Absorbing layers
for shallow water models

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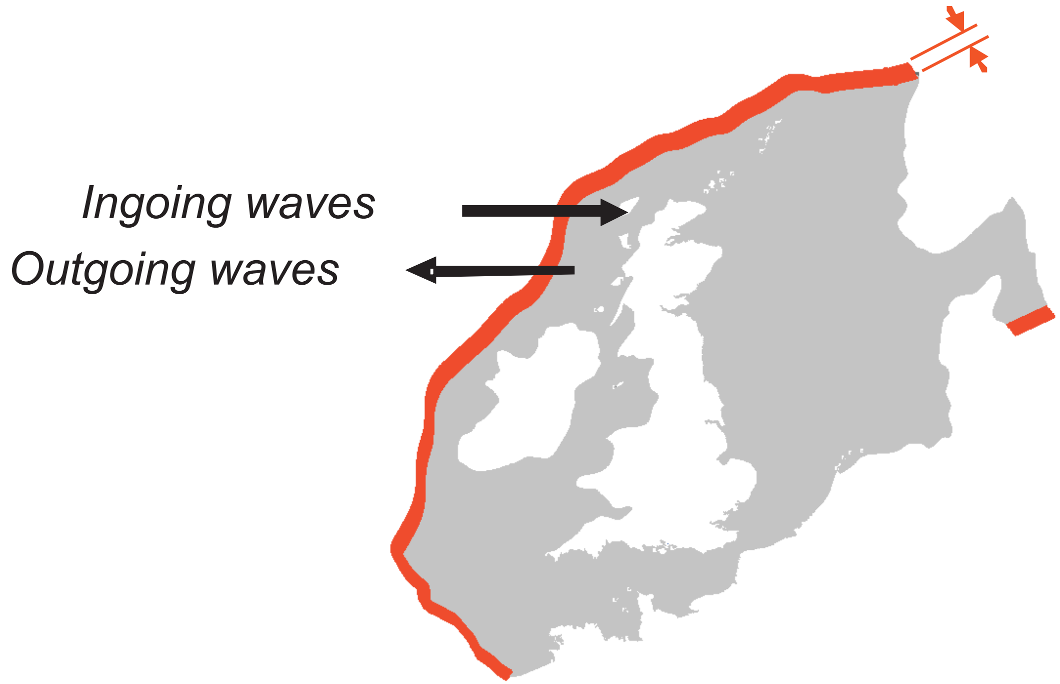
Absorbing layer

as open boundary treatment



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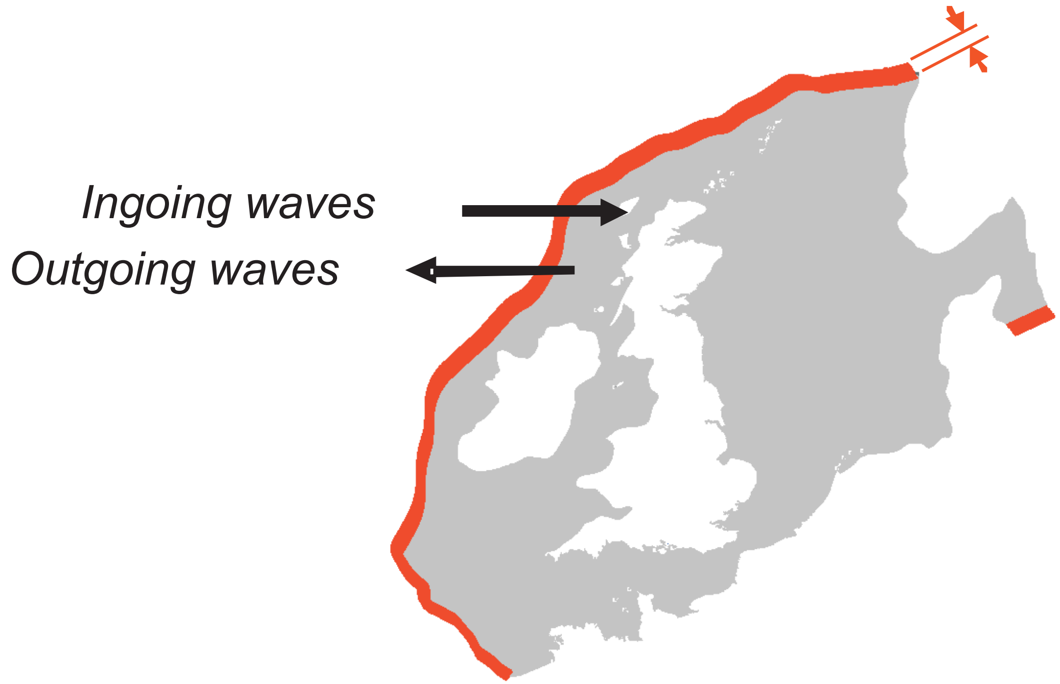

- ✔ Extension of the computational domain
  The fields are subject to a particular treatment
- ✔ Prescribe progressively the external forcing
- ✔ Minimize the reflection of outgoing waves

The best layer for linear gravity wave
Different kinds of layers
Comparison of layers

