Improving Wave Boundary Conditions for Nearshore Delft3D Simulations

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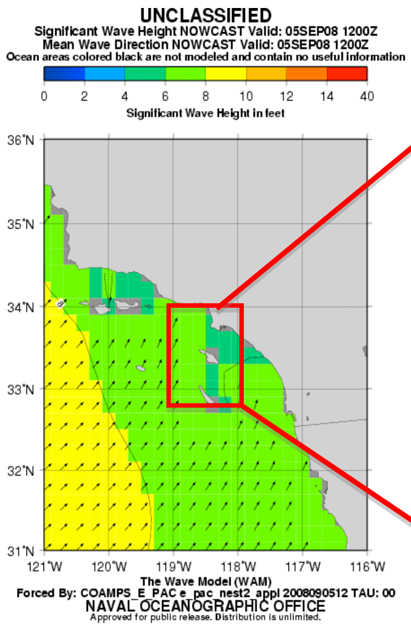
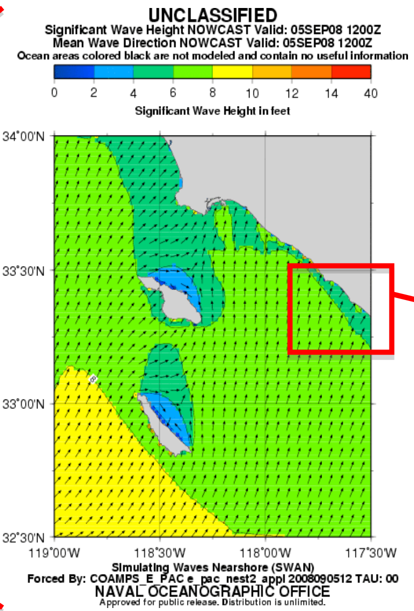
Why improve boundary conditions?

- Spectral wave models (WaveWatch III/WAM/SWAN) used in nested mode to provide forecasts of wave heights, periods, and directions
  - In hindcast mode, re-analyzed wind is used
- Wave conditions used by Delft3D (circulation/transport model)
  - predict magnitude & directions of currents

Regional WAM



SWAN

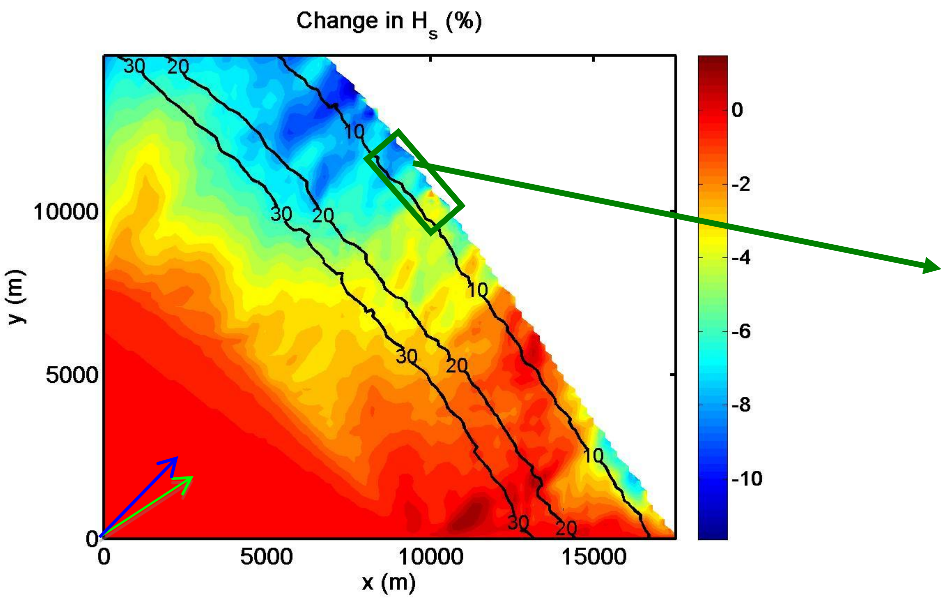


SWAN

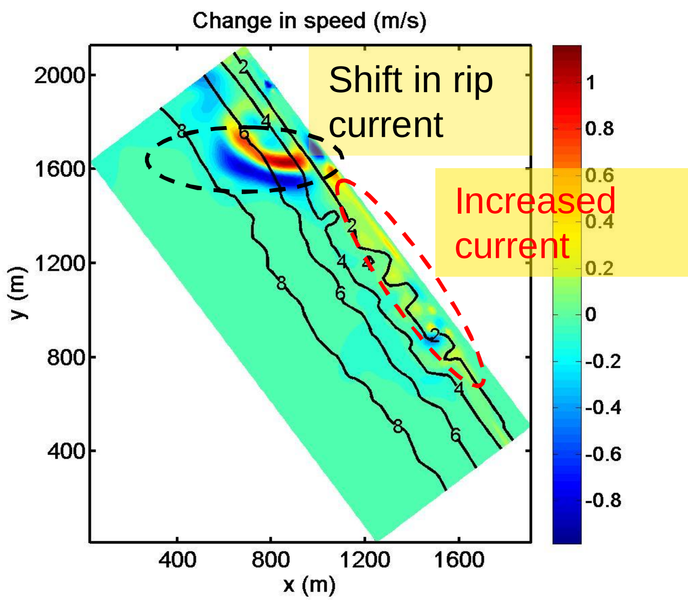
The Problem

- Errors in wave condition at boundary cause errors in wave heights (H) closer to shore
- Currents in the near-shore are driven by gradients in wave stress (proportional to H^2) => very sensitive to the wave conditions

