Tracking of sediment particles in a tidal dominated area

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Introduction

Time evolution of concentration field (biology, pollutants, sediment,…) described by partial differential equations (PDE)

$$\partial_t C = -\nabla \cdot (u\,C - K \cdot \nabla C)$$
$$\nabla \cdot u = 0$$

Numerical problems: - positiveness [Stijn et al. 1987, Yang et al 1998]

- excessive numerical disperison [Zheng et al. 1999]
- artificial oscillations [Zheng et al 2002]

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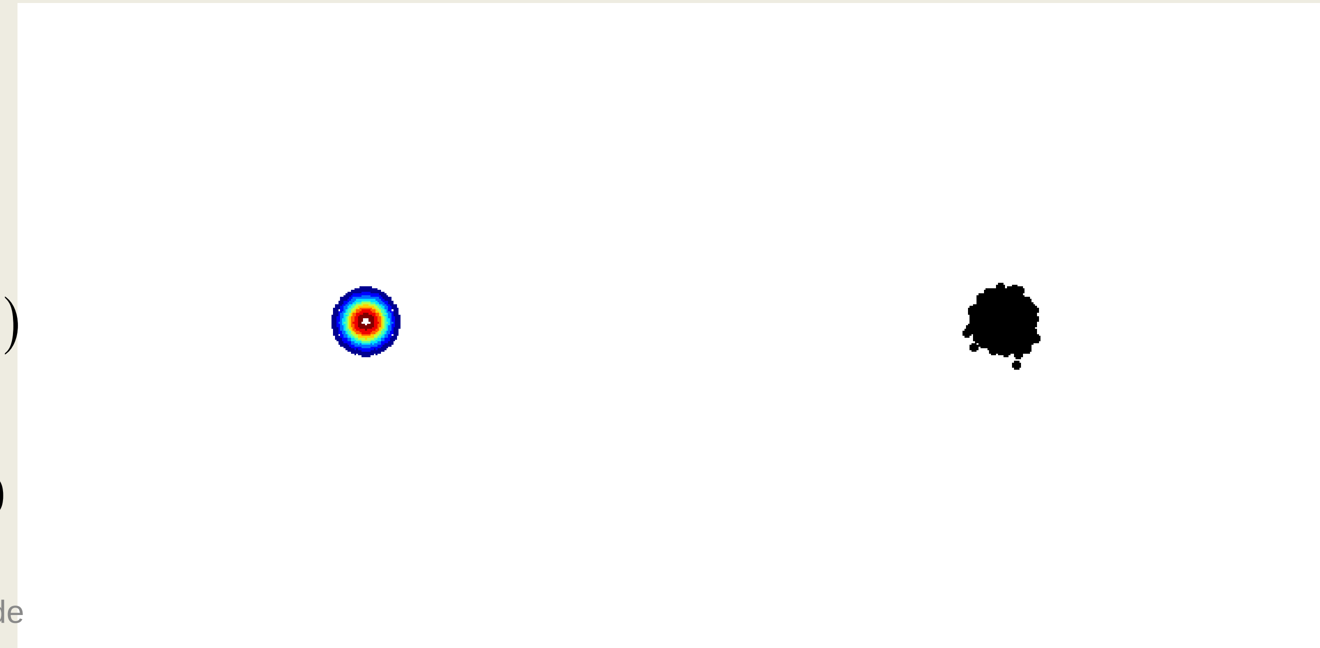
$$\frac{\partial C}{\partial t} = \nabla \bullet \left(K_H \nabla C(x,y,t) \right)$$

$$IC: \quad C(x,y,0) = \delta(0,0)$$



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Why the Lagrangian framework

Pros:

- Straightforward physical interpretation
- Assessment of integrated properties (residence time, trajectories, …)
- Aggregation/fragmentation
- Release of particle size distributions

Cons:

- Post processing statistics
- Computational expense

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From Euler to Lagrange and back

Euler:

$$\partial_t C(z,t) = -\partial_z \left(wC - K_z \partial_z C \right)$$

$$K = \begin{bmatrix} K_H & 0 & 0 \\ 0 & K_H & 0 \\ 0 & 0 & K_Z \end{bmatrix}$$

Lagrange:

$$dZ_i(t) = w dt + \sqrt{2K_z}\, dW_i(t)$$

$$\langle W(t) \rangle = 0 \, ; Std\left(W(t) - W(s) \right) = \sqrt{|t-s|}$$

$$if \qquad K_z = K_z(z)$$

$$dZ_i(t) = (w + \partial_z K) dt + \sqrt{2K_z}\, dW_i(t)$$

$$histogram(Z_i(t)) = C(z,t)$$

Lagrange ⟷ Euler

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Numerical schemes

$$b = \sqrt{2K_Z(z)}$$

$$dZ(t) = a\,dt + b\,dW(t)$$

$$a = w + \partial_Z K$$

EULER: $O(\Delta t)$

$$Z_{n+1} = Z_n + a\Delta t + b\,\Delta W_n \qquad with \qquad \Delta W_n = N\left(0, \sqrt{\Delta t}\right)$$

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$$\tilde{Z} = Z_n + a\Delta t + b\,\Delta W_n$$

$$Z_{n+1} = Z_n + 0.5\left(a(Z_n) + a(\tilde{Z})\right)\Delta t + b\,\Delta W_n$$