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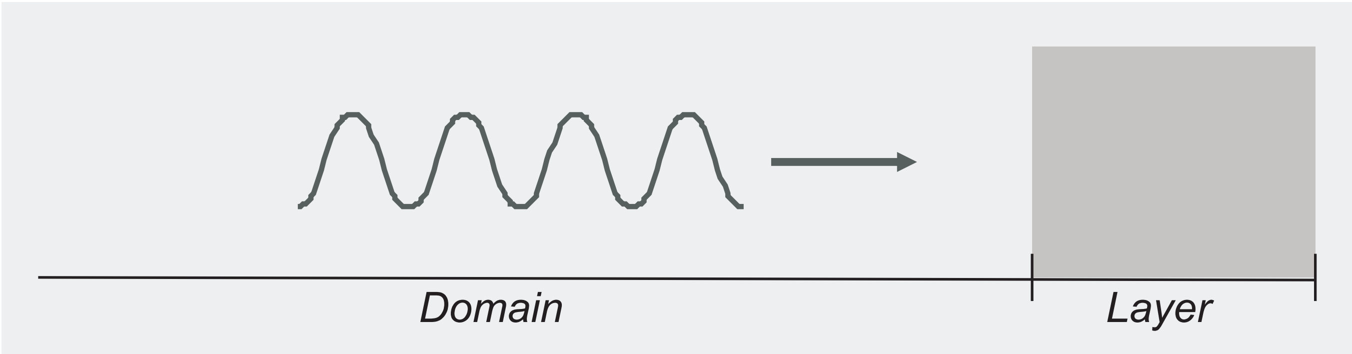
Absorbing layer

for linear gravity wave

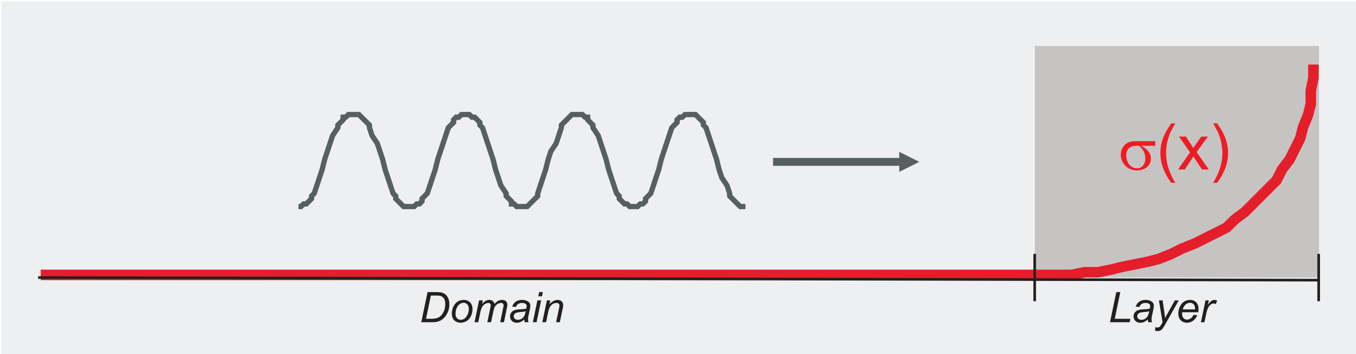


Basic equations

$$\frac{\partial \eta}{\partial t} + h\frac{\partial u}{\partial x} = 0$$
$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0$$

Absorbing layer

for linear gravity wave

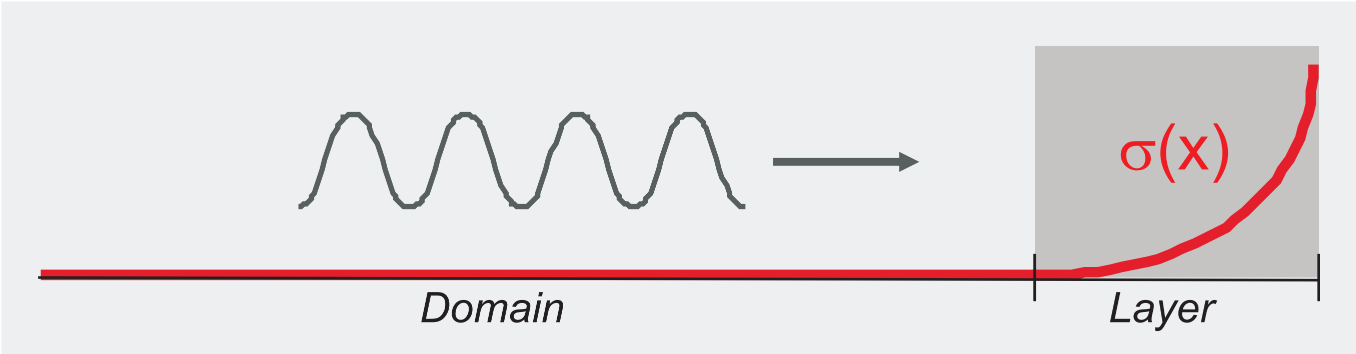


Equations with absorption terms

$$\frac{\partial \eta}{\partial t} + h\frac{\partial u}{\partial x} = -\sigma\ \eta$$
$$\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = -\sigma\ u$$

