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IAP Program

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Does turbulence help sinking phytoplankton species to survive ?

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Introduction

In the literature

✔ Turbulence *increases* the residence time in the surface layer : *e.g.* Lande and Wood, 1987, Fung, 1993; Ruiz, 1996;

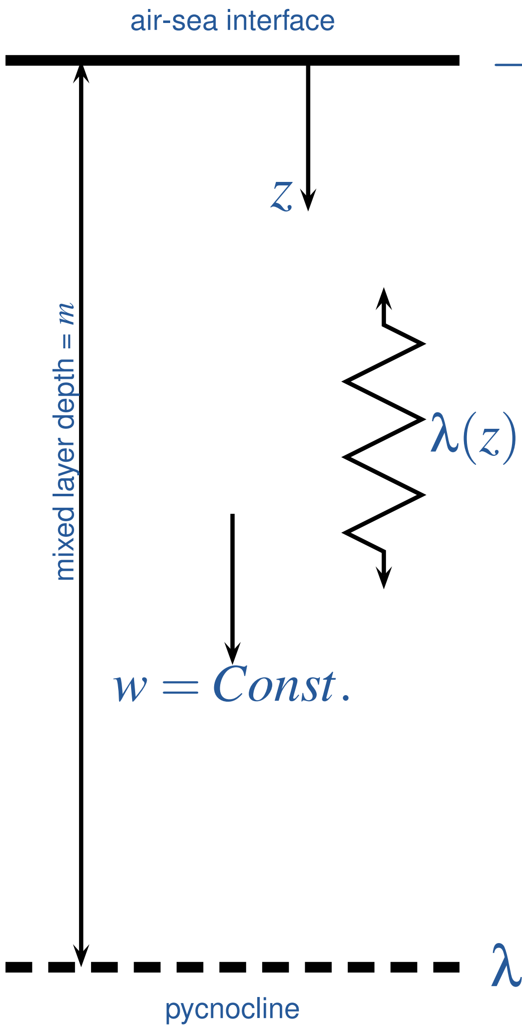
✔ Turbulence *decreases* the residence time in the surface layer : *e.g.* Maxey, 1987; Wang and Maxey, 1993; Franks, 2001;

✔ It depends. . . : *e.g.* Ross, 2006; Spivakovskaya et al., 2007

Who's wrong ? Who's right ?

Residence time

A first model



$$-wC + \lambda \frac{\partial C}{\partial z} = 0$$

$$\begin{cases} \dfrac{\partial C}{\partial t} = \dfrac{\partial}{\partial z}\left[-wC + \lambda(z)\dfrac{\partial C}{\partial z}\right] \\ C(0,z) = \delta(z - z_0) \end{cases}$$

$$\lambda \frac{\partial C}{\partial z} = 0$$

Inventory : $\mathcal{M}(t) = \int_0^m C(t,z)\,dz$

(Mean) Residence time :

$$\theta(z_0) = \int_0^\infty \mathcal{M}(t)\,dt$$

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Operator formulation

$$\mathcal{M}(t;t_0,z_0) = \int_0^m C(t,z)dz = < C(t,z), \delta_\omega(z) > = < \mathcal{A}_{t,t_0}\delta(z-z_0), \delta_\omega(z) >$$

- ✔ $\mathcal{A}_{t,t_0}$ = forward operator, *i.e.* $C(t,z) = \mathcal{A}_{t,t_0}C(t_0,z) = \mathcal{A}_{t,t_0}\delta(z-z_0)$;
- ✔ δ_ω = characteristic function of control domain $\omega =]0,m[$,

$$\delta_\omega(x) = \begin{cases} 1 & \text{if } z < m, \\ 0 & \text{if } z \geq m \end{cases}$$

