On the modeling of gas diffusion by turbulence in hydraulic jumps

I.W.M. (Ivo) Pothof[1,2] (IAHR member) and F.H.L.R. (Francois) Clemens[2]

[1]Deltares | Delft Hydraulics, Department of Industrial Hydrodynamics, P.O. Box 177, 2600 MH Delft, The Netherlands; PH (+31) 88 335 8448; FAX (+31) 15 285 8582; email: ivo.pothof@deltares.nl
[2]Sanitary Engineering, Delft University of Technology, Stevinweg 1, 2628 CN Delft, The Netherlands; PH (+31) 15 278 4734; FAX (+31) 15 278 4918; email: F.H.L.R.Clemens@tudelft.nl

ABSTRACT

Pressurised wastewater mains are subject to gas pockets that may accumulate in downward slopes. Such gas accumulations cause an additional head loss that may result in a dramatic capacity reduction; especially in delta areas with negligible static head. The available literature on the required velocity, to transport gas pockets to the bottom of an inverted siphon, is limited (Pothof 2008). Furthermore, literature on the rate of gas transport is hardly available (Lubbers 2007a), (Lubbers and Clemens 2006). The modelling of gas transport in hydraulic jumps requires a suspended bubble model that is valid both in fully developed boundary layer flows and in decelerating jet flows. This paper proposes a more generalised approach for the eddy viscosity model and explores the most appropriate turbulence variable to model the eddy viscosity profile in hydraulic jumps. Such an eddy viscosity model is considered essential for a reliable predictive model of the gas transport processes in hydraulic jumps in closed conduits.

1. INTRODUCTION

We aim to develop a physically-based predictive model for the mitigation of capacity reducing gas pockets in downward sloping (waste)water pipelines. At the downstream end of the gas pocket a hydraulic jump is present, which entrains gas bubbles. The turbulence in the hydraulic jump helps to keep the bubbles in suspension and prevents coalescence to some extent, such that a portion of the entrained gas is transported to the bottom of the downward slope. If the downward slope is long enough, bubbles rising to the pipe soffit may accumulate to a new gas pocket with its own gas entraining hydraulic jump. If gas is supplied continuously, a series of gas pockets and hydraulic jumps will occur at certain liquid flow rates as illustrated in Figure 1. The two-phase flow gas transport model will be validated against lab and field data from Deltares | Delft Hydraulics (Lubbers and Clemens 2005; Tukker 2007; Stegeman 2008).

A hydraulic jump is a well-known liquid flow phenomenon including gas entrainment and highly turbulent flow. A number of properties of hydraulic jumps, like the sequent depth ratio, velocity profiles and local void fraction have been investigated

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over the last century; overviews are provided in a number of books (Rajaratnam 1967; Hager 1993; Chanson 1996).

Figure 1: Subsequent hydraulic jumps in test rig (length 40 m, height 7 m) at wastewater treatment plant; water flows from right to left.

The transport of gas bubbles from the impingement point through the hydraulic jump is essentially an advection-diffusion problem(Chanson 2004), for which we should be able to rely on the vast amount of literature on mixing problems in turbulent flows. However, we need a bubble mixing model that is valid both in fully developed boundary layer flows and in decelerating jet flows. In order to build such a model, we will propose a generalised eddy viscosity model, such that it becomes applicable in friction dominated flows and in local losses, like hydraulic jumps, backward facing steps and bends (Section 2. Generalised eddy viscosity).