Absorbing layer

for linear gravity wave



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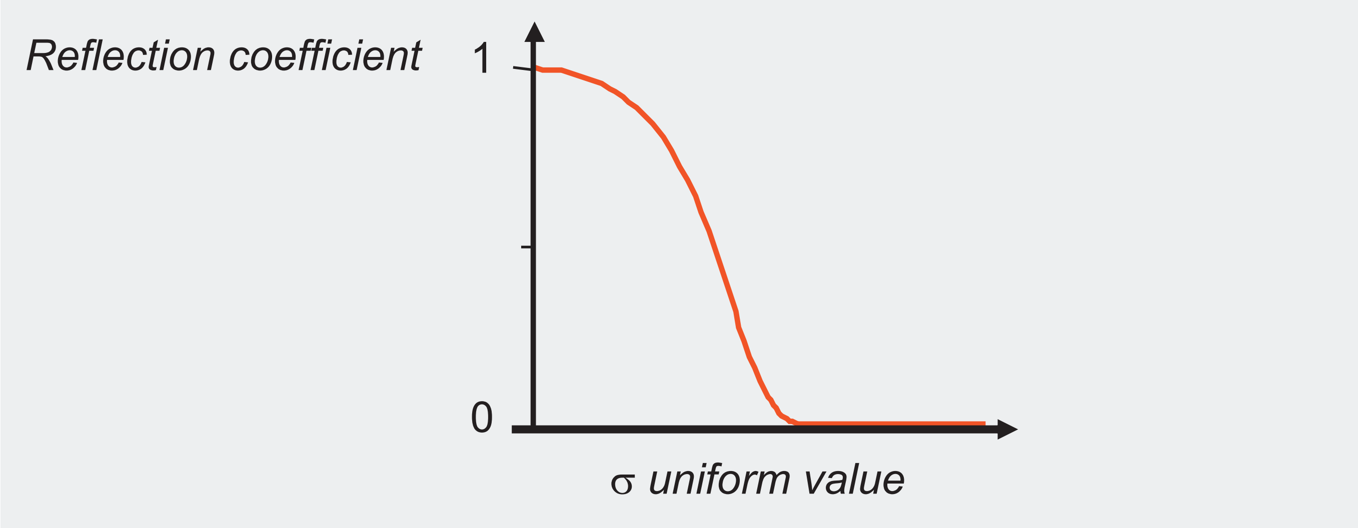
Which absorption coefficient ?

