

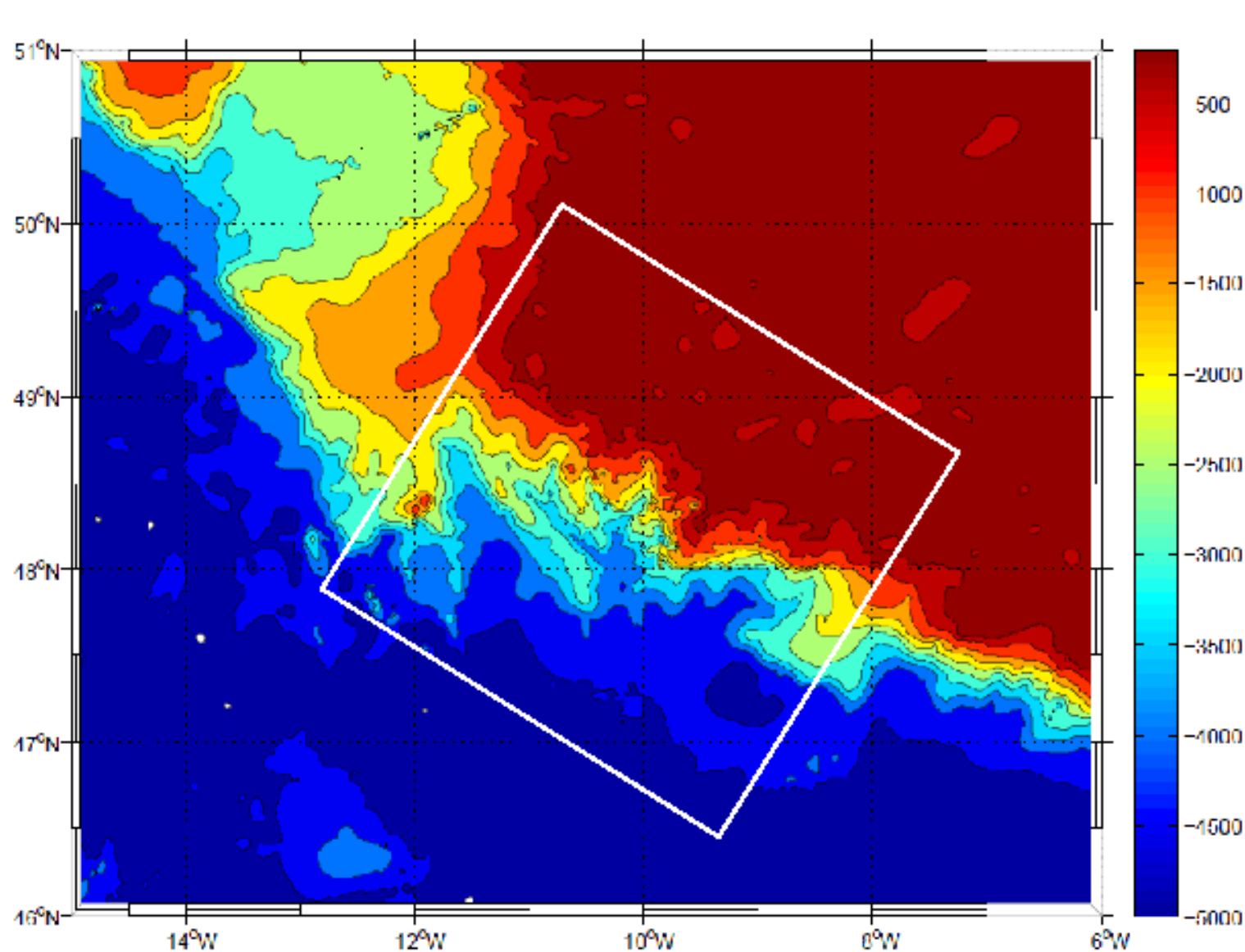
# 3D Numerical Simulation of Baroclinic Tides in the Celtic Sea

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# OUTLOOK:

1. Preliminary analysis
  - Bottom topography and stratification
  - Tidal forcing
  - Linear analysis of the “body force”
2. Model set up
3. Simulation of barotropic tide
  - Analysis of tidal ellipses
  - Setting the barotropic tidal forcing in MITgcm
4. Modelling of baroclinic tide
  - “Averaged bottom” experiment
  - “Flat-shelf” experiment
  - Experiments with real topography
5. Investigation of transport between deep and shallow parts
6. Conclusions

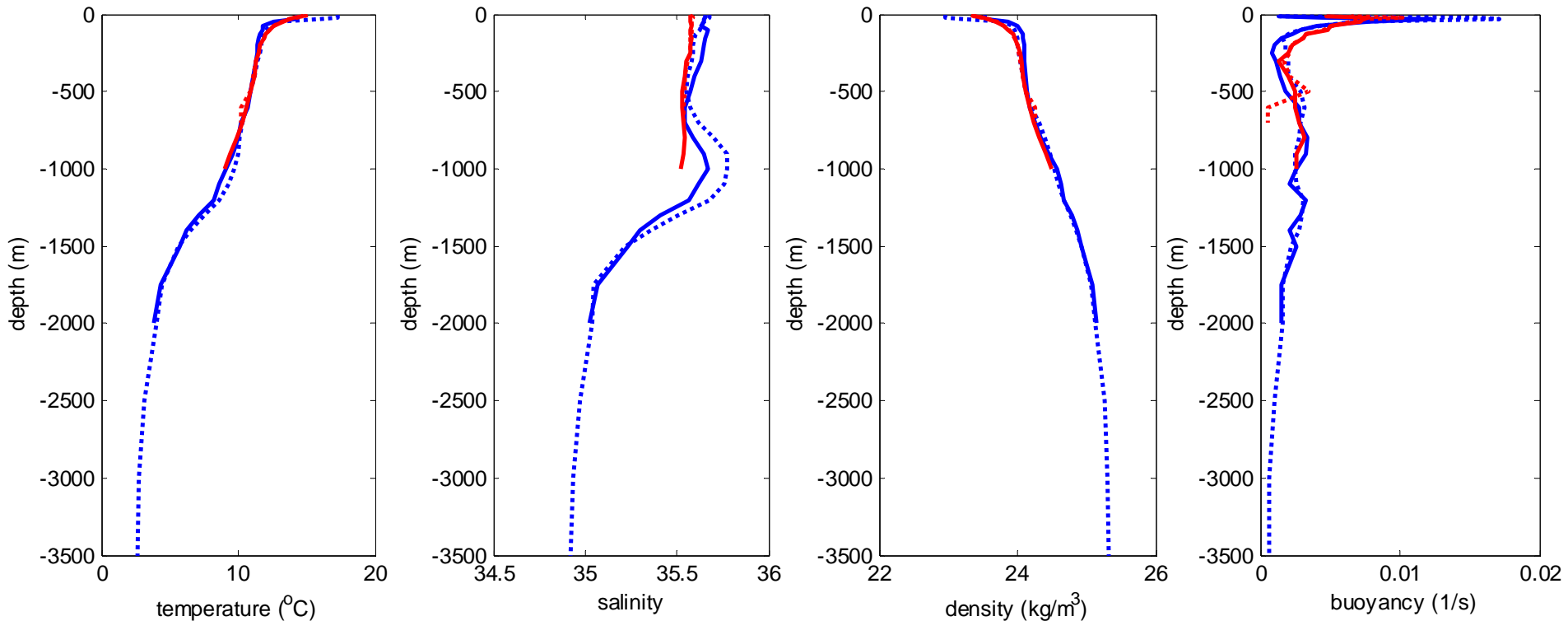
# 1. Preliminary analysis: Bottom topography



Bottom topography of the Celtic Sea.  
The model domain is shown by a rectangular box.

# 1. Preliminary analysis: Stratification

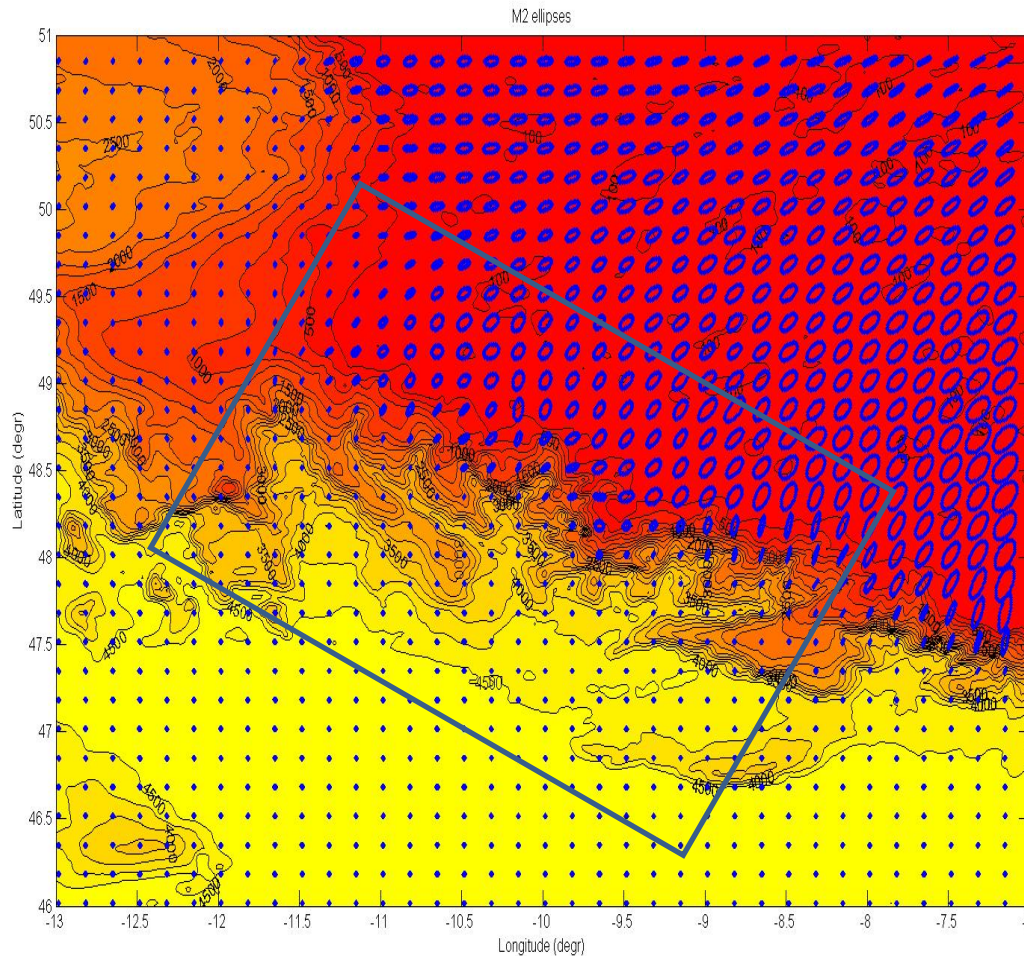
Four temperature (T) and salinity (S) profiles were used for setting background stratification. Summer (June) deep-water stations were taken from the Levitus's data base. **Averaged** profiles T(z) and S(z) were used for setting background stratification.



Calculated density and buoyancy frequency profiles reveal seasonal and main pycnoclines at the depths of ~40 m and ~1000 m, respectively.

# 1. Preliminary analysis: Tidal forcing

A Matlab code was written to build tidal ellipses from TPXO7.1 model<sup>1</sup> predictions for 11 tidal constituents. The dominant M2 tidal constituent is presented below. The ratio of major to minor axes, as well as inclinations of ellipses vary over the model domain.



