# On gas transport in downward slopes of sewerage mains 

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#### Abstract

Wastewater pressure mains are subject to gas pockets in declining sections. These gas pockets cause an additional head loss and an associated capacity reduction, which cannot be predicted with sufficient accuracy. This paper includes a critical review of the literature on gas transport by flowing water in downward sloping pipes. This review shows that subtle misinterpretations of the original data have caused the wide spread in the correlations for the clearing velocity, as reported by various investigators. Finally, the paper proposes a new dimensionless velocity parameter, which seems a more appropriate scaling parameter than the existing velocity scaling.


## KEYWORDS

channel flow; gas transport; sewerage; surface tension; two-phase flow; waste water pipelines.

## Nomenclature

| A | pipe cross section ( $\mathrm{m}^{2}$ ) | $\theta$ | downward pipe angle (rad) |
| :---: | :---: | :---: | :---: |
| D | internal pipe diameter (m) | $\rho$ | density ( $\mathrm{kg} / \mathrm{m}^{3}$ ) |
| Eo | Eötvös number, Eo $=\rho g D^{2} / \sigma(-)$ | $\sigma$ | surface tension ( $\mathrm{N} / \mathrm{m}$ ) |
| $F$ | Flow number (-) |  |  |
| Fr $g$ | Froude number (-) gravitational acceleration ( $\mathrm{m} / \mathrm{s}^{2}$ ) | $\sim$ | normalized quantity |
| $n$ | dimensionless gas volume as the number of filled pipe diameters (-) |  |  |
| $Q$ | discharge ( $\mathrm{m}^{3} / \mathrm{s}$ ) |  |  |
| $S_{E}$ | Energy slope (-) |  |  |
| $v$ | average velocity in pipe cross section ( $\mathrm{m} / \mathrm{s}$ ) |  |  |
| $y$ | water depth (m) |  |  |

## Subscripts

| $B$ | buoyancy |  |  |
| :--- | :--- | :--- | :--- |
| $c$ | clearing (velocity) i.e. for gas <br> pocket removal <br> effective | $g$ <br> $i$ | gas / air <br> initiation of downward gas <br> transport |
| $e$ | $w$ | $w$ | water |
| $c f$ | clearing velocity including friction |  |  |

## INTRODUCTION

Gas pockets in pipelines originate from a number of sources. Air may entrain continuously as bubbles in case the sewer outflow is a free-falling jet into the pump pit. Air may also entrain discontinuously after pump stop if the pump inertia is sufficient to drain the pit down to the bell-mouth level. This kind of discontinuous air entrainment occurs mainly in wastewater systems with a marginal static head. If the pipeline is subject to negative pressures during normal operation or during transients, then air may leak into pipeline or may enter intentionally via air valves. Another transient phenomenon, however more unlikely, is a pump trip in a dendritic pressurised wastewater system. The induced transient may suck some wastewater from an idle pumping station, causing air entrainment in the idle pumping station. Another cause of gas pocket development consists of biochemical processes in the pipeline, mainly producing carbon dioxide $\left(\mathrm{CO}_{2}\right)$, nitrogen $\left(\mathrm{N}_{2}\right)$ and methane $\left(\mathrm{CH}_{4}\right)$.


Figure 1. Gas entrainment in the hydraulic jump in a downward sloping pipe

## Physical processes

Pressurised wastewater mains are characterized by an intermittent operation. Gas will accumulate in elevated sections of the pipeline during shut down periods and dry weather flow conditions. If an air pocket is present in the top of a declining section and liquid flows through the conduit, then a hydraulic jump will develop at the tail of the gas volume. The hydraulic jump entrains gas bubbles into the flowing liquid (Figure 1). The pumping action of the hydraulic jump transports a fraction of the suspended bubbles down to the bottom of the downward slope, as schematized in Figure 2.