% Change in wave height due to 5° change in incident wave angle



Corresponding change in current (from Delft3D)



Wave data assimilation: Background

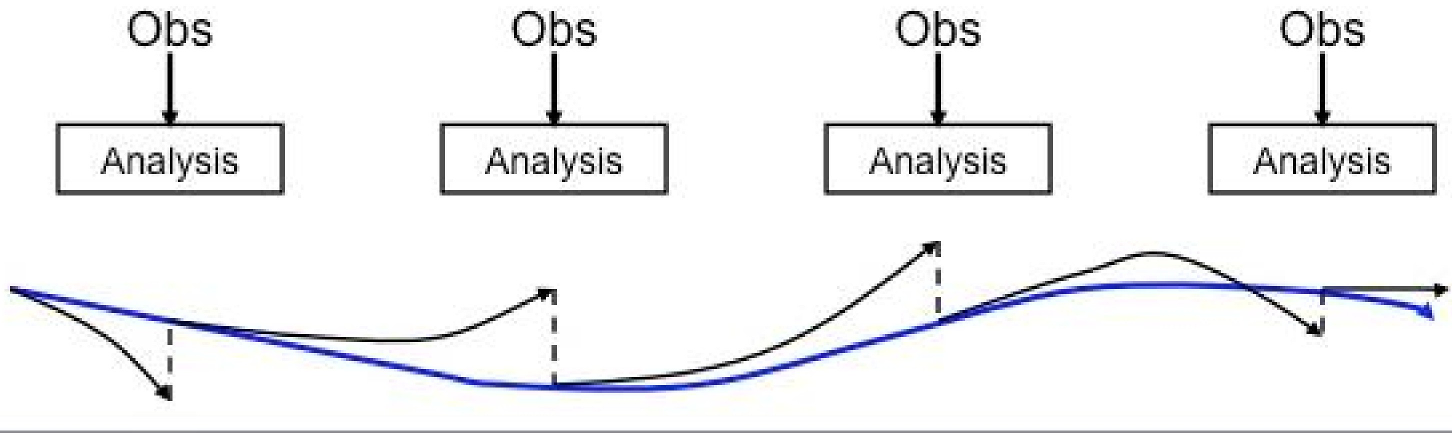
- Optimal Interpolation methods (e.g. Voorrips et al. [1997], Hasselmann et al. [1994]
  - Interpolation of integral parameters (H_s, T_p) or individual wave systems (wind sea, swell) at observation locations and times

    Advantage:

    - Computationally inexpensive

    Disadvantages:

    - No model input



Wave Data Assimilation: Walker (2001)

Developed adjoint to SWAN governing equations and combined with the full SWAN model to create assimilation system

Model equation:

$$\frac{\partial N}{\partial t} + \frac{\partial c_x N}{\partial x} + \frac{\partial c_y N}{\partial y} + \frac{\partial c_\sigma N}{\partial \sigma} + \frac{\partial c_\theta N}{\partial \theta} = \frac{S_{tot}}{\sigma}$$

$$S_{tot} = S_{ds,b} + S_{ds,w} + S_{ds,br} + S_{in} + S_{nl}$$

$S_{ds,b}$	: Bottom friction (sink)	N	: Wave action ($E(\mathbf{x},\mathbf{s})/\sigma$)
$S_{ds,w}$	: White capping (sink)	**x**	: t,x,y
$S_{ds,br}$	: Depth-limited breaking (sink)	**s**	: σ, θ
S_{in}	: Wind input (source)	$c_x, c_y, c_\sigma, c_\theta$	: Propagation speed
S_{nl}	: Non-linear interactions	S_{tot}	: Source/Sink terms

Assimilation methodology

GOAL: Estimate wave spectrum *E(**x**,s)* in the region which minimizes error compared to a set of observed spectra

Constraints to create adjoint:

- Stationary in time
- Most error is propagated into the domain
- Boundary spectrum is spatially uniform

Develop objective function that reflects the constraints

Assimilation methodology

Objective function:

$$J = \int_{\mathcal{R}} \int_{\mathcal{S}} \left\{ \frac{1}{M} \sum_{i=1}^{M} (E - \widehat{E}_i)^2 \delta(\mathbf{x} - \mathbf{x}_i) + A\, \tilde{\tilde{\nabla}} \cdot (\tilde{\mathbf{C}} N) \right\} ds\, d\mathbf{x} + \phi \int_{\mathcal{S}} E_b^2\, ds$$

M	: Number of observation locations
E(**x**,**s**)	: Energy spectrum
A(**x**,**s**)	: Lagrange multiplier - adjoint wave action spectrum

Penalty on the control variable (second integral) ensures unique solution (Bennet and Miller, 1991)

Minimize J with respect to E, A (first variation of J vanishes)

Assimilation methodology

Collecting and organizing:

Cost function (diagnostic):

$$J = \frac{1}{M}\sum_{i=1}^{M}\int_{\mathcal{S}}\left[E(\mathbf{x}_i,\mathrm{s}) - \widehat{E}_i(\mathrm{s})\right]^2 d\mathrm{s} + \phi\int_{\mathcal{S}} E_b(\mathrm{s})^2\, d\mathrm{s}$$