<sup>1</sup>Egbert, G.D., and S.Y. Erofeeva, 2002: Efficient inverse modeling of barotropic ocean tides, *J. Atmos. Oceanic Technol.*, **19**(2), 183-204.

# 1. Preliminary analysis: Linear analysis of the “Body Force”

Linear theory can predict potential areas of internal waves (IWs) generation. Presenting the tidal wave field as a superposition of barotropic  $(u_b, v_b, w_b, P_b)$  and baroclinic  $(u_s, v_s, w_s, \rho_s, P_s)$  tidal components, one can split the problem of baroclinic tidal generation into two sub-tasks (2D case is presented here):

For barotropic tide,

$$(u_b)_t - fv_b = -(P_b)_x / \rho_0,$$

$$(v_b)_t + fu_b = 0,$$

$$(w_b)_t = -(P_b)_z / \rho_0,$$

$$(u_b)_x + (w_b)_z = 0,$$



$$w_b = -u_b Hz \left( \frac{1}{H} \right)_x \exp(i\sigma t);$$



For baroclinic tide,

$$(u_s)_t - fv_s = -(P_s)_x / \rho_0,$$

$$(v_s)_t + fu_s = 0,$$

$$(\rho_s)_t + w_s \rho_{0z} = -w_b \rho_{0z},$$

$$(u_s)_x + (w_s)_z = 0,$$



$$g\rho_s + i\rho_0 w_s \frac{N^2(z)}{\sigma} = i\rho_0 u_b Hz \frac{N^2(z)}{\sigma} \left( \frac{1}{H} \right)_x.$$

The RHS of this inhomogeneous PDE gives the formula for the “Body Force”, which controls the location and intensity of the isopycnal displacements, viz.

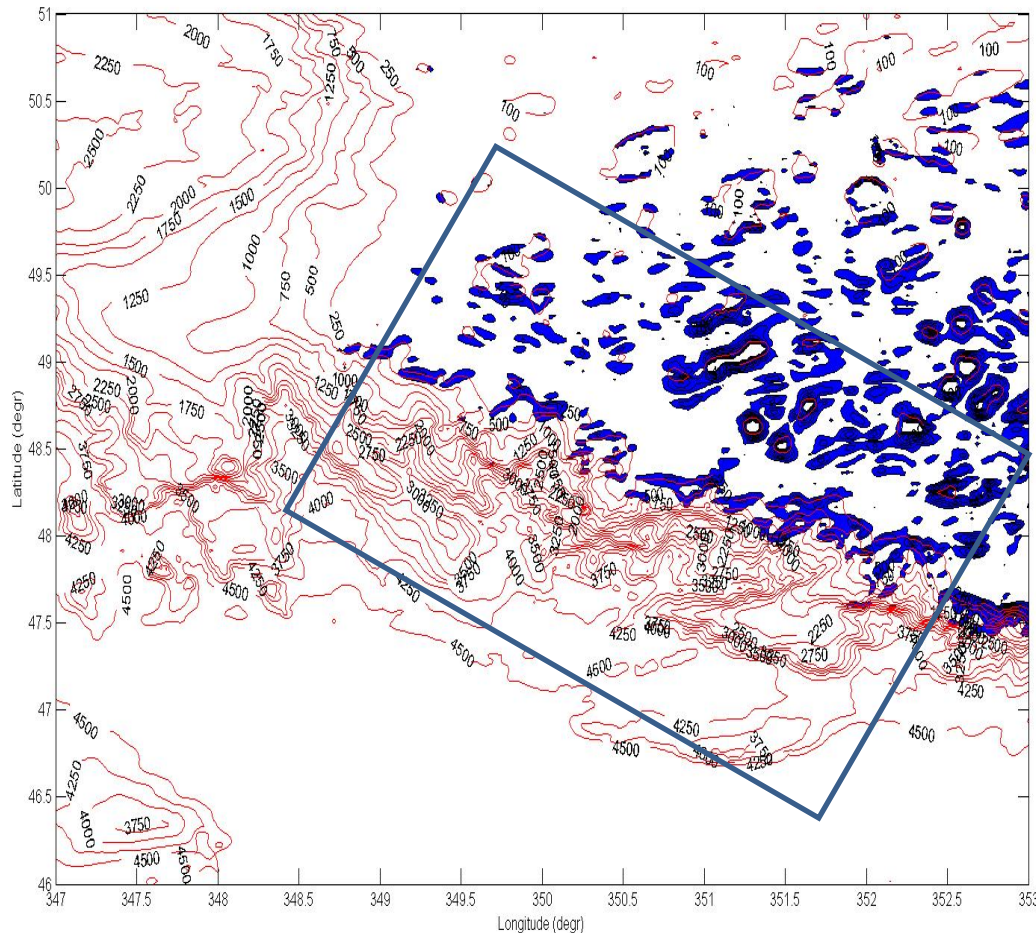
$$F = \rho_0 u_b Hz \frac{N^2(z)}{\sigma} \left( \frac{1}{H} \right)_x.$$

# 1. Preliminary analysis: Linear analysis of the “Body Force”

Substitution of the stratification and tidal velocities into the body force and its integration through the whole water column gives an “integral body force” which in a 3D case reads:

$$\Phi = \int_{H(x,y)}^0 F(x,y,z) dz = \rho_0 / \sigma [(u_b H dH^{-1} / dx)^2 + (v_b H dH^{-1} / dy)^2] \int_{H(x,y)}^0 N^2(z) z dz$$

The largest values of  $F$  are shown below in blue. They indicate the potential places of IWs generation.



## CONCLUSION:

### Linear theory predicts:

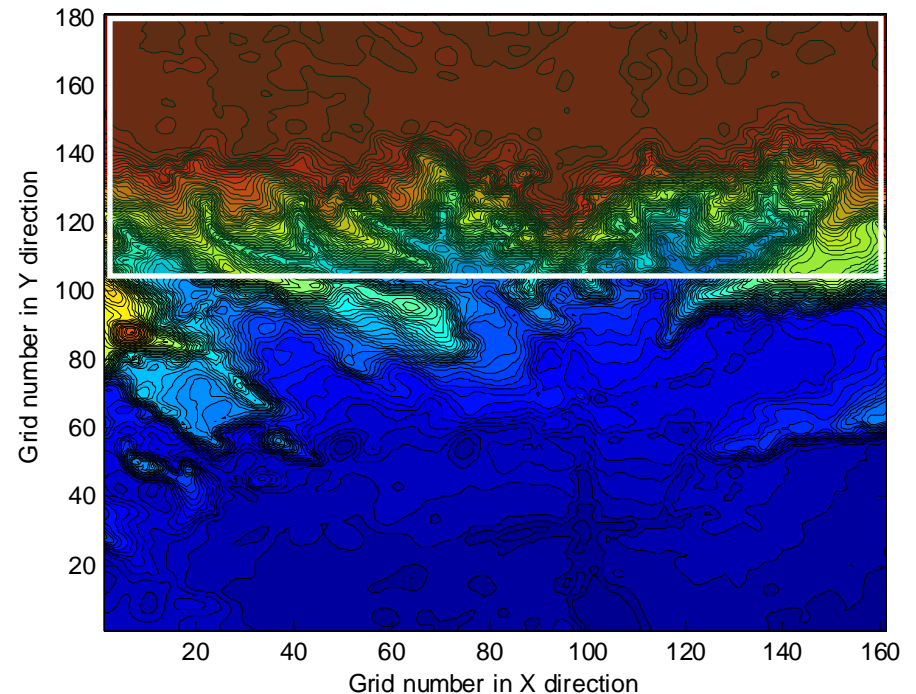
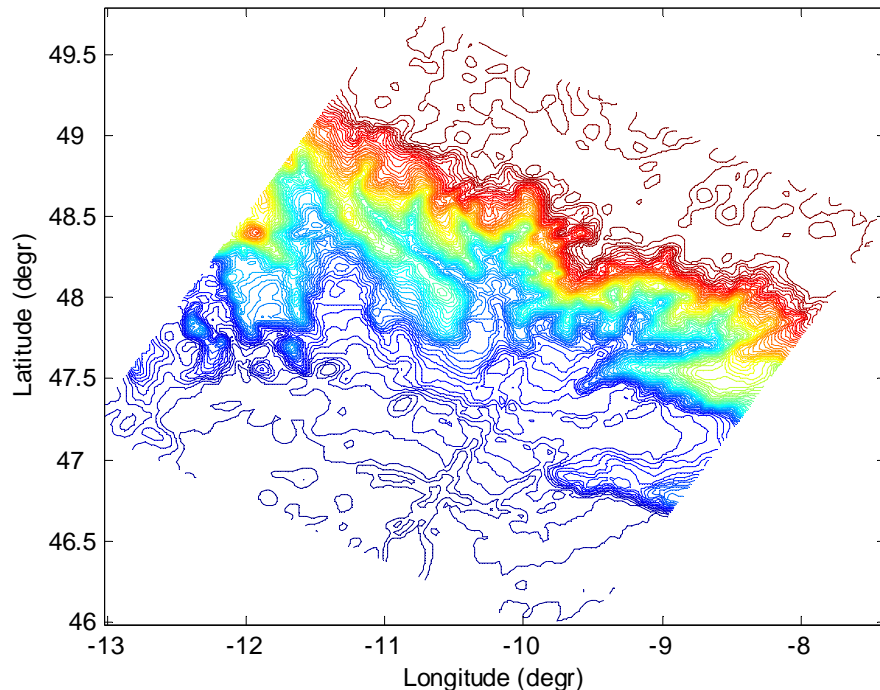
- 1) The generation area is not localised. There are a number of local “hot spots” randomly distributed across the area;
- 2) the largest value of the “body force” is not necessarily located in the slope area. Most of them are predicted on the shelf (expectation of complex of 3D structure in modelling and observations.) 7

## 2. Model set up

The model domain was rotated anticlockwise by 30 deg.

The “coarse-grid” runs were conducted on a grid  $161 \times 181$  with horizontal resolution  $\Delta x = \Delta y = 1852\text{m}$  (1 nautical mile).

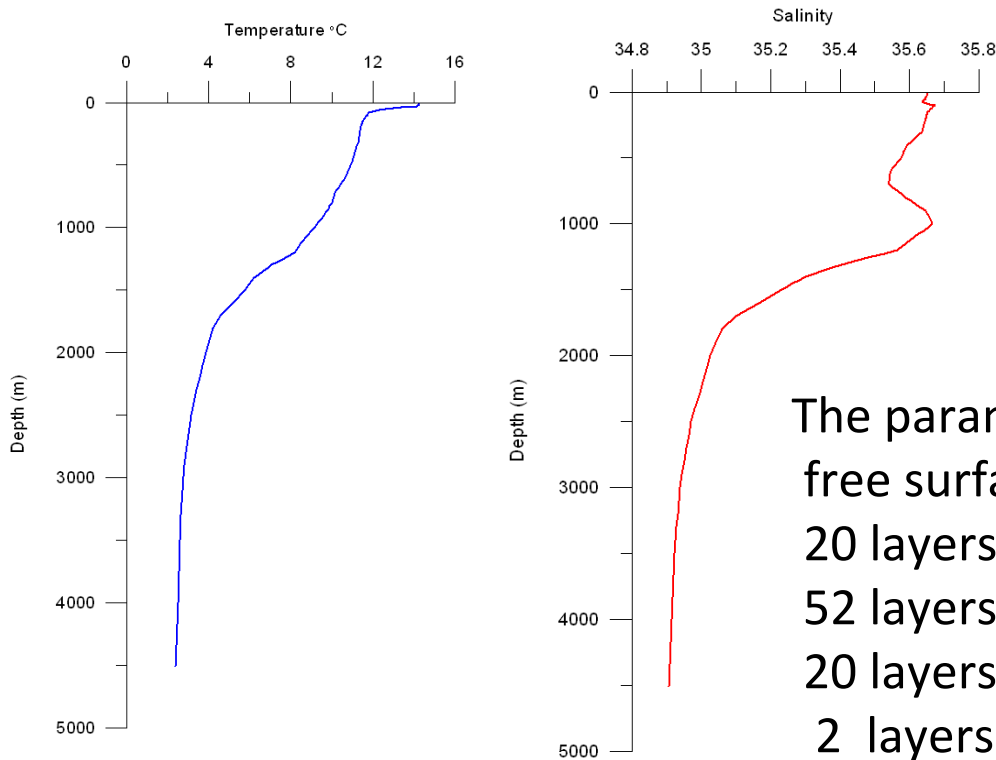
The “fine-grid” runs were conducted on a grid  $1281 \times 561$  (that partly covers initial domain) with horizontal resolution  $\Delta x = \Delta y = 234\text{m}$ .