- ✔ $< f,g > = \int f(z)g(z)dz$

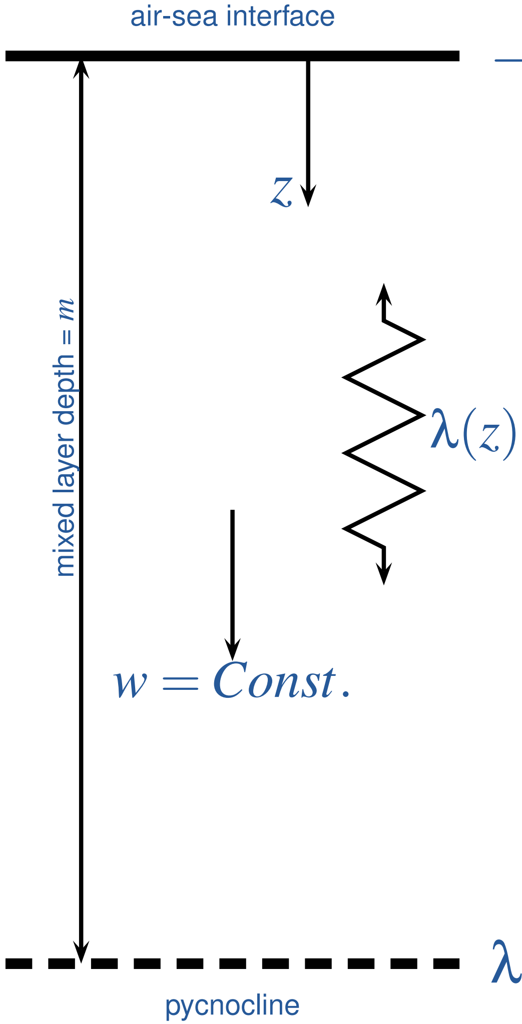
$$\mathcal{M}(t;t_0,z_0) = < \mathcal{A}_{t,t_0}\delta(z-z_0), \delta_\omega(z) > = < \delta(z-z_0), \mathcal{A}^*_{t_0,t}\delta_\omega(z) >$$

where $\mathcal{A}^*_{t_0,t}$ = adjoint operator of $\mathcal{A}_{t,t_0}$.

Delhez et al., ECSS, 2004

http://www.climate.be/CART

Adjoint problem



$$-wC + \lambda \frac{\partial C}{\partial z} = 0 \quad \longrightarrow \quad \lambda \frac{\partial \theta}{\partial z} = 0$$

$$\begin{cases} \dfrac{\partial C}{\partial t} = \dfrac{\partial}{\partial z}\left[-wC + \lambda(z)\dfrac{\partial C}{\partial z}\right] \\ C(0,z) = \delta(z - z_0) \end{cases}$$

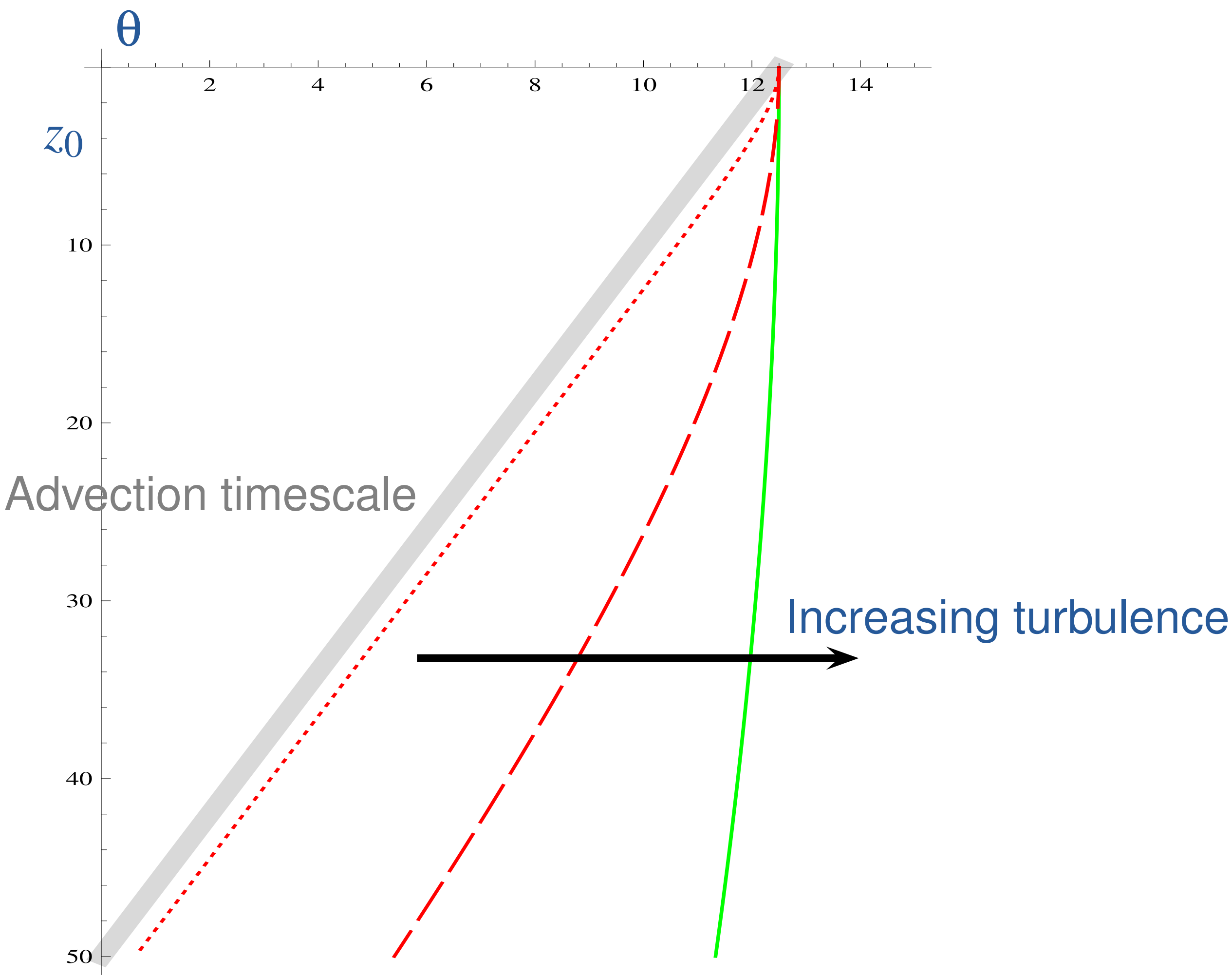
$$w\frac{d\theta}{dz} + \frac{d}{dz}\left[\lambda(z)\frac{d\theta}{dz}\right] + 1 = 0$$