Forward model
(Stationary SWAN):

$$\widetilde{\nabla}\cdot(\widetilde{\mathrm{C}}N) = \frac{S_{tot}}{\sigma}$$

Adjoint SWAN model:

$$\widetilde{\mathrm{C}}\,'\cdot\widetilde{\nabla}A = -\frac{2\sigma}{M}\sum_{i=1}^{M}(E-\widehat{E}_i)\delta(\mathbf{x}-\mathbf{x}_i)$$

Gradient of *J* (to calculate
corrected boundary spectrum):

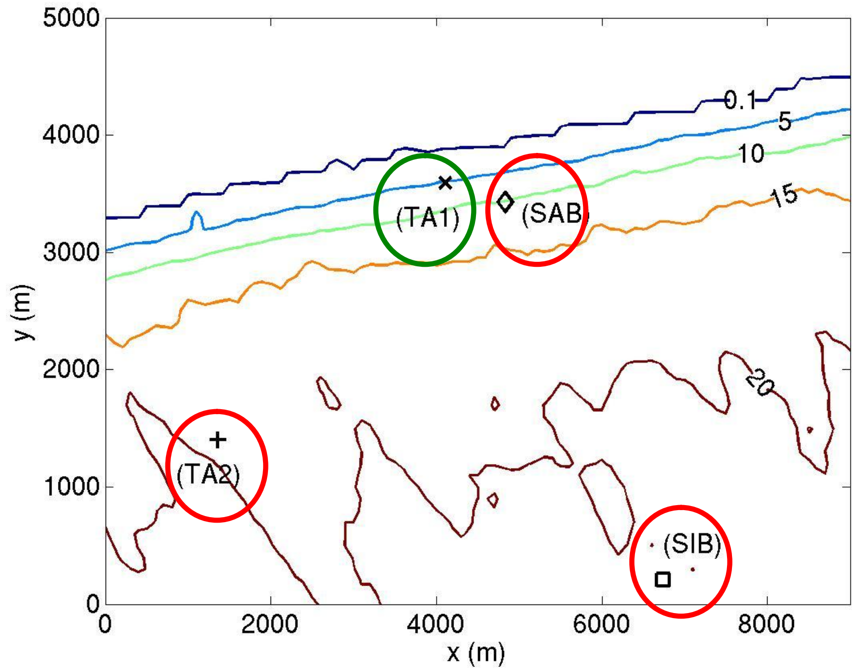
$$\frac{\partial J}{\partial E_b} = \int_{\partial\mathcal{R}}\frac{A}{\sigma}\,\widetilde{\mathrm{C}}\cdot\widehat{n}\,d\xi + 2\phi E_b$$

Application of algorithm

Test area – Santa Rosa Island, FL, Jan 2009

- Mild wave conditions: Typical significant wave height < 1.5m
- Waves enter domain from southeast

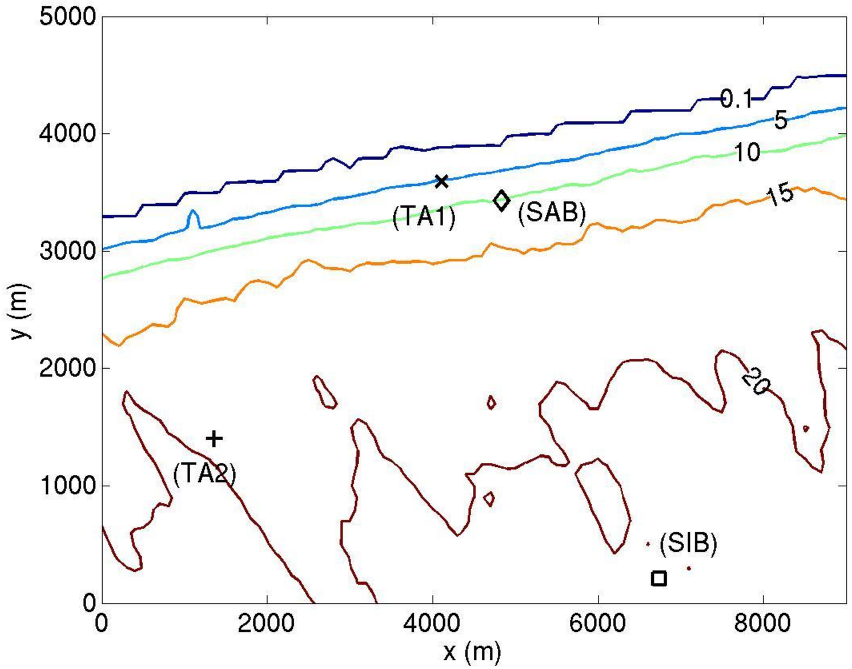
Buoy Type (name)	Water depth (m)	Duration of Operation
Triaxys #1 (TA1)	5.3	Jan 27, 5PM - Jan 31, 7PM
Sentry ADCP (SAB)	10.2	Jan 27, 3PM - Feb 5, 9AM
Triaxys #2 (TA2)	17.8	Jan 28, 1AM - Jan 29, 5AM
Sentry (SIB)	20.6	Jan 27, 2PM - Feb 5, 9AM



- Data from TA1 used to correct boundary conditions
- Initial guess of zero energy in the domain

Verification of assimilation system

- Use synthetic data to generate wave conditions in domain using full SWAN model
- Take results at assimilation location as "observed spectra"
- Using "observed spectra", try to recreate the boundary spectra
- Compare results using recreated boundary spectra to that from using the synthetic data at the boundary