## 2. Model set up

The following “background” temperature and salinity profiles were used in modelling. They were found by averaging of four summer profiles taken from the Levitus’s data set



### VERTICAL GRID

The parameters of irregular vertical grid from the free surface to the bottom were as follows:

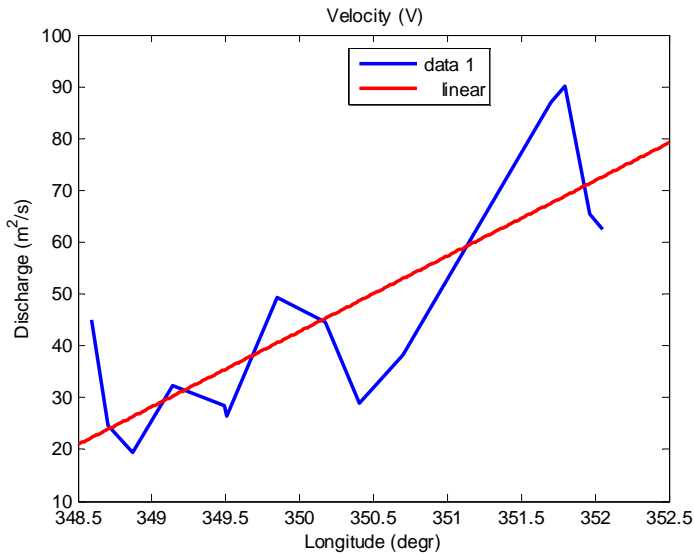
- 20 layers from 0m to 200m with  $\Delta z=10\text{m}$ ;
- 52 layers from 200m to 1500m with  $\Delta z=25\text{m}$ ;
- 20 layers from 1500m to 3500m with  $\Delta z=100\text{m}$ ;
- 2 layers from 3500m to 4500m with  $\Delta z=500\text{m}$ .

**Tidal forcing** was incorporated into MITgcm as an external forcing into RHS of the momentum balance equations. The new Fortran code based on the MITgcm allows using variety of form of barotropic tidal ellipses: different values of ratio of major to minor axis and inclinations.

### 3. Simulation of barotropic tides: Analysis of tidal ellipses

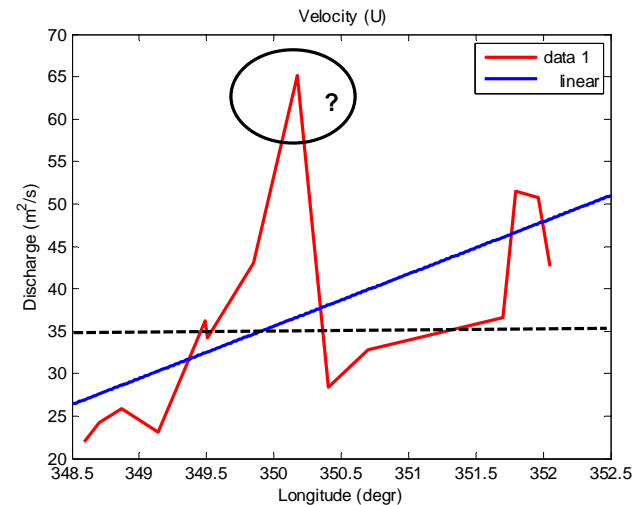
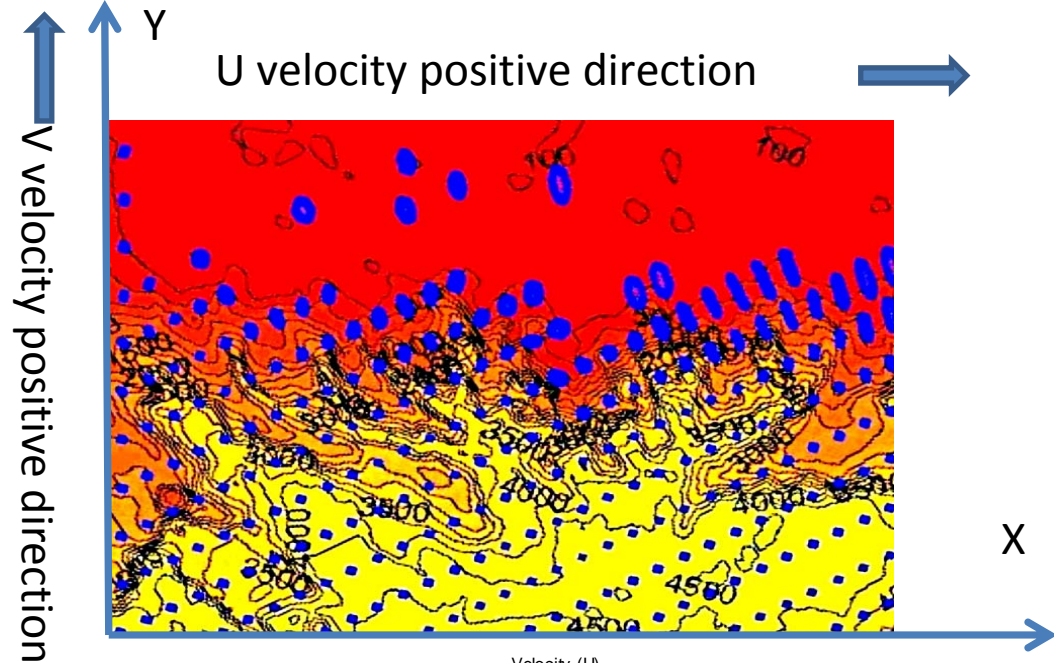
In order to initialize the numerical model correctly, tidal ellipses predicted by TPXO7.1 were analysed. Their structure is shown in the top-right figure (only ellipses at the depth greater than 200m are shown).

Amplitude of the across-shelf tidal water discharge (y-direction) increases eastward.

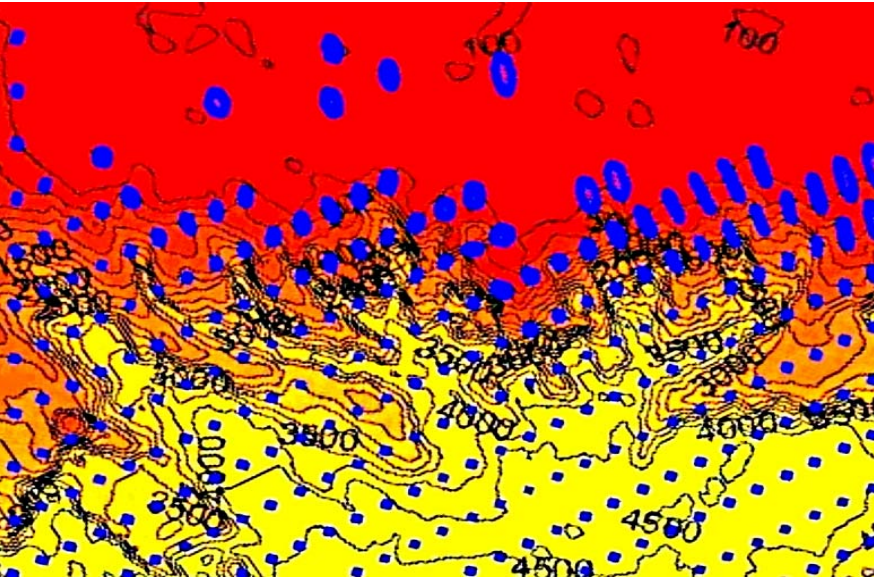


Amplitude of the along-shelf tidal water discharge (x-direction) reveals some spikes but can be taken as constant.

U velocity positive direction

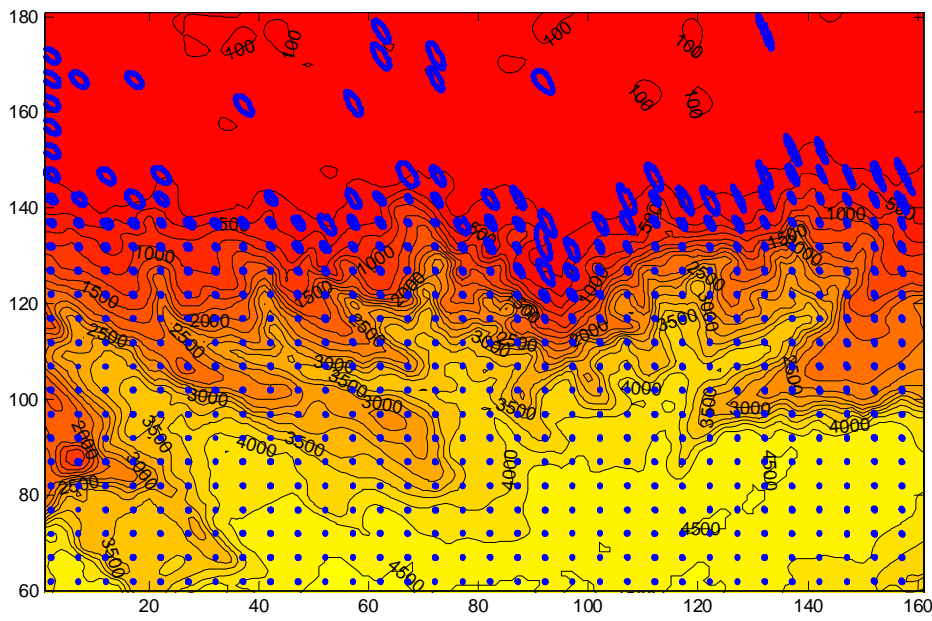


### 3.Simulation of barotropic tide: Setting the barotropic tidal forcing in MITgcm



M2 tidal ellipses predicted by TPXO7.1

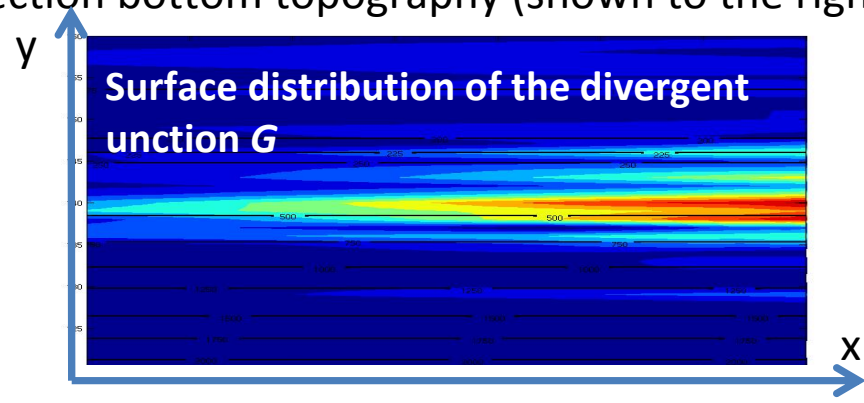
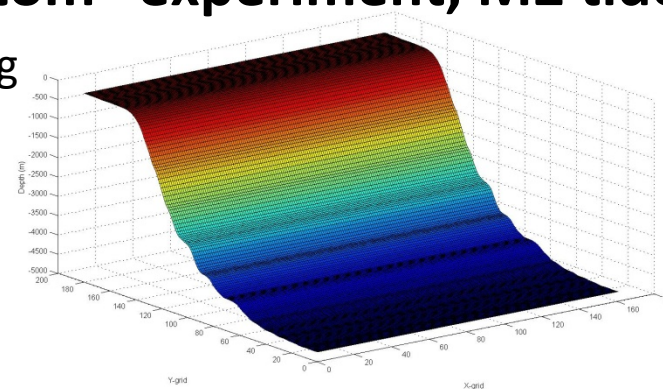
*(Presentation in both figures is restricted by 200 m isobath.)*