$$\lambda \frac{\partial C}{\partial z} = 0 \quad \longrightarrow \quad w\theta + \lambda \frac{\partial \theta}{\partial z} = 0$$

Deleersnijder et al., EFM, 2006

RT in the mixed layer

$$\theta(z) = \frac{m-z}{w} + \frac{1}{w}\int_0^z \exp\left[-w\int_\zeta^z \frac{du}{\lambda(u)}\right] d\zeta$$



RT in the mixed layer

$$\theta(z) = \frac{m-z}{w} + \frac{1}{w}\int_0^z \exp\left[-w\int_\zeta^z \frac{du}{\lambda(u)}\right] d\zeta$$

In this model, turbulence increases the residence time in the surface mixed layer :

$$\frac{m-z}{w} \leq \theta(z) \leq \frac{m}{w}$$

No diffusion — Infinite mixing

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Behavior at the bottom of the s.m.l.

$$\theta(z) = \frac{m-z}{w} + \frac{1}{w}\int_0^z \exp\left[-w\int_\zeta^z \frac{du}{\lambda(u)}\right] d\zeta$$

In general

$$\lim_{z \to m^-} \theta(z) \neq 0$$

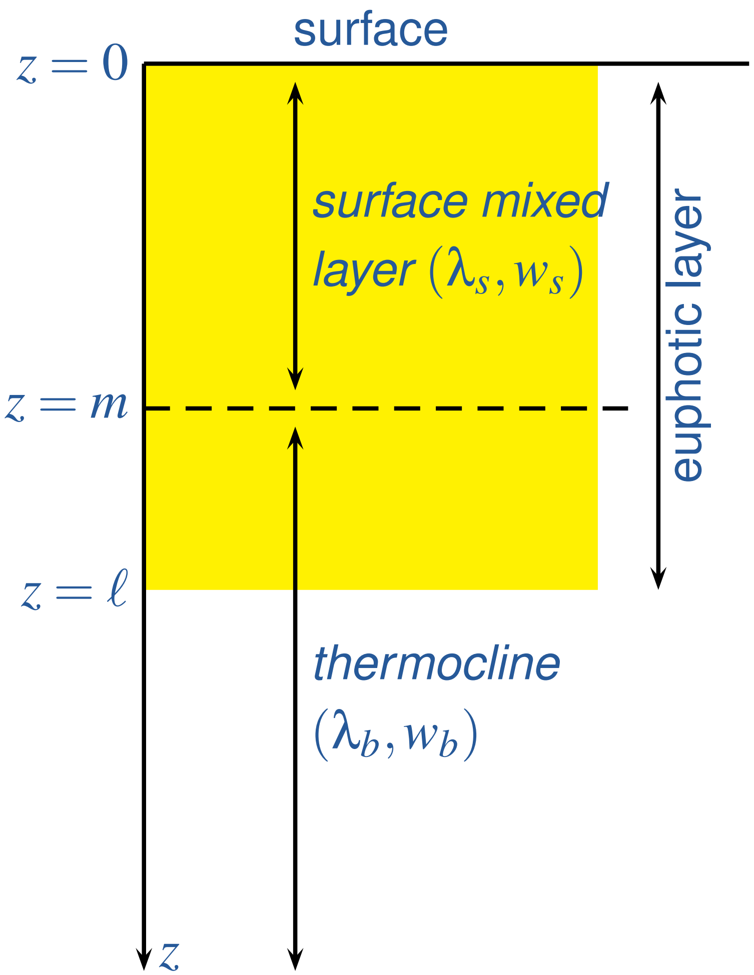
The residence time does not vanishes at the bottom of the mixed layer if