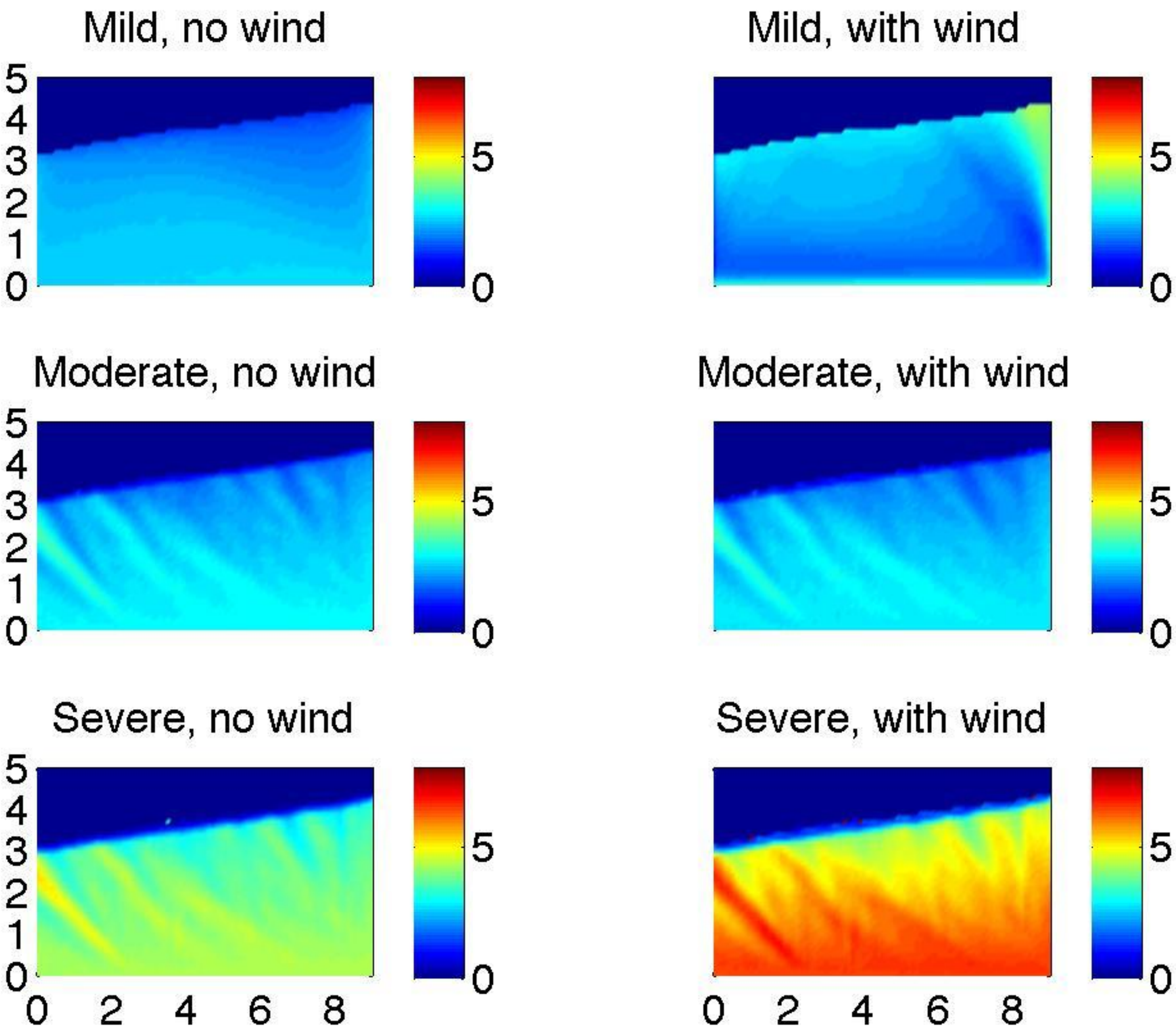
Setup of synthetic data

- Pierson-Moskowitz spectrum at boundary
- Three wave conditions:

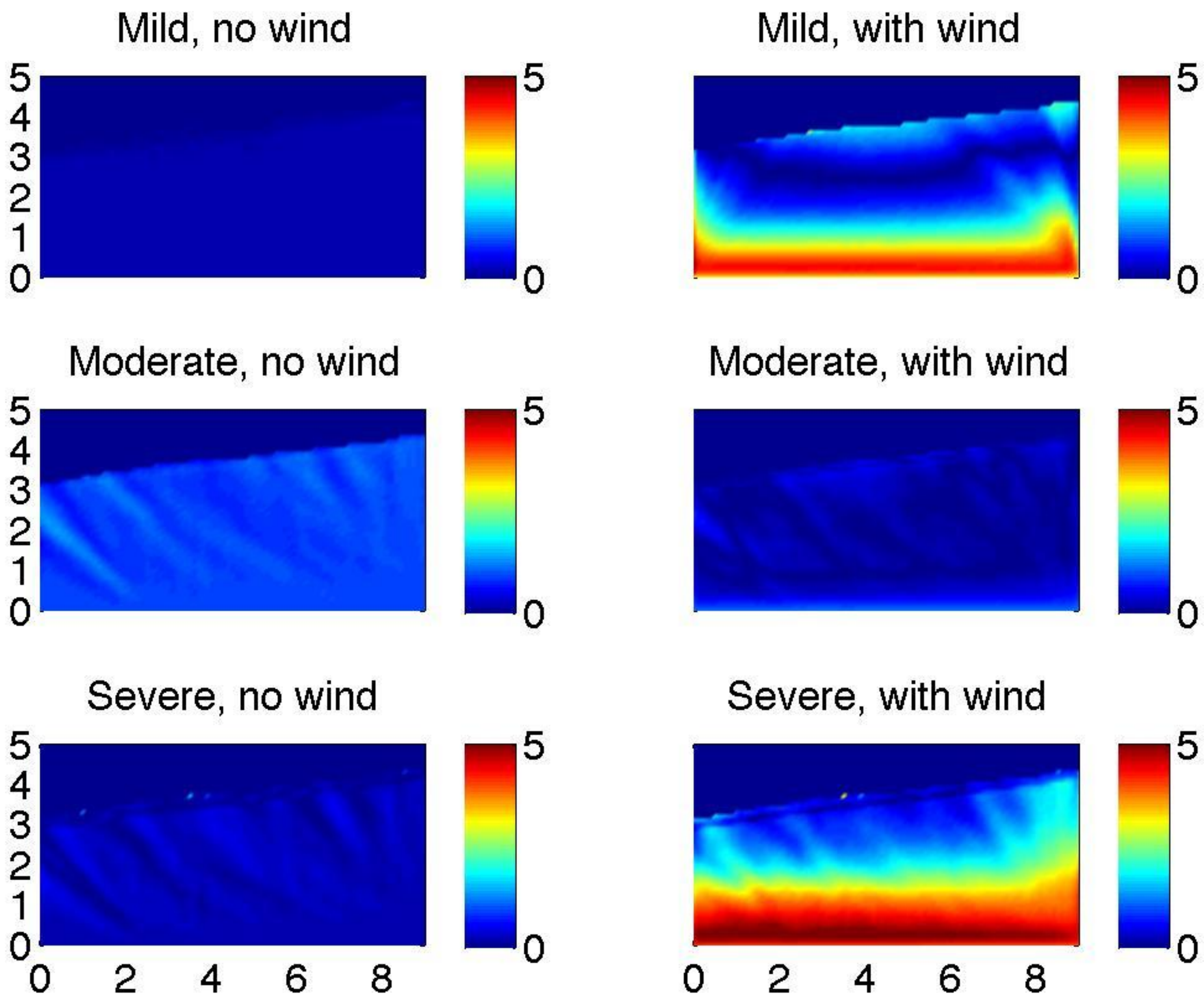
	Mild	Moderate	Severe
U_{10} (m/s)	5	10	12
Mean Dir	100	100	100
H_{sig} (m)	0.6	2.1	3.2

- Two scenarios each in assimilation system – with & without winds in the domain for forward run
- Number of frequencies: 33
  Number of directions: 72