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Milstein: $O(\Delta t^2)$

$$Z_{n+1} = Z_n + a\,\Delta t + b\,\Delta W_n + \frac{1}{2}bb'\left(\Delta W_n^{\ 2} - \Delta t\right) + \frac{1}{2}\left((ab)' + \frac{1}{2}b''b^2\right)\Delta W\,\Delta t + \frac{1}{2}\left(aa' + \frac{1}{2}a''b^2\right)\Delta t^2$$

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Test case: vertical diffusion

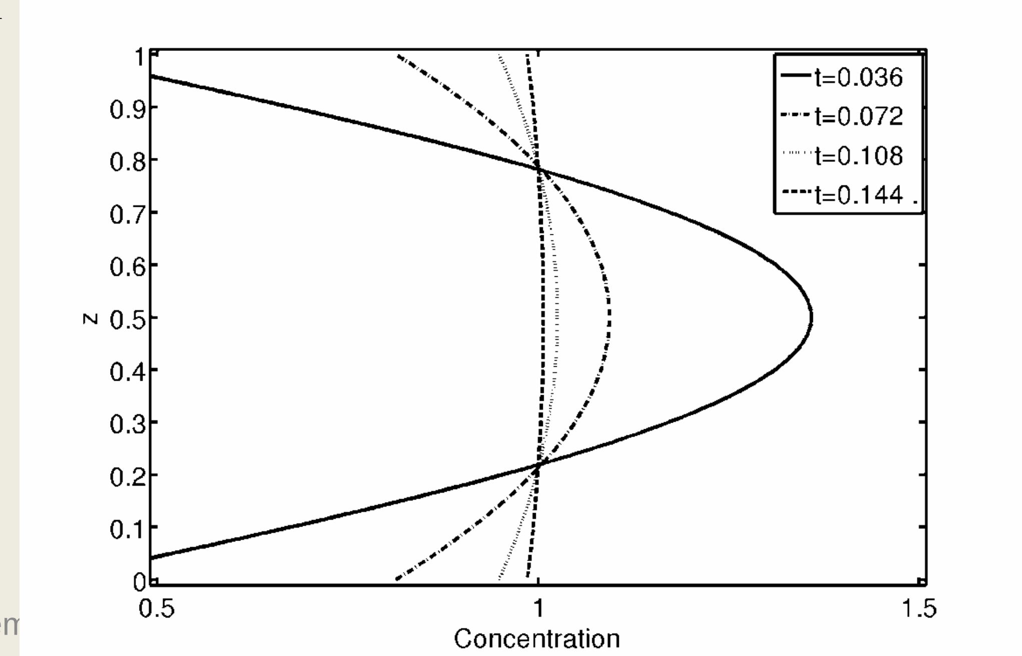
$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left(K_Z(z) \frac{\partial C}{\partial z} \right)$$

$$K_Z(z) = 6\, z\, (1 - z)$$

$$BC: \quad \left[K_Z(z) \frac{\partial C}{\partial z} \right]_{z=0,1} = 0 \quad ; \quad IC: \quad C(0,z) = \delta(z - z_0)$$

$$C(t,z) = 1 + \sum_{n=1}^{\infty} (2n+1) P_n(2z-1) P_n(2z_0 - 1) \exp\{-6n(n+1)t\}$$

[Spivakovskaya et al. 2007]



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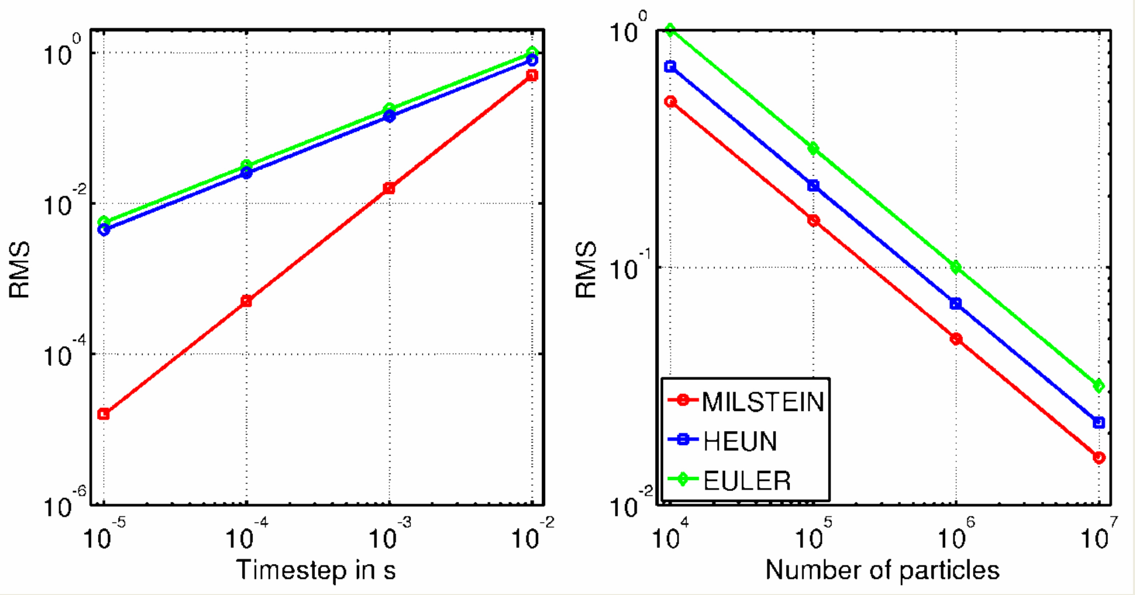
$$dZ(t) = \partial_z K_Z(z)\,dt + \sqrt{2K_Z(z)}\,dW(t)$$