- ✔ $\lim_{z \to m^-} \lambda(z) \neq 0$, or
- ✔ $\lambda(z) \sim \lambda_0 (z-m)^\alpha, \quad z \to m^-$ with $0 < \alpha < 1$

Deleersnijder et al., EFM, 2006

$\Rightarrow$ The behavior of $\lambda(z)$ **in and around** the pycnocline must be considered.

A two layer model

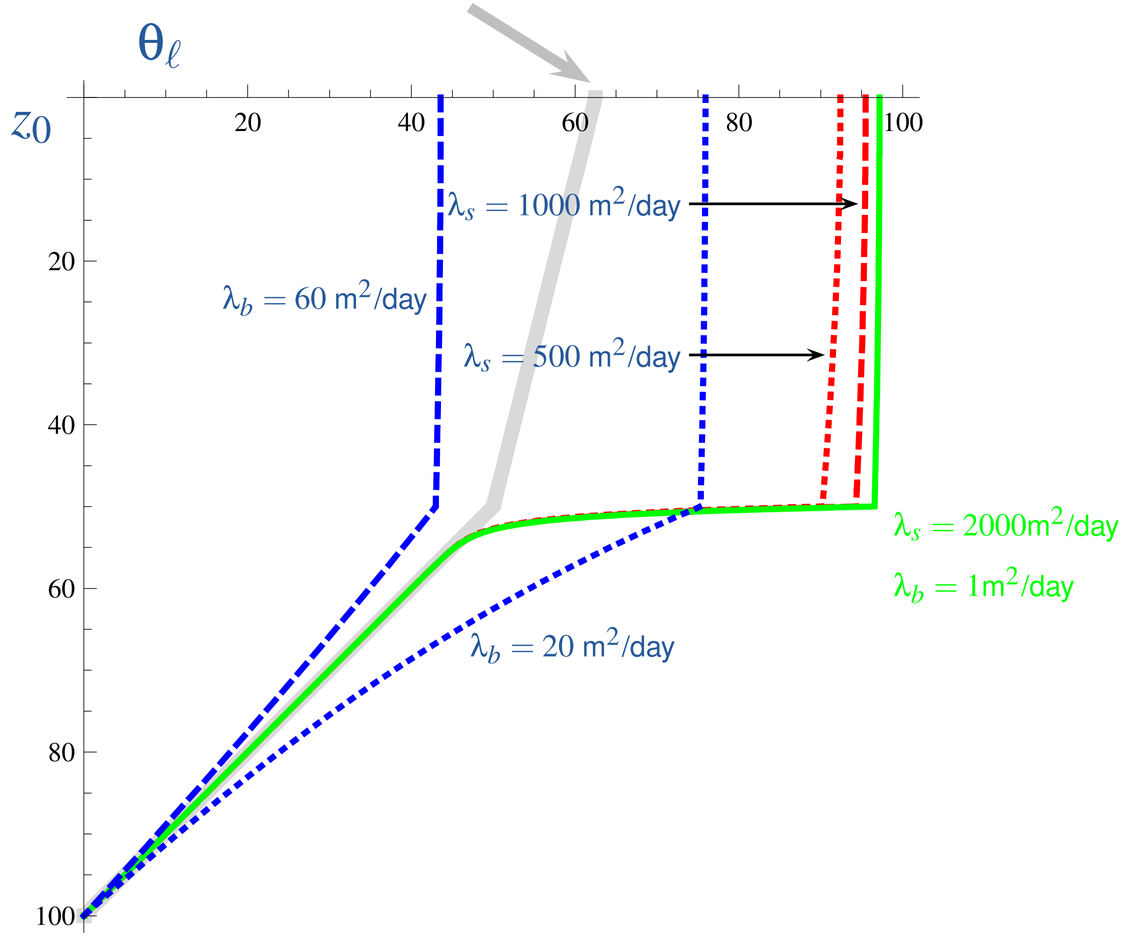


- ✔ $m = 50$ m, $\ell = 100$ m;
- ✔ $\lambda_s = 2000\text{m}^2$/day, w_s = 4 m/day;
- ✔ $\lambda_b = 1\text{m}^2$/day, w_s = 1 m/day;