Since dissipation and skin friction are not negligible in most local losses, it is useful to determine the longitudinal eddy viscosity profile, depending on profiles of Reynolds stresses and turbulent kinetic energy. Unfortunately, turbulent kinetic energy profiles and Reynolds shear stress profiles in hydraulic jumps are scarce, despite its practical relevance. Rouse (1959) has performed turbulence measurements and derived the dominant terms in the turbulence equations. Rouse's experiments were performed in an air duct, which causes differences with proper hydraulic jumps due to the absence of a free surface and gas entrainment. Further measurements of turbulent shear stress and turbulent kinetic energy were performed in the 1970s (Resch and Leutheusser 1972; Resch, Leutheusser et al. 1976). Svendsen et al. (2000) have investigated weak hydraulic jumps without gas entrainment. Only very recently, other turbulence characteristics have been investigated (Kucukali and Chanson 2008; Murzyn and Chanson 2008). Several properties of the entrained gas in hydraulic jumps have been measured in channels (Chanson 2004) and downward sloping pipes (Lubbers 2006). Section 3 explores which turbulence characteristics —production P, dissipation ε or turbulent kinetic energy k— determine the gas transport properties in hydraulic jumps. Section 4 presents conclusions and recommendations.

2. GENERALISED EDDY VISCOSITY

The eddy viscosity is a key parameter in mixing and diffusion problems in fluid dynamics; scour, erosion, brine dispersion, dilution of river pollution and aeration to name a few examples in hydraulic engineering. Air bubbles diffuse in the turbulent shear layer in a hydraulic jump (Chanson and Brattberg 2000). The current approach to suspended bubble modeling in hydraulic jumps is based on the vertical mixing equation (1), neglecting the bubble rise velocity in the hydraulic jump (Chanson 2004).

$$u_1 \frac{\partial C}{\partial x} = \varepsilon_y \frac{\partial^2 C}{\partial y^2} \tag{1}$$

Chanson has analysed the air bubble diffusion and found (Chanson 2004):

$$\varepsilon_y \approx 0.04 \cdot d_1 \cdot u_1 \approx 0.7 \cdot d_1 \cdot u_{*1} \tag{2}$$

The increased mixing coefficients are due to the local deceleration losses in the hydraulic jump. This paper proposes to model the eddy viscosity in a more generalized way by focusing on the a priori knowledge of the macroscopic energy loss.

Scaling of turbulent mixing. The eddy viscosity and bubble diffusion coefficient should be derived from both the skin friction losses and local deceleration losses. Since the friction velocity u_* represents the energy losses due to skin friction, it is possible to rephrase the friction velocity in terms of the hydraulic grade line:

$$u_* = \sqrt{\frac{\lambda}{8}} u = \sqrt{gR_h} \sqrt{\frac{\partial H}{\partial x}} = \sqrt{\frac{gA}{S}} \sqrt{\frac{\partial H}{\partial x}} \tag{3}$$

where

U	[m/s]	Advective velocity
λ	[-]	White-Colebrook friction factor for pipe or channel flow
X	[m]	Flow direction
$\partial H/\partial x$	[-]	Hydraulic grade line, $H(x) = d + u^2/2g$
R_h	[m]	Hydraulic radius
A	[m^2]	Wet cross section
S	[m]	Shear exerting surface per length unit

Now, the diffusion coefficient can be expressed as a function of the hydraulic gradient, that may be affected by skin friction and local losses.

$$\varepsilon_y \sim d \cdot u_* \sim \sqrt{gd^3 \frac{\Delta H}{L}} \tag{4}$$

where

d	[m]	Cross section length scale, typically the upstream water depth
$\Delta H/L$	[m]	Hydraulic gradient, which involves the deceleration length of local losses

The integrated macroscopic deceleration loss for a 1D duct flow or hydraulic jump are the well known Borda-Carnot and hydraulic jump head loss equations, derived from the overall momentum and energy balance.

Borda-Carnot
$$\Delta H = \left(1 - \frac{A_2}{A_1}\right)^2 \frac{u_2^2}{2g} = \left(\frac{A_1}{A_2} - 1\right)^2 \frac{u_1^2}{2g} \quad (5)$$

Hydraulic jump (rectangular channel)
$$\Delta H = \frac{d_1^2}{4d_2}\left(\frac{d_2}{d_1} - 1\right)^3 \quad (6)$$

Similar expressions may be derived for other local losses in which a jet suddenly decelerates from some vena contracta into a wider cross section.