LogLog plot of RMS of the dispersion test: A) for fixed particle number N=10^6 and B) for fixed time step 10^{-4}s

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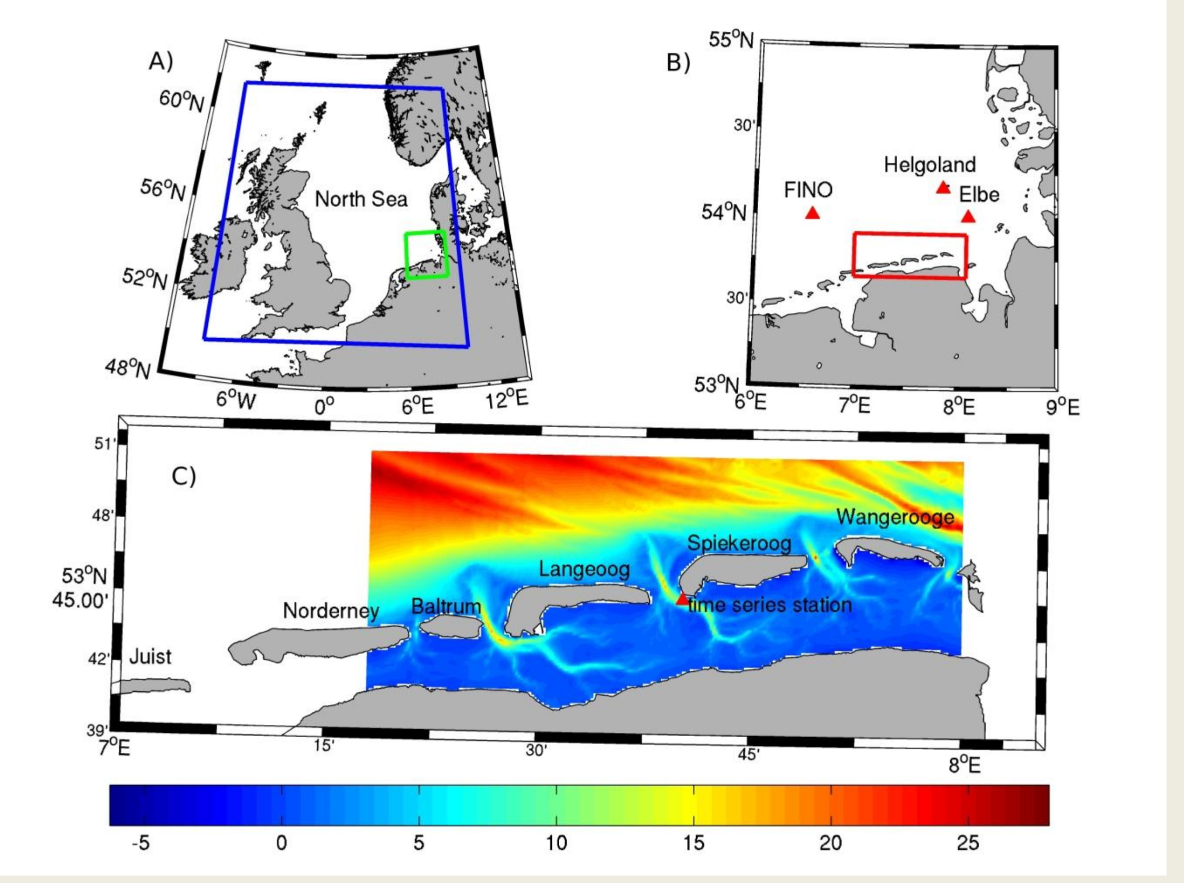
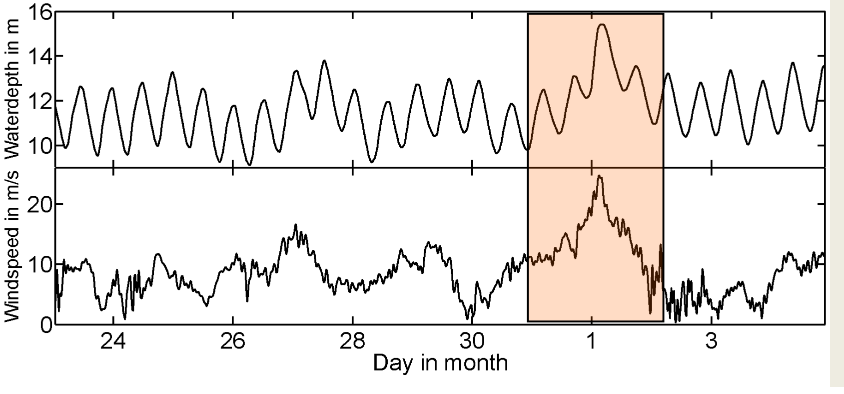
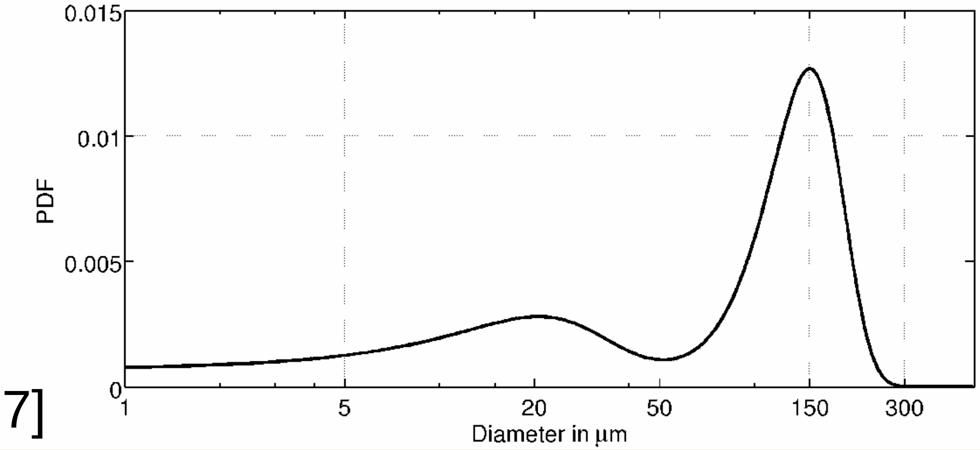
A winter storm