M2 tidal ellipses reproduced by MITgcm

# 4. Modelling of baroclinic tide: “Averaged bottom” experiment, M2 tide

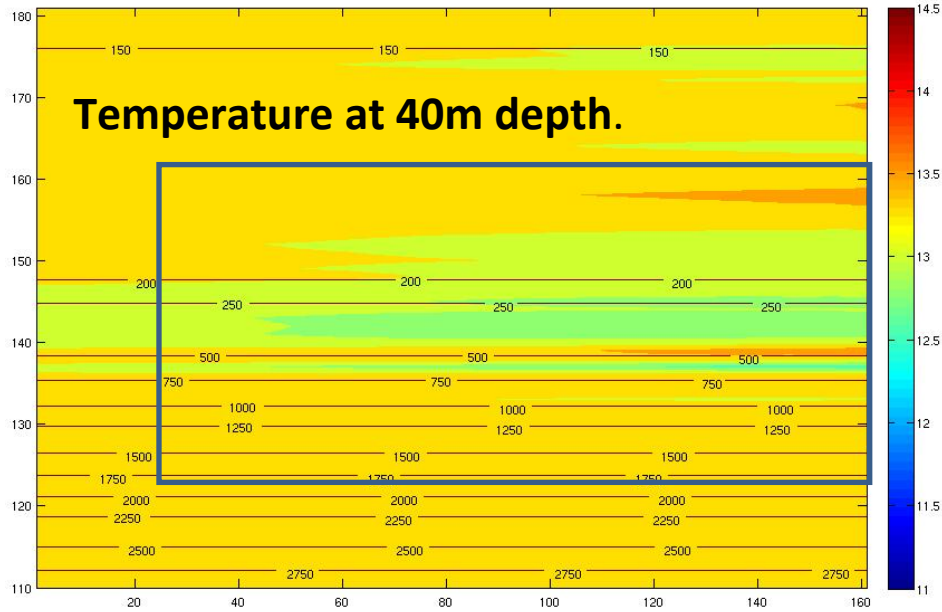
Cross-slope/shelf energy/mass transport can be assessed using a 2D model approach (without account of local 3D effects). In “averaged bottom” numerical experiment an averaged in x-direction bottom topography (shown to the right) was used.



The divergent function

$$G(x, y, 0) = \sqrt{(du / dx)^2 + (dv / dy)^2}$$

is used for interpretation of SAR images.



## CONCLUSION:

Spatial distribution of function G (left top panel) shows that the slope area with the depth less than 500 m is the place of the strongest baroclinic tidal signal.

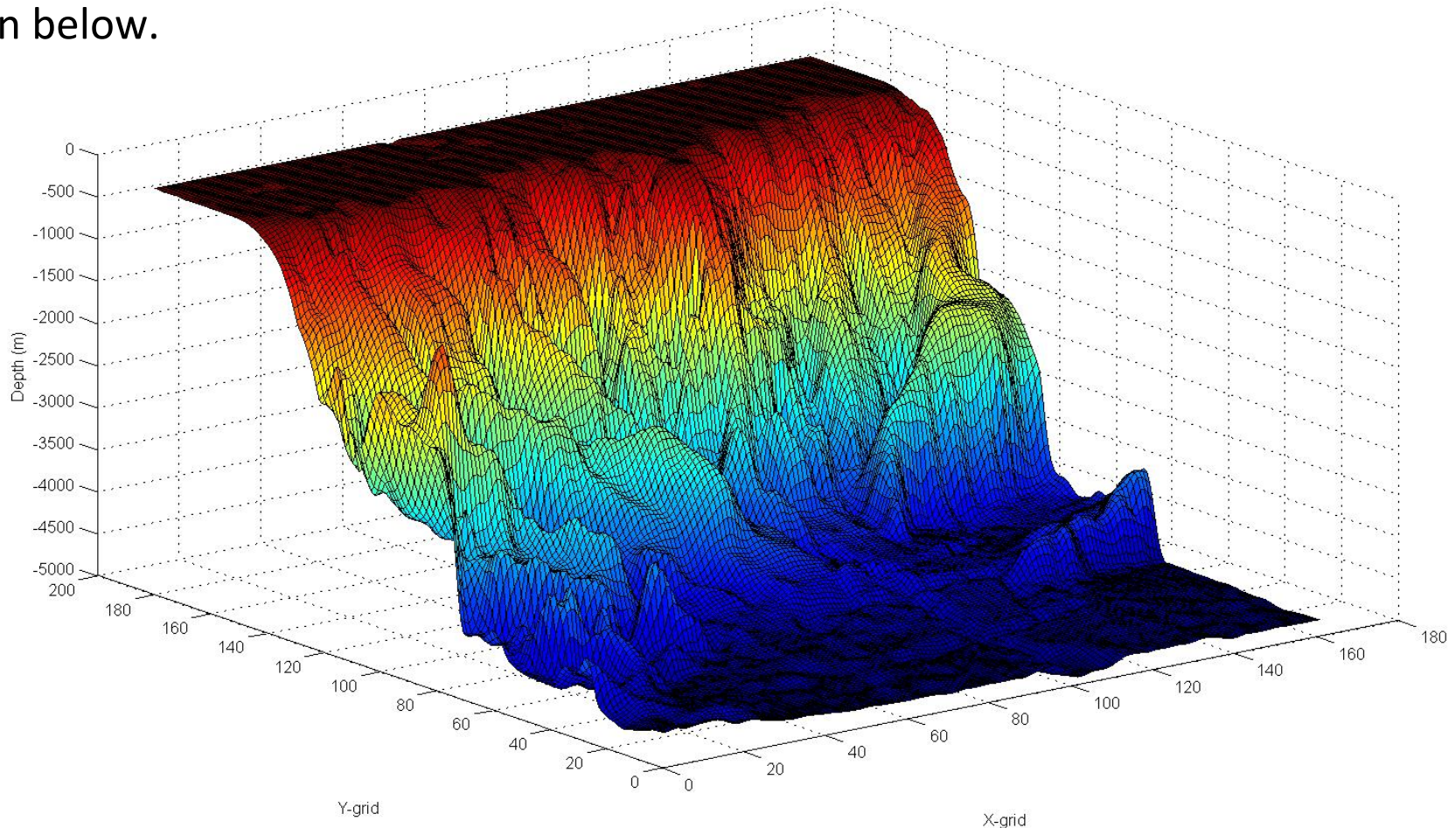
## EVIDENCE

- Its position coincides with the generation area predicted by the linear theory (slide 7)
- Greatest vertical displacements appeared in fully-nonlinear numerical modelling (left panel) also match the position of maximum  $G(x, y, 0)$ .

Blue colours correspond to isotherms elevations (cold water), red colours to isotherms depression (warm water).

## 4. Modelling of baroclinic tide: “Flat shelf” experiment

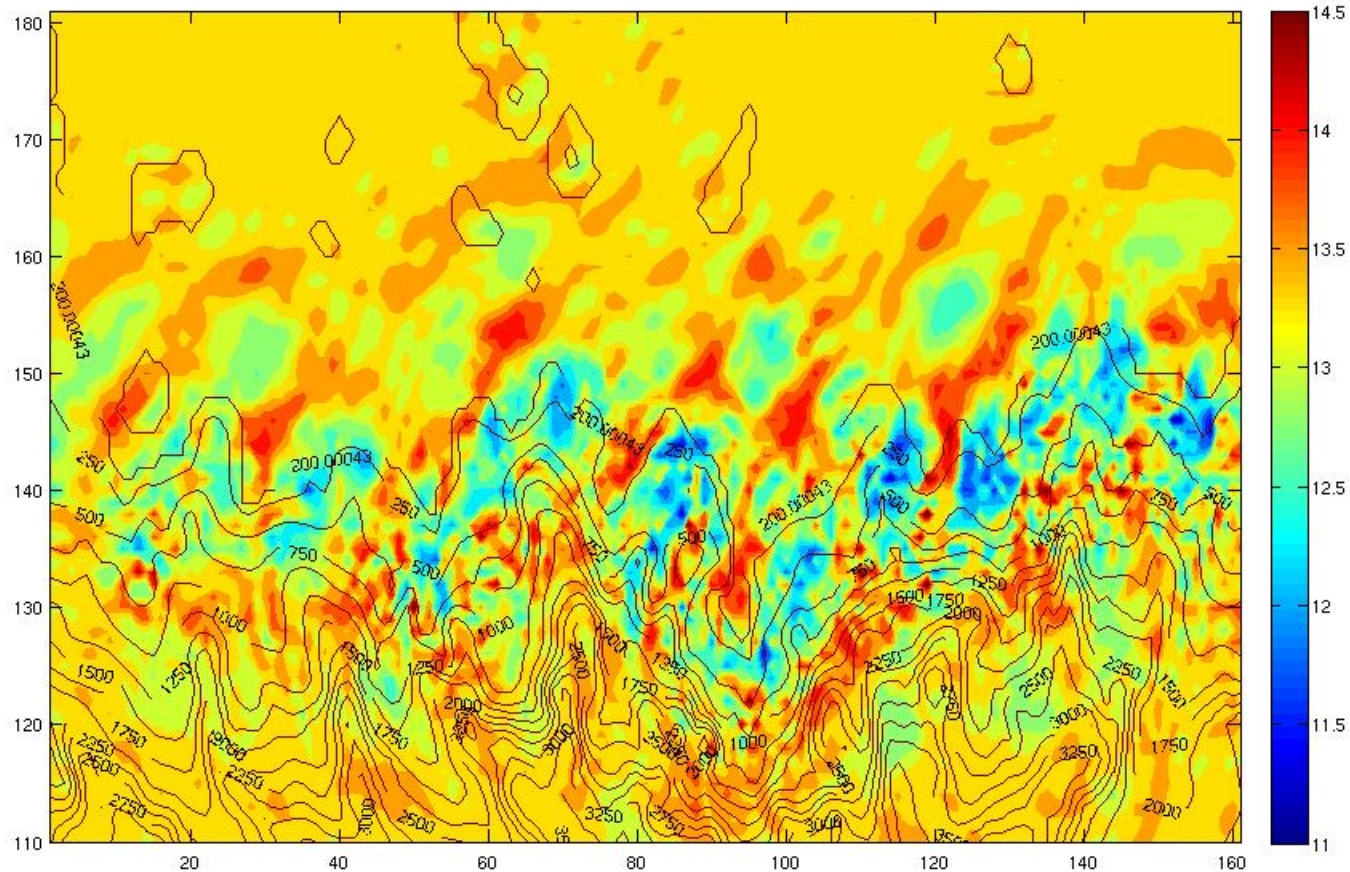
Next series of experiments was conducted to specify the role of corrugated slope topography in the generation mechanism. The influence of local sources of generation on the shelf was excluded by setting a formal restriction on the depth: all local bottom elevation with depth less than 200 m were eliminated. The “truncated” bottom is shown below.



