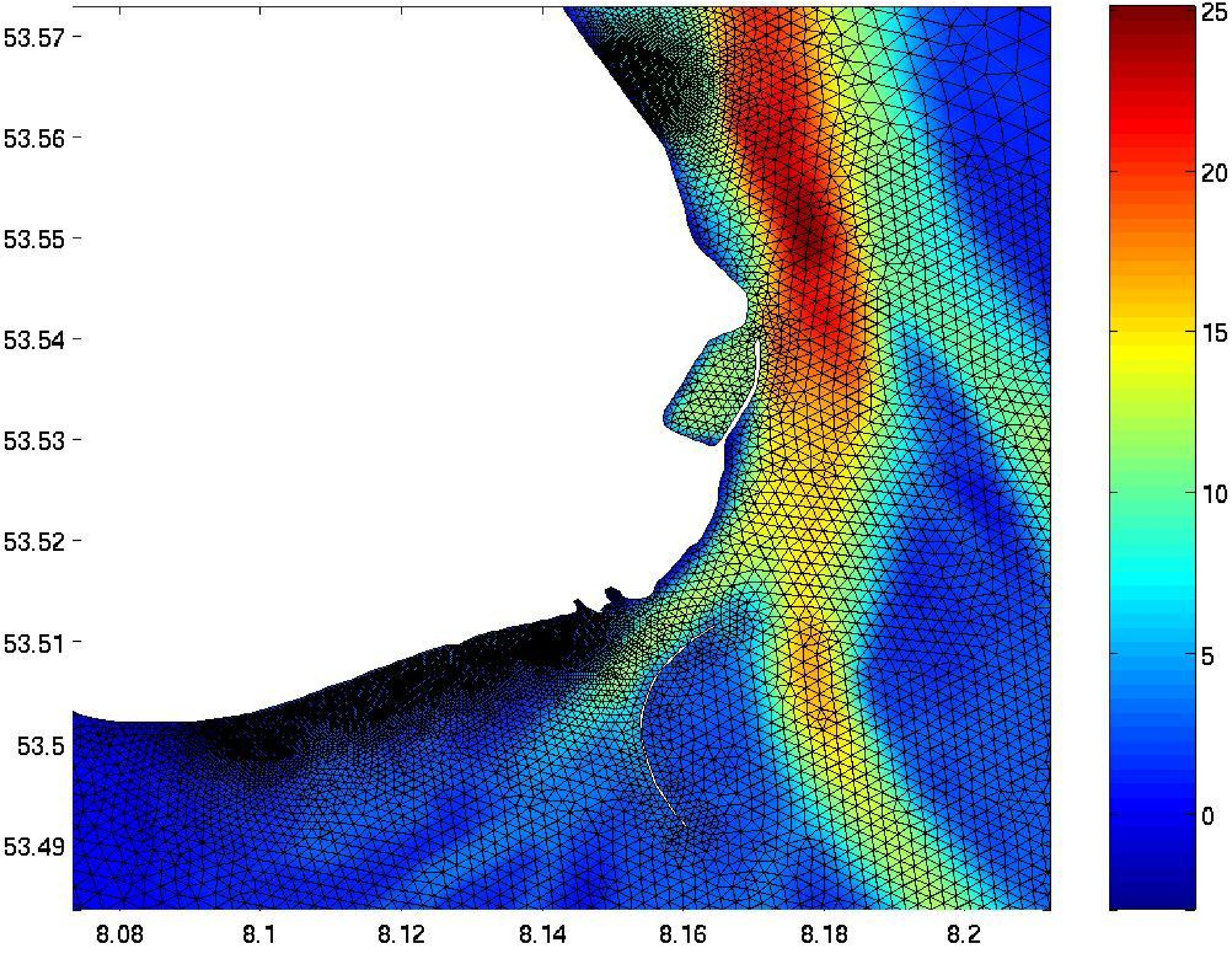
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
time [min]
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0
0.2
0.4
0.6
0.8
1
time [min]
T_1
T_2
T_3

The used mesh





53.65
53.6
53.55
53.5
53.45
53.4
8
8.05
8.1
8.15
8.2
8.25
8.3
8.35
25
20
15
10
5
0

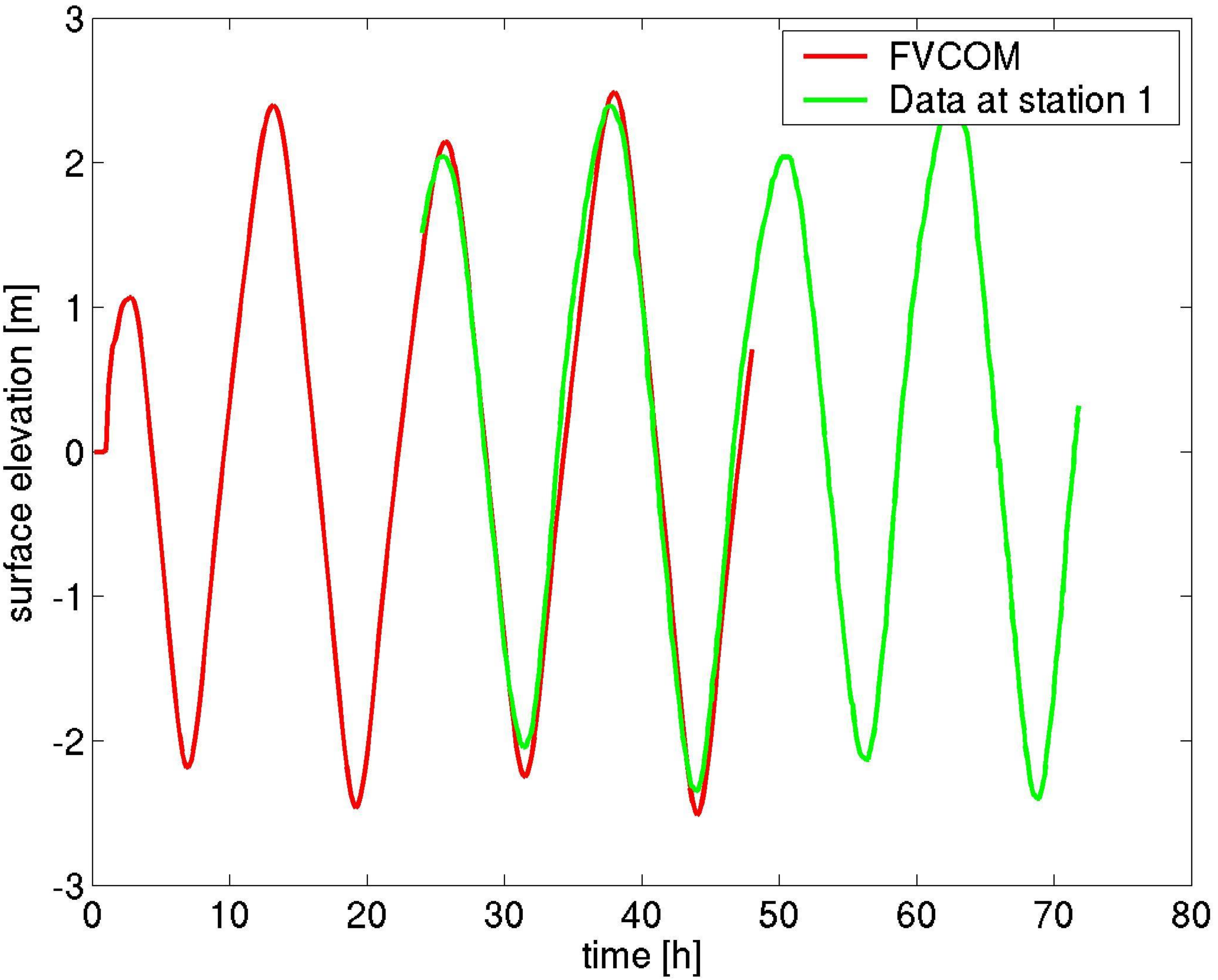

53.57
53.56
53.55
53.54
53.53
53.52
53.51
53.5
53.49
8.08
8.1
8.12
8.14
8.16
8.18
8.2
25
20
15
10
5
0