The behaviour of the suspended bubbles has been investigated in a large-scale experimental facility, erected at the wastewater treatment plant Nieuwe Waterweg in Hoek van Holland, The Netherlands (Stegeman 2008). This facility, including a 40 m long $10^{\circ}$ downward slope with an elevation difference of 7 m , has been designed and operated by Deltares | Delft Hydraulics and Delft University of Technology. The pipe internal diameter is 192 mm to eliminate scale effects. Co-current air-water flow has been investigated at low air flow rates from $0.7 \mathrm{l} / \mathrm{min}$ up to $10 \mathrm{l} / \mathrm{min}$ and at water flow rates up to $50 \mathrm{l} / \mathrm{s}$. The experimental results from this facility are currently being analysed and will be presented in detail in future publications. The observations on the co-current flow of water and air reveals the following flow regimes. If the flow number F , defined as $F \equiv v / \sqrt{g D}$, is smaller than 0.54 , then only one hydraulic jump is present in the downward slope near the bottom of the slope. If the flow number F is greater than 0.54 , then the suspended bubbles coalesce to the pipe soffit and grow to a new gas volume with its own hydraulic jump. The maximum number of steady
consecutive hydraulic jumps in this 40 m downward slope was seven. The consecutive hydraulic jumps moved upward very slowly or remained on a fixed position. A further increase of the liquid flow rate up to a certain critical value reduces the length of the first gas pocket and causes the suspended bubbles to coalesce into stable plugs that travel along the pipe soffit at a smaller velocity than the average liquid velocity. These plugs are called stable, because no gas bubbles are ejected from the plugs. The transition from consecutive hydraulic jumps to stable plugs marks the clearing of gas pockets and reduces the additional gas pocket head loss to zero.


Figure 2. Schematic overview of downward gas transport by flowing water.

## Set-up of paper

This paper includes a review of the literature on gas transport by flowing water in declining pipes. This review will show that the wide spread in the correlations for the clearing velocity, as reported by (Wisner, Mohsen et al. 1975) and (Escarameia 2006), is mainly caused by a subtle misinterpretation of the original data. Furthermore, new information from a few old references will be presented and integrated into a synthesis of the available literature.

## GAS TRANSPORT IN DOWNWARD SLOPING PIPES

This section provides an overview of available literature on gas transport in downward sloping pipes. The literature contains intercomparable data on three different processes in downward sloping pipes, namely:

1. The breakdown and transport of a large gas volume in the top of a downward sloping pipe (Bliss 1942), (Kalinske 1943), (Lubbers 2007a) and (Stegeman 2008);
2. The transport of an individual gas pocket from which air bubbles are entrained into the flowing liquid (Kent 1952), (Gandenberger 1957) and (Escarameia 2006) ; and
3. The transport of a stable plug (Veronese 1937), (Kent 1952) and (Wisner, Mohsen et al. 1975).
The available literature to date does not explicitly distinguish these different processes and the clearing velocity has been associated with all of these processes. The following paragraphs discuss the literature on the three processes mentioned above.

## Gas volume clearing

Among the oldest available research works in the field of liquid driven gas transport in downward sloping pipes are the publications by Kalinske and Bliss (1943) and Kalinkse and

Robertson (1943). They determined the dimensionless flow rate $Q_{i}^{2} / g D^{5}$ at which gas bubbles are ripped off from the gas volume by the hydraulic jump at the end of the gas volume and start to move downward to the bottom of the slope. Kalinske and Bliss (1943) propose the following relation for this incipient downward gas transport, based on experiments in a 100 mm and 150 mm pipe:

$$
\begin{equation*}
\frac{Q_{i}^{2}}{g D^{5}}=\frac{\sin \theta}{0.71} \tag{1}
\end{equation*}
$$

where Kalinske et al. defined the pipe slope as the sine of the angle with the horizontal plane.
Kalinske and Bliss described three distinct flow regimes (Kalinske 1943):

- A blow-back flow regime at relatively small discharges, although still above the critical discharge in (1). In this flow regime, the bubbles coalesce and periodically blow back upward, which limits the net gas transport. The net gas transport is controlled by the flow characteristics below the hydraulic jump, which are described by the flow number.
- A full gas transport flow regime at higher discharges, at which all entrained gas bubbles are transported to the bottom of the downward slope. The gas transport becomes almost independent of the dimensionless velocity in this flow regime. Kalinske et al. (1943) conclude that the Froude number upstream of the hydraulic jump determines the gas transport. This Froude number is defined as: $F_{1}=v_{1} / \sqrt{g y_{e}}$, where $v_{1}$ is the water velocity upstream of the hydraulic jump and $y_{e}$ is the effective depth, i.e. the water area divided by the surface width.
- At the transition from the blow-back flow regime to the full gas transport flow regime, a series of 2 to 4 stationary gas pockets and hydraulic jumps were observed in the downward slope. The length of the downward slope of their test rig was 10.5 m long. Blow-back did not occur any more in this transitional flow regime.