The a priori deceleration length is 6D for any deceleration loss in a pipe geometry (bend, valve, etc.); see e.g. standards for control valve capacity measurements (ANSI/ISA-75.02 1996). The deceleration length of a hydraulic jump can be assessed more accurately, but it is around $6d_2$ over a wide range of inflow Froude numbers (Hager 1993). Consequently a deceleration length $L = 6d_2$ seems a widely applicable deceleration length. Now, the eddy viscosity and diffusivity can be assessed for local losses. Equation (5) and (6) neglect the influence of skin friction. It is desirable to include the effect of skin friction and fully developed inflow conditions on the hydraulic jump properties. Since normal depth is reached in about 9D in downward pipe slopes between 5° and 30° (Lubbers 2007a), the inflow to the hydraulic jump is generally fully developed, which does affect the total head loss and the turbulent diffusion coefficient. Section 3 explores which turbulence characteristics determine the gas transport properties in hydraulic jumps by analyzing the literature on the influence of inflow conditions on various longitudinal hydraulic jump properties.

3. TURBULENCE IN HYDRAULIC JUMPS

This section focuses on the physical processes that have to be modeled to explain the observed differences in turbulence characteristics between three types of experiments on hydraulic jumps:

- Single phase experiments on an expanding air duct with non-developed inflow conditions, e.g. (Rouse, Siao et al. 1959).
- Two-phase experiments (water-air) with developed inflow conditions, e.g. (Resch, Leutheusser et al. 1976).
- Two-phase experiments (water-air) with non-developed inflow conditions, e.g. (Rajaratnam 1967; Hager and Bremen 1989; Chanson 1995)

The head loss in any decelerating jet is caused by the (turbulent) shear stresses exerted along the envelop surface of the jet; i.e. the bottom surface and the surface of the dividing streamline between the jet and the roller. It is remarkable to note that only a few authors explicitly focus their analysis on the dividing streamline (Resch, Leutheusser et al. 1976; Svendsen, Veeramony et al. 2000), despite its relevance for many hydraulic jump properties. The influence of the inflow condition and skin friction are discussed with respect to the sequent depth ratio, the length of the roller and jump and the turbulent kinetic energy.

Sequent depth ratio. The influence of skin friction on Hager (1993) provides detailed experimentally validated velocity profiles in the decelerating jet and the roller above the jet of a hydraulic jump with non-developed (ND) inflow conditions. The dividing streamline (DS) has been derived from these velocity profiles for an inflow Froude number of 7.3 (labels *DS, ND* and *d/d1, ND* in Figure 2), which is the same Froude number as in (Resch, Leutheusser et al. 1976), based on the local depth-averaged velocity (labels *DS, D* and *d/d1, D* in Figure 2). Resch and Leuttheusser have reported cross section averaged upstream velocities and depth averaged velocities on the centerline; their reported Froude numbers of 2.85 and 6.0 are based on the cross section averaged velocities. This paper uses the Froude numbers, derived from the depth-averaged centerline velocities, which equals 7.3 in stead of 6.0.

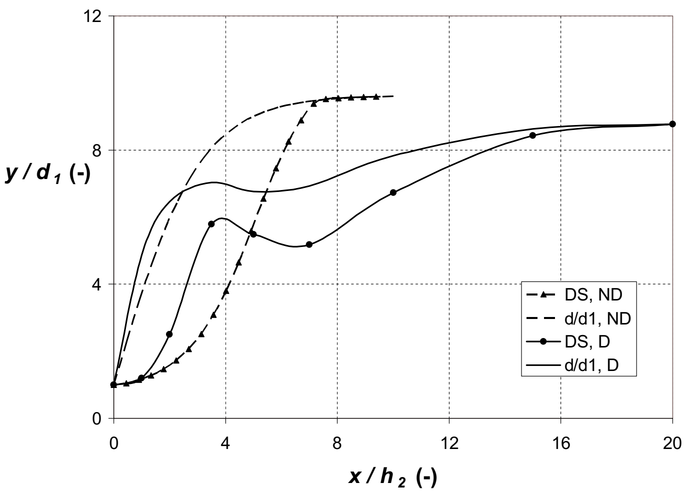

Figure 2: Dividing streamlines (DS, marked lines) and water level in hydraulic jump for non-developed (ND, dotted lines) and developed (D, solid lines) inflow conditions at $Fr_1=7.3$.

