OpenDA

Deltares

Enabling Delta Life

www.openda.org

Kalman filtering in OpenDA

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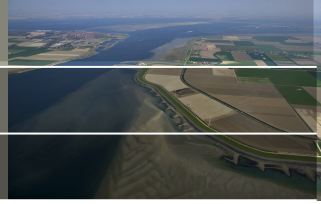
Outline



- Introductory example: storm surge forecasting
- Statistics and probability
- Kalman filter example
- Configuration of Kalman filters in OpenDA

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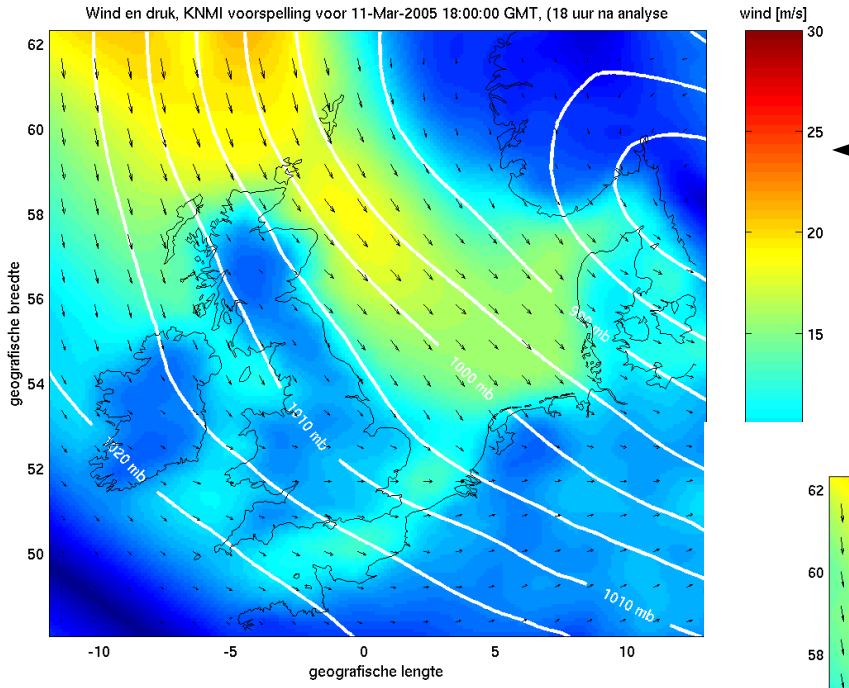
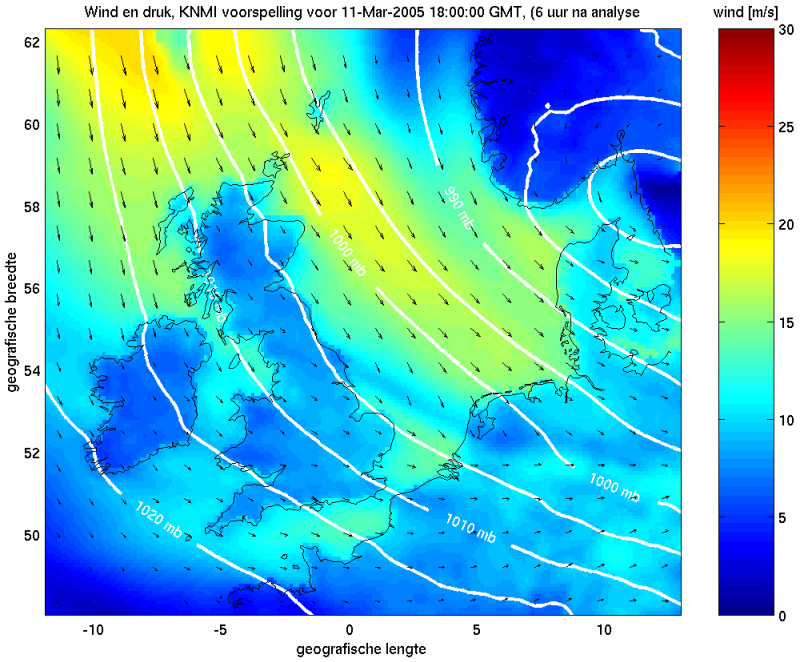
Storm March 11 2005