Validation of the hydrodynamic module

- Using surface elevation data from a nearby time series station
- Using trajectories of surface drifters

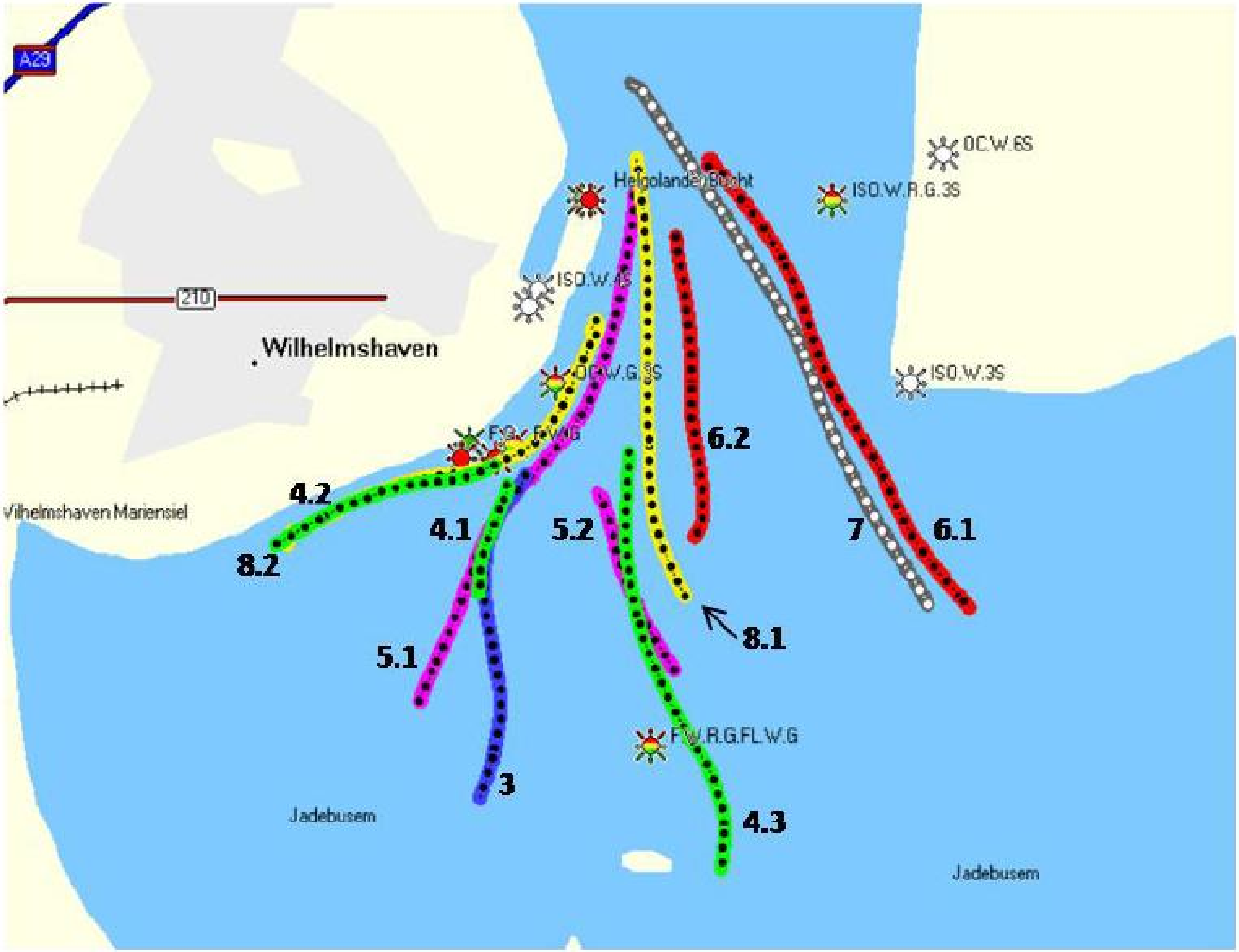
The tidal forcing at the open boundary was taken from the FES2004 tidal atlas (Lyard et al., 2006).