As a **RESULT**, all underwater banks on the shelf were excluded from the analysis to show more clearly the evolution of the “slope-generated” waves.

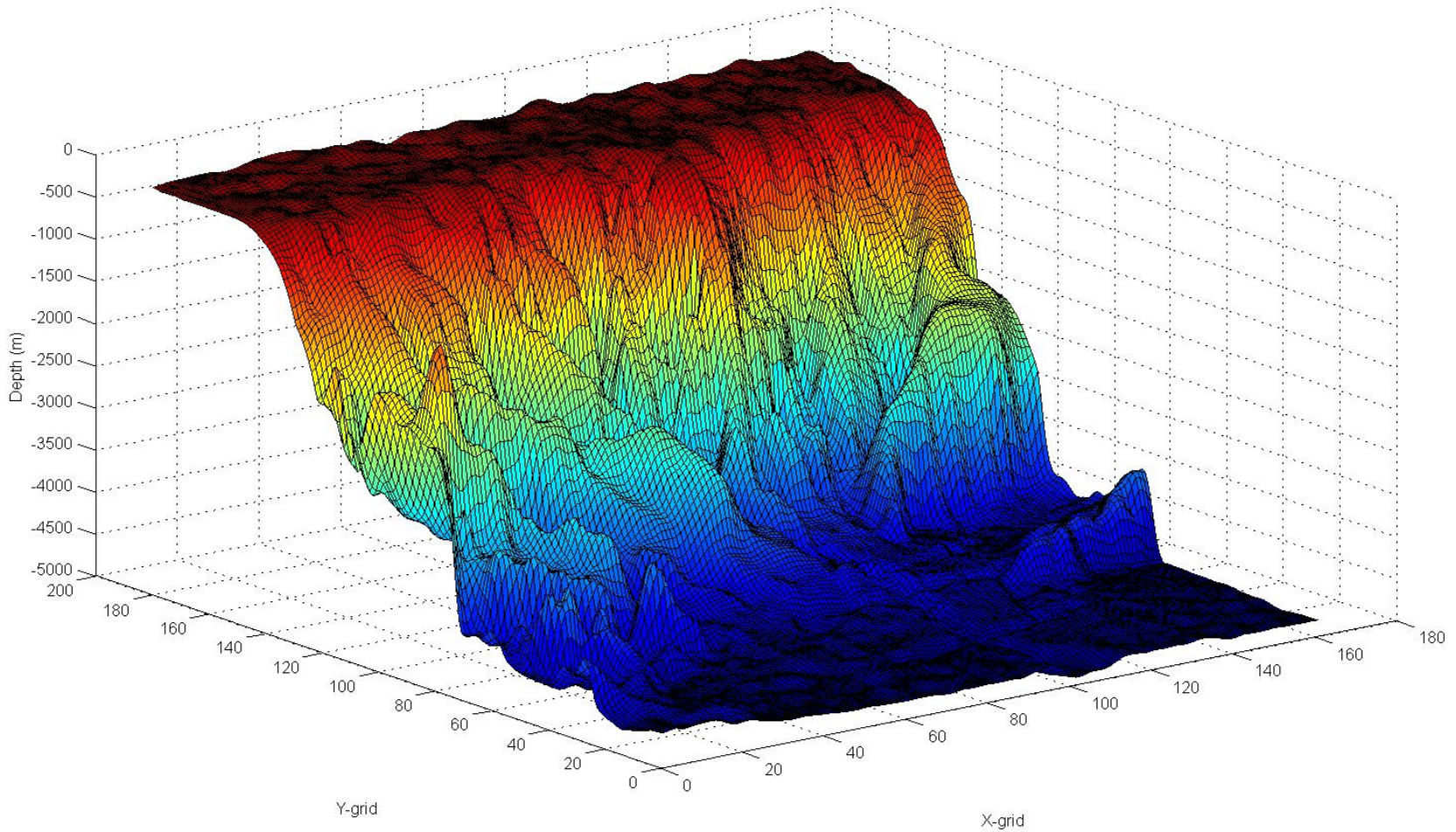
## 4. Modelling of baroclinic tide: “Flat shelf” experiment, M2 tide

Temperature field at the depth of main thermocline ( $\sim 40$  m).



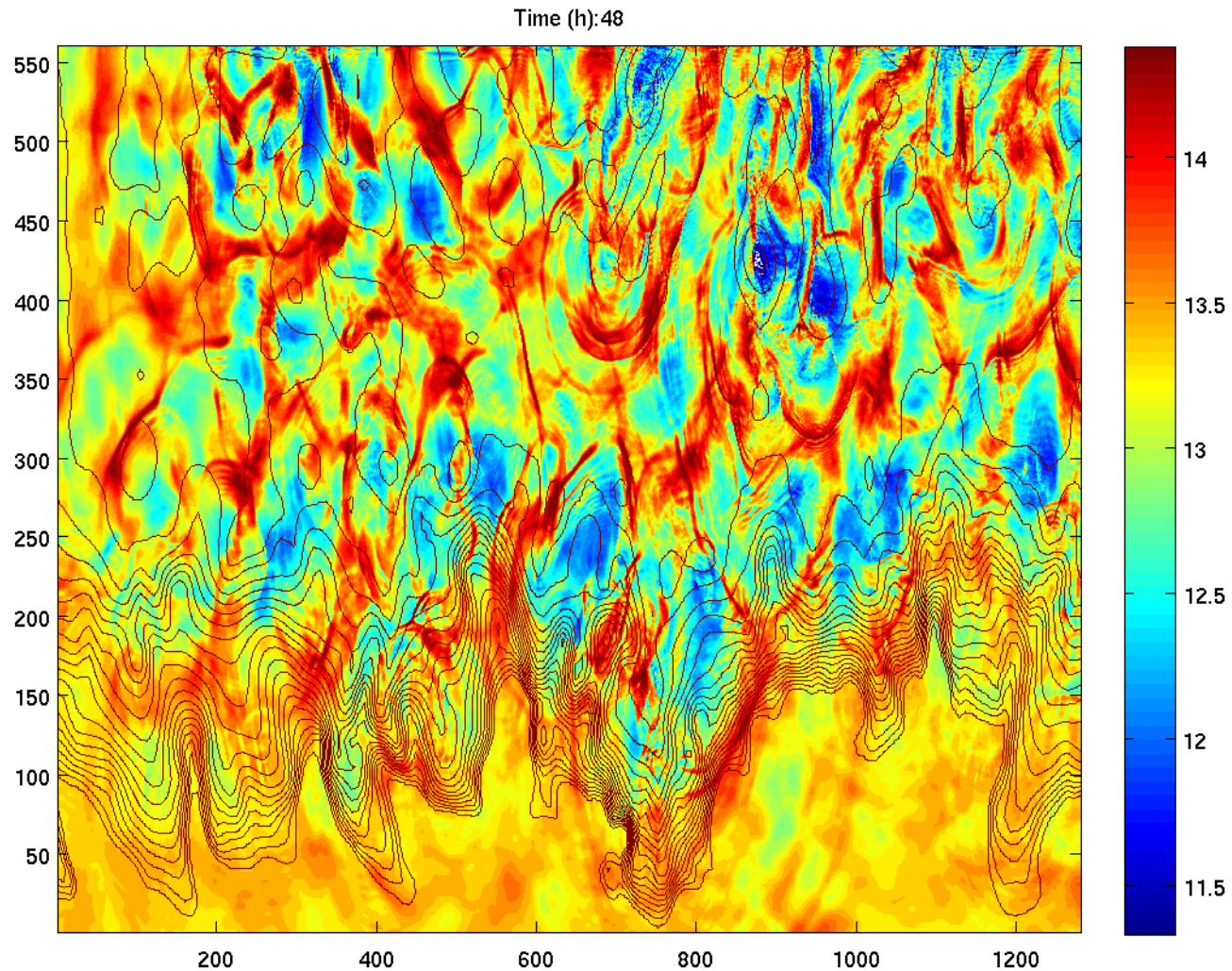
Blue colour shows isotherms elevations (cold water), red colours correspond to isotherms depression (warm water). Movie starts after the first tidal cycle was completed (model spin-up). The movie shows five tidal cycles with one hour temporal step.

## 4. Modelling of baroclinic tide: Experiments with real topography



The bottom topography used for “real topography” experiments.  
Source: ETOPO1 data base (1 minute spatial resolution).

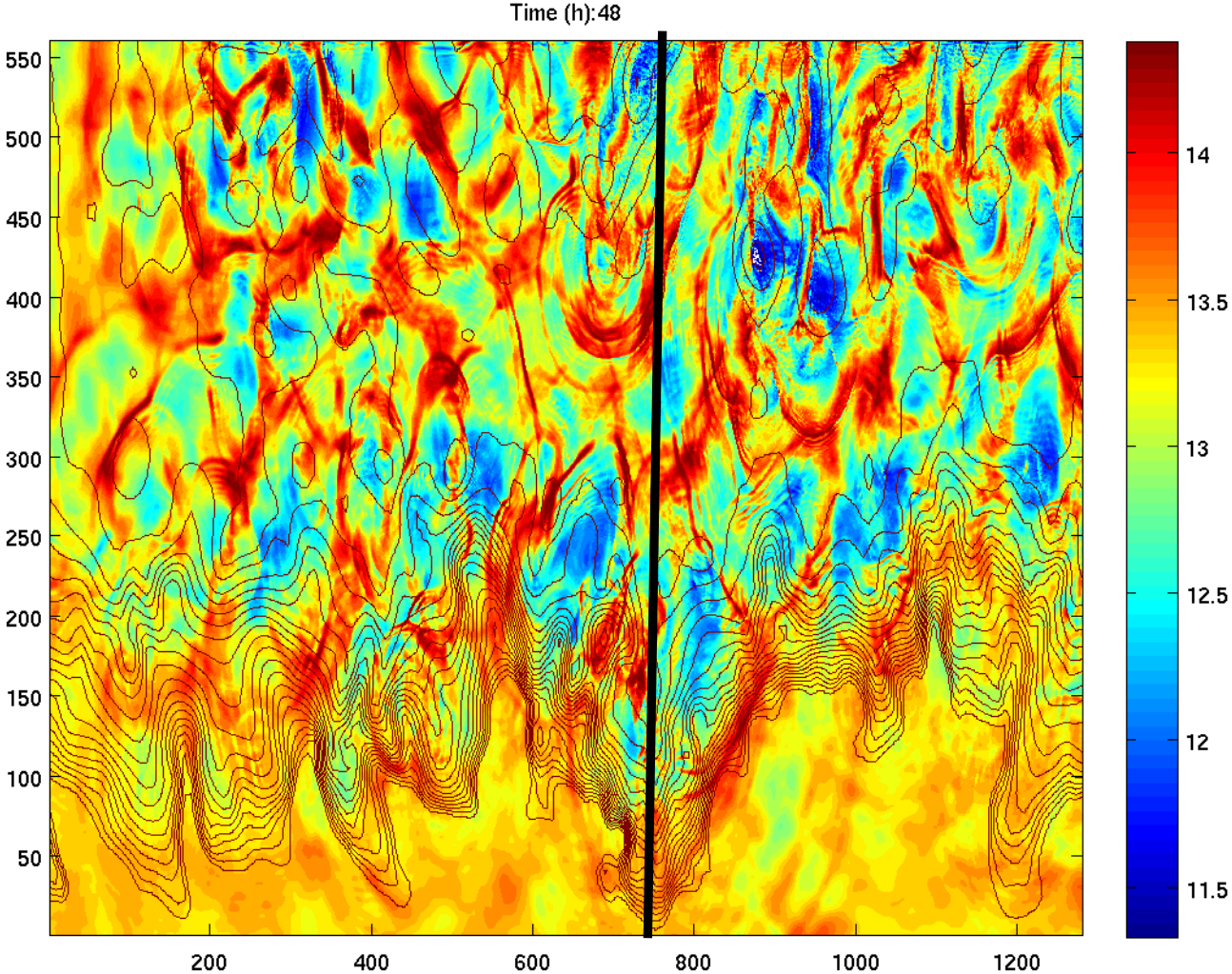
## Temperature field at the depth of main thermocline (~ 40 m).