11 March 2005 18:00h UTC

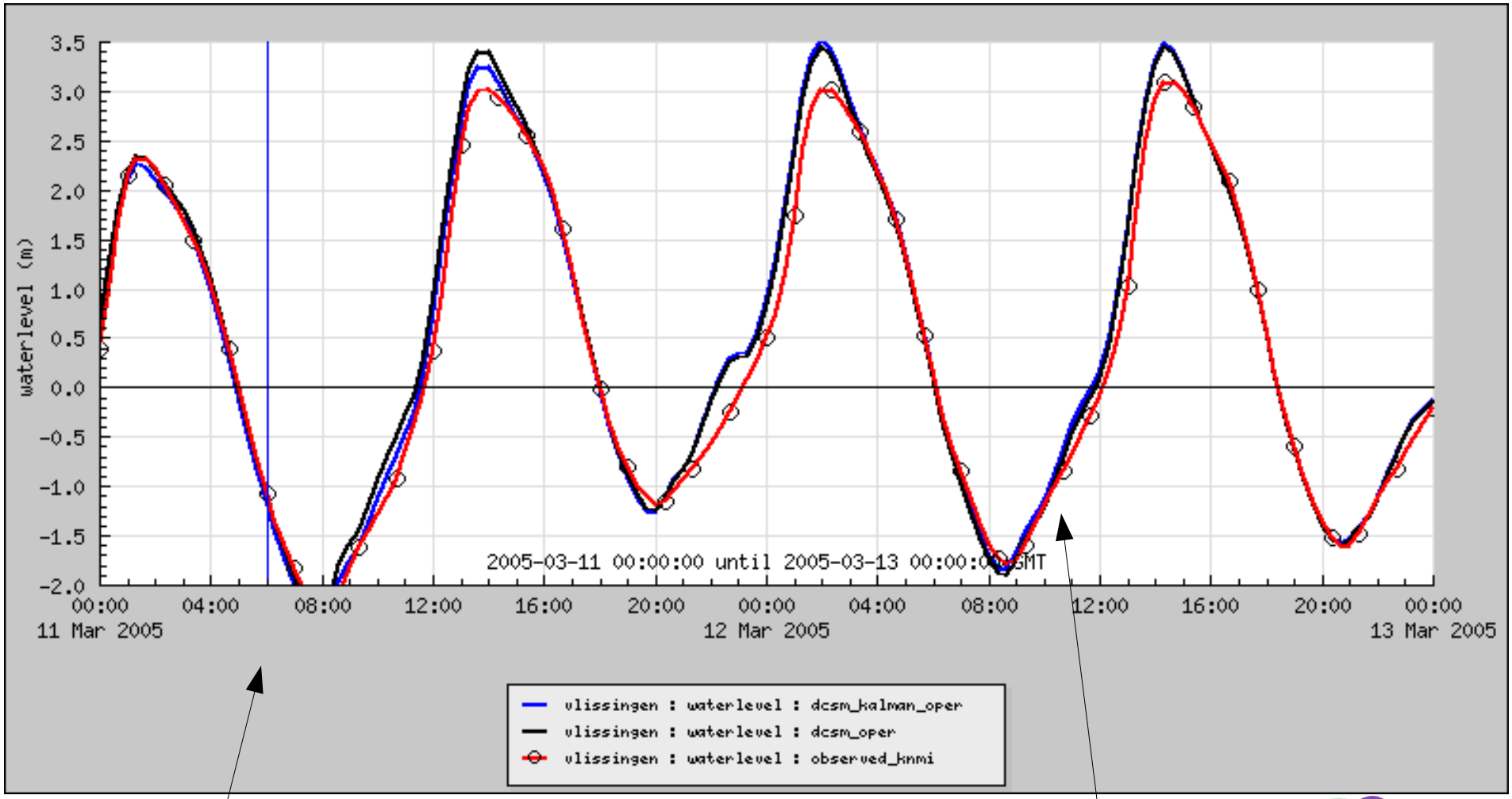
Forecast 10-MAR-2005 00:00h

Forecast 10-MAR-2005 12:00h



Weather forecasts are adjusted to match observations → Forecasts for same time change as that time comes closer.

