Numerical studies of dispersion due to tidal flow through Moskstraumen, northern Norway

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Outline

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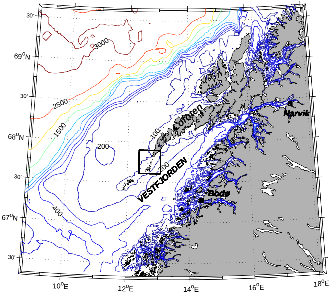
Introduction - Motivation

- The objective was to investigate the sensitivity of the horizontal dispersion of particle pairs to the grid size $\Delta x, \Delta y$ and stratification in a numerical ocean model
- Accurate model prediction of transports in complex current fields can be of considerable value for several practical purposes
  - oilspill
  - sea lice from fishfarming
  - pollutants
- Tidal currents dominates the current field in many coastal areas in Norway
- Tidal effects on dispersion and transports are hence of particular interest

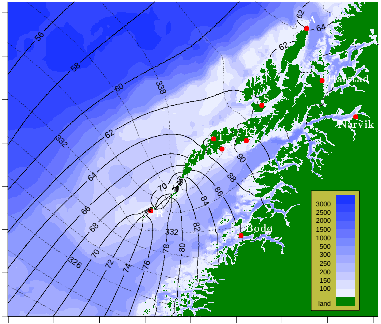
The tidal current Moskstraumen in Lofoten area

- Our focus: horizontal relative dispersion of particle pairs in tidal currents
  - on a short time scale (one tidal cycle)
  - when small scale flow features are important
- The site for our investigation is the Moskstraumen Maelstrom outside Lofoten on the northern coast of Norway
- Moskstraumen is known for its strong tidal current (3-5 ms^{-1}) and whirlpools

Location of Moskstraumen and the model area



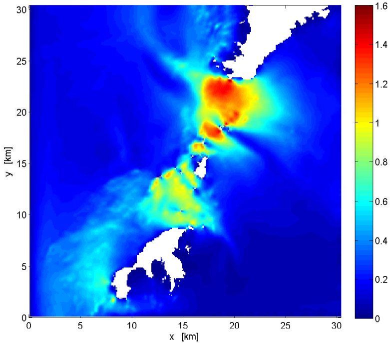
Model area



Amplitude and co-tidal lines for M_2 (major semi-diurnal tide)

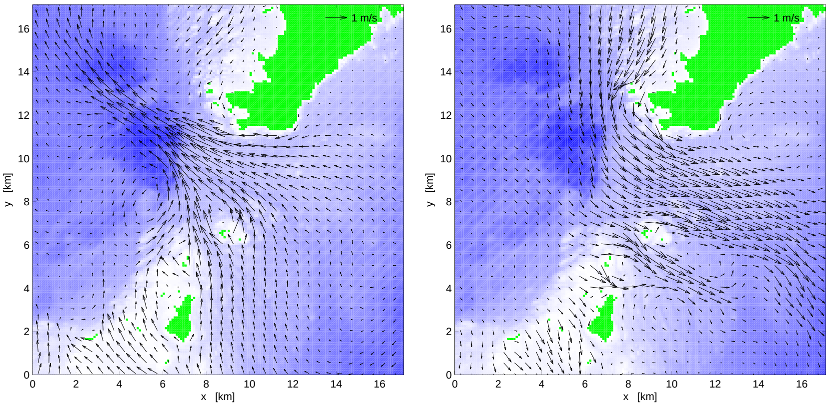
Strong tidal current in Moskstraumen

SAR image (ERS1-SAT) (*Wahl* 1995)



Semi-major current axis for M_2-tide)

Strong tidal current in Moskstraumen



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Bergen Ocean Model

- Bergen Ocean model (BOM)
- Three dimensional (x,y,z) σ-coordinate model
- Include non-linear terms, assume hydrostatic pressure and Boussinesq approximation
- Mode split used to split the 3-D velocity field into its baroclinic part and its depth integrated part

Bergen Ocean Model

- Horizontal grid resolution ranging from 50 to 800 meters ($\Delta x = \Delta y$)
- 10 equidistant σ-layers
- Boundary conditions: tidal elevation represented by the main diurnal constituent M_2 (from Bjørn Gjeviks tidal model - on 500 meters grid), imposed by FRS
- Simulations with both homogenous conditions and stratification