% errors in H_{sig}

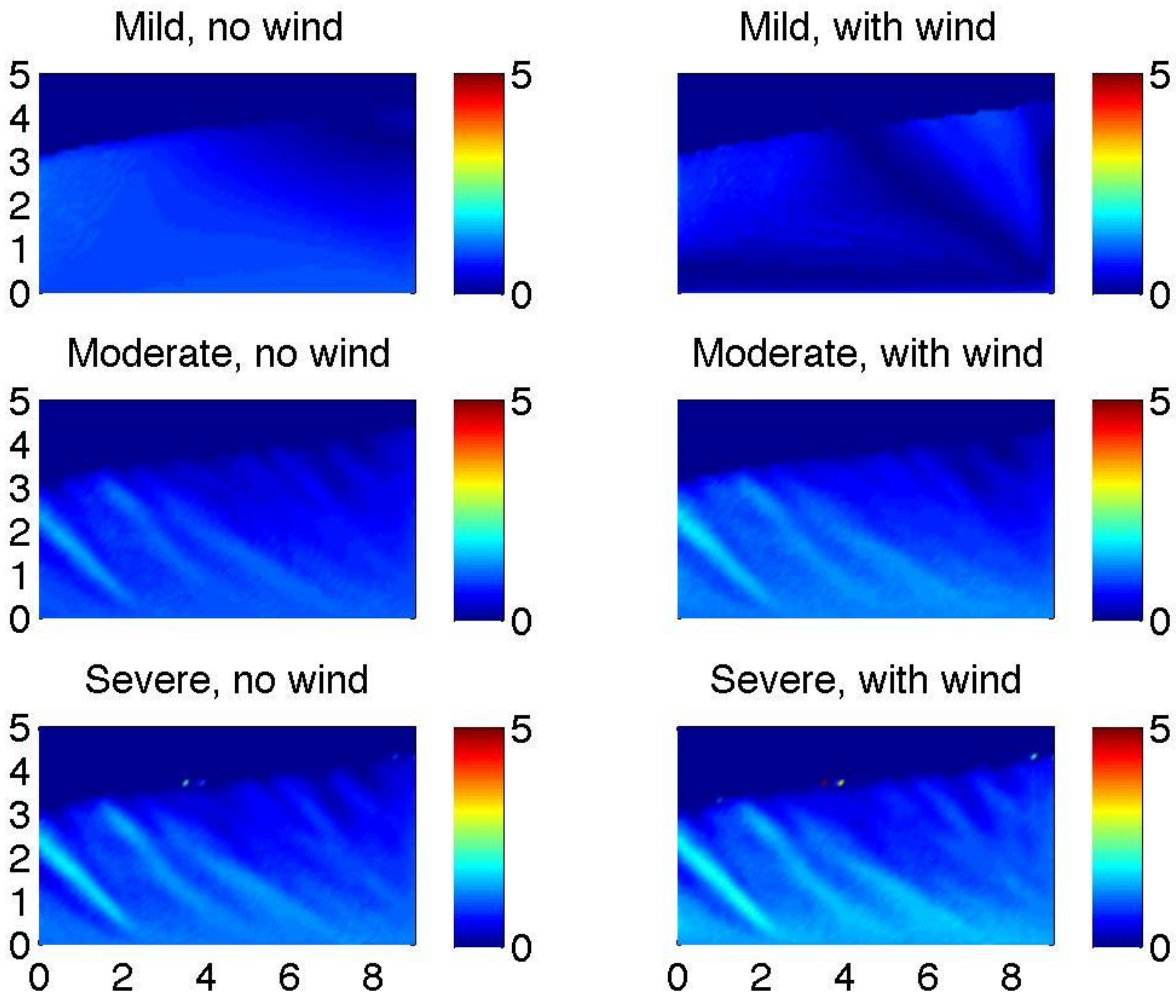


% errors in T_m

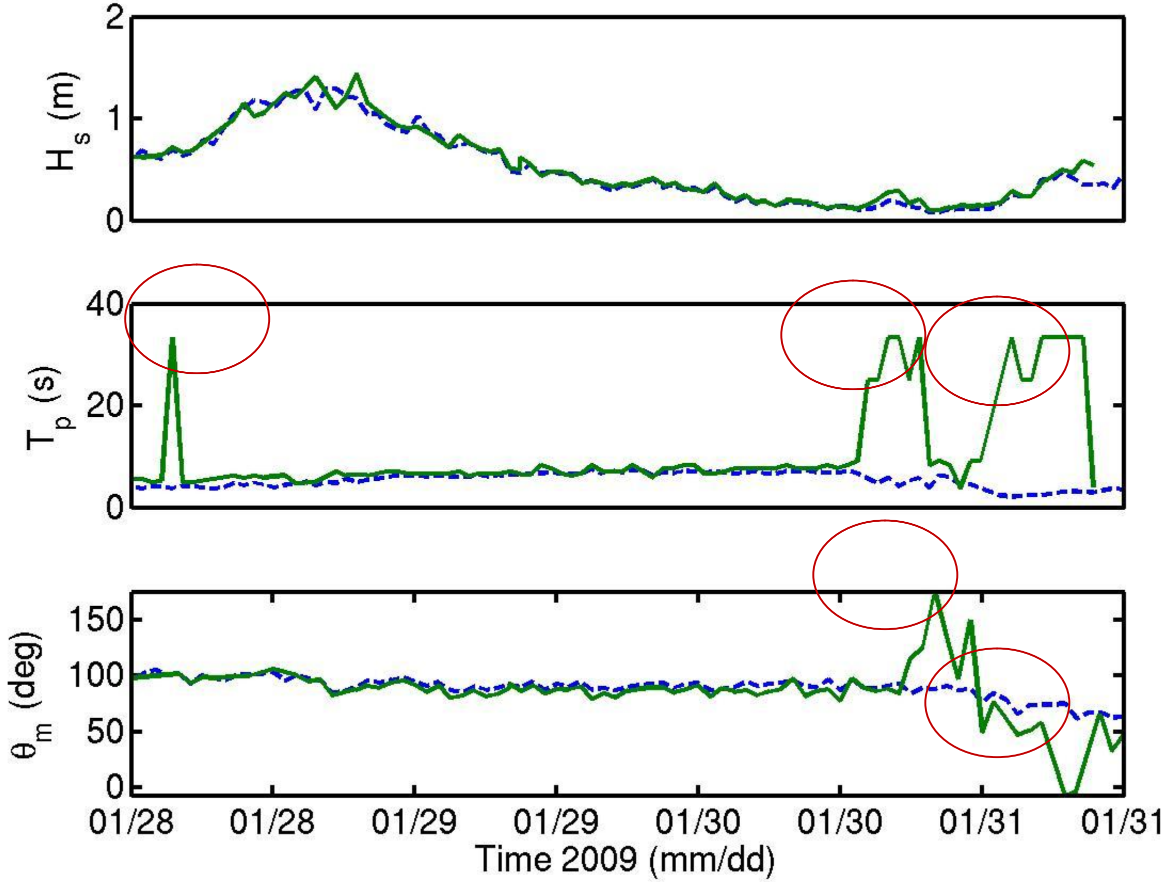


Mild, no wind
Mild, with wind
Moderate, no wind
Moderate, with wind
Severe, no wind
Severe, with wind
5
4
3
2
1
0
0
2
4
6
8