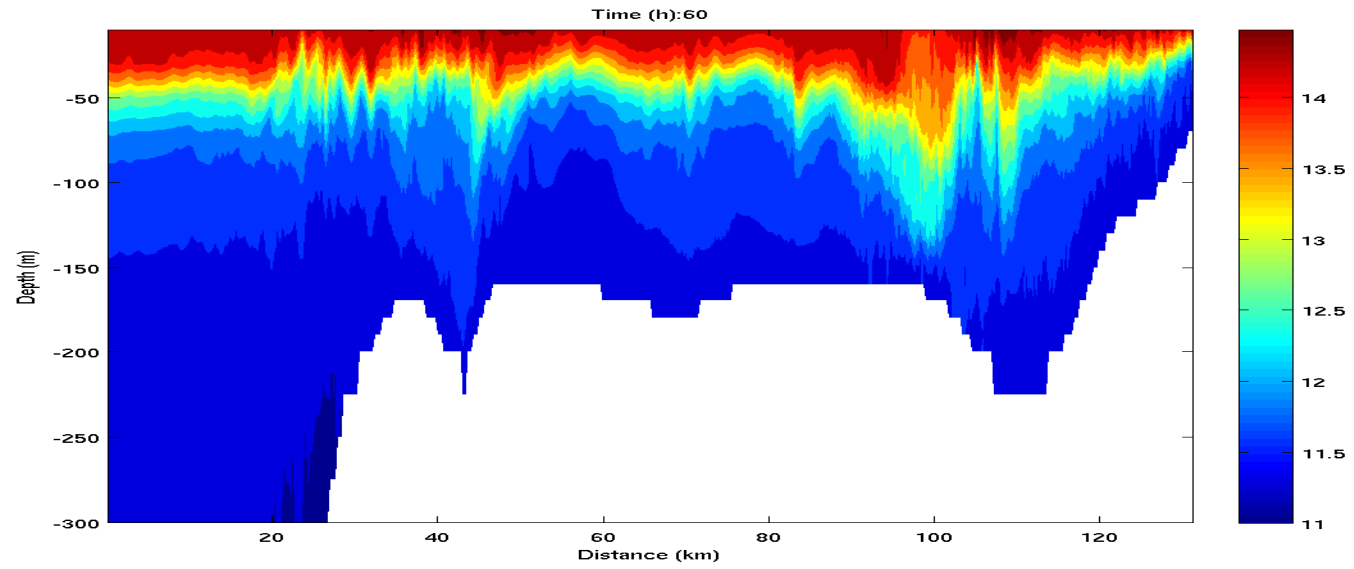
Blue colour shows isotherms elevations (**cold water**), red colours correspond to isotherms depression (**warm water**).



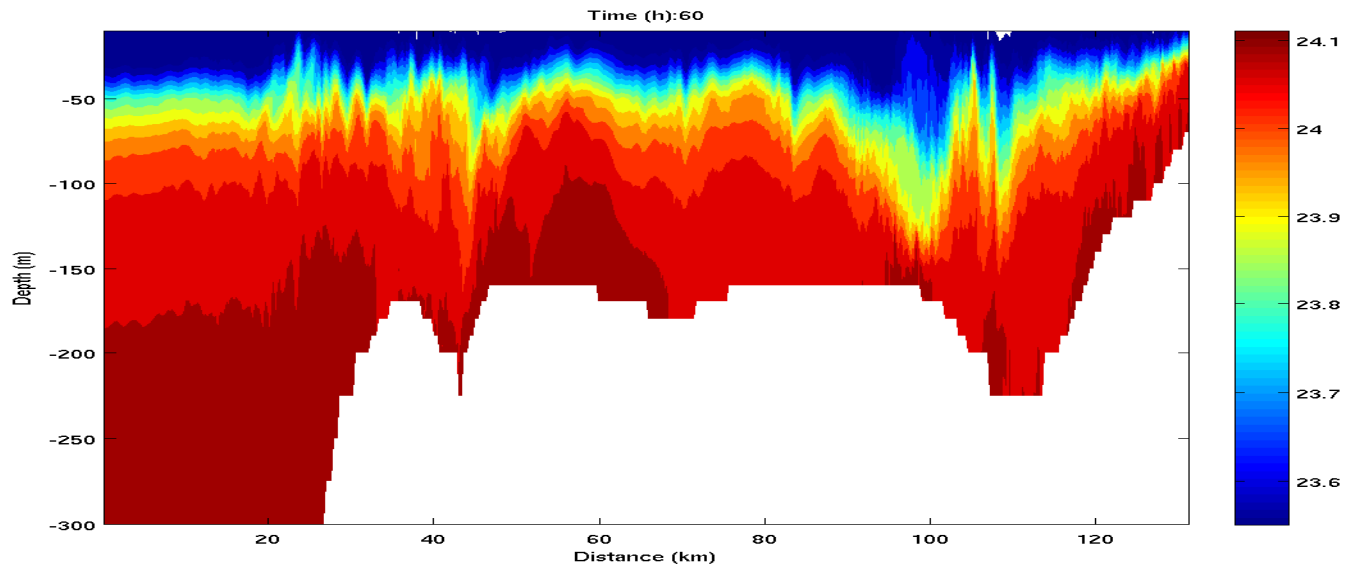
# Investigation of parameters of solitary waves