Equation (1) cannot be considered an equation for the clearing velocity, as explained by Kalinske and Bliss: "..to maintain proper air removal, the actual value of the water discharge should be appreciably larger than $\mathrm{Q}_{\mathrm{i}}$ " (Kalinske 1943). Nevertheless, equation (1) has been used as a correlation for the clearing velocity. In fact, equation (1) may depend on the length of the downward slope, which was 10.5 m (i.e. $105^{*} \mathrm{D}$ for the 100 mm pipe and $70 * \mathrm{D}$ for the 150 mm pipe). Equation (1) is equivalent to a flow number criterion:

$$
\begin{equation*}
F_{i}=\frac{v_{i}}{\sqrt{g D}}=\frac{Q_{i}}{\left(0.25 \pi D^{2}\right) \sqrt{g D}}=\frac{4}{\pi} \frac{Q_{i}}{D^{2} \sqrt{g D}}=\frac{4}{\pi} \sqrt{\frac{\sin \theta}{0.71}} \tag{2}
\end{equation*}
$$

The transition to the full gas transport flow regime is highly relevant from a practical point of view, because this transition represents the clearing velocity that breaks down gas pockets (almost) completely and that minimises the head loss by the gas pockets. Bliss has reported this transition in his research report only (Bliss 1942). The transition to full gas transport requires a significantly larger dimensionless velocity than the more widely reported 'start of downward gas transport' according to equation (1). Figure 3 shows both correlations.

Bliss reported that the full gas flow occurred at lower water flow rates, if the gas pocket was not held in position by the roughness of a joint or a projecting point gage. The results in Figure 3 apply to the required velocities with a projecting point gage at the beginning of the
downward slope, because Bliss anticipated that any prototype pipe would contain sufficient roughness elements to hold a gas pocket in position in the top of the downward slope. Finally, Bliss noted that the 4 " pipe required smaller clearing velocities than the $6^{\prime \prime}$ pipe.


Figure 3. Required dimensionless velocities for 'start of gas transport' (Kalinske 1943) and clearing velocity in full gas flow regime (Bliss 1942) and Lubbers (2007a).

Lubbers (Lubbers 2007a), Tukker (2007) and Stegeman (2008) have performed experiments in different pipe sizes from 80 mm to 500 mm at various downward slopes from $5^{\circ}$ to $30^{\circ}$ and $90^{\circ}$. We have injected air upstream of the downward slope in a horizontal section and measured the extra head loss due to the gas volume at different combinations of co-current air water flow rates. The clearing velocity (or critical velocity) is reached when the extra head loss due to the presence of the gas flow attains a minimum. This minimum value is close to zero. Lubbers found that the largest clearing velocity is required at downward slopes of $10^{\circ}$ to $20^{\circ}$. Figure 3 shows a gradual drop in clearing velocity at slopes above $20^{\circ}$. The dimensionless clearing velocity drops to about 0.4 for a vertical pipe with 220 mm internal diameter. An inter-comparison of the clearing velocities at a downward slope of $10^{\circ}$ and the three pipe sizes, revealed that the clearing velocity becomes constant for pipe diameters above 200 mm Figure 4.


Figure 4. Clearing velocities at a downward slope of $10^{\circ}$ from Lubbers, Tukker and Stegeman. The three points in the bubble clearing velocity line are from Gandenberger (1957), Mosvell (1976, extrapolated) and Escarameia (2006).

## Individual gas pocket clearing

Kent (Kent 1952) has performed detailed experiments in a 33 mm pipe and a 102 mm (4") pipe on stationary gas pockets in declining pipes with downward angles varying between $15^{\circ}$ and $60^{\circ}$. The length of the downward slope was $5.5 \mathrm{~m}(18 \mathrm{ft}$ or $54 * \mathrm{D})$ for the 102 mm pipe. Since the gas pockets are stationary, the flow regime is similar with Kalinske's transitional flow regime between blow-back and full gas transport.

Kent focused on the determination of the drag coefficient, $C_{D}$, as a function of the plug length, $L_{b}$ and the maximum projected plug area, $A_{b}$. Kent's gas pockets were so large that a hydraulic jump formed at the tail of the gas pocket. In order to maintain a constant gas volume, Kent continuously injected air in the stationary pocket in the downward slope. He measured the total gas volume in the downward slope by rapidly closing two valves at the beginning and end of the slope simultaneously, after which he could measure the total gas volume in the slope. This enabled Kent to determine the total buoyant force, which must balance the drag force, because the gas pocket remained stationary. Kent established the following relation from his experiments:

$$
\begin{equation*}
\frac{v_{c}}{\sqrt{g D}}=1.23 \cdot \sqrt{\sin \theta} \tag{3}
\end{equation*}
$$

A clearly better curve fit on Kent's data, which allows for a non-zero offset is given in equation (4); see (Mosvell 1976), (Lauchlan 2005) and (Wisner, Mohsen et al. 1975), who first published the systematic deviation between Kent's data and equation (3); see Figure 5.