Model validation - surface eleva

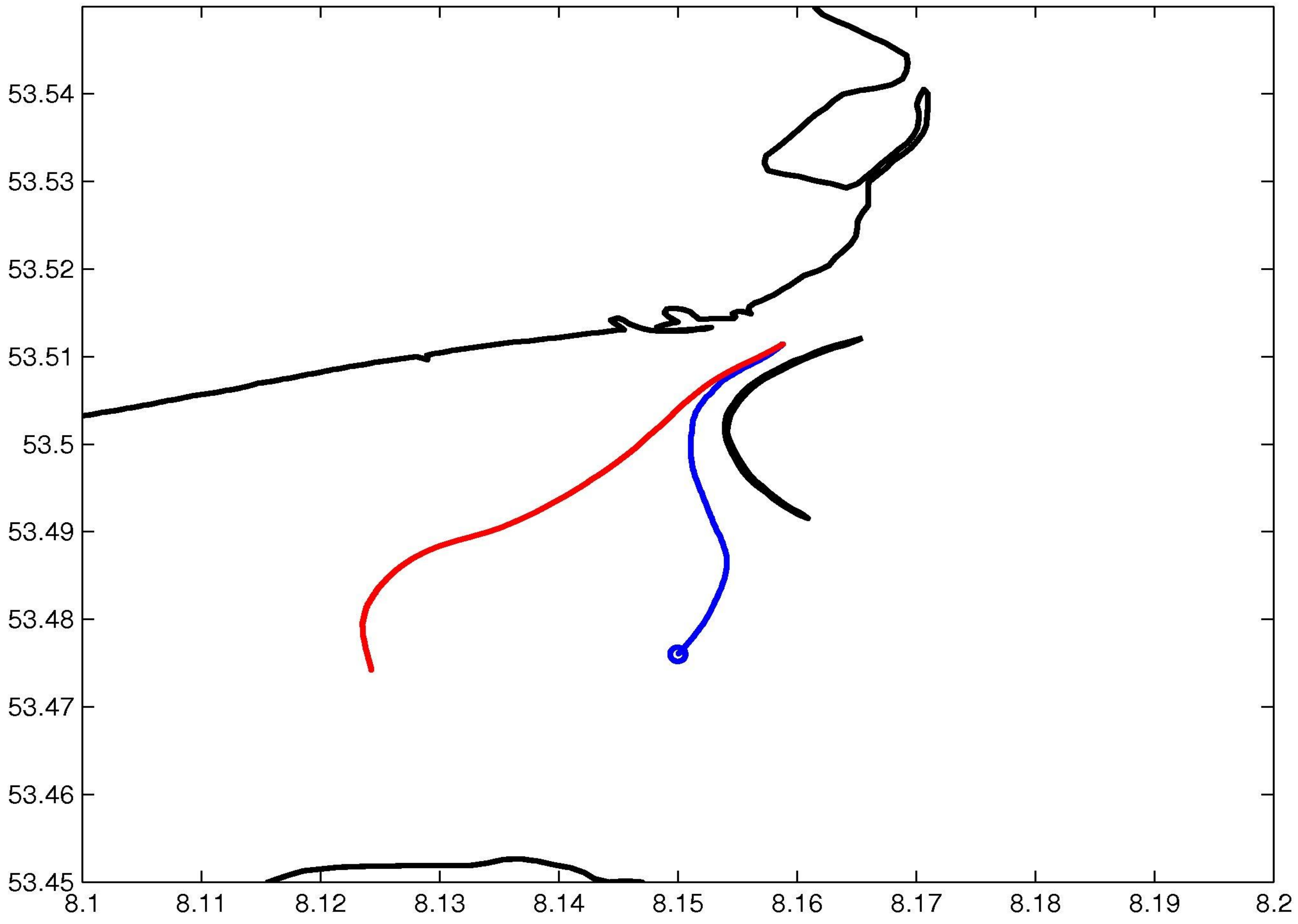


FVCOM
Data at station 1
surface elevation [m]
time [h]
3
2
1
0
-1
-2
-3
0
10
20
30
40
50
60
70
80