Reynolds momentum equations

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \vec{\nabla} u + w\frac{\partial u}{\partial z} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}(A_v \frac{\partial u}{\partial z}) + F_x,$$

$$\frac{\partial v}{\partial t} + \vec{u} \cdot \vec{\nabla} v + w\frac{\partial v}{\partial z} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\partial}{\partial z}(A_v \frac{\partial v}{\partial z}) + F_y,$$

$$\rho g = -\frac{\partial p}{\partial z}$$

$\vec{u} = (u, v, w)$	Velocity field
F_x, F_y	Horizontal eddy viscosity terms (Smagorinsky)
A_v	Veritical viscosity coeffisient
g	Acceleration of gravity
f	Coriolis parameter

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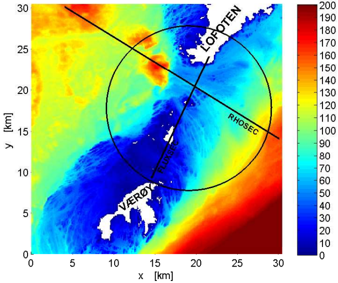
Particle tracking

- Lagrangian tracers where released in the middle of each horizontal cell at 5 m depth
- Lagrangian tracers where advected passively with the flow over one M_2 period
- The relative horizontal dispersion is defined by :
  $r_{i,j} = (x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2$
- $r^2(t)$ has been calculated for a varying initial distance (δ) between the particles

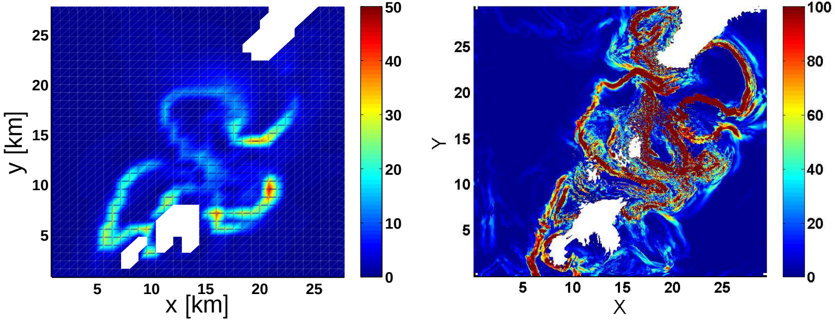
Mean relativ dispersion in Moskstraumen

- Mean relative horizontal dispersion is computed by:
  - $R^2(t) = \frac{1}{P}\sum_{i \neq j} r_{i,j}^2(t)$,
  - where P is number of particle pairs initially released inside a circle with radius 10 km



Outline

Spatial variability of relative dispersion



Δ=800 m Δ=50 m

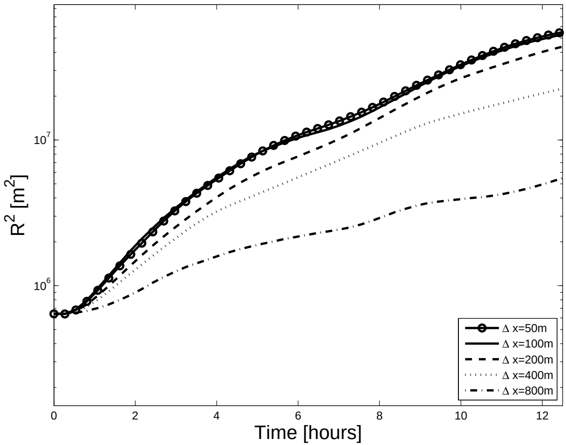
Mean relativ dispersion [km^2] for each grid cell after one M_2 cycle

Mean relative dispersion in Moskstraumen

- Mean relative dispersion $R^2(T)(\cdot 10^{-6}[m^2])$ after one tidal cycle (T)
- -calculated for different initial displacement of particles (δ)
- Larger $R^2(T)$ for smaller grid size Δx
- Relative difference gets smaller for larger δ

	δ [m]							
Δx [m]	50	100	200	400	800	1600	2400	3200
50	12.787	18.915	27.517	38.964	56.257	81.186	104.540	125.955
100		15.580	24.525	37.187	54.683	79.980	98.009	111.842
200			14.755	27.010	44.983	71.437	92.568	107.611
400				11.589	23.118	45.051	65.133	84.470
800					5.689	15.036	26.928	38.455