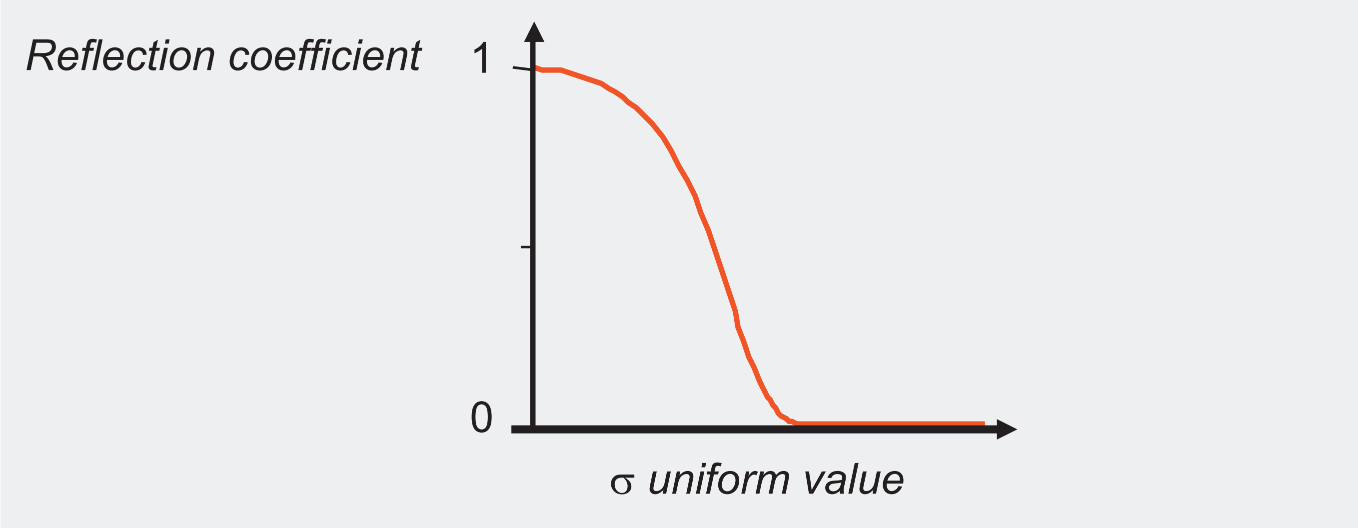
The best absorption coefficient

A discrete problem



The best absorption coefficient

A discrete problem

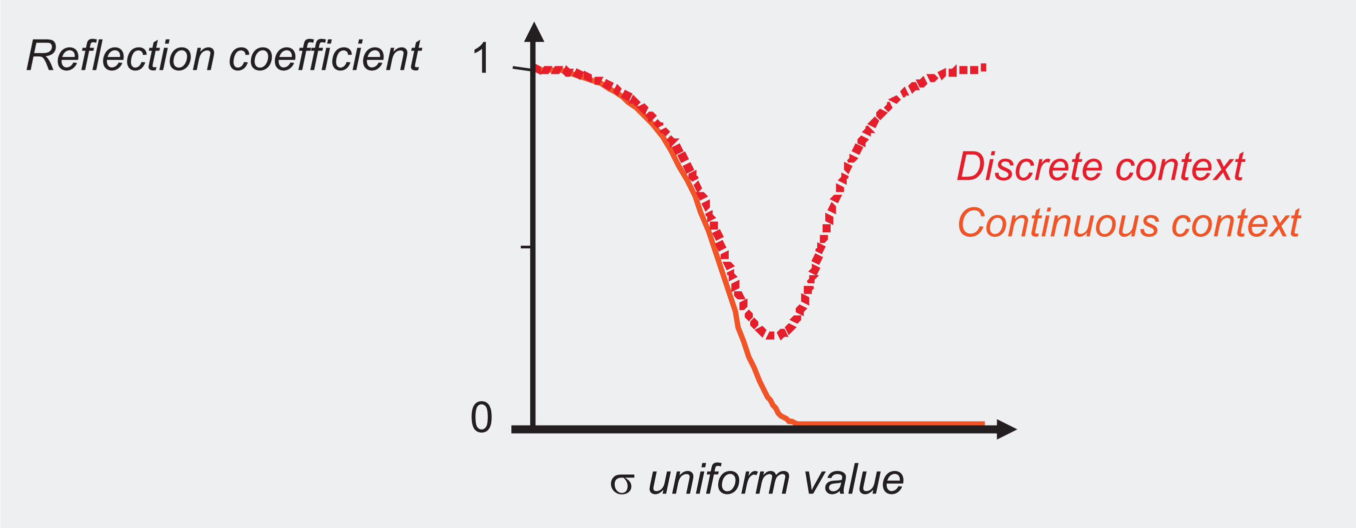


In the continuous context, the reflection coefficient is 0 if

$$\int_{\text{Layer}} \sigma(x)\, dx = +\infty$$

The best absorption coefficient

A discrete problem



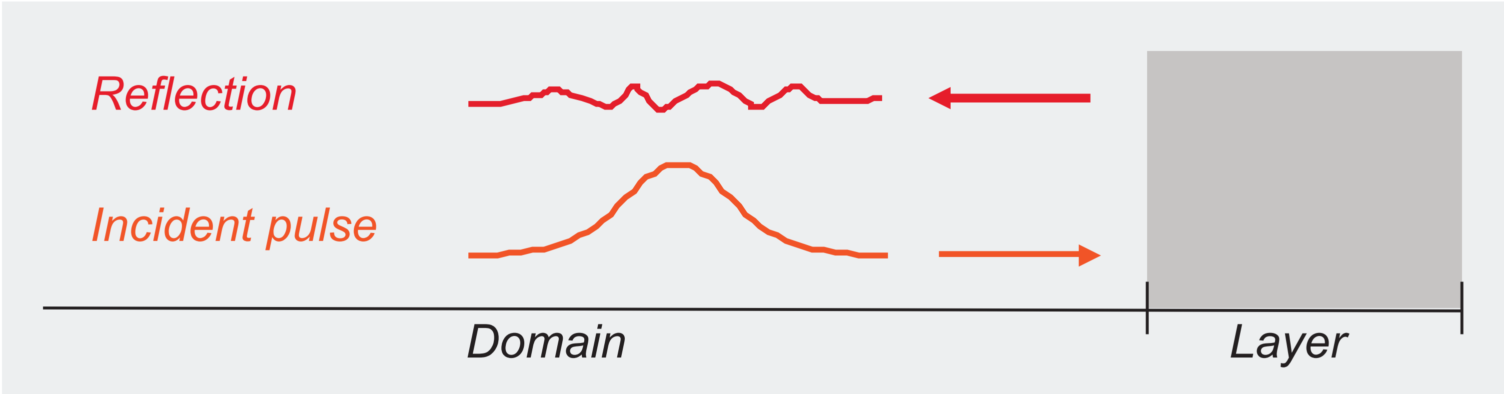
In the continuous context, the reflection coefficient is 0 if

$$\int_{\text{Layer}} \sigma(x)\, dx = +\infty$$

In the discrete context, the fields variations must also be represented by the discrete scheme