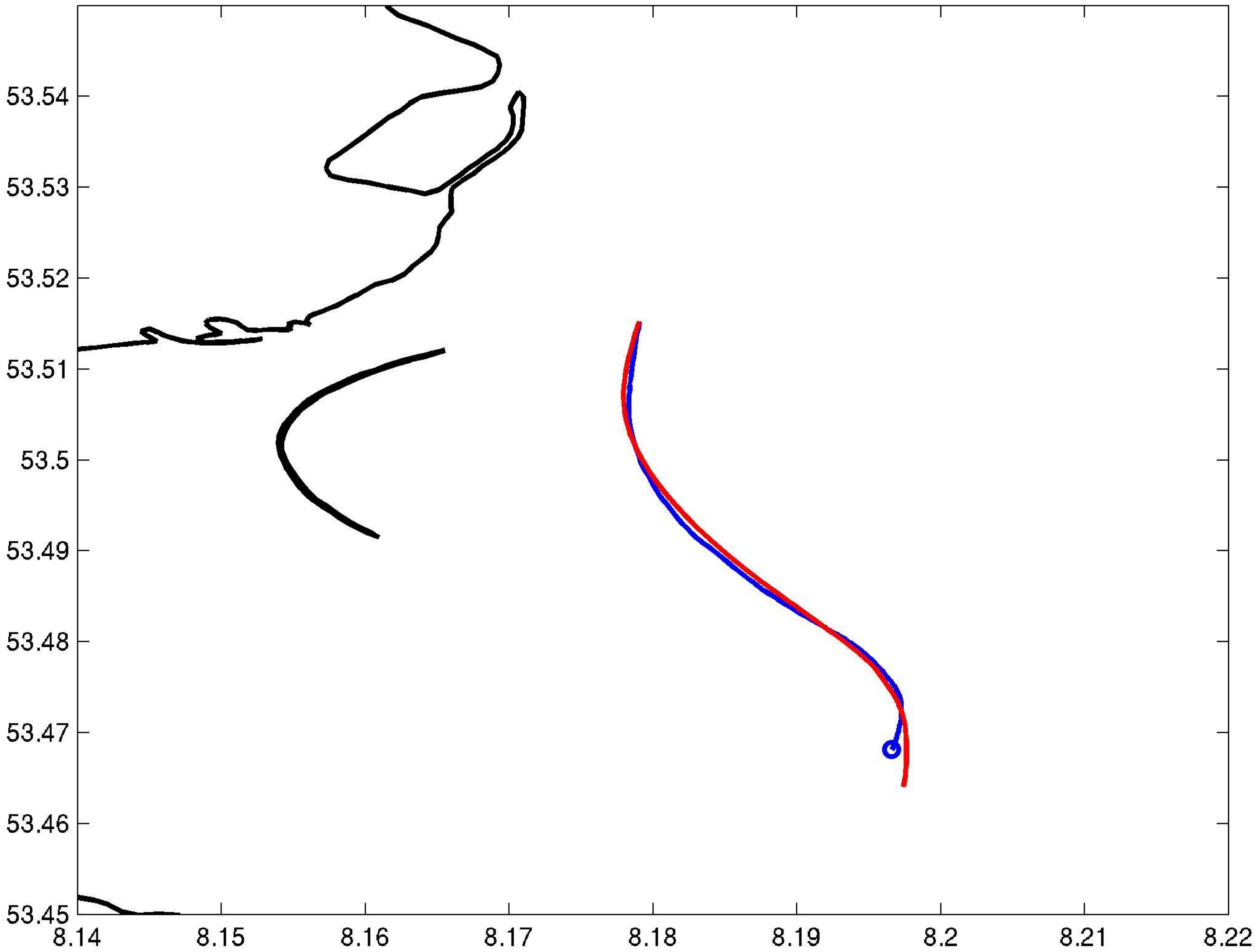
Trajectories of surface drifter

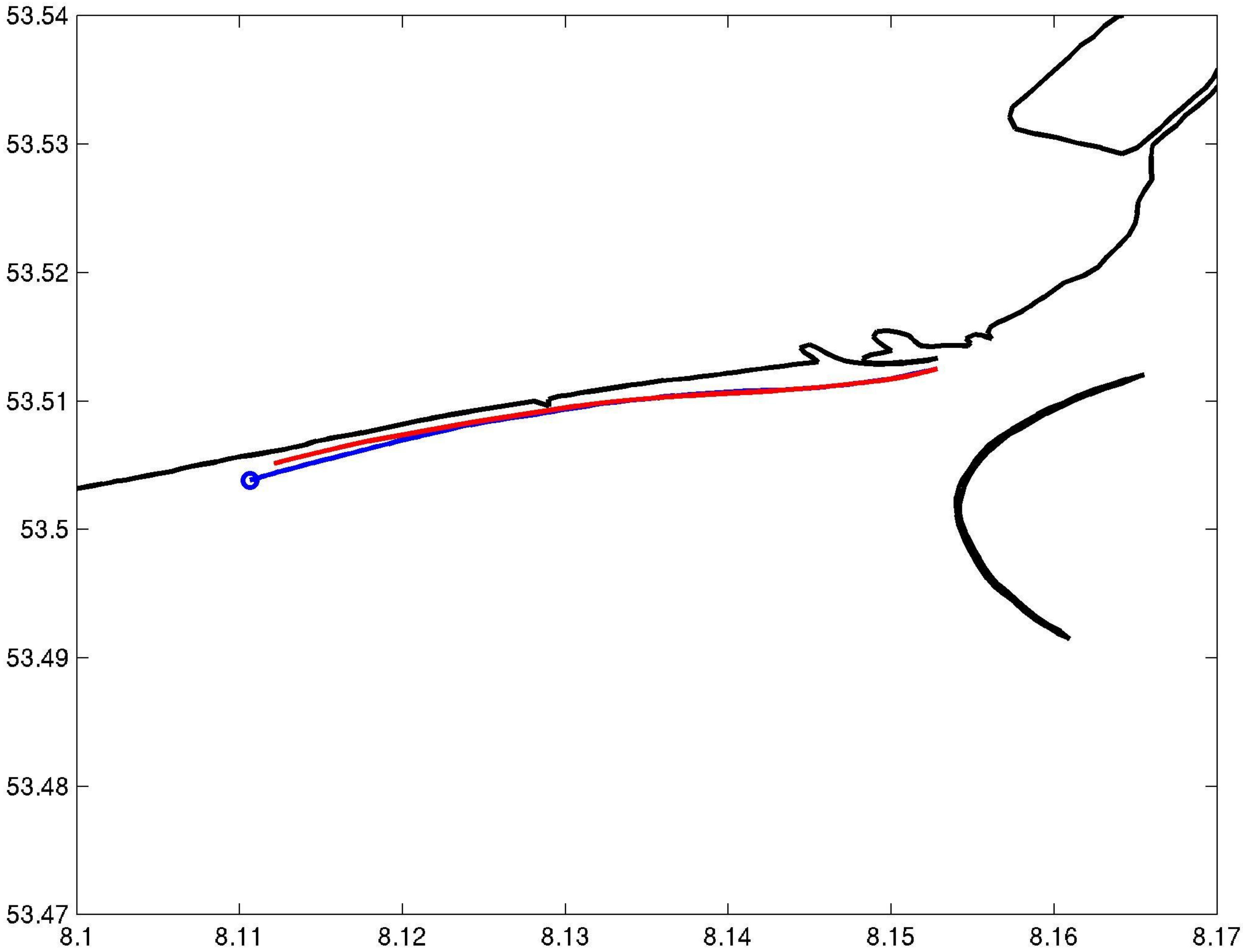


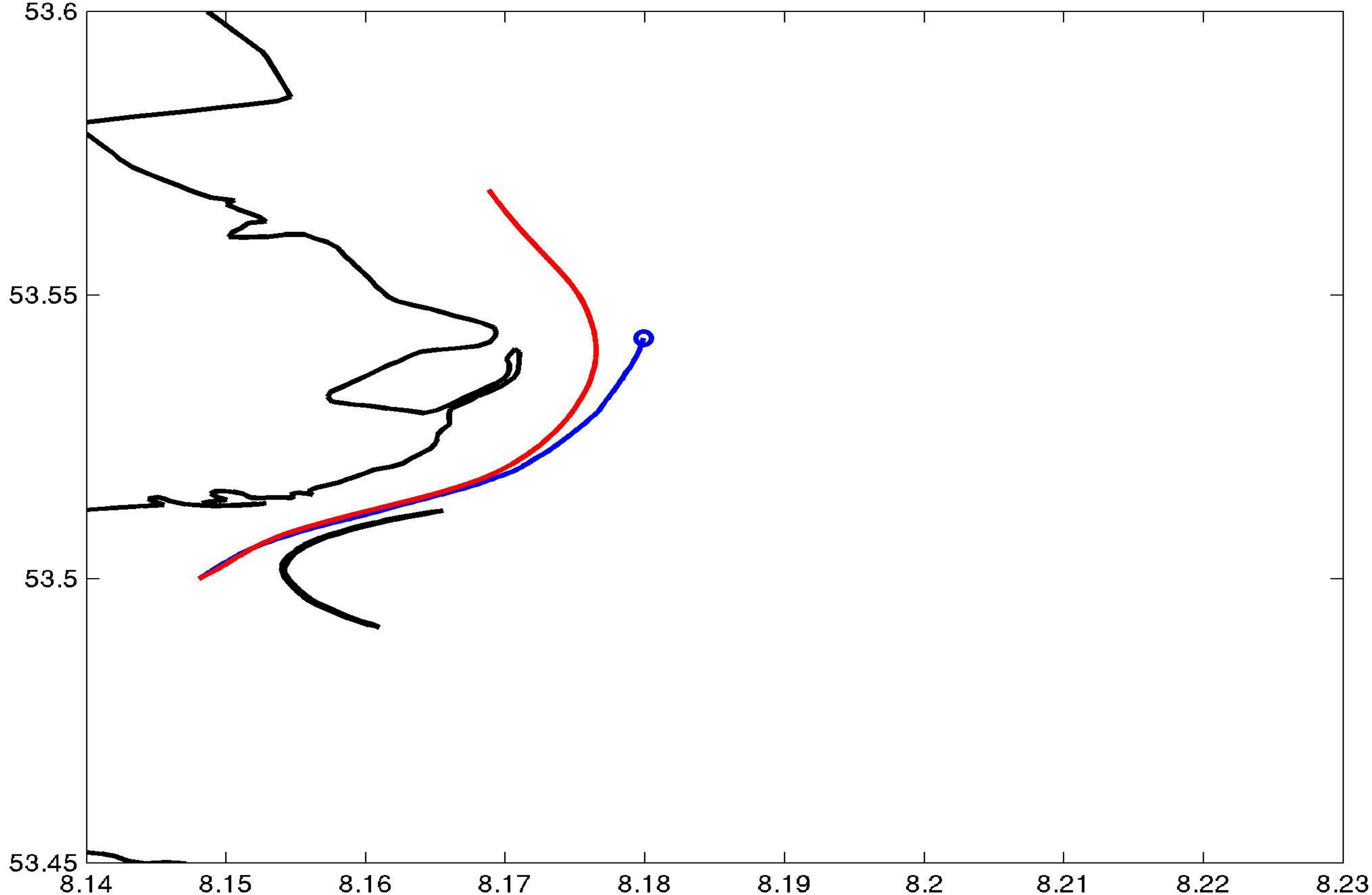
Cruises done by done by Christopher Dibke, student at ICBM