$$
\begin{equation*}
\frac{v_{c}}{\sqrt{g D}}=0.55+0.5 \cdot \sqrt{\sin \theta} \tag{4}
\end{equation*}
$$

Equation (4) is valid for gas pockets with a dimensionless length exceeding 1.5D, which coincides with a gas pocket volume exceeding 0.55D in full pipe diameters ( $\mathrm{n}>0.55$ ). Unfortunately, Equation (3) has been used frequently in the design of Dutch sewerage mains; also for pipe angles smaller than $15^{\circ}$. The significant difference between Kent's equation (3) and Mosvell's curve fit (eq. (4)) is caused by Kent's dimensional analysis that did not include an offset parameter. The difference is particularly pronounced, if Kent's equation is extrapolated to pipe angles smaller than $15^{\circ}$; see Figure 5.


Figure 5. Required dimensionless clearing velocities for large pockets with air entraining hydraulic jumps and for stable pockets without air entrainment. The maximum stable pocket length is indicated as well.
Figure 5 shows that the clearing velocity for stable individual plugs is smaller than the clearing velocity for large pockets with hydraulic jumps and air entrainment. Figure 5 suggests that the clearing velocity for stable plugs and large pockets might become equal at a downward slope of approximately $10^{\circ}$, if Mosvell's line and the stable plug line are extrapolated to $10^{\circ}$.

Gandenberger (1957) has performed measurements on individual stationary pockets and pockets moving downward in various pipes with internal diameter between 10 mm and 100 mm at downward angles between $5^{\circ}$ and $90^{\circ}$. Most of Gandenberger's results are based on measurements in a 45 mm glass pipe. Gandenberger measured a maximum clearing velocity at $40^{\circ}$. He investigated gas pocket volumes up to $\mathrm{n}=1.5$ and found that the gas pocket velocity becomes constant if n exceeds 0.5 at all pipe angles, which confirms Kent's results. The applicability of Gandenberger's measurements for downward gas transport in water pipelines has the same limitations as Kent's results. A further limitation of Gandenberger's measurements is the fact that Gandenberger did not inject air to maintain a constant pocket volume. Hence, the bubbly flow at the tail of the hydraulic jump could not be maintained during the experiments. Gandenberger's clearing velocities are slightly smaller
than Kent's velocities, which may be attributed to the smaller pipe diameter or to the lack of air injection in the pocket.

Escarameia et al. (2006) have performed experiments in a 150 mm pipe with gas pockets up to 5 litres $(\mathrm{n}=1.9)$ and pipe slopes up to $22.5^{\circ}$. Escarameia proposed the following formula for the dimensionless clearing velocity, which extends Kent's data, eq. (4), to angles smaller than $15^{\circ}$ :

$$
\begin{equation*}
\frac{v_{c}}{\sqrt{g D}}=0.61+0.56 \cdot \sqrt{\sin \theta} \tag{5}
\end{equation*}
$$

The same limitations as mentioned for other individual gas pocket investigators apply to these results.

Wisner (Wisner, Mohsen et al. 1975) developed an envelope curve from the available data from Veronese (1937), Gandenberger, Kent and Kalinkse and found:

$$
\begin{equation*}
\frac{v_{c}}{\sqrt{g D}}=0.825+0.25 \cdot \sqrt{\sin \theta} \tag{6}
\end{equation*}
$$

It must be noted that Wisner et al. have misquoted Kalinske and Bliss in equation (1) by using a coefficient 0.707 instead of the correct coefficient $1 / 0.71$, which (incorrectly) resulted in $30 \%$ smaller dimensionless clearing velocities.

## Stable plug clearing

Kent (1952) also performed measurements on relatively small gas plugs from which no bubbles were entrained by the hydraulic jump. These measurements reveal that both the maximum stable plug length and the clearing velocity reduce at steeper slopes; see Figure 5.

Wisner (Wisner, Mohsen et al. 1975) recognized the large spread in proposed critical liquid velocities for downward gas transport in the literature up to 1975 including Gandenberger (Gandenberger 1957), Kalinske et al. (Kalinske 1943) and (Kent 1952). Wisner assumed that scale effects could have caused the large spread and therefore performed experiments in a large 245 mm pipe at a fixed downward angle of $18^{\circ}$. Wisner focused on stable plugs rather than the clearing velocity in his experiments.