Hydrodynamic setup

- GETM [Burchard et al. 2007] provides velocity fields (u,v,w) and vertical diffusivity K_z (based on k-eps model)
- Constant horizontal diffusivity (1 m^2/s)
- 22 sigma level, horizontal resolution 200 m
- Including wave effects – SWAN [Booij et al. 1999]
- Modification of bottom stress and vertical diffusivity [Pleskachevsky et al. 2005, Gayer et al. 2006]
- Validated setup [Lettmann et al. 2009]



[Hild, 1997]

Lagrangian setup

Erosion:

$$F_{ero} \propto \begin{cases} \left(\frac{\tau_b}{\tau_c} - 1\right) & \tau_b > \tau_c \\ 0 & \tau_b < \tau_c \end{cases}$$

for PDEs [Partheniades, E.: 1965]

Modification:

$$\Delta z = \begin{cases} \sqrt{\tau_b - \tau_c}\,\Delta t & \tau_b > \tau_c \\ 0 & \tau_b < \tau_c \end{cases}$$

$$u_* = \sqrt{\tau_b}$$

$$\Delta z = \begin{cases} \sqrt{\tau_b - \tau_c}\,\Delta t\,\theta(\tau_c, \tau_b) & \tau_b > \tau_c \\ 0 & \tau_b < \tau_c \end{cases}$$

$$\theta(\tau_c, \tau_b) = \begin{cases} 1 & U > \frac{\tau_c}{\tau_b} \\ 0 & else \end{cases}$$

with U=R[0,1]

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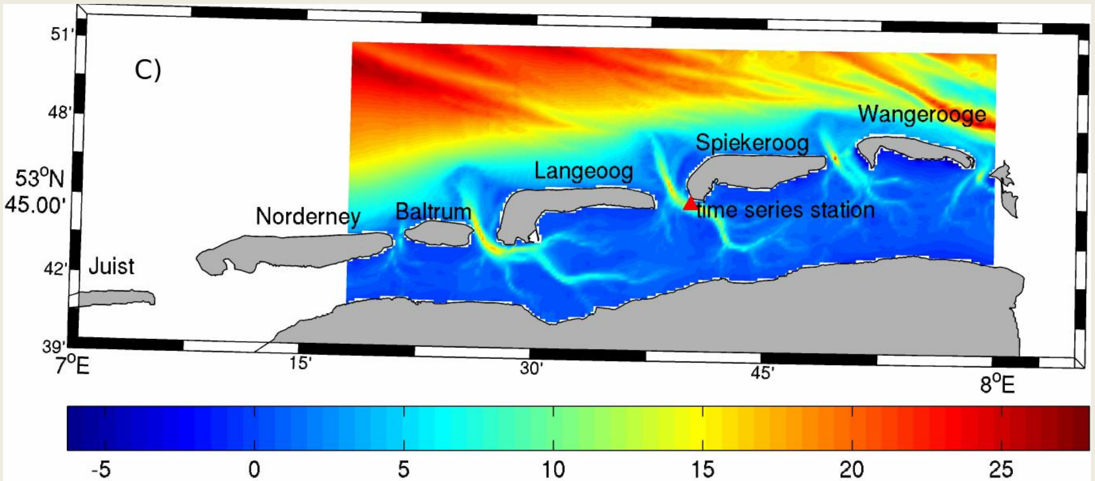
Lagrangian setup

- $50*10^6$ particles
- Periodic east-west boundary conditions
- Sea surface and land boundaries are reflective
- Horizontal time step 60 s, Heun scheme
- Vertical time step 3 s, Euler/Milstein scheme
- Vertical diffusivity fitted by cubic splines