Lande and Wood, DSR, 1987

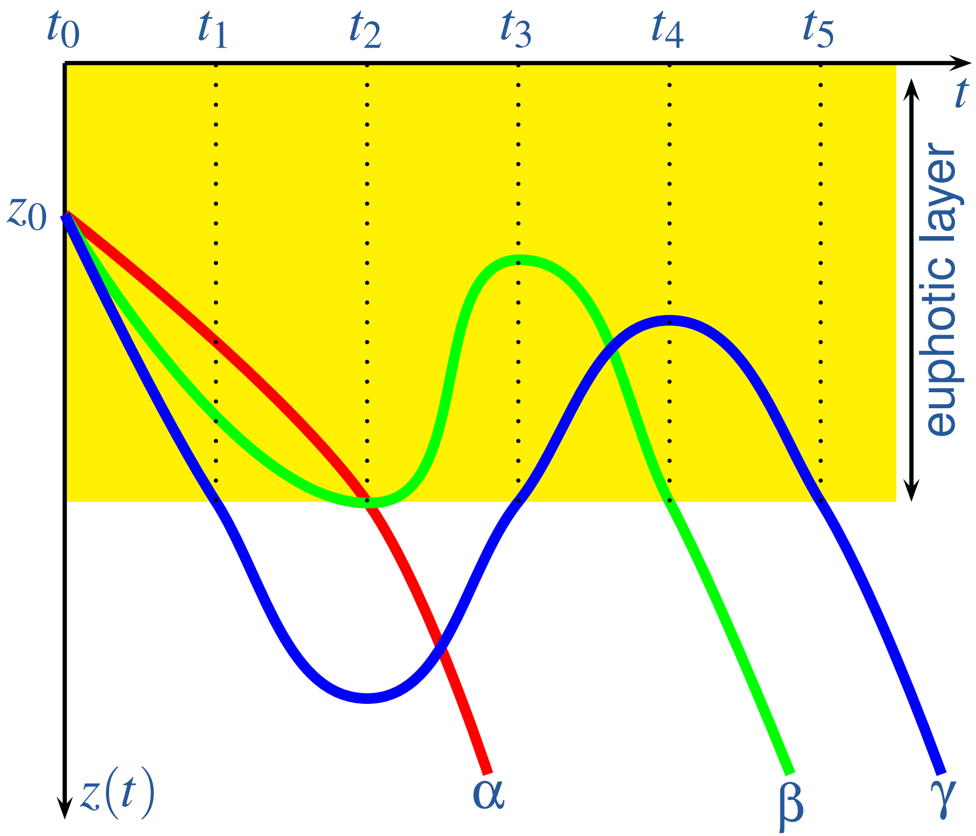
RT in the two layer model

Advection time scale : $\lambda_s = \lambda_b = 0$



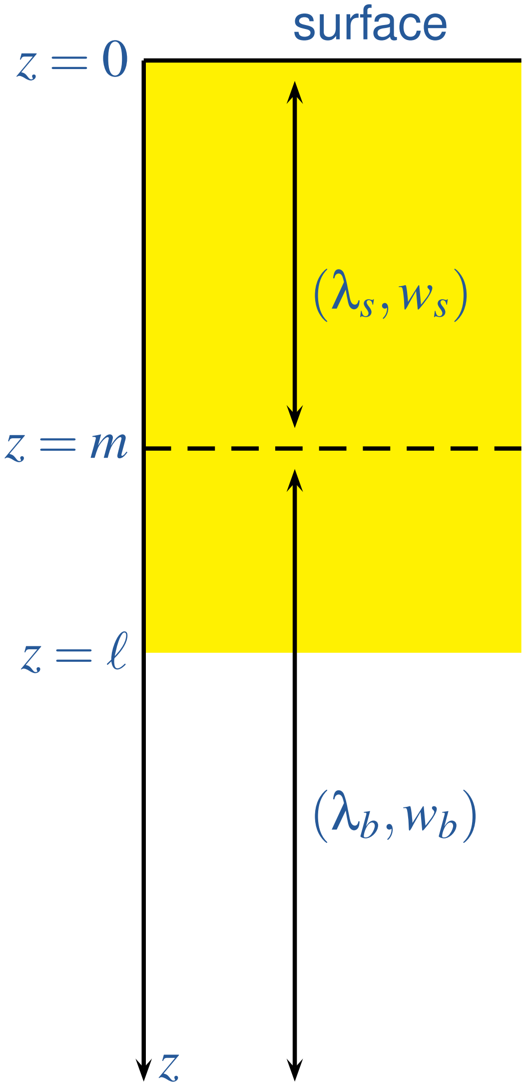
Exposure time

Concept



	α	β	γ
Residence time θ	$t_2 - t_0$	$t_2 - t_0$	$t_1 - t_0$
Exposure time Θ	$t_2 - t_0$	$t_4 - t_0$	$(t_1 - t_0) + (t_5 - t_3)$