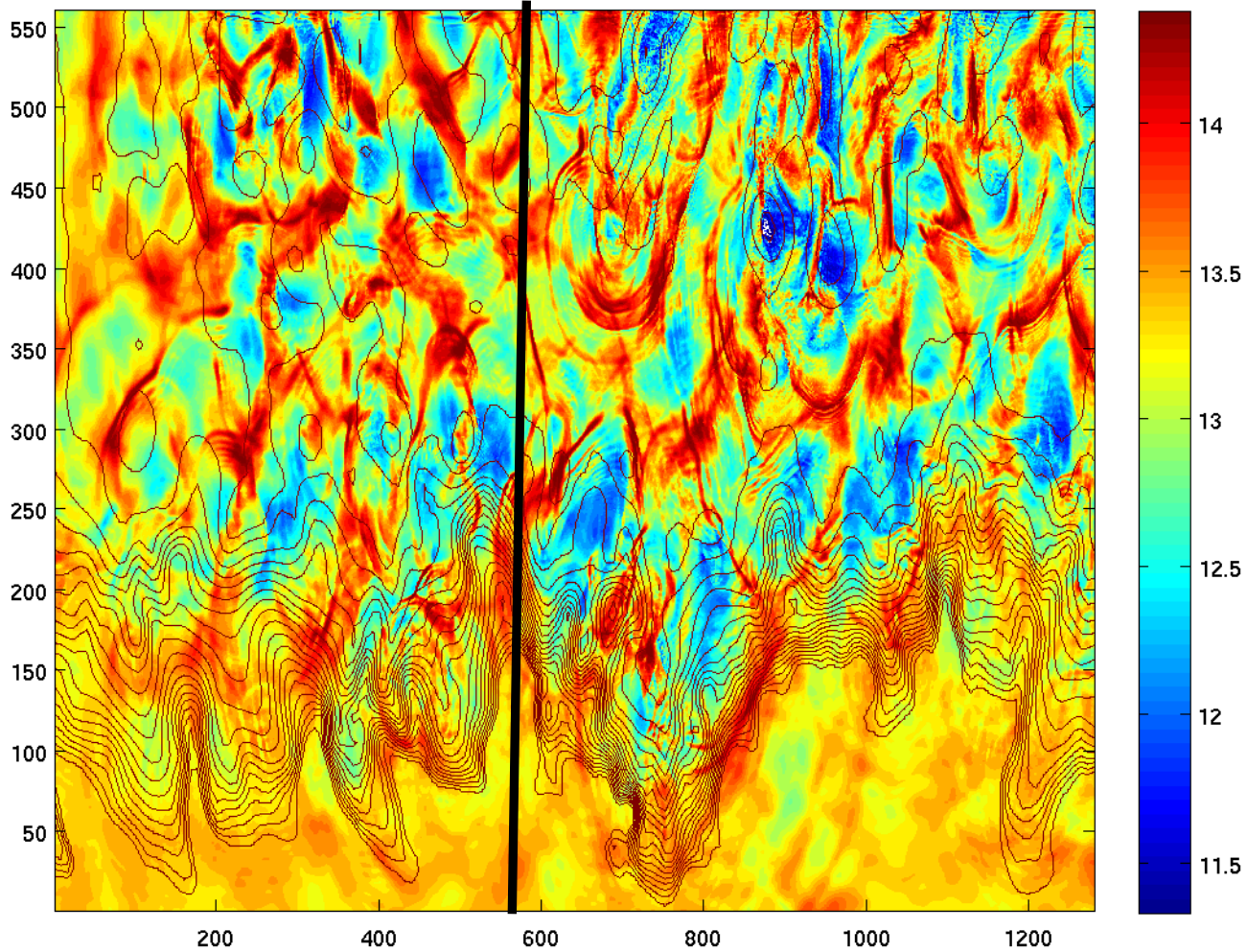
# Temperature



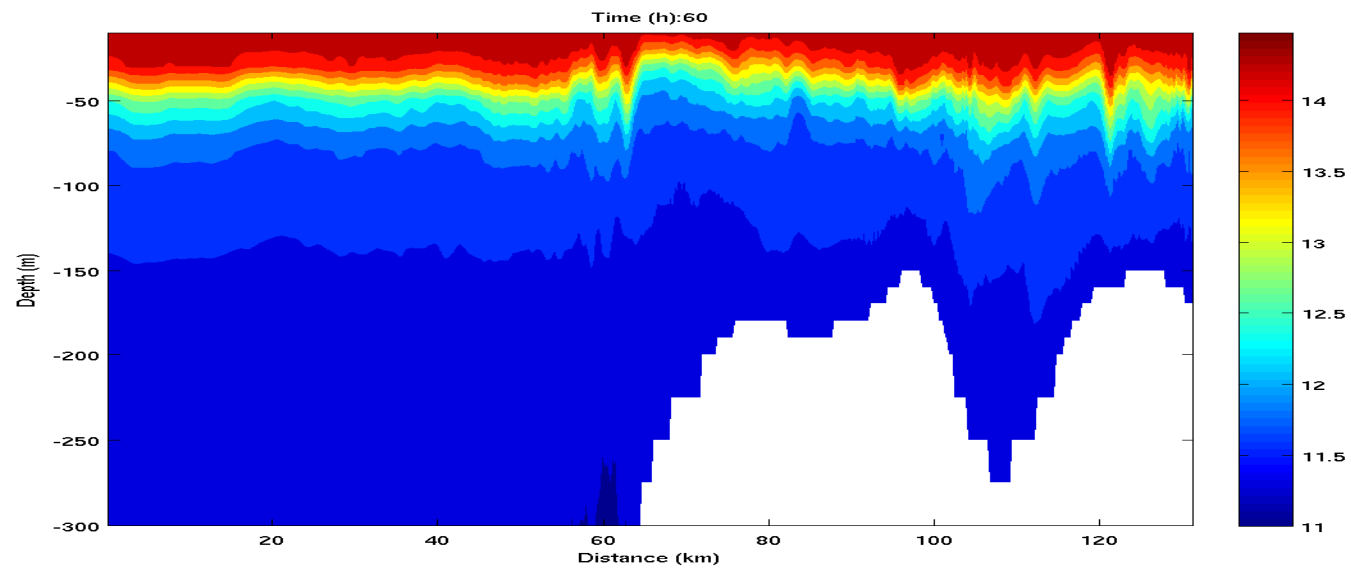
# Density



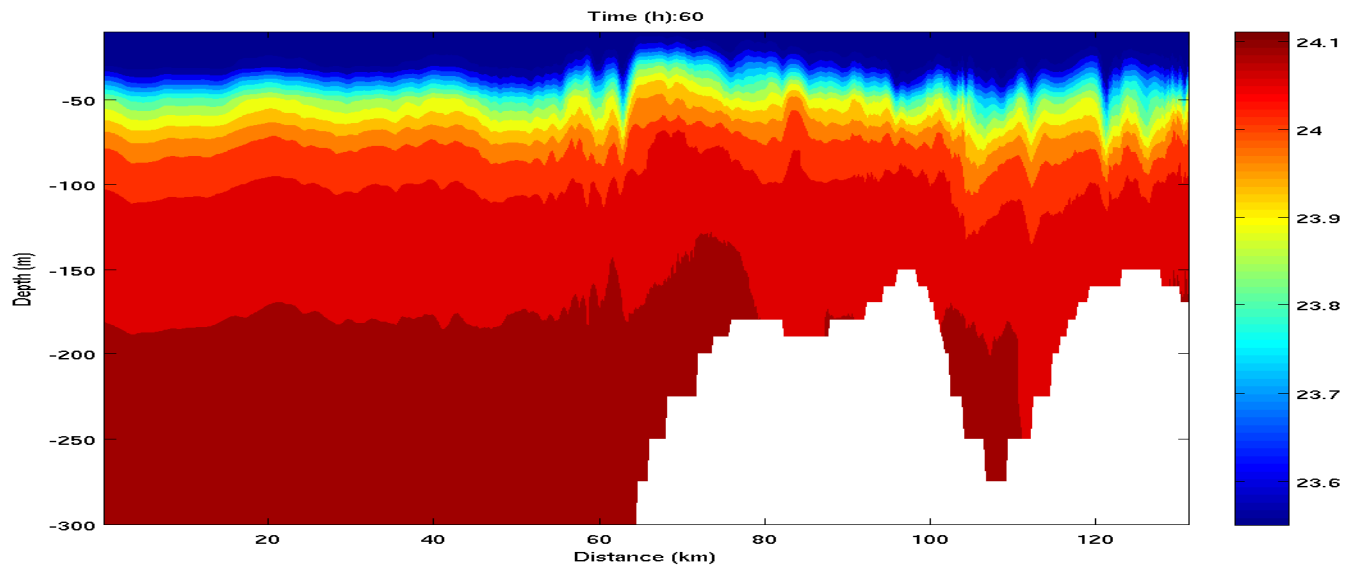
Time (h): 48



# Temperature



# Density



## 4. Transport of substances. Experiments with tracers

- 1. On-shelf transport of slope water lenses within the seasonal pycnocline

J. Hopkins,<sup>1</sup> J. Sharples,<sup>12</sup> J. M. Huthnance,<sup>1</sup>

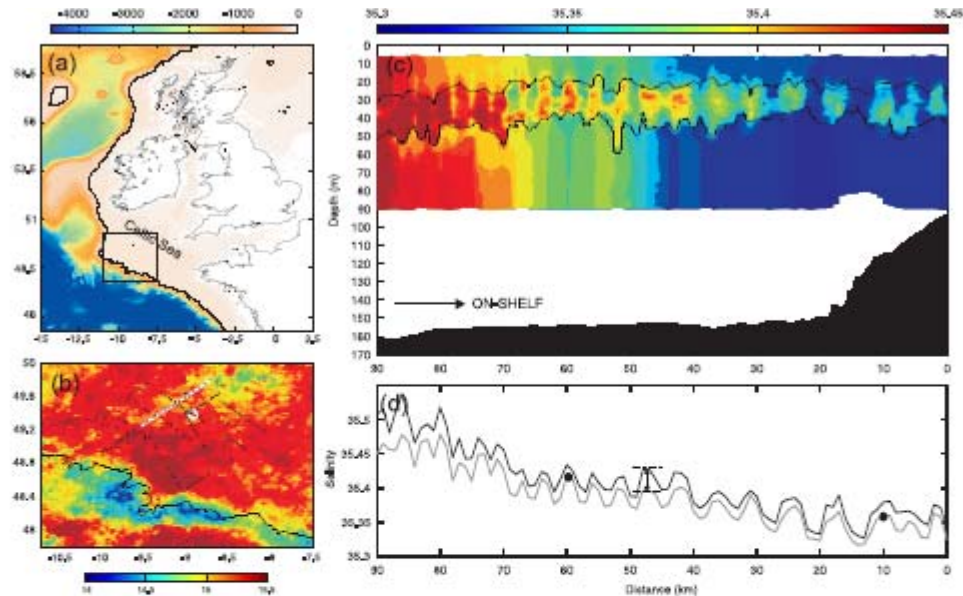
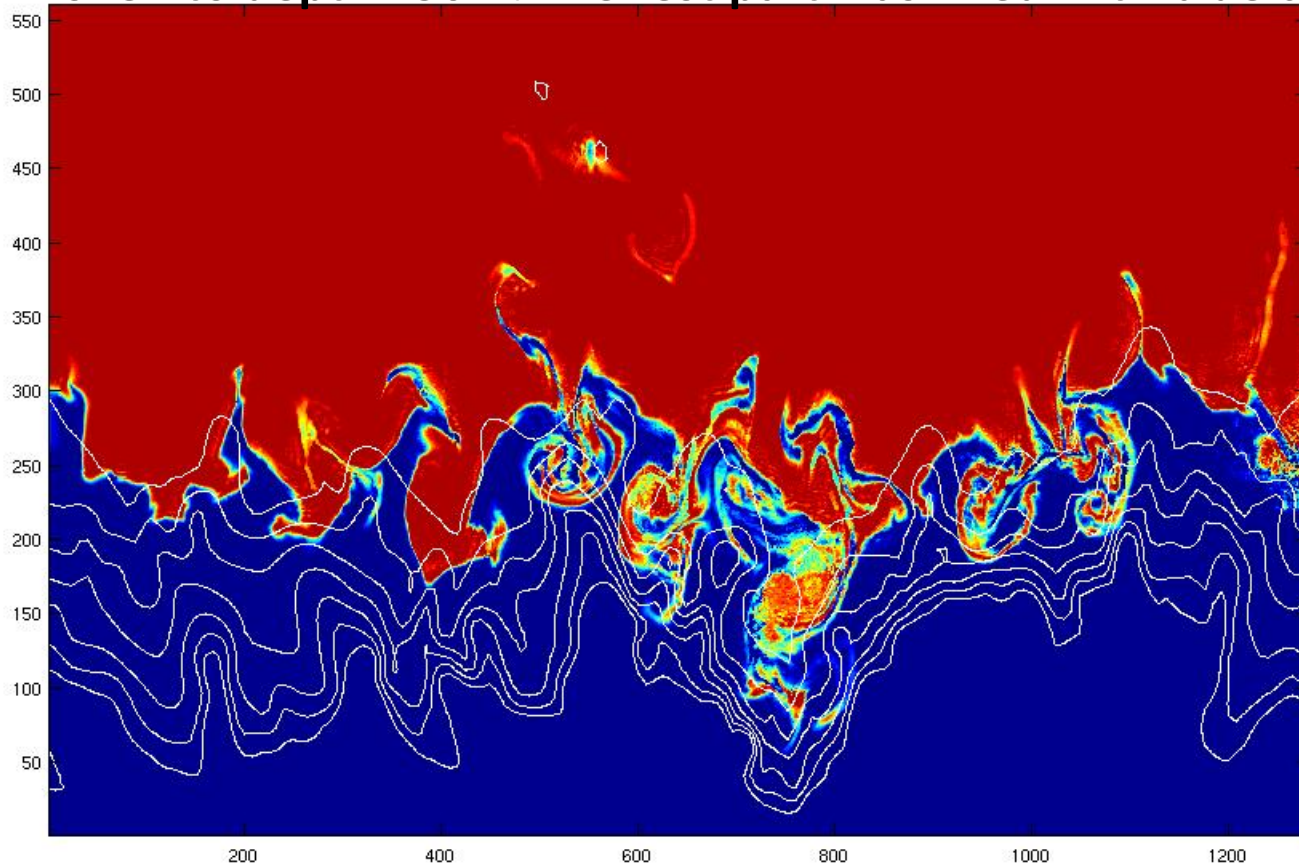


Figure 1. (a) Bathymetry of the NW European shelf (m) with 200 m shelf edge isobath contoured. (b) SST (°C) composite for June 2010 around survey area (box in (a)). Thick white line marks the Scanfish transect shown in (c). Black dots show the location of lenses over the survey area. IM1 and IM3 moorings are located within the open white circle. (c) Salinity along Scanfish transect. Contours are the top ( $\sigma_\theta=26.4$ ) and bottom ( $\sigma_\theta=27.2$ ) of the pycnocline. (d) Maximum (black) and mean (gray) salinity within the pycnocline. Vertical arrow indicates the range of peak pycnocline salinities from the CTD casts. Values used to estimate the salt flux are taken from the two dots.