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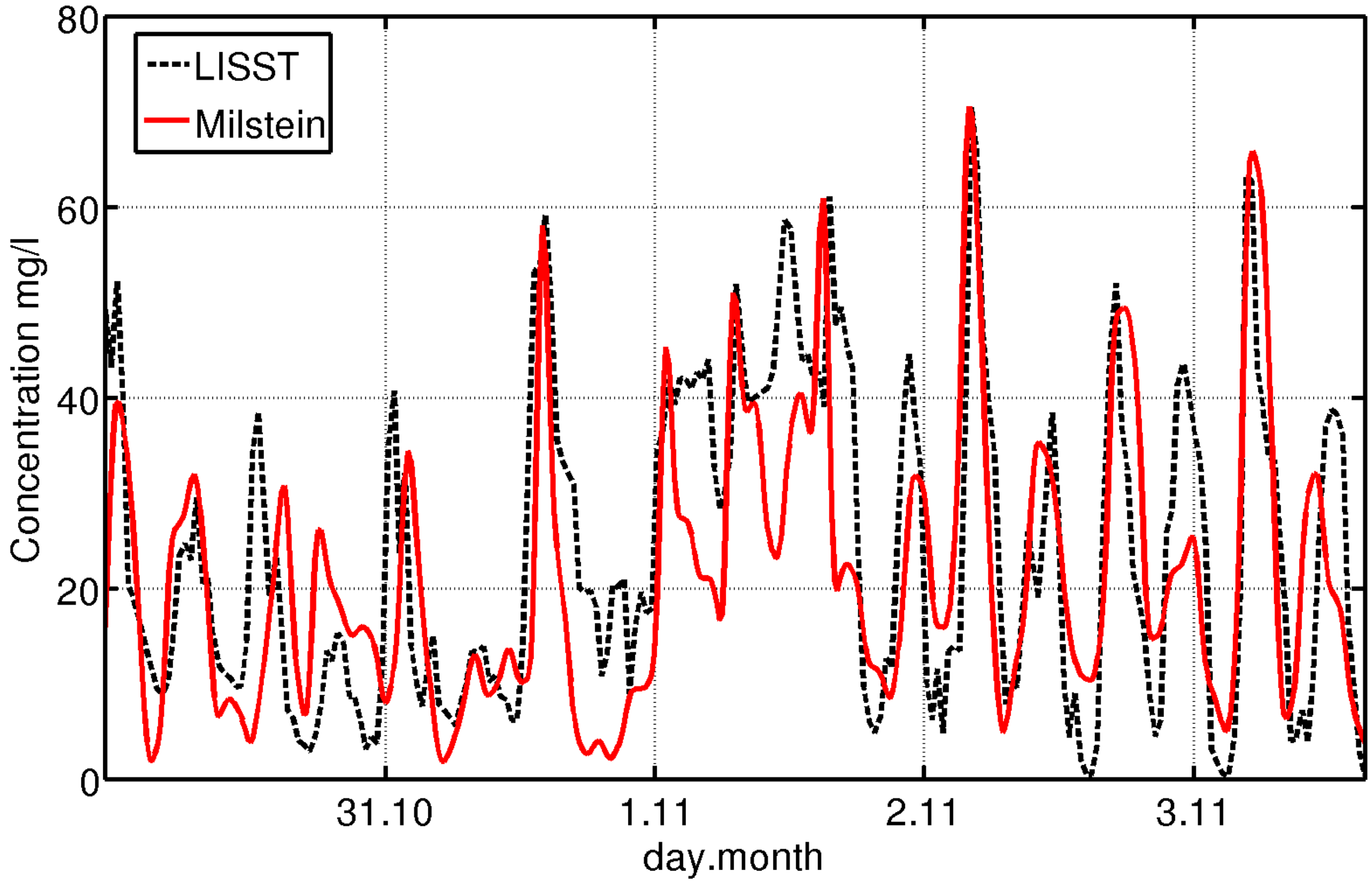
Validation



Modelled and measured concentrations of suspended matter at the pile station during storm *Britta* in Nov. 2006. Measured data are obtained by using a multispectral transmissiometer

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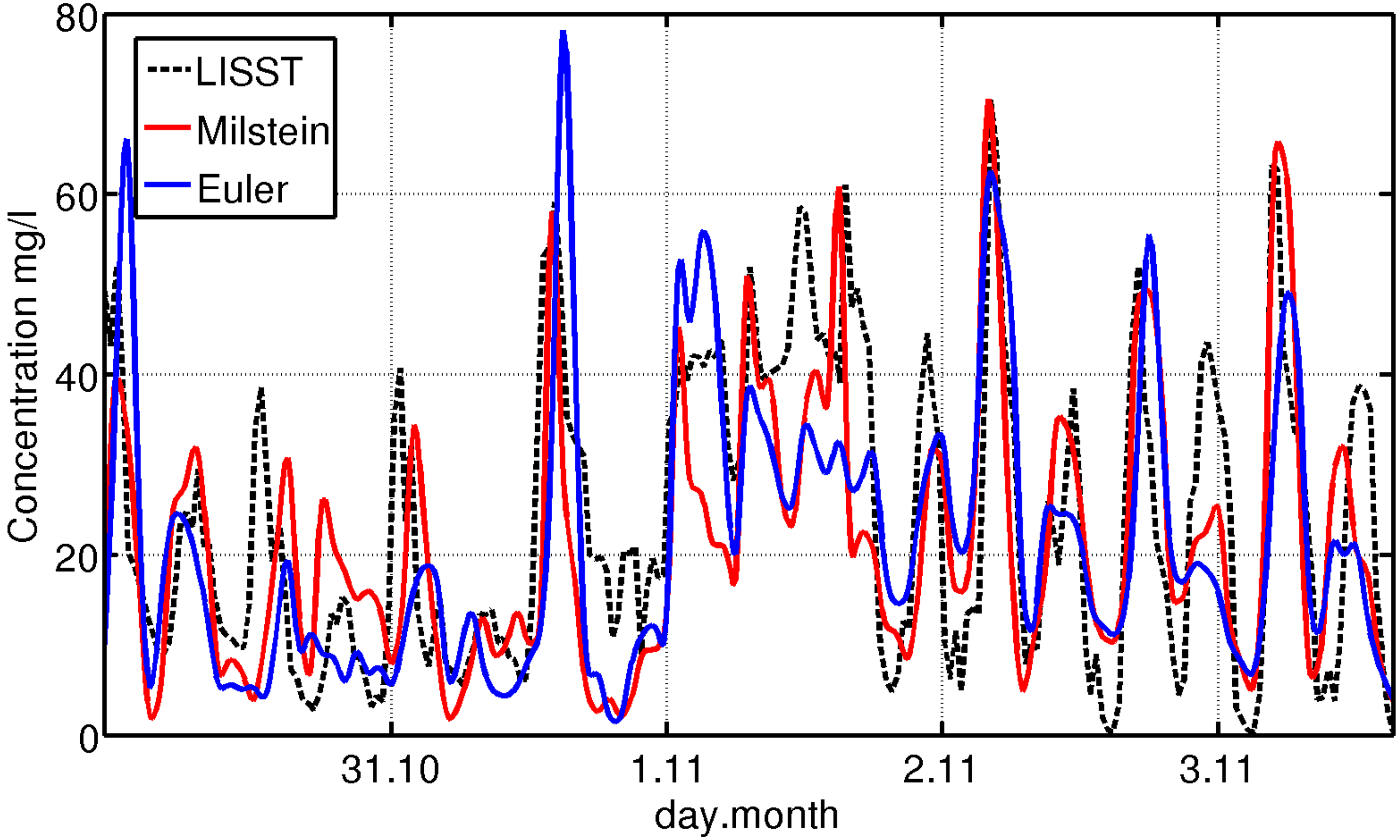