The influence of skin friction on the sequent depth ratio is limited to about 3% of the Bélanger equation, $d_2/d_1 = 0.5\left[\sqrt{1+8Fr_1^2}-1\right]$, for non-developed inflow conditions (Hager and Bremen 1989). Resch and Leutheusser have measured similar skin friction effects in their non-developed inflow experiments. However, in fully developed inflow, the sequent depth is 10% smaller than predicted by the Bélanger equation (Resch, Leutheusser et al. 1976), as illustrated in Figure 2. Further inspection of the dividing streamlines in Figure 2 shows that the fully developed inflow decelerates more quickly between the impingement point and $x = 4*d_2$. This initial deceleration is triggered by the fluid properties at the free surface of the inflowing jet. On one hand, the initial local shear stress at the free surface is zero for both the developed and non-developed inflow condition. Consequently the free surface shear stress does not discriminate the developed inflow condition from the non-developed inflow condition. On the other hand, the turbulent kinetic energy at the free surface is non-zero for a developed inflow, $k = 0.65 \cdot u_*^2$ (Nezu 1993), and zero for a non-developed inflow. These observations indicate that the upstream turbulent kinetic energy k_1 affects the water level and dividing streamline profiles in the hydraulic jump.

Roller, jump and aeration length. Resch and Leutheusser's data shows indicative streamlines in the roller, but their data does not show the length of the roller; i.e. the location where the bubbles rise vertically upward. The length of the jump with developed inflow is assessed from Resch, Leutheusser et al. (1976); it is determined from the first measuring station with a horizontal free surface and the dividing streamline at the free surface (label *Lj, D* in Figure 3). Rouse has included a graph on the roller length (Rouse, Siao et al. 1959) for the air duct experiments (label *Lr; ND(air)* in Figure 3). Hager (1993) provides equations for the roller and jump length with non-developed inflow (labels *Lr, ND* and *Lj, ND* in Figure 3).

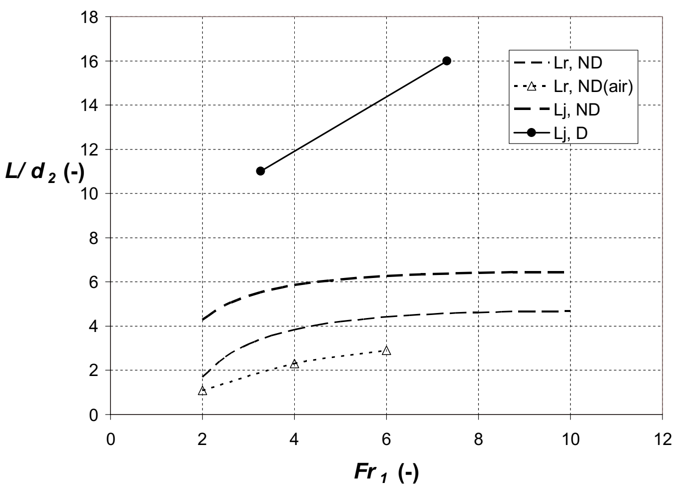

Figure 3: Roller and jump length data at various Froude numbers.

Figure 3 shows that the air duct data has systematically smaller roller lengths than the hydraulic jump data. Since the presence of entrained gas is the main difference between the air duct data and hydraulic jump data, the void fraction appears to extend the length of the roller and the jump. This trend is confirmed by the large jump length with developed inflow conditions, because then gas bubbles remain suspended much longer, although the void fraction in a fully developed hydraulic jump is not considerably greater than the void fraction in a hydraulic jump with non-developed inflow. A number of physical processes may explain the longer aeration length with fully developed inflow: 1) a larger overall head loss causing greater turbulence k, 2) a stronger deceleration up to $x = 4*d_2$ causing a smaller average bubble size or 3) a larger void fraction may all increase the aeration length. Unfortunately, no data has been found on the average bubble size or bubble count rate with a fully developed inflow. The available experimental data is insufficient to assess the relevance of each of the physical processes listed above.