Exposure time in the two-layer model



$$-wC + \lambda \frac{\partial C}{\partial z} = 0 \quad \longrightarrow \quad \lambda \frac{\partial \Theta}{\partial z} = 0$$

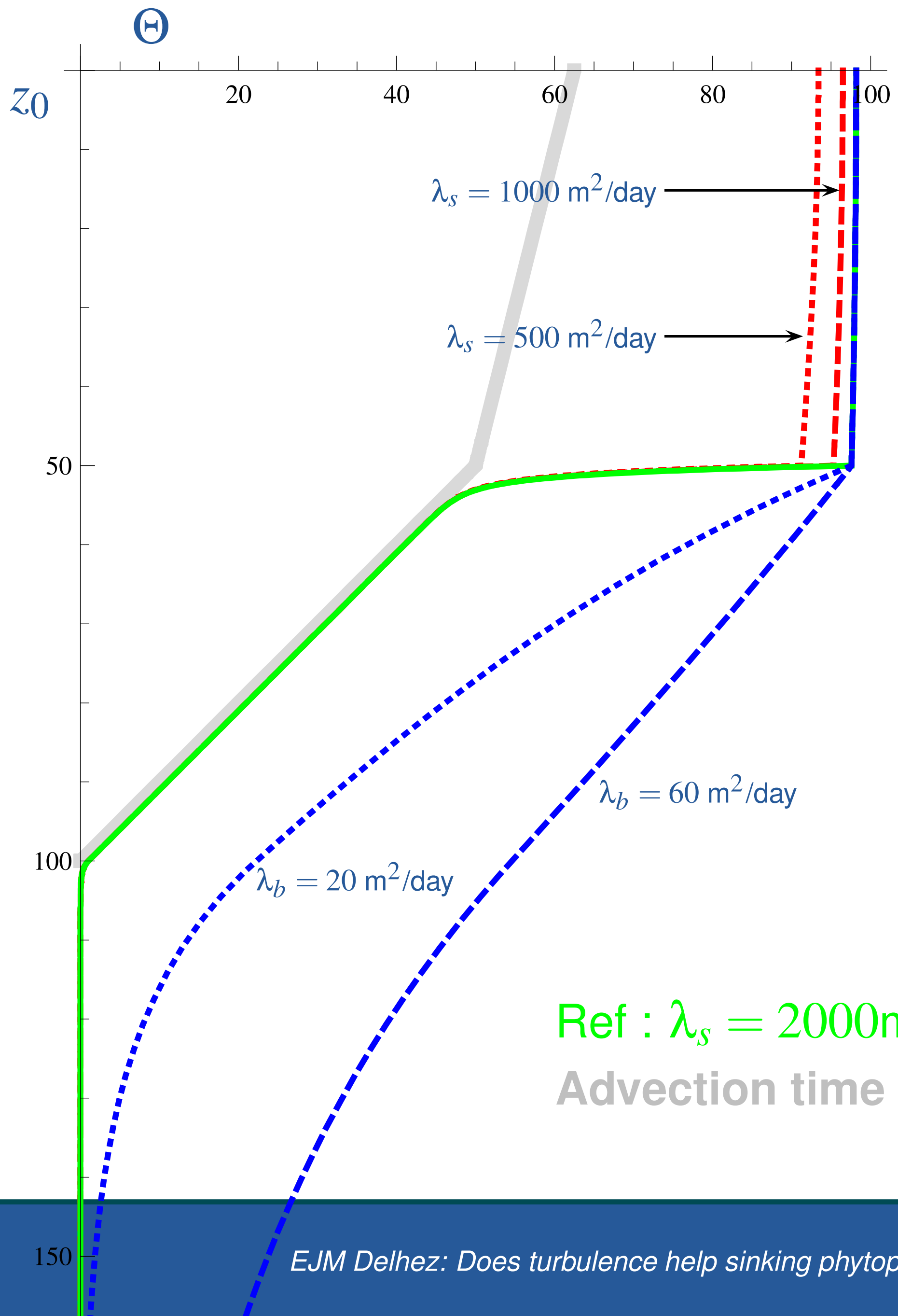
$$\begin{cases} \dfrac{\partial C}{\partial t} = \dfrac{\partial}{\partial z} \left[-wC + \lambda(z) \dfrac{\partial C}{\partial z} \right] \\ C(0,z) = \delta(z - z_0) \end{cases}$$