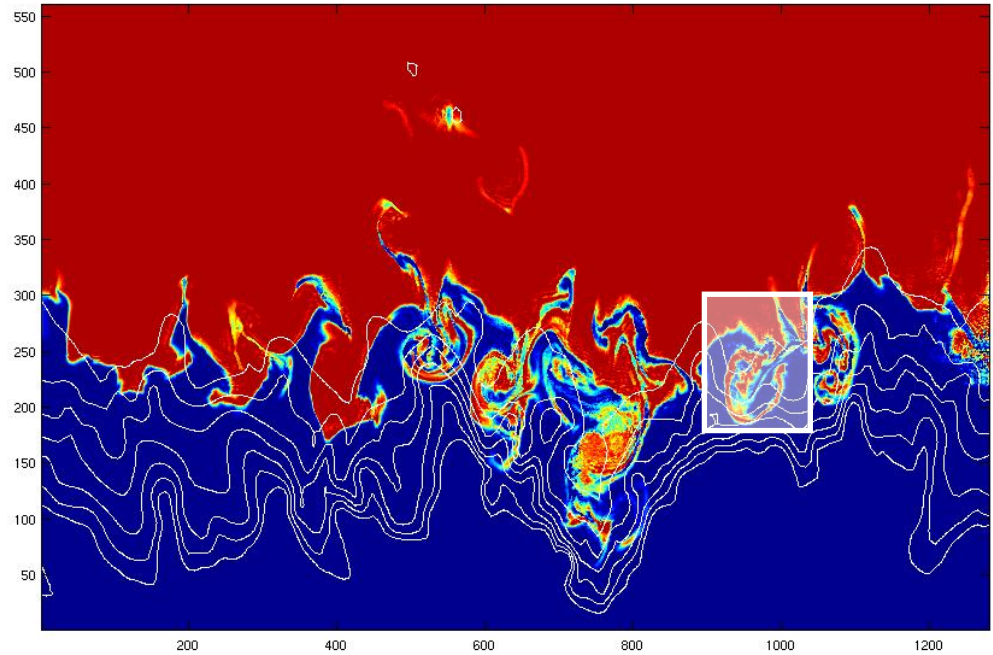
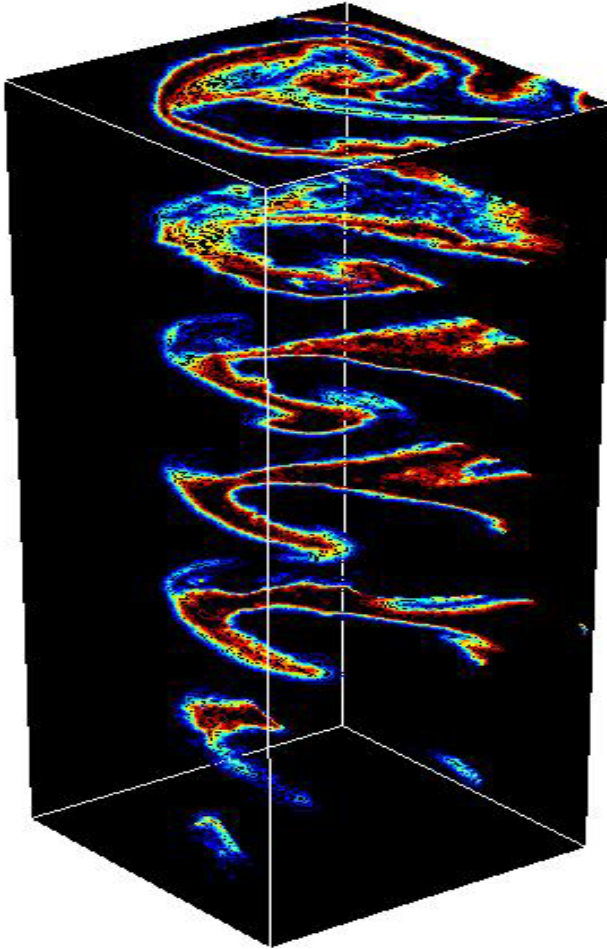
# Experiment with tracer .

Water was filled with red color from top to bottom at shelf to depth 250m. The rest part was filled with blue color.



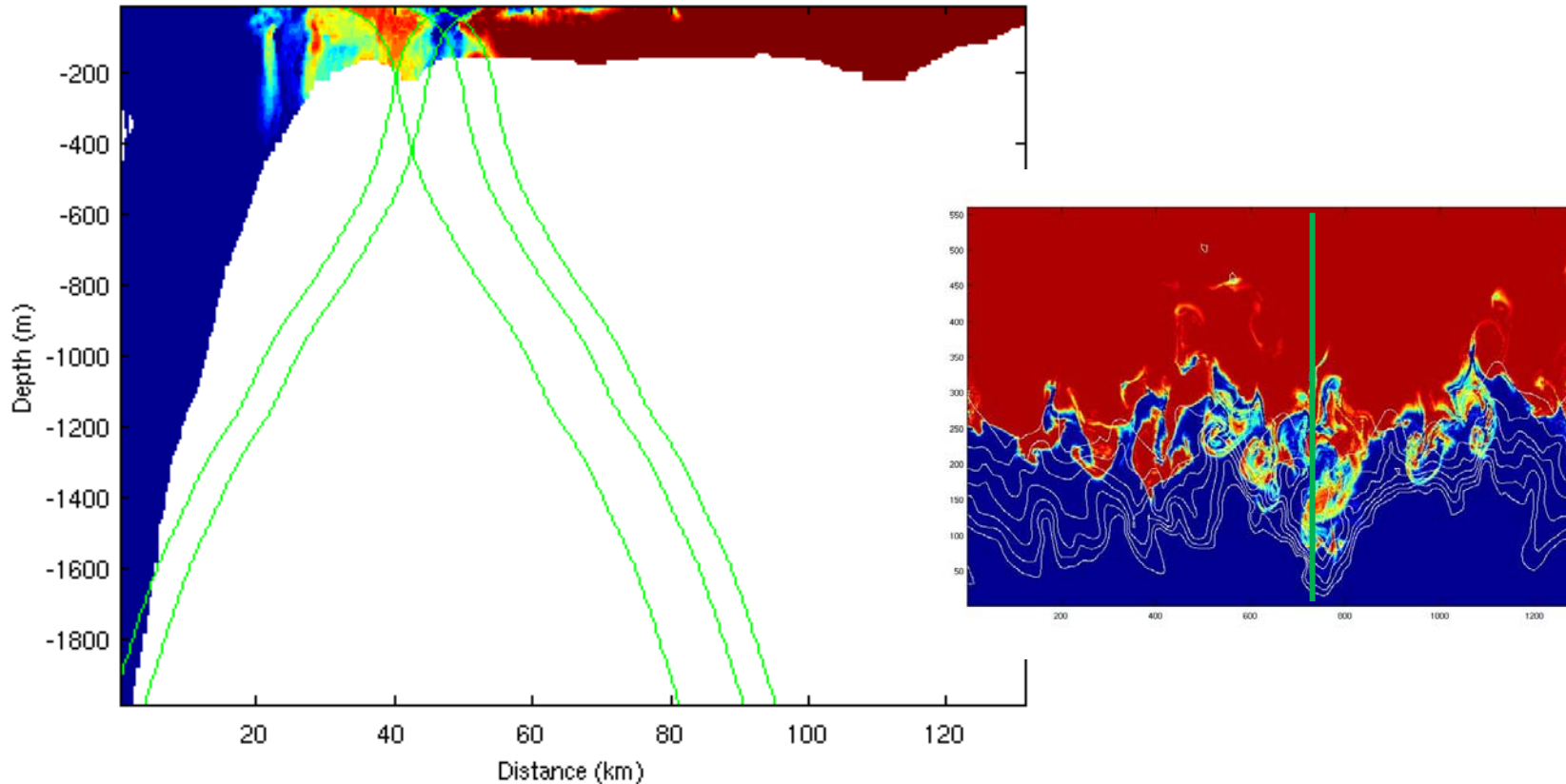
Evolution of tracer field at the surface  
Initial boundary of the tracer coincides with  
the bathymetry line 250 m

## Three – dimensional structure of eddies



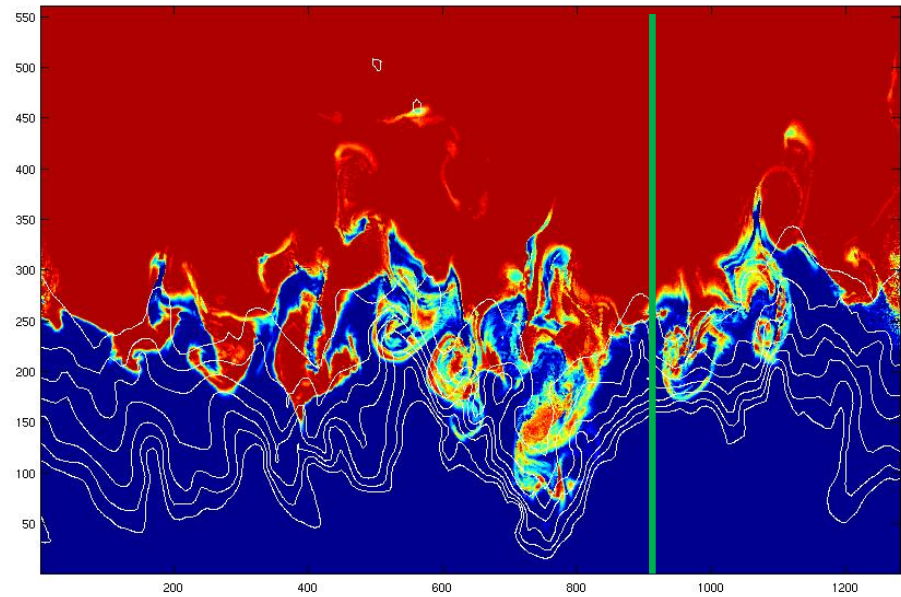
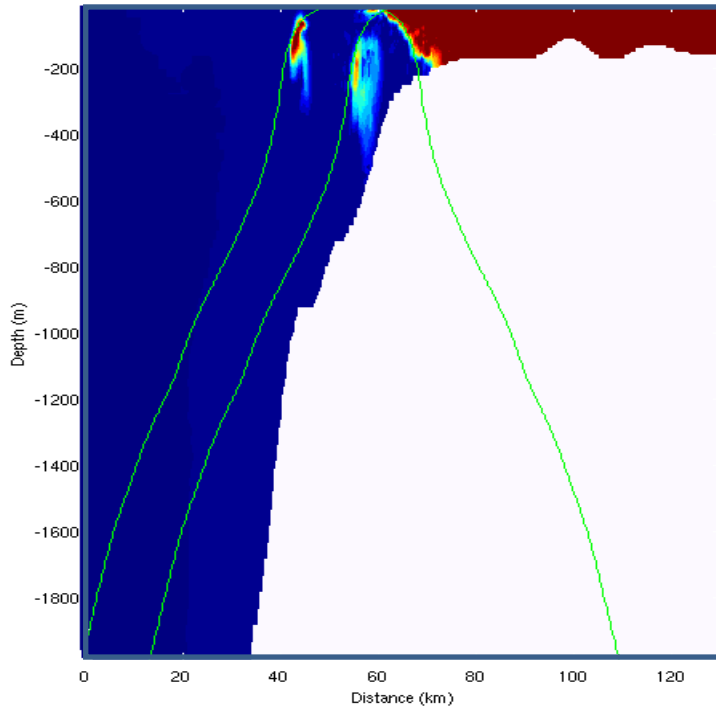
# Vertical section of tracer field.

The connection between the tracer distribution and the characteristic lines  
Characteristic lines show the path of energy propagation of internal waves.





# Vertical structure of the characteristic lines and tracer



# Conclusions

“Fine-grid” experiment (grid step 234 m) showed generation of internal solitary waves with amplitudes up to 50 m (horizontal scale  $\sim 1$  km) ;

Internal waves were generated in the areas of banks and some part of the continental slope and present 3D structure;

Experiments with tracers showed that transport in the area present eddy-structure;

The eddies are 3D and have connection with the generated wave field;

The fields of internal waves and eddies are controlled by bottom topography that consists with banks and headlands;