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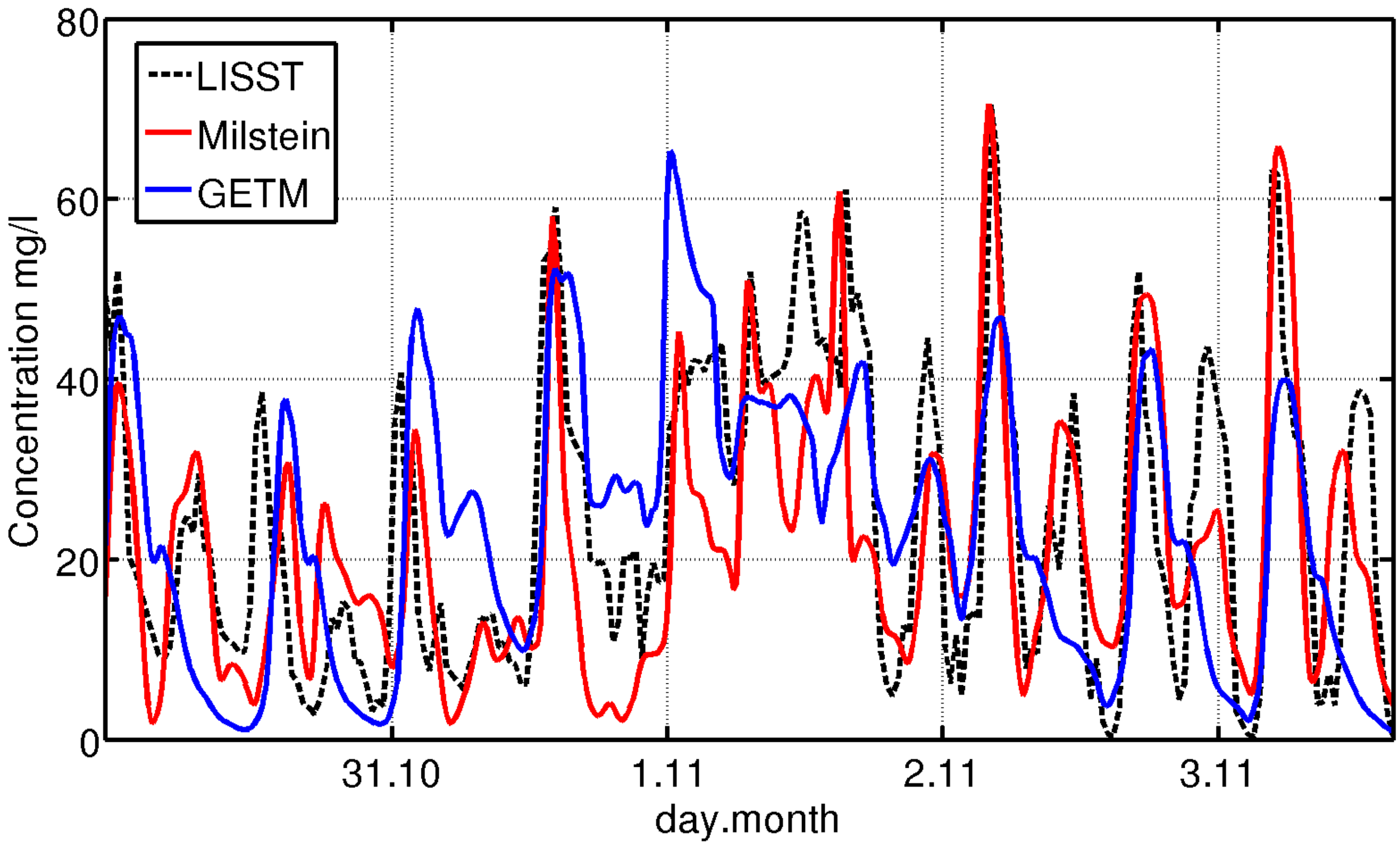