$$\Theta = \int_0^\infty \int_0^\ell C(t,z)\, dz\, dt$$

$$w \frac{d\Theta}{dz} + \frac{d}{dz} \left[\lambda(z) \frac{d\Theta}{dz} \right] + \delta_{]0,\ell[}(z) = 0$$

$$\boxed{\lambda \frac{\partial C}{\partial z} = 0} \quad \longrightarrow \quad w\Theta + \lambda \frac{\partial \Theta}{\partial z} = 0$$

Exposure time in the two-layer model



Ref : $\lambda_s = 2000$m^2/day, $\lambda_b = 1$m^2/day

Advection time scale : $\lambda_s = \lambda_b = 0$

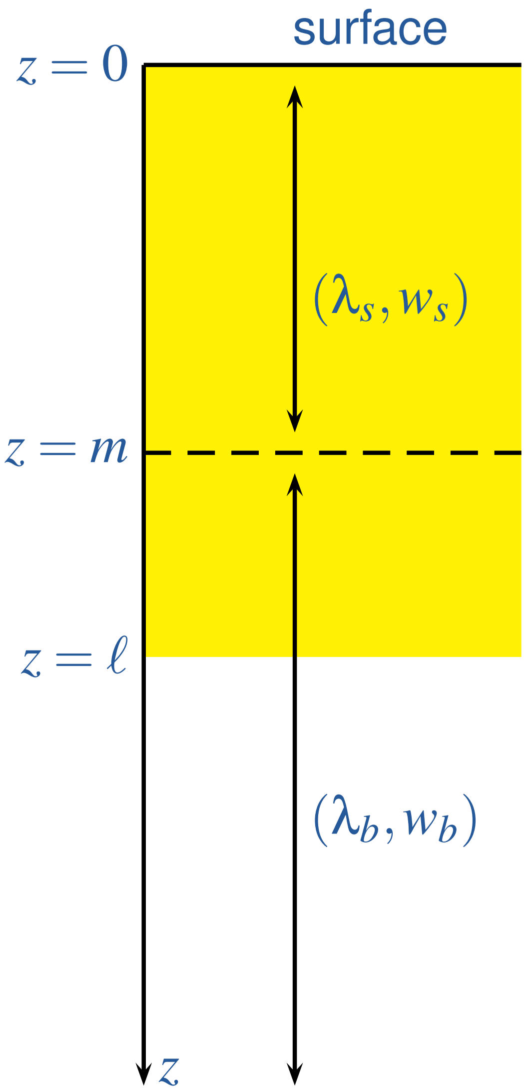
Properties of the exposure time

If the settling velocity does not increase with depth, the exposure time at a given depth z_0 is an increasing function of the diffusion coefficient at shallower depths.
If the settling velocity increases with depth, no general conclusion can be drawn.

P.S. : The value of the exposure time at the surface does not depend on mixing intensity !!!

Light exposure

Light exposure in the two-layer model



$$-wC + \lambda \frac{\partial C}{\partial z} = 0 \quad \longrightarrow \quad \lambda \frac{\partial \Theta_f}{\partial z} = 0$$

$$\begin{cases} \dfrac{\partial C}{\partial t} = \dfrac{\partial}{\partial z}\left[-wC + \lambda(z)\dfrac{\partial C}{\partial z}\right] \\ C(0,z) = \delta(z - z_0) \end{cases}$$