Filter impact on time-series



Present time is crucial

Future observations do not exist

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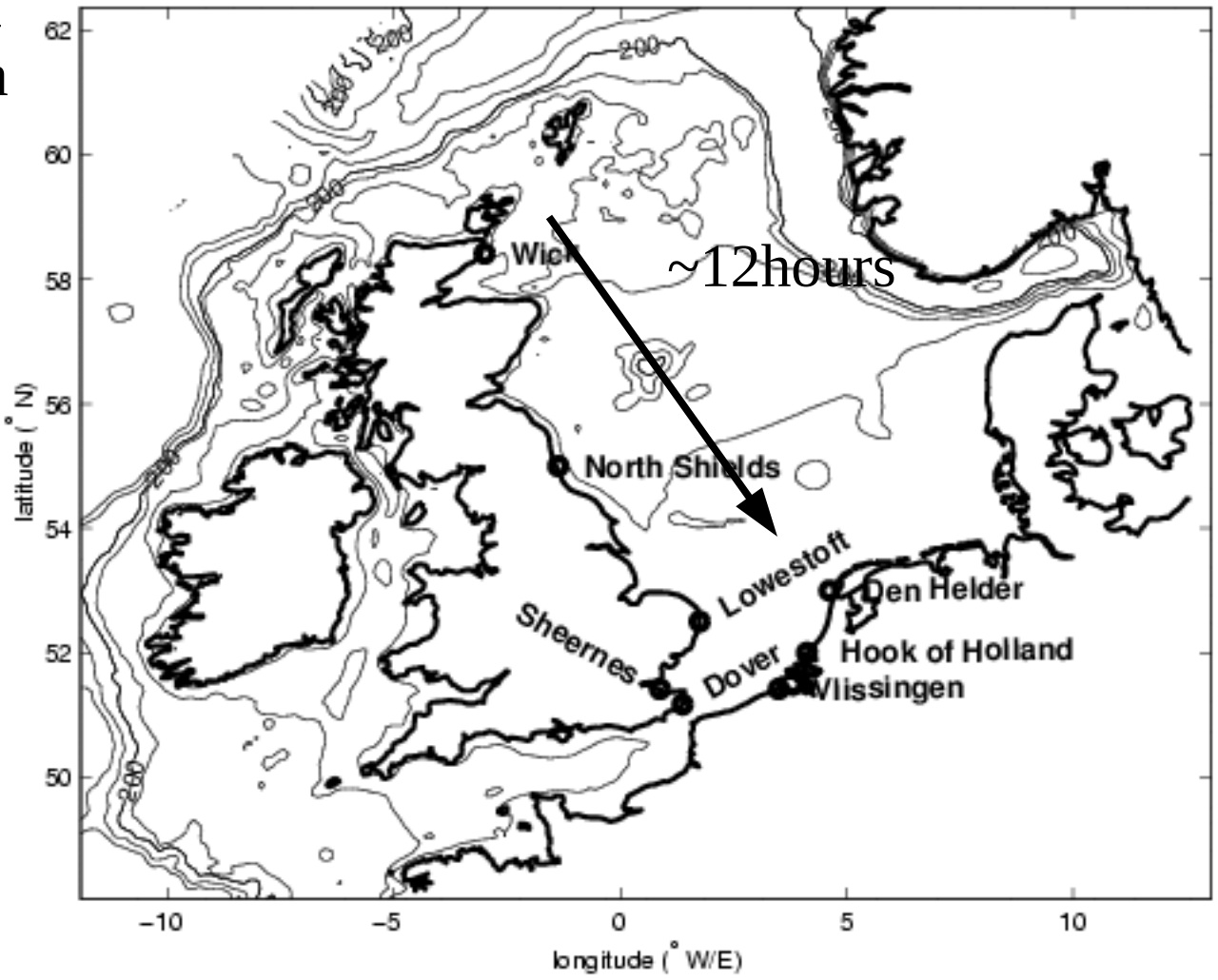
Operational Kalman filter DCSM-V5