% errors in θ_m

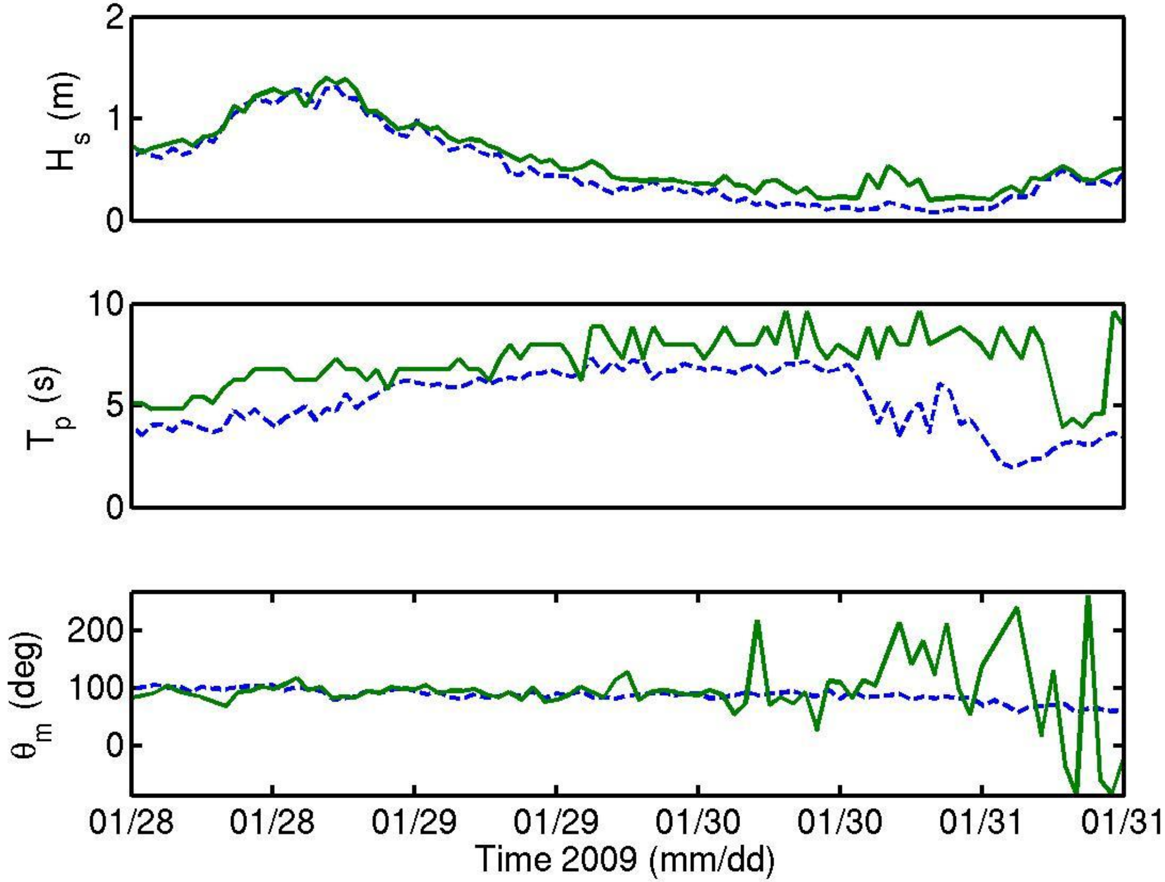


Results at assimilation location



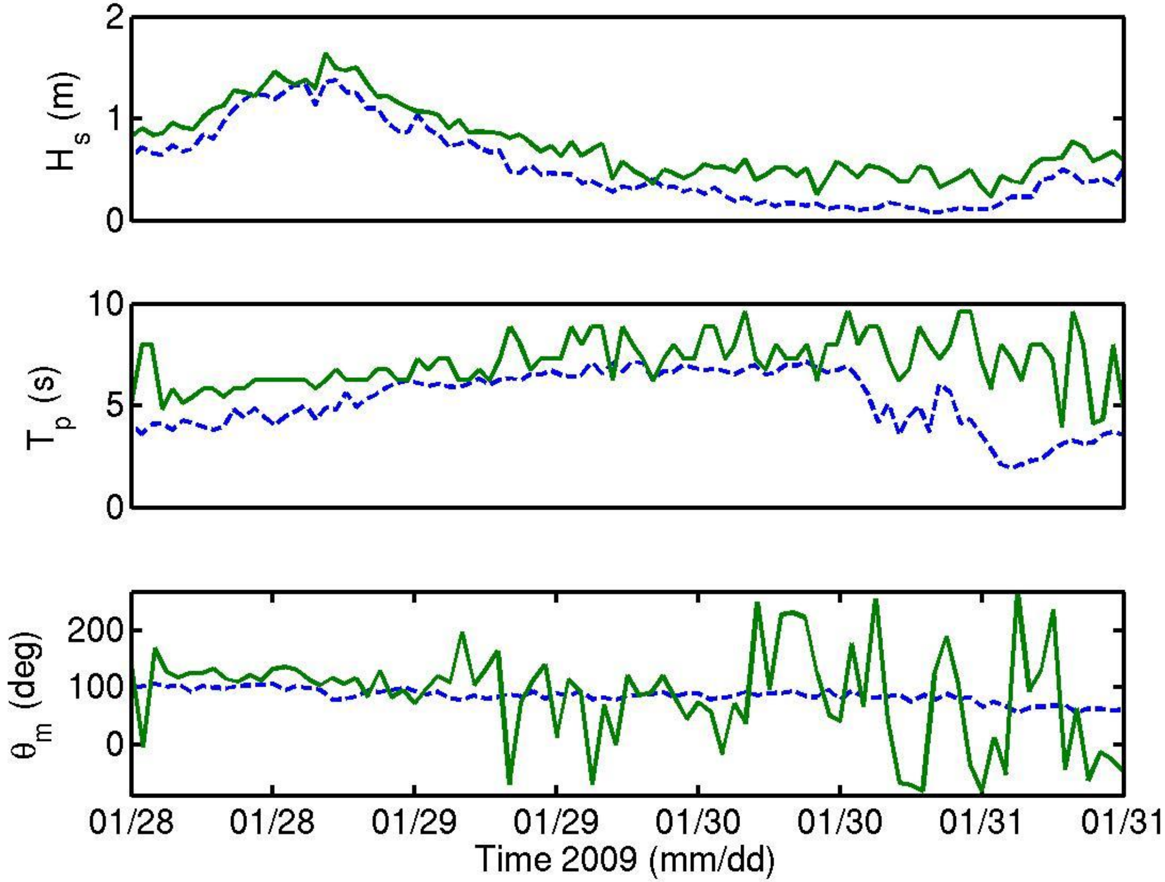
Comparison of averaged quantities at the assimilation location – model results (blue line) from the full forward model (SWAN) with the corrected boundary conditions