$$\Theta_f = \int_0^\infty \int_0^\infty C(t,z) f(z) dz dt$$

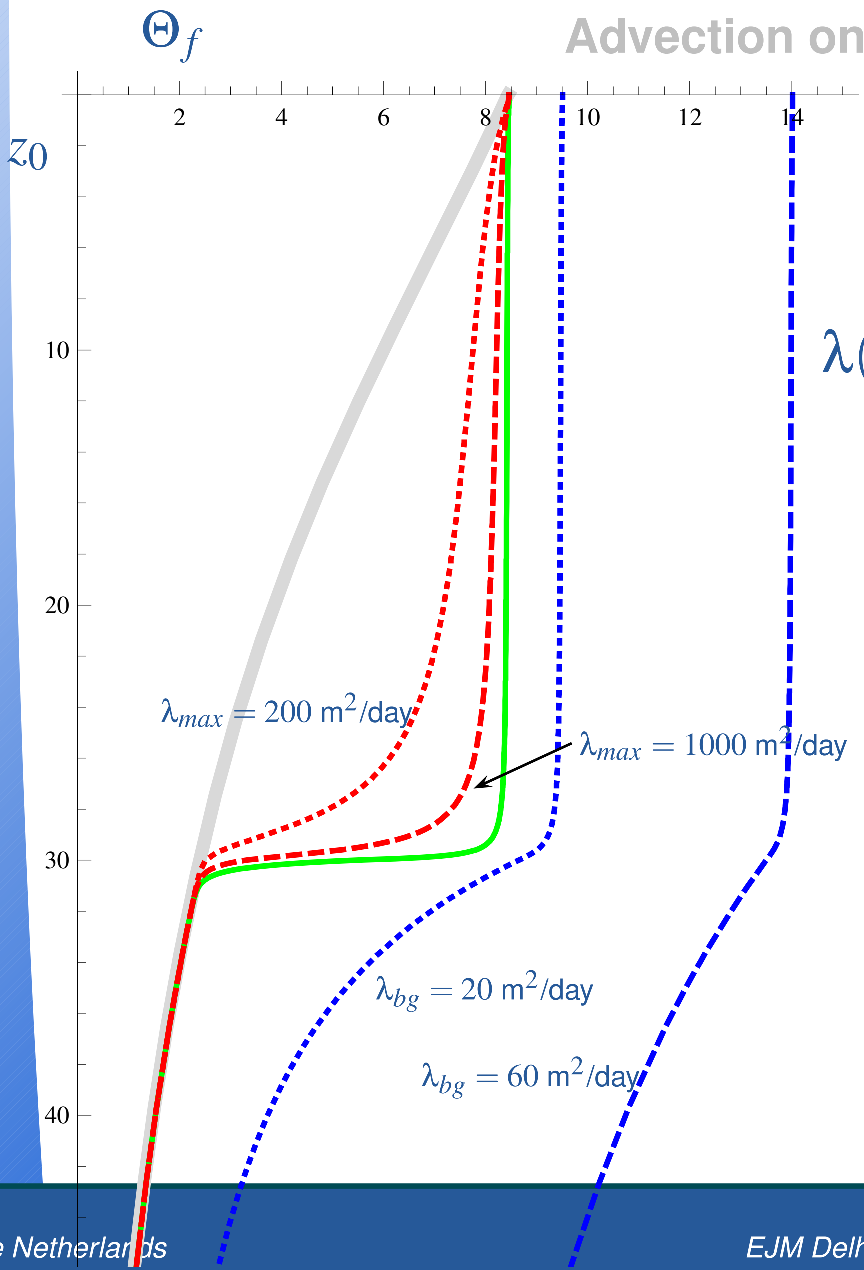
where $f(z) = \frac{I(z)}{I_{opt}} \exp\left(1 - \frac{I(z)}{I_{opt}}\right)$

$$w(z)\frac{d\Theta_f}{dz} + \frac{d}{dz}\left[\lambda(z)\frac{d\Theta_f}{dz}\right] + f(z) = 0$$

$$\lambda \frac{\partial C}{\partial z} = 0 \quad \longrightarrow \quad w\Theta_f + \lambda \frac{\partial \Theta_f}{\partial z} = 0$$

Light exposure in the two-layer model

Advection only : $\lambda_s = \lambda_b = 0$



$$\lambda(z) = \begin{cases} \lambda_{bg} + \dfrac{\lambda_{max}}{2}\left(1 - \cos\dfrac{2\pi z}{m}\right) & z \leq m \\ \lambda_{bg} & z > m \end{cases}$$

$$I(z) = I_0\, e^{-\alpha z}$$

If w does not increase with depth, the light exposure increases with turbulence.

Conclusions

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Conclusions

- ✔ Turbulence helps sinking phytoplankton cells to spent more time in the surface mixed layer and to receive more light energy.
- ✔ The concepts of exposure time and light exposure must be preferred to the residence time to diagnose the time spent in the surface mixed layer.
- ✔ The residence time, exposure time and light exposure can be efficiently computed using an adjoint model.

See also :

`http://www.climate.be/CART`

Delhez and Deleersnijder, 2010. *Journal of Theoretical Biology.*