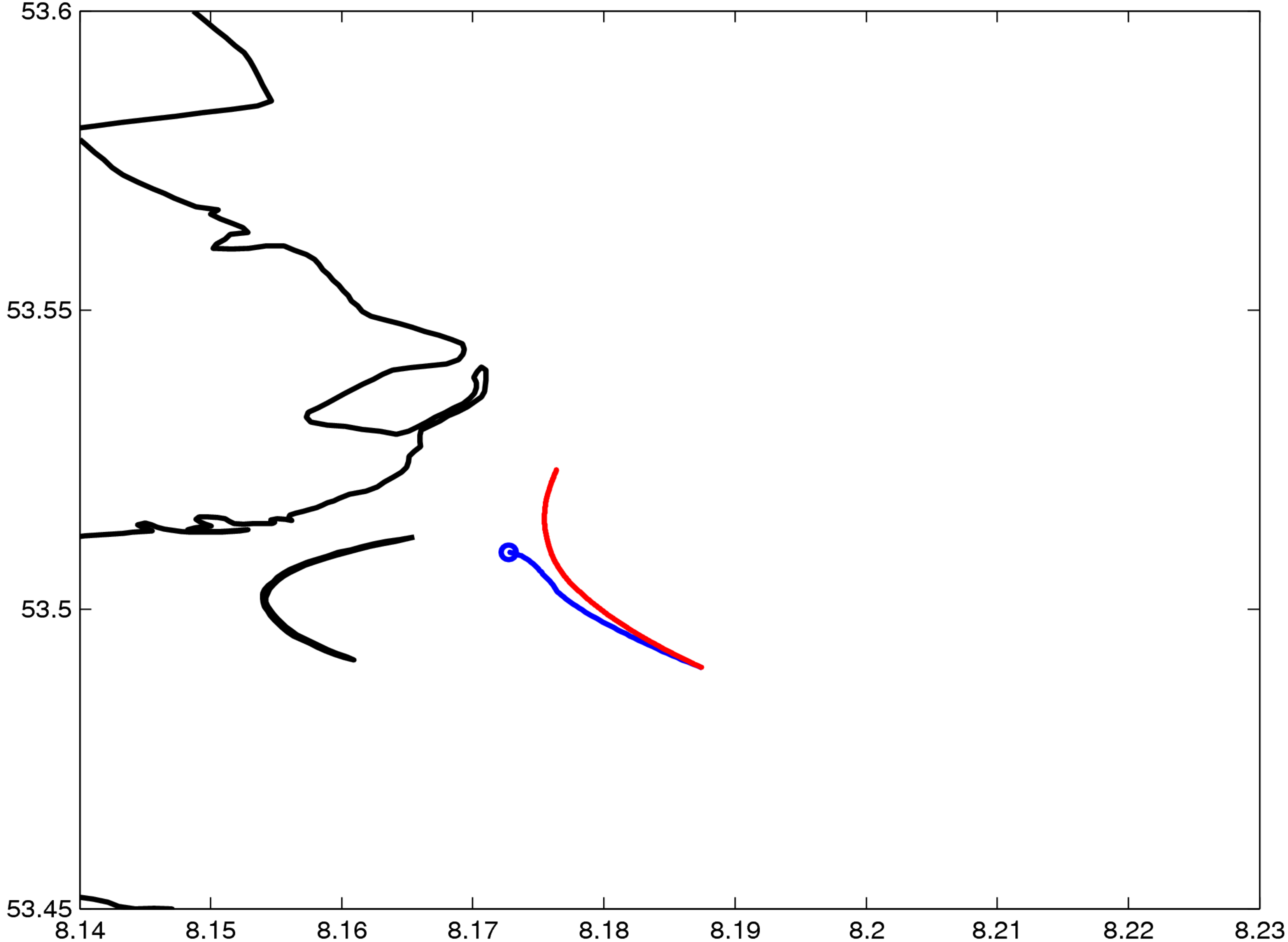


53.54
53.53
53.52
53.51
53.5
53.49
53.48
53.47
53.46
53.45
8.1
8.11
8.12
8.13
8.14
8.15
8.16
8.17
8.18
8.19
8.2


53.54
53.53
53.52
53.51
53.5
53.49
53.48
53.47
53.46
53.45
8.14
8.15
8.16
8.17
8.18
8.19
8.2
8.21
8.22


53.54
53.53
53.52
53.51
53.5
53.49
53.48
53.47
8.1
8.11
8.12
8.13
8.14
8.15
8.16
8.17


53.6
53.55
53.5
53.45
8.14
8.15
8.16
8.17
8.18
8.19
8.2
8.21
8.22
8.23


53.6
53.55
53.5
53.45
8.14
8.15
8.16
8.17
8.18
8.19
8.2
8.21
8.22
8.23

Discharge event in Feb. 2007

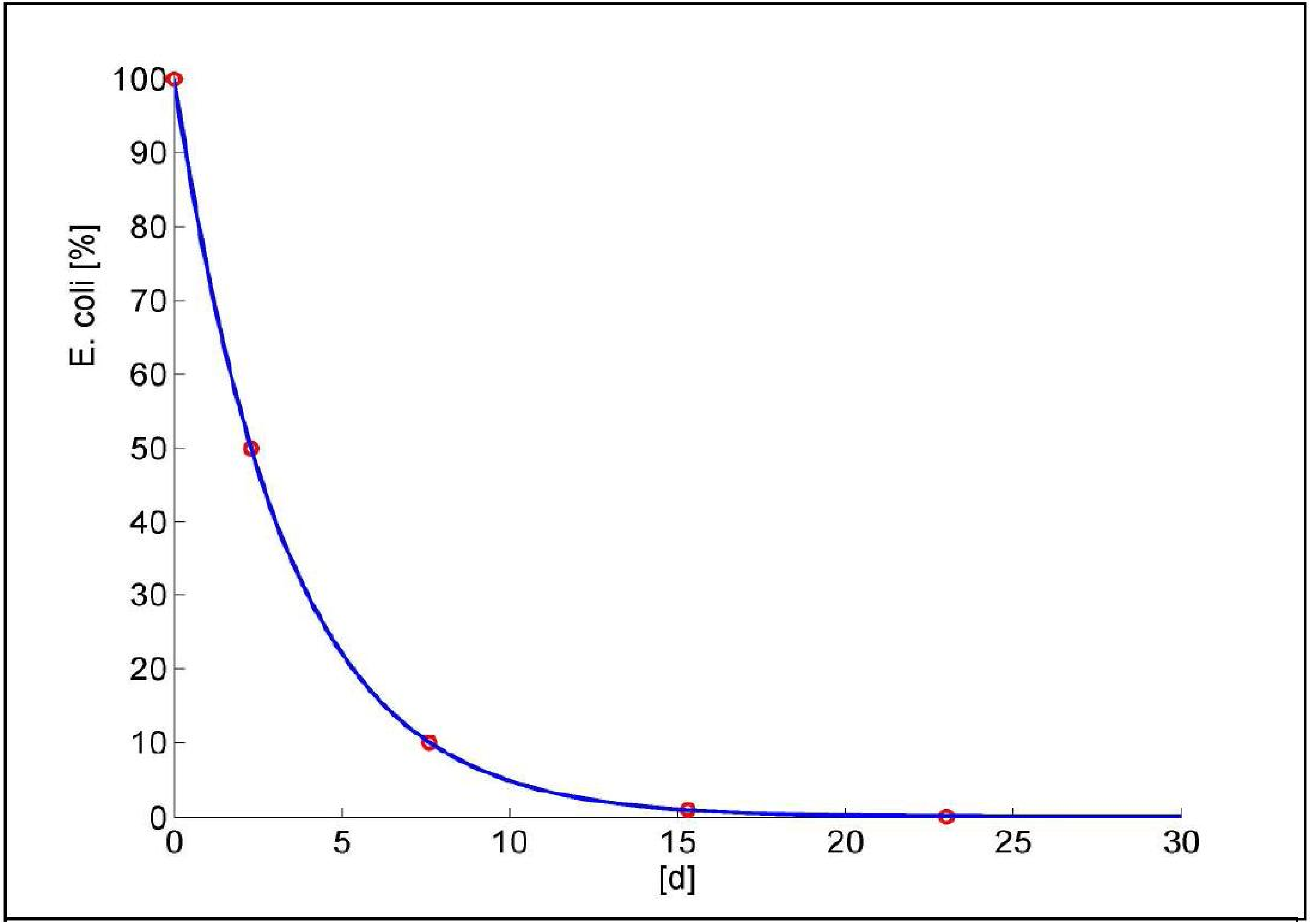
We simulated a large discharge event of about 17.000 m^3 of waste water input over a time interval of about 2 hours shortly before high tide.