## DISCUSSION

## Individual gas pocket clearing

If the results from the single-gas-pocket-investigators on their largest gas pockets are compiled into one graph, then the trends are similar; see Figure 6. The only differences between the results are caused by diameter differences, which induce a Reynolds number influence and possibly a Eötvös number influence. The Reynolds number influence is included in the friction factor. A logical extension of the dimensionless clearing velocity is the clearing velocity multiplied with $\sqrt{f}$ yielding:

$$
\begin{equation*}
\tilde{v}_{c f} \equiv \sqrt{\frac{f}{g D}} v_{c} \tag{7}
\end{equation*}
$$

In fact, this definition implies that the hydraulic grade lines are identical if the dimensionless clearing velocities including friction, $\tilde{v}_{c f}$, are identical, as illustrated for pipe diameters $\mathrm{D}_{1}$ and $D_{2}$ in equation (8):

$$
\begin{align*}
& S_{E}\left(D_{1}\right)=S_{E}\left(D_{2}\right) \Leftrightarrow \frac{f}{D_{1}} \frac{v_{D_{1}}^{2}}{2 g}=\frac{f}{D_{2}} \frac{v_{D_{2}}^{2}}{2 g} \Leftrightarrow  \tag{8}\\
& \Leftrightarrow \quad \tilde{v}_{c f}\left(D_{1}\right)=\sqrt{\frac{f}{g D_{1}}} v_{D_{1}}=\sqrt{\frac{f}{g D_{2}}} v_{D_{2}}=\tilde{v}_{c f}\left(D_{2}\right)
\end{align*}
$$



Figure 6. Clearing velocities of large individual bubbles.


Figure 7. Clearing velocities of large individual bubbles including friction factor.

Assuming a relative wall roughness of $10^{-4}$ for all transparent pipes, the bubble clearing velocity including friction as a function of the downward angle, shown in Figure 7, becomes nearly independent of the pipe diameter. The analysis of these observations may proceed from two perspectives. One possibility is that the result in Figure 7 is a coincidence and the surface tension or Eötvös number is still the dominant factor to explain the observed variation in the clearing velocity at different pipe diameters. The other possibility is that the hydraulic grade line is the dominant factor to explain the variation in the clearing velocity. A numerical analysis of the applicable momentum balance should confirm whether the friction or the surface tension explains the clearing velocity variation.

## Gas volume clearing

The results on clearing velocities of gas volumes, Bliss (1942) and Lubbers (2007a), are intercomparable, as illustrated in Figure 3.

A remarkable difference between Figure 6 and Figure 3 is the decreasing trend, measured by Lubbers (2007) at pipe angles steeper than $10^{\circ}$. The flow regime at the clearing velocity explains this apparent inconsistency. If the liquid velocity exceeds Lubbers' clearing velocity, then the gas is transported as stable plugs downstream of the aeration zone of the hydraulic jump. These stable plugs are significantly smaller than the large injected pockets. Since the stable plug size and plug clearing velocity both reduce with the pipe angle, Lubbers' clearing velocity reduces as well. This process is not included in the experiments with individual gas pockets, injected in the slope. Hence the experimental facilities with gas injection upstream of the downward slope provide a more complete picture of the transport processes in the pipe slope.

## CONCLUSIONS

A critical review of the existing literature on the clearing velocity of gas pockets in downward sloping pipes has revealed new information on the gas pocket clearing velocity (Bliss 1942) and the stable plug clearing velocity (Kent 1952).

The paper has shown that the wide spread in available correlations for the clearing velocity is caused by:

- different diameters smaller than 200 mm . A pipe diameter of 200 mm or greater is required to eliminate the diameter influence,
- wrongful comparison of different processes, i.e.
- individual gas pocket transport has been compared with gas volume clearing and
- initiation of gas transport has been compared with complete gas clearance.

The existing literature has been analyzed and integrated to explain the opposing trends of the clearing velocity at increasing pipe angle. If the gas pockets are initiated upstream of the downward slope, then the clearing velocity drops at pipe angles greater than $10^{\circ}$ to $20^{\circ}$. This reduction in clearing velocity is caused by the flow regime with stable plug flow. These stable plugs become smaller at steeper downward slopes, requiring smaller velocities to transport the stable plugs along the pipe soffit.

An intercomparison of the single gas pocket clearing velocities shows a systematic increase of the clearing velocity at a pipe diameter increase from 45 mm to 150 mm . An alternative
dimensionless clearing velocity, defined as $\tilde{v}_{c f} \equiv \sqrt{\frac{f}{g D}} v_{c}$, seems to explain the clearing velocity variation with pipe diameter, but a numerical analysis still has to confirm this analysis from the available experimental results.

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