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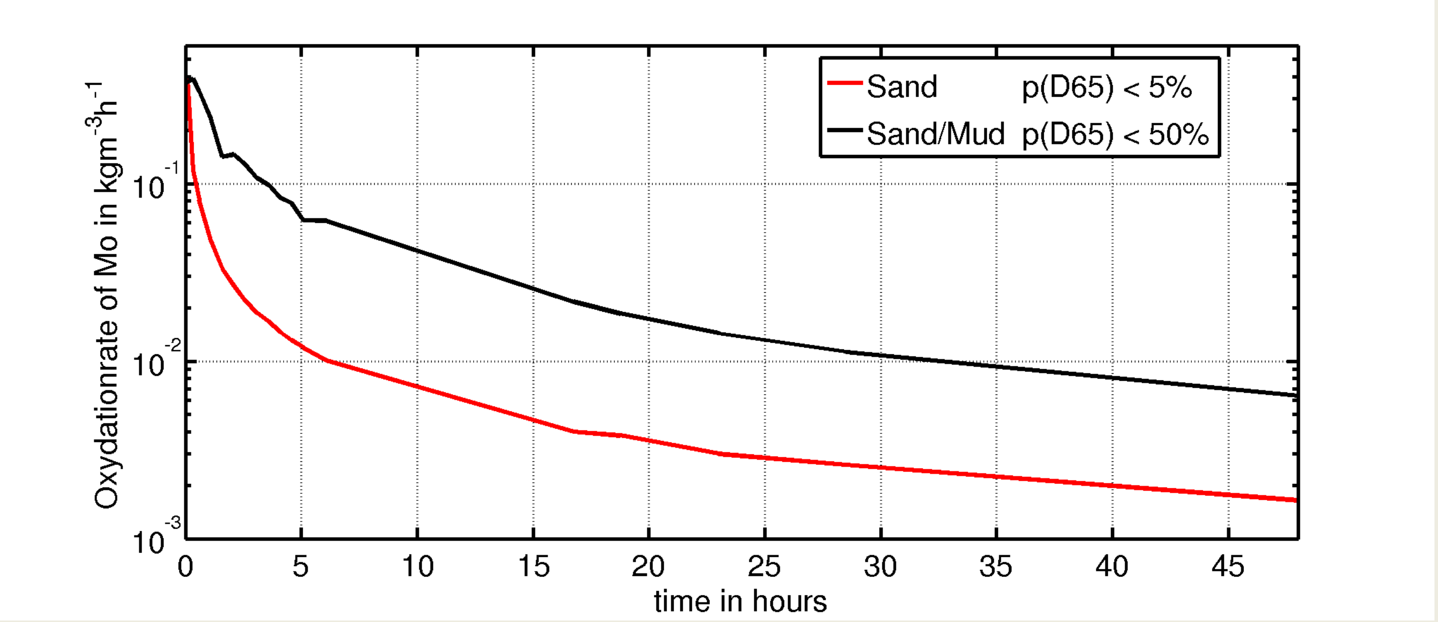
Application

- Estimation of residence time in the water column
- Oxidation of trace elements
- Excursion during tidal cycle



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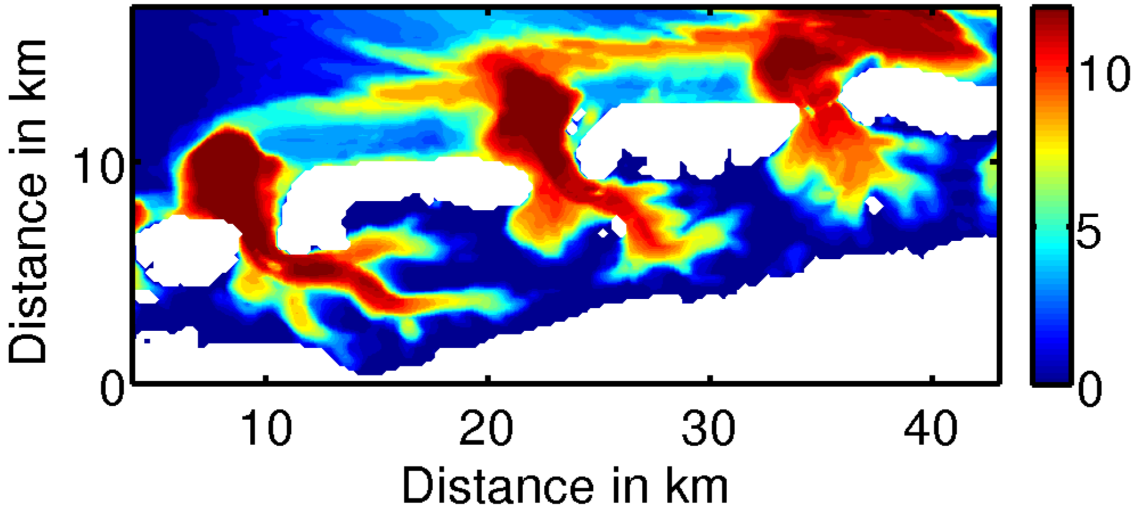
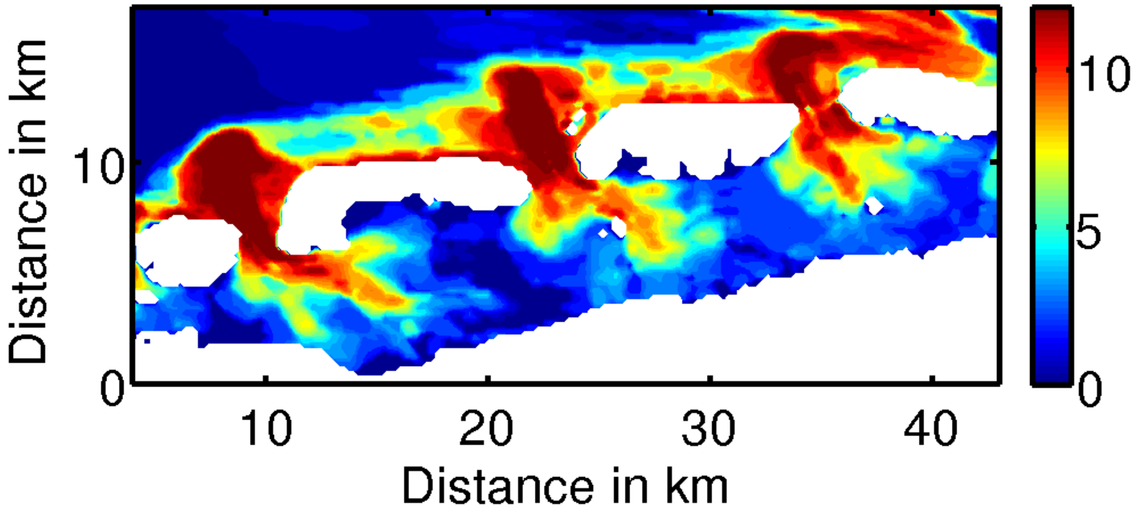
Application (residence time)

Calm conditions



Storm conditions
'Britta'



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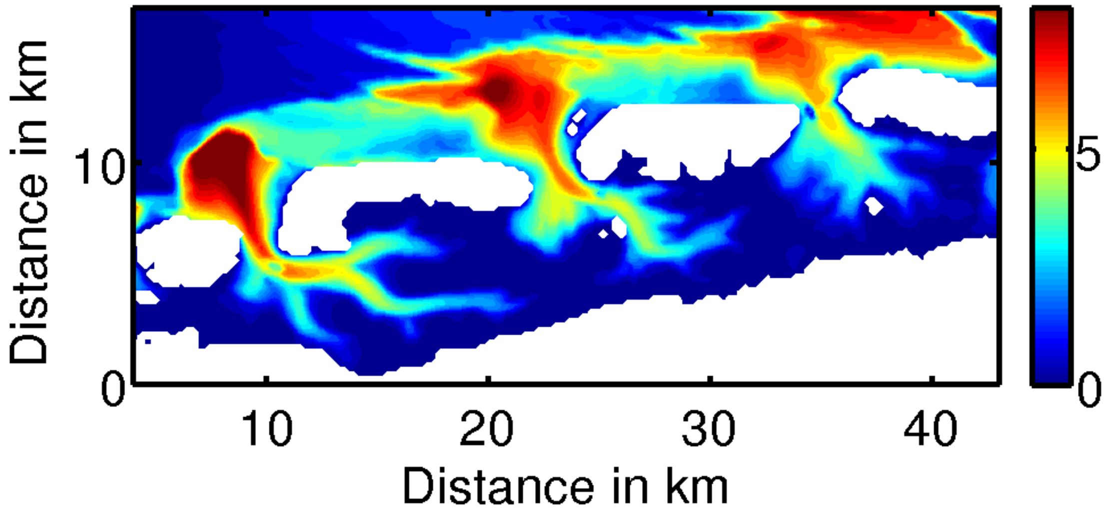
Application (tidal excursion)

Calm conditions



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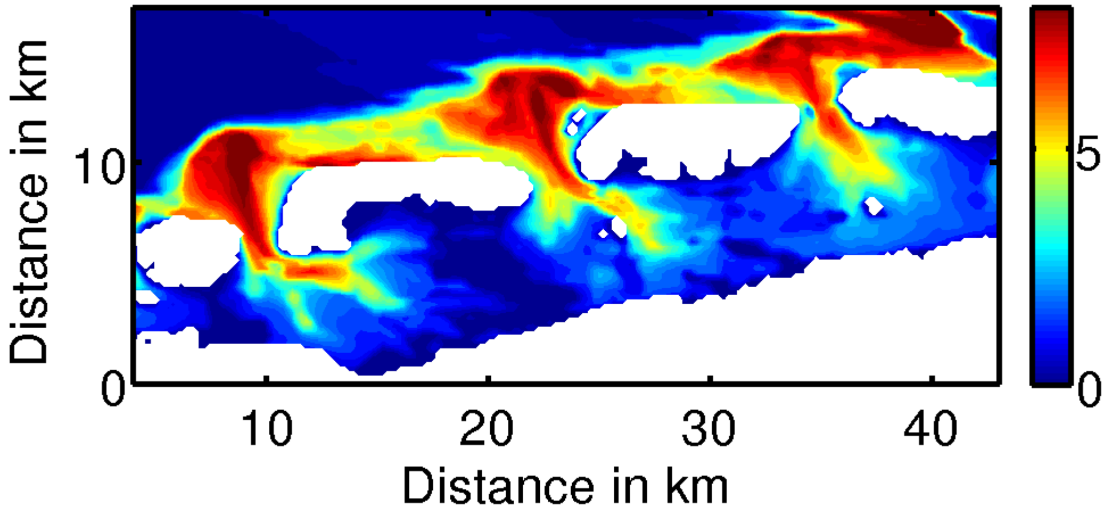

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Conclusion

- Three particle tracking algorithms (Euler, Heun, Milstein)
- SPM dynamics in Lagrangian framework needs modification in boundary conditions
- Erosion depends on ratio of critical and bottom shear stress
- Lagrangian SPM module captures the SPM dynamics during storm *Britta* (2006)
- Connecting biology and physics

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