Mean relative dispersion $R^2(t)$ in Moskstraumen, fixed δ=800 m

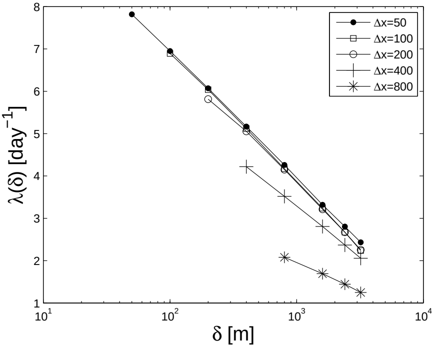


Lyapunov- and power law exponent, $\delta = 800m$

- For analysing separation statistics
  - The Lyapunov exponential model - $R_L^2(t) \sim R^2(0)e^{2\lambda t}$
  - The power law model - $R_P^2(t) \sim R^2(0) + t^{c_1}$
- Estimate the Lyapunov exponent λ and the exponent c_1
- The table shows the sensitivity of λ and c_1 to grid size Δx for δ=800 m

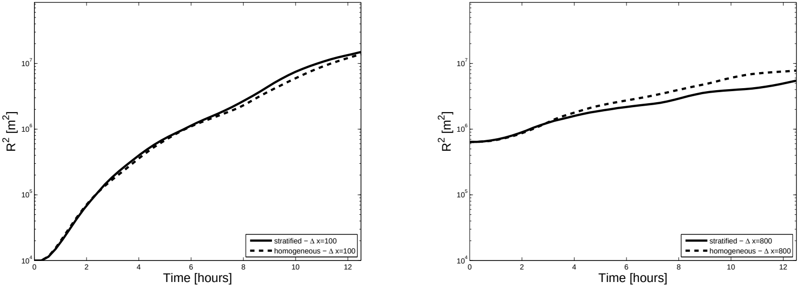
Δx	δ	λ		c_1	
		strat	hom	strat	hom
$[m]$	$[m]$	$[day^{-1}]$	$[day^{-1}]$		
50	800	4.260	3.932	1.611	1.591
100	800	4.171	4.135	1.606	1.604
200	800	4.151	4.114	1.579	1.571
400	800	3.518	3.895	1.538	1.541
800	800	2.079	2.610	1.415	1.441

The Lyapunov exponent λ as a function of Δx and δ



- The rate of growth of mean relative dispersion increases with finer grid resolutions and as $\delta \to 0$
- Convergence of λ for the different Δx as δ increase

The sensitivity of $R^2(t)$ to the effects of stratification

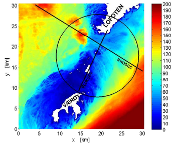


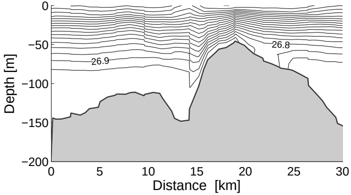
$\Delta x = \delta = 100\,m$ $\Delta x = \delta = 800\,m$

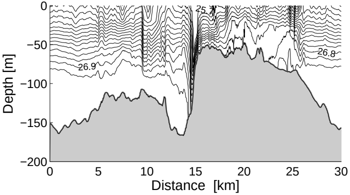
- $R^2(t)$ (and λ) tend to be somewhat larger under stratified conditions for smaller grid size
- While for larger Δx stratification tend to act against dispersion

Density field

Density field along a cross-section (RHOSEC) at maximum outflow, for $\Delta x = 800\,\mathrm{m}$ and $\Delta x = 50\,\mathrm{m}$







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Conclusion

- The horizontal mean relative dispersion in Moskstraumen, on a time scale of one tidal cycle (T), is highly dependent on grid resolution Δx
- The finer grid resolutions gives the largest $R^2(T)$ and hence also largest growth of $R^2(t)$ shown by the Lyapunov exponet λ and the power law exponent c_1
- We need to resolve the small scale eddies and complexity of the current field with grid resolution of at least 50-100 m
- Dispersion is less sensitive to stratification than to grid resolution for the range of grid sizes applied here

Conclusion

- Simulations with stratification gives somewhat increased dispersion for the finer grid resolutions
- The increased dispersion may be explained by increased vertical mixing due to stratification, resolved for the finer grid sizes
- With higher spatial resolution small scale features, such as internal waves and flow separation, may be represented
- There are still unresolved processes that may be important for the small-scale mixing and the dispersion of particles
- In future studies with smaller grid size, it may be necessary to include non-hydostatic pressure effects