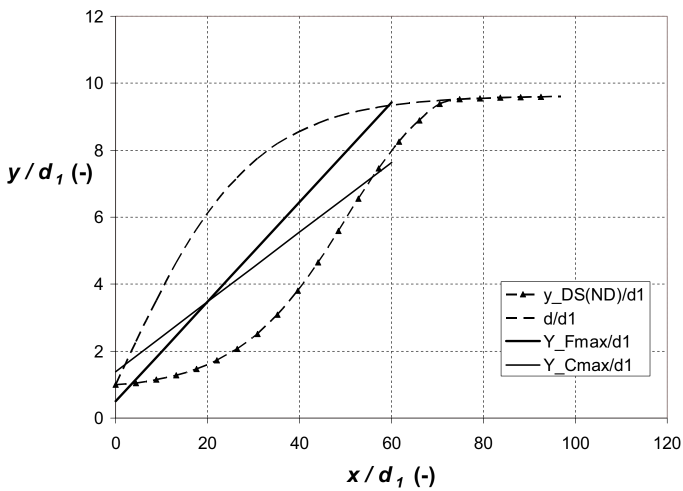

Figure 4: Vertical location of maximum void fraction and bubble count rate.

Suspended bubbles. Chanson has extensively investigated the air transport processes in hydraulic jumps with non-developed inflow conditions; e.g. (Chanson 1995; Gualtieri and Chanson 2007). The two-phase flow properties provide further insight into the modeling of the gas transport processes by turbulence characteristics. The maximum bubble count rate drops exponentially in longitudinal direction and the decay coefficient is smaller for larger inflow Froude numbers, which confirms that bubbles remain suspended over a longer distance at larger inflow Froude numbers (Gualtieri and Chanson 2007). Unfortunately, similar data with fully developed inflow conditions are not yet available to the authors' knowledge. Figure 4 shows the vertical location of the maximum void fraction (Y_{Cmax}) and maximum bubble count rate (Y_{Fmax}) plotted together with the dividing streamline and water level for non-developed inflow conditions, derived from Hager (1993). Figure 4 clearly shows that the maximum bubble count rate occurs at the centerline of the roller. The maximum void fraction occurs in the bottom half of the roller. Consequently, only a small portion of the entrained gas remains suspended in the jet.

Inspection of the air duct data (Rouse, Siao et al. 1959) and derivation of the specific energy profiles from Hager (1993) shows that the reduction of specific energy occurs mainly within the roller length of the jump L_r. Recognising that the turbulence production equals the specific energy reduction, implies that the production of turbulent kinetic energy has practically stopped at $x = L_r$. Inspection of suspended bubble measurements (Resch, Leutheusser et al. 1974) shows that bubbles remain suspended considerably longer in a hydraulic jump with developed inflow, especially at a distance greater than $6*d_2$. Since the turbulence production has diminished at $6*d_2$, the bubble dispersion beyond this location must be driven by the turbulent kinetic energy k. The longer aeration length may also be explained by a smaller average bubble size in hydraulic jumps with fully developed inflow, but the available experimental is not conclusive on this issue.

4. CONCLUSIONS AND RECOMMENDATION

The arguments listed in the paper support the formulation of a k-based eddy viscosity model in order to explain the strong deceleration and long aeration length in hydraulic jumps with fully developed inflow. Since dissipation in the hydraulic jump cannot be neglected, it is necessary to develop longitudinal profiles of turbulence production P, dissipation ε and the turbulent kinetic energy k.

If friction in hydraulic jumps or other local losses can be neglected, then we propose to model the turbulent diffusion coefficient as a function of the a priori known local head loss, following equation (4).

New experimental data on the bubble count rate in hydraulic jumps with partially to fully developed inflow conditions would improve the modeling of the gas diffusion.

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