Results nearshore



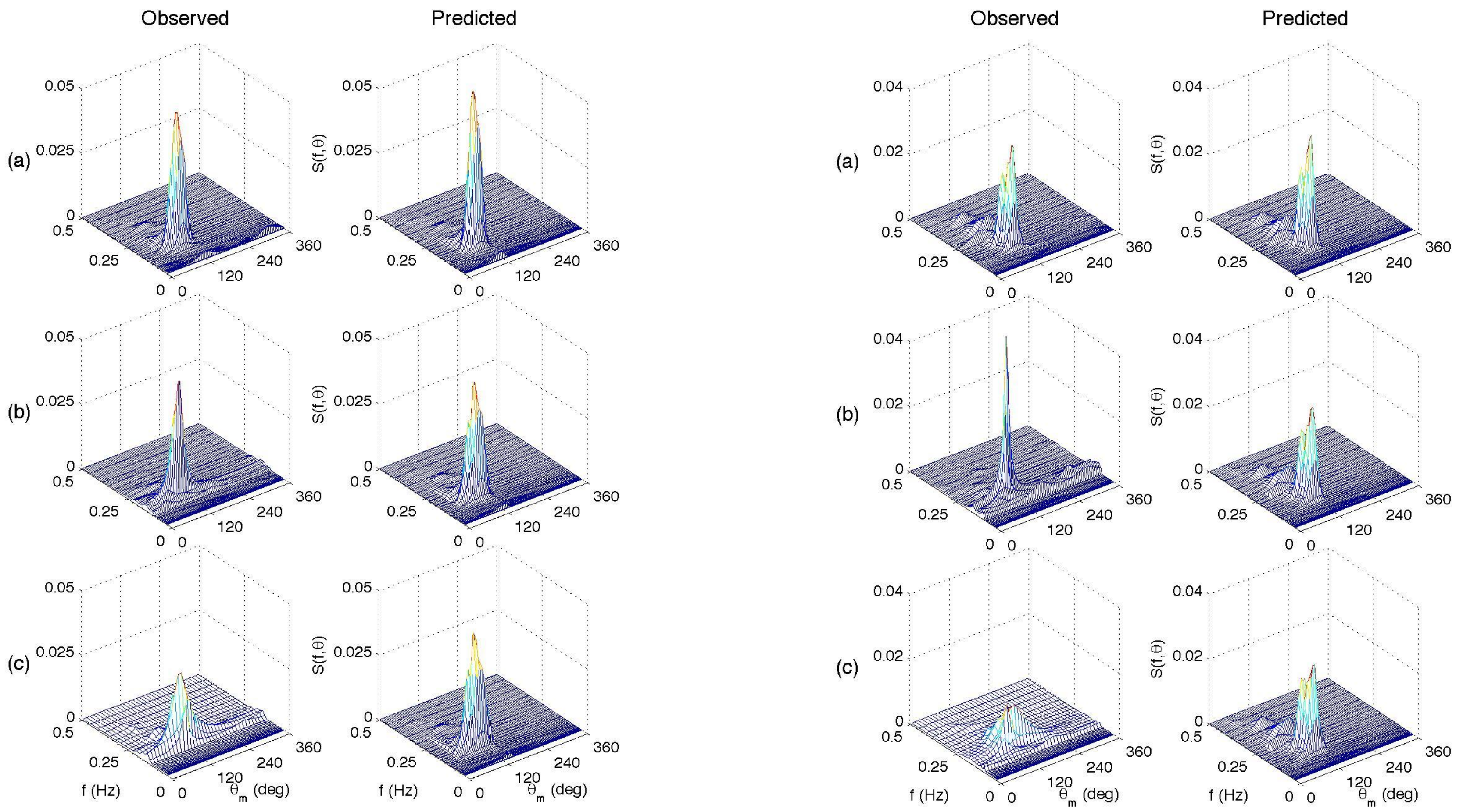
Comparison of averaged quantities at the other nearshore location – model results (blue line) from the full forward model (SWAN) with the corrected boundary conditions

Results offshore



Comparison of averaged quantities at the offshore location – model results (blue) from the full forward model (SWAN) with the corrected boundary conditions

Results of wave spectra



Comparison of spectra at the different locations – smallest difference in averaged quantities (left), significant difference in averaged values

Conclusions

- Adjoint technique used to correct boundary conditions works well in improving predictions in the entire region

- Agreement degrades as distance between assimilation location and comparison point increases – interactions and sources and sinks ignored

- Data at assimilation location(s) have to be influenced by the conditions at the boundary

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