Tidal wave propagates counter-clockwise

Waterlevel at Each location Assimilated Every 10min



Soon to be replaced by DCSM-v6

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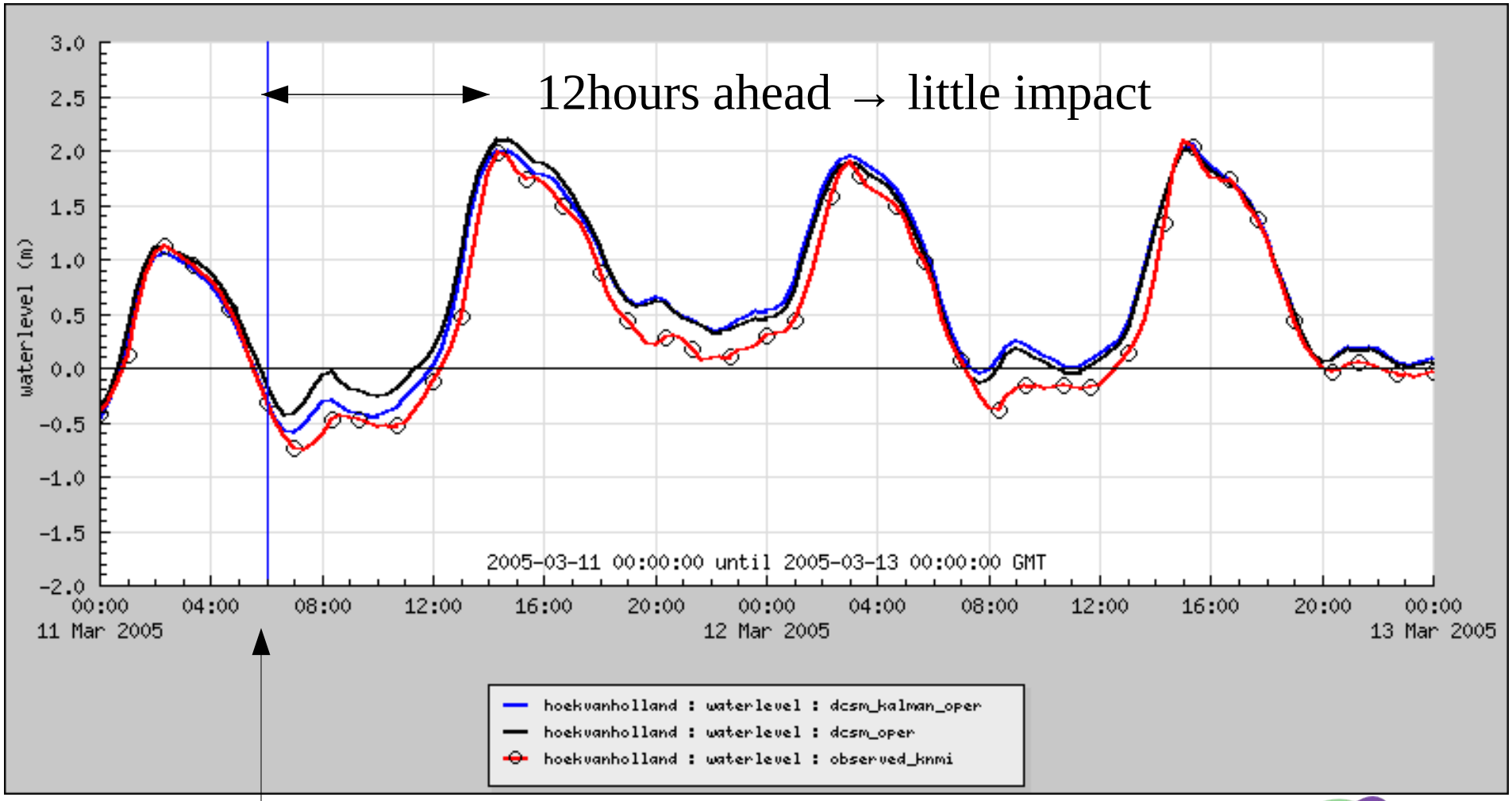