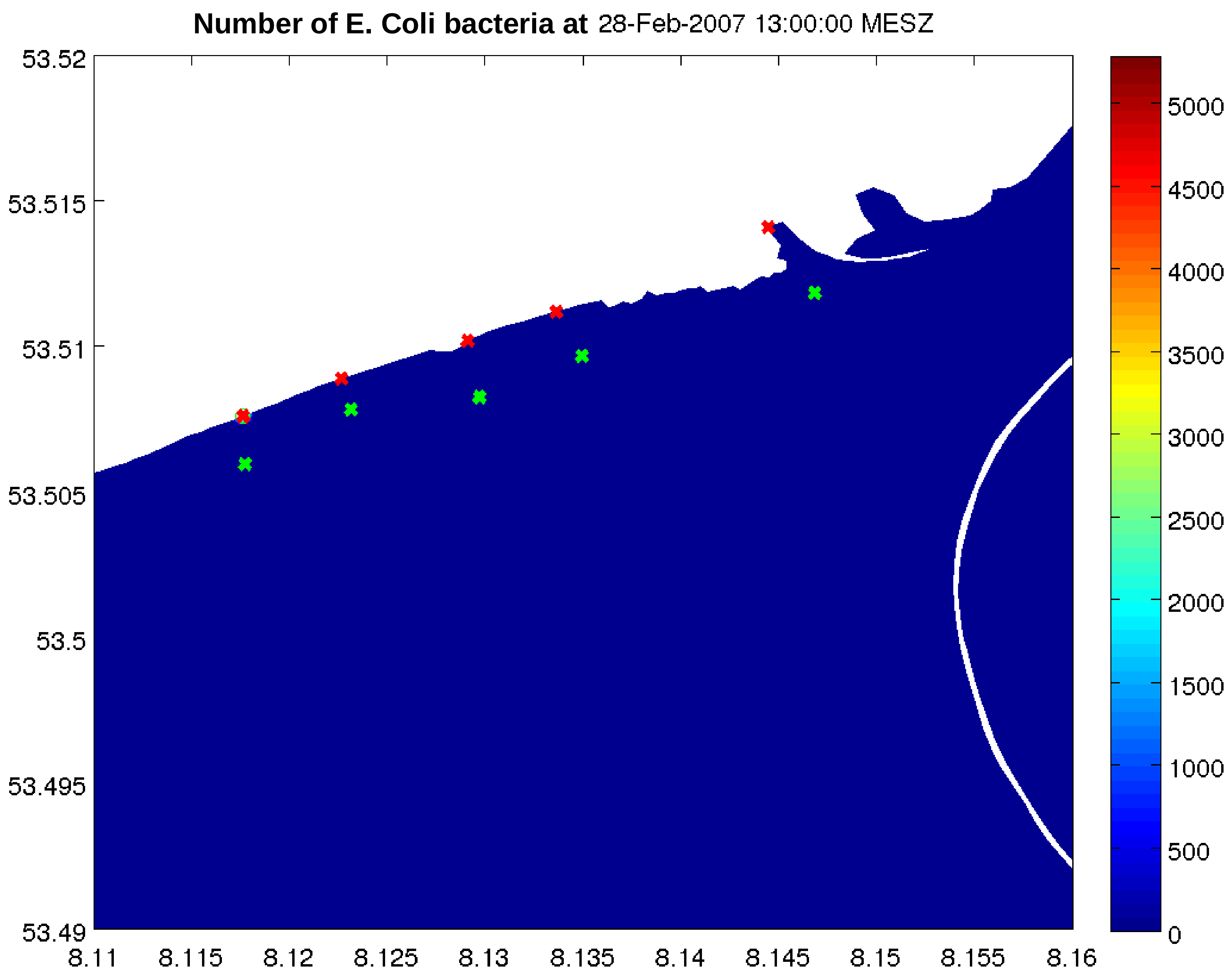
two different bacteria classes:
- a free swimming species,
- sinking species fixed to the waste water particles,

Constant sinking speed of about 0.1 mm/s.

Dying of bacteria within the sea water by a first order decay rate with a decay constant of 3.5e-6 s.