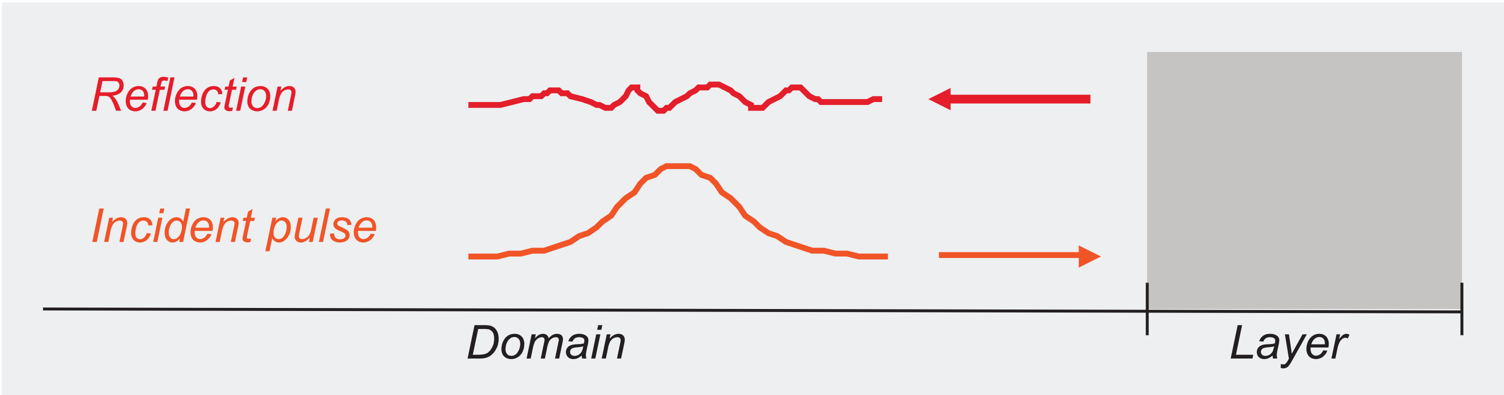
The best absorption coefficient

Optimization procedure



The best absorption coefficient

Optimization procedure



Minimize Energy norm of the reflected signal

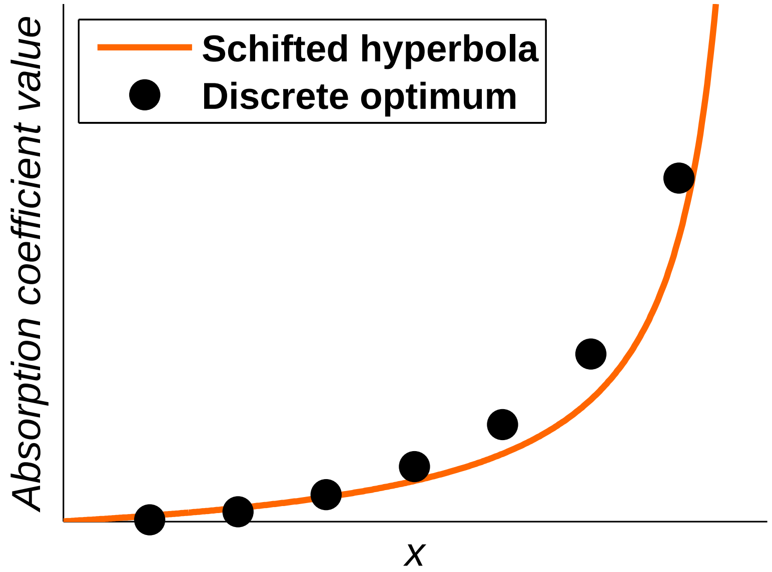
$$\int_{\text{Domain}} \left[\frac{1}{2} g \left(\eta - \eta_{\text{reference}} \right)^2 + \frac{1}{2} h \left(u - u_{\text{reference}} \right)^2 \right] \, dx$$

Function of the absorption coefficient

The best absorption coefficient

Discrete optimum

Modave and al. Ocean Dynamics (2010)



Shifted hyperbola : $\dfrac{\sqrt{gh}}{\delta}\dfrac{x}{x-\delta}$

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The best layer for linear gravity wave

Different kinds of layers

Comparison of layers

Flow relaxation scheme (FRS)

An easy-used layer

Martinsen and al. Coastal Eng. (1987)

$$\frac{\partial H}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = -\sigma(H - H^{ext})$$

$$\frac{\partial (Hu)}{\partial t} + \frac{\partial (gH^2/2)}{\partial x} + \frac{\partial (Hu^2)}{\partial x} + \frac{\partial (Huv)}{\partial y} - fHv = -\sigma(Hu - Hu^{ext})$$

$$\frac{\partial (Hv)}{\partial t} + \frac{\partial (gH^2/2)}{\partial y} + \frac{\partial (Huv)}{\partial x} + \frac{\partial (Hv^2)}{\partial y} + fHu = -\sigma(Hv - Hv^{ext})$$

Relaxation terms

Adapted FRS

An other easy-used layer

Lavelle and al. Ocean Modell. (2008)

$$\frac{\partial H}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = -(\sigma_x + \sigma_y)(H - H^{ext})$$

$$\frac{\partial (Hu)}{\partial t} + \frac{\partial (gH^2/2)}{\partial x} + \frac{\partial (Hu^2)}{\partial x} + \frac{\partial (Huv)}{\partial y} - fHv = -\sigma_x(Hu - Hu^{ext})$$

$$\frac{\partial (Hv)}{\partial t} + \frac{\partial (gH^2/2)}{\partial y} + \frac{\partial (Huv)}{\partial x} + \frac{\partial (Hv^2)}{\partial y} + fHu = -\sigma_y(Hv - Hv^{ext})$$

Other

relaxation terms

Perfectly matched layer (PML)