Current time

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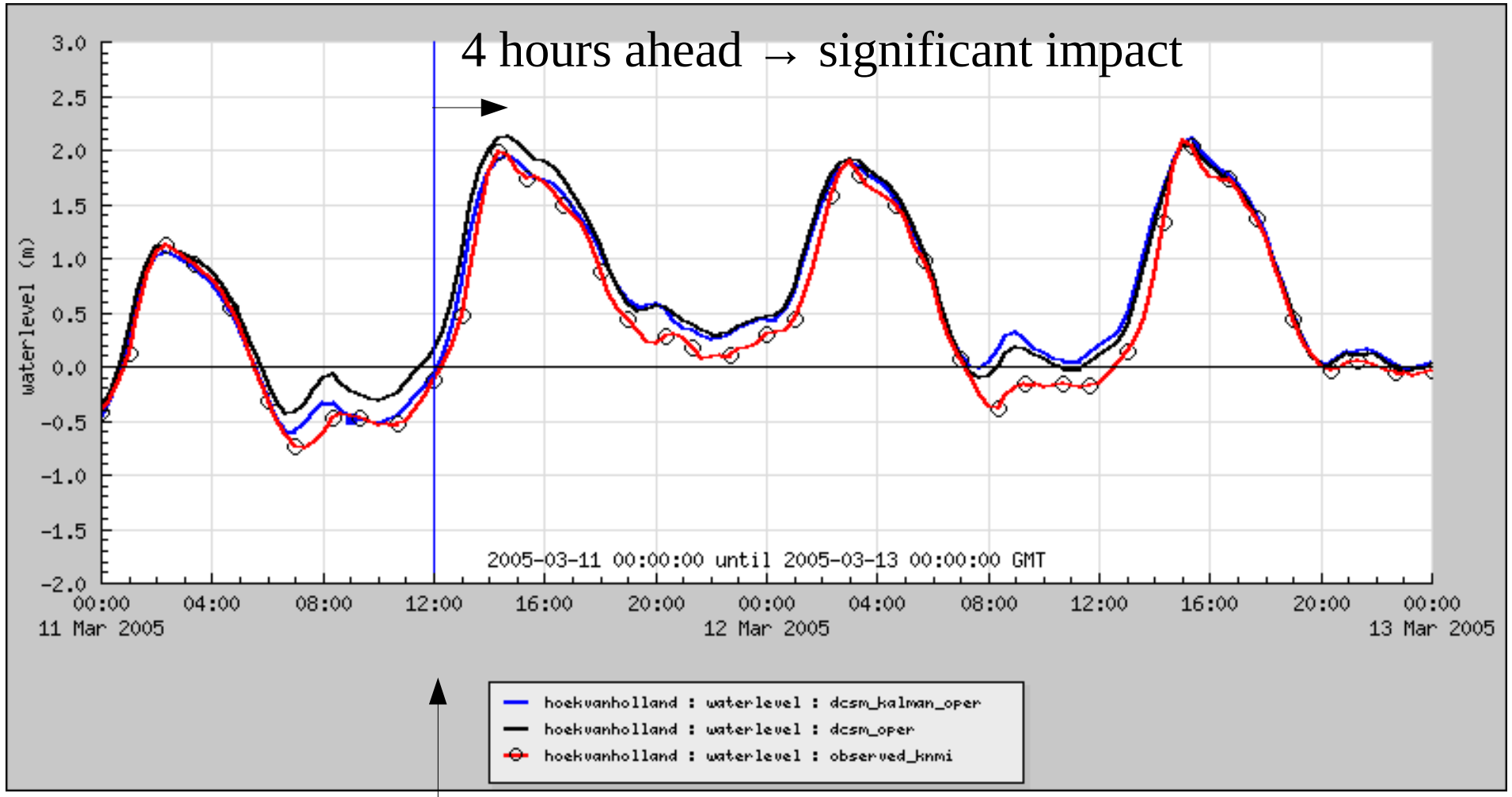

At a later time

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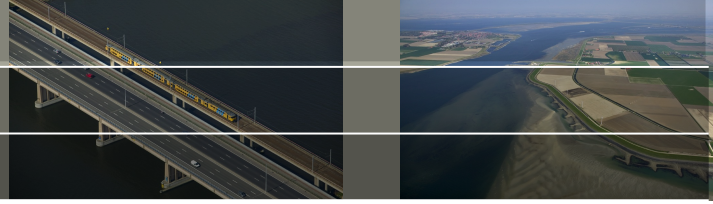