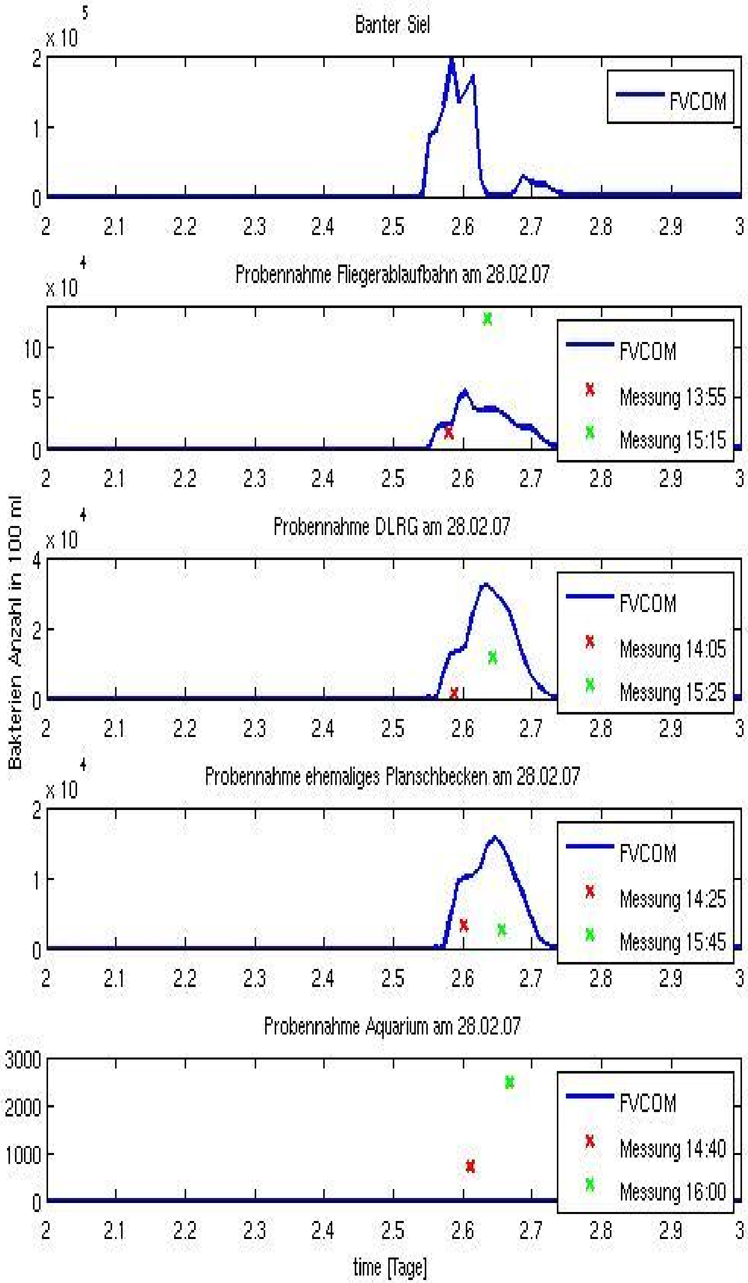
Dying curve of E. coli bacteria



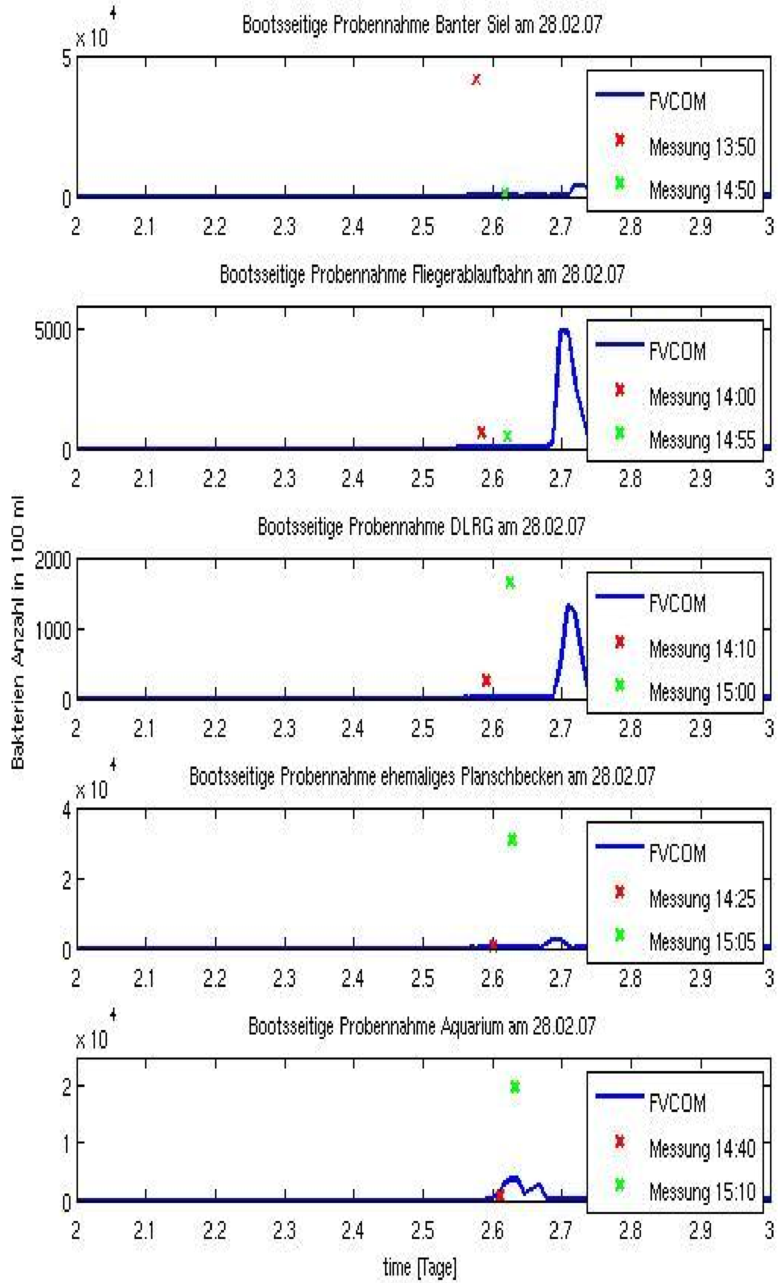


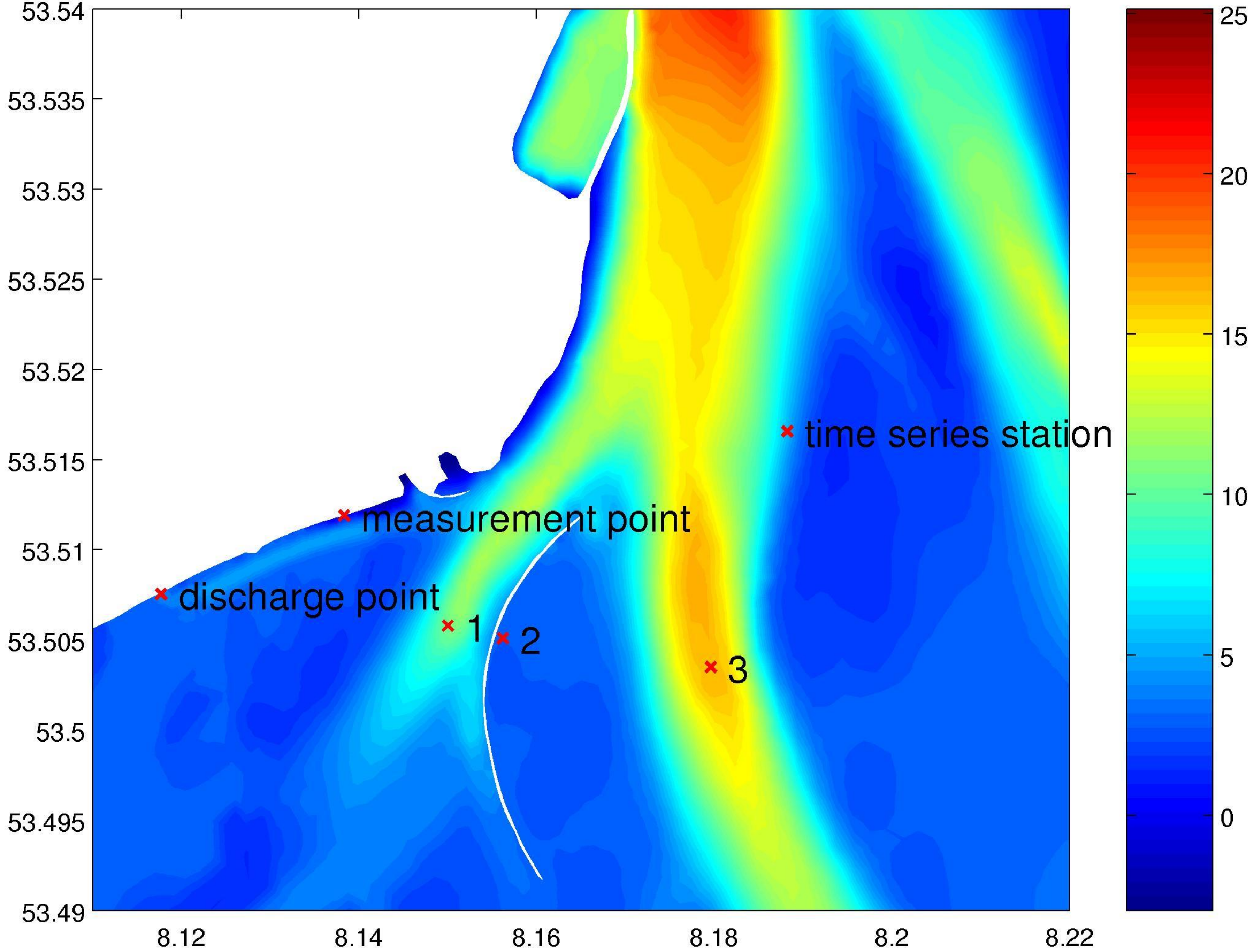
Number of E. Coli bacteria at 28-Feb-2007 13:00:00 MESZ
53.52
53.515
53.51
53.505
53.5
53.495
53.49
8.11
8.115
8.12
8.125
8.13
8.135
8.14
8.145
8.15
8.155
8.16
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0