A layer with theoretical justification

Hu Comput. Fluids (2008)

$$\frac{\partial H}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = -(\sigma_x + \sigma_y)(H - H^{ext}) - q_H$$

$$\frac{\partial (Hu)}{\partial t} + \frac{\partial (gH^2/2)}{\partial x} + \frac{\partial (Hu^2)}{\partial x} + \frac{\partial (Huv)}{\partial y} - fHv = -(\sigma_x + \sigma_y)(Hu - Hu^{ext}) - q_{Hu}$$

$$\frac{\partial (Hv)}{\partial t} + \frac{\partial (gH^2/2)}{\partial y} + \frac{\partial (Huv)}{\partial x} + \frac{\partial (Hv^2)}{\partial y} + fHu = -(\sigma_x + \sigma_y)(Hv - Hv^{ext}) - q_{Hv}$$

With the additional equations:

$$\frac{\partial q_H}{\partial t} = \sigma_x \sigma_y (H - H^{ext}) + \sigma_y \frac{\partial (Hu - Hu^{ext})}{\partial x} + \sigma_x \frac{\partial (Hv - Hvext)}{\partial y}$$

$$\frac{\partial q_{Hu}}{\partial t} = (\sigma_x + \sigma_y)(-fHv + (fHv)^{ext}) + \sigma_x \sigma_y (Hu - (Hu)^{ext} + \hat{q}_{Hu})$$
$$+ \sigma_y \frac{\partial \left[(gH^2/2 + Hu^2) - (gH^2/2 + Hu^2)^{ext}\right]}{\partial x} + \sigma_x \frac{\partial \left[(Huv) - (Huv)^{ext}\right]}{\partial y}$$

$$\frac{\partial q_{Hv}}{\partial t} = (\sigma_x + \sigma_y)(fHu - (fHu)^{ext}) + \sigma_x \sigma_y (Hv - (Hv)^{ext} + \hat{q}_{Hv})$$
$$+ \sigma_y \frac{\partial \left[(Huv) - (Huv)ext\right]}{\partial x} + \sigma_x \frac{\partial \left[(gH^2/2 + Hv^2) - (gH^2/2 + Hv^2)^{ext}\right]}{\partial y}$$

$$\frac{\partial \hat{q}_{Hu}}{\partial t} = -fHv + (fHv)^{ext} \qquad \frac{\partial \hat{q}_{Hv}}{\partial t} = fHu - (fHu)^{ext}$$

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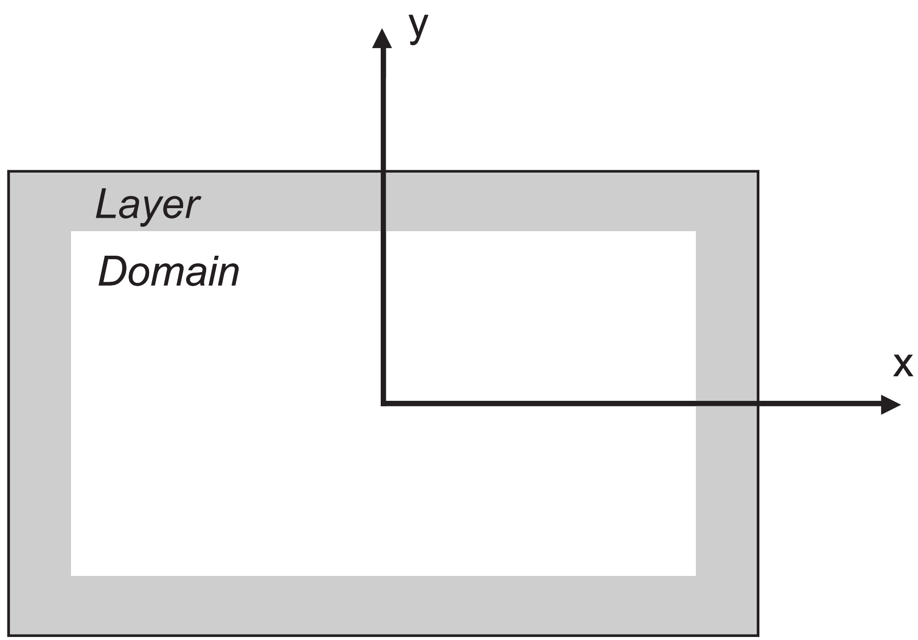
Test case

Collapse of the Gaussian-shaped mound of water



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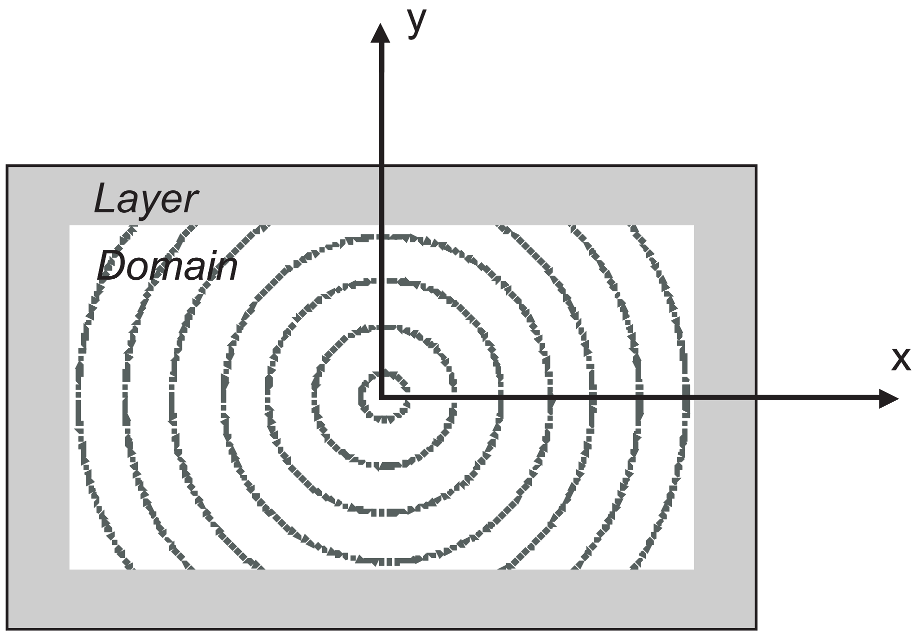
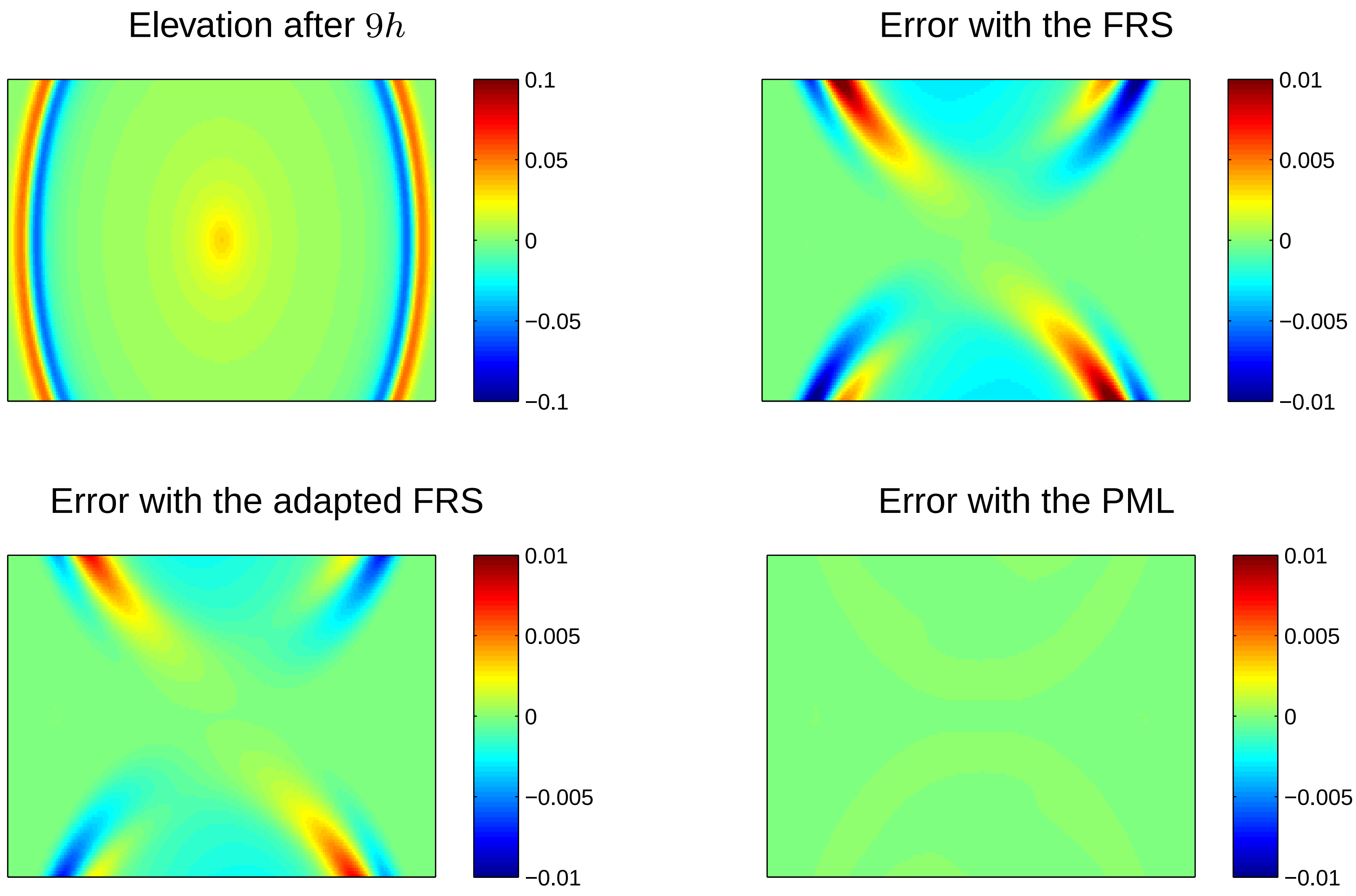


Elevation and error

using the shifted hyperbola



Reflection ratio

A measure of the layer efficiency

$$\textit{Reflection ratio} = \frac{\textit{Energy norm of the reflected signal}}{\textit{Energy norm of the initial fields}}$$

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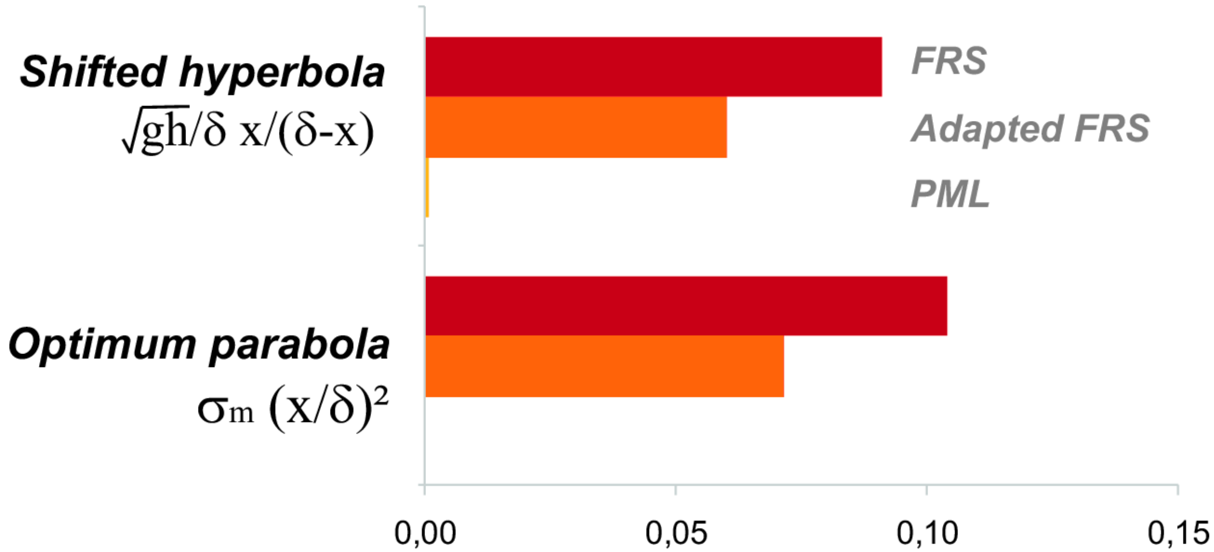

Reflection ratio

A measure of the layer efficiency

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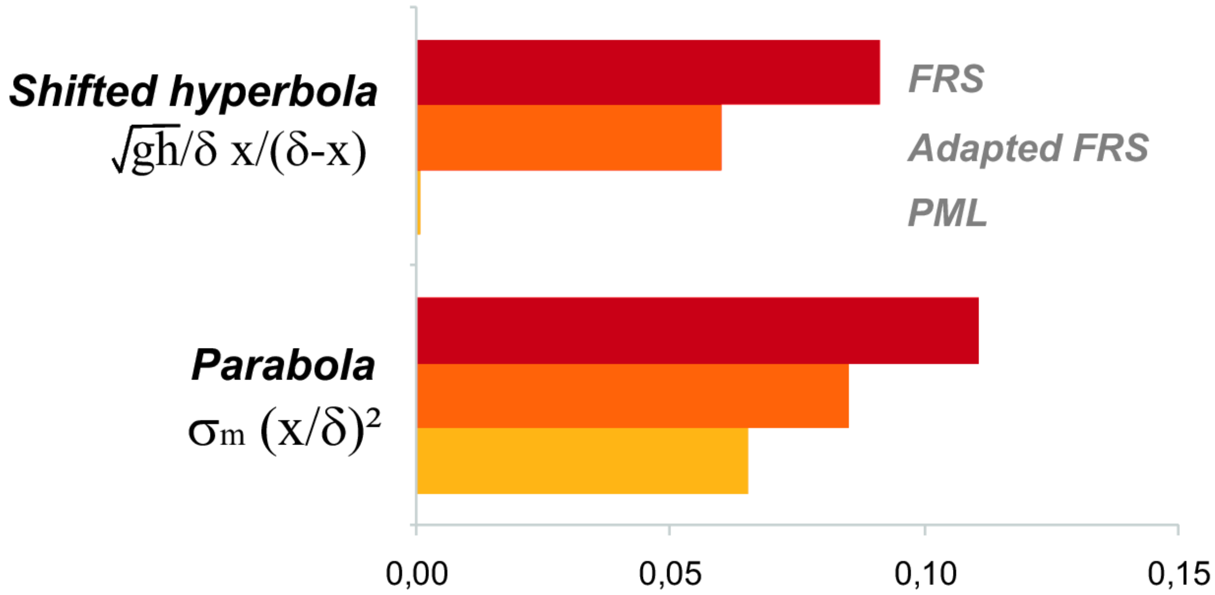

Summary and conclusion

- ✔ *Absorbing layer*
  - ▷ The PML gives the best results
    A layer with a theoretical justification
    Additional fields and equations
  - ▷ The adapted FRS is better than the FRS
    Easy to use
- ✔ *Absorption coefficient*
  - ▷ The choice of this coefficient is a discrete problem
  - ▷ The shifted hyperbola is the best for gravity wave
    No additional parameters to adjust

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