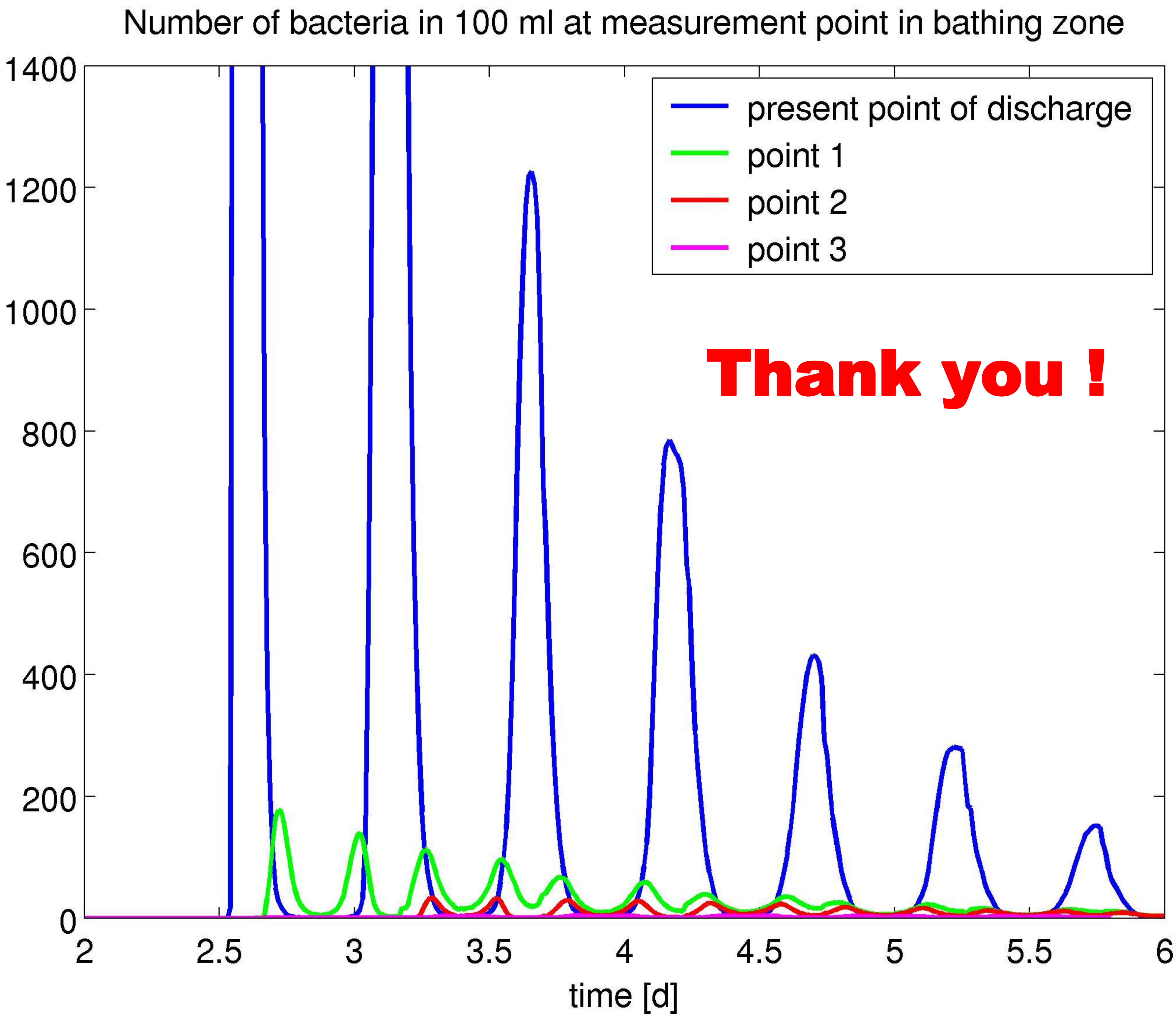
At coast line



From ship




53.54
53.535
53.53
53.525
53.52
53.515
53.51
53.505
53.5
53.495
53.49
8.12
8.14
8.16
8.18
8.2
8.22
25
20
15
10
5
0
time series station
measurement point
discharge point
1
2
3



Number of bacteria in 100 ml at measurement point in bathing zone
1400
1200
1000
800
600
400
200
0
2
2.5
3
3.5
4
4.5
5
5.5
6
time [d]
present point of discharge
point 1
point 2
point 3
Thank you !

Diffusion on a triangle

$$\frac{\partial T}{\partial t} = \nabla \cdot (A_h \nabla T)$$

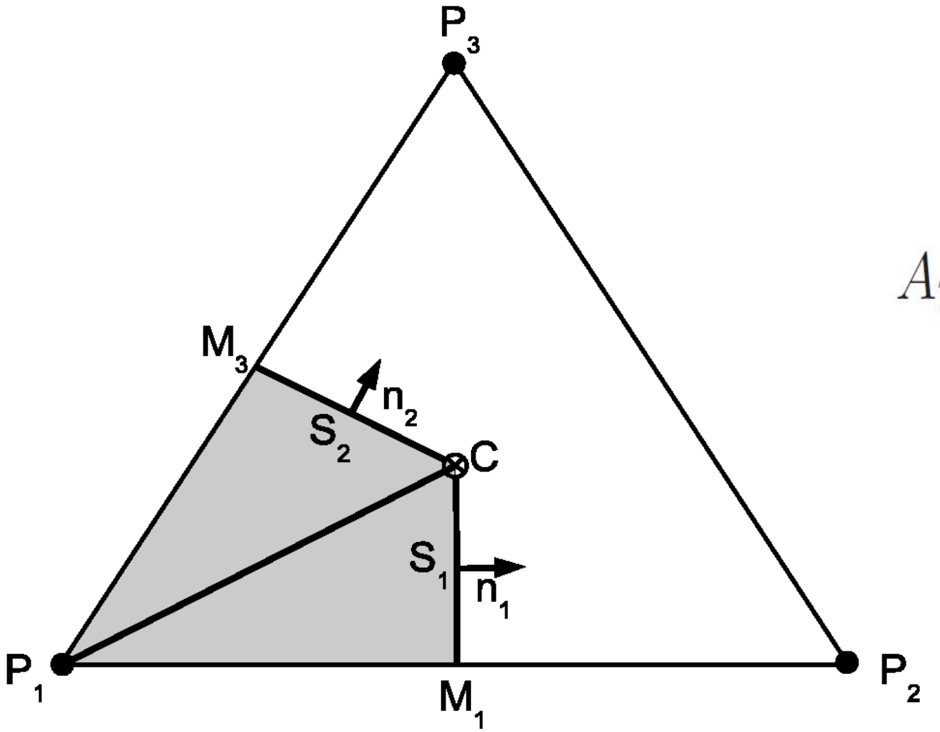
$$\int\limits_{A_{TCE}} \frac{\partial T}{\partial t}\, dA = \int\limits_{A_{TCE}} \nabla \cdot (A_h \nabla T)\, dA$$

$$A_{TCE} \frac{\partial T_n}{\partial t} = \oint\limits_{\Omega_{TCE}} (A_h \nabla T) \cdot \boldsymbol{n}\, ds$$

(n = 1,2,3)

$$= \sum_i \int\limits_{S_i} (A_h \nabla T) \cdot \boldsymbol{n}\, ds$$

$$A_{TCE} \frac{\partial T_n}{\partial t} = \sum_i \Delta S_i\, \left[(A_h \nabla T) \cdot \boldsymbol{n} \right]_{\text{center of side } S_i}$$



How to approximate the gradient?

Approximation for the gradient

$$T(x,y) = \sum_{i=1}^{3} N_i(x,y) T_i$$

$$N_1(x,y) := \frac{1}{2\,A}(a_1 + b_1 x + c_1 y)$$

$$N_2(x,y) := \frac{1}{2\,A}(a_2 + b_2 x + c_2 y)$$

$$N_3(x,y) := \frac{1}{2\,A}(a_3 + b_3 x + c_3 y)$$

$$A = \frac{1}{2}\,[(x_2 - x_1)(y_3 - y_1) - (x_3 - x_1)(y_2 - y_1)]$$

$$a_1 = x_2 y_3 - x_3 y_2, \quad b_1 = y_2 - y_3, \quad c_1 = x_3 - x_2$$

$$a_2 = x_3 y_1 - x_1 y_3, \quad b_2 = y_3 - y_1, \quad c_2 = x_1 - x_3$$

$$a_3 = x_1 y_2 - x_2 y_1, \quad b_3 = y_1 - y_2, \quad c_3 = x_2 - x_1$$

$$\frac{\partial T}{\partial x} = \sum_{i=1}^{3} \frac{\partial N_i}{\partial x} T_i = \frac{1}{2A} \sum_{i=1}^{3} b_i T_i = const.$$

$$\frac{\partial T}{\partial y} = \sum_{i=1}^{3} \frac{\partial N_i}{\partial y} T_i = \frac{1}{2A} \sum_{i=1}^{3} c_i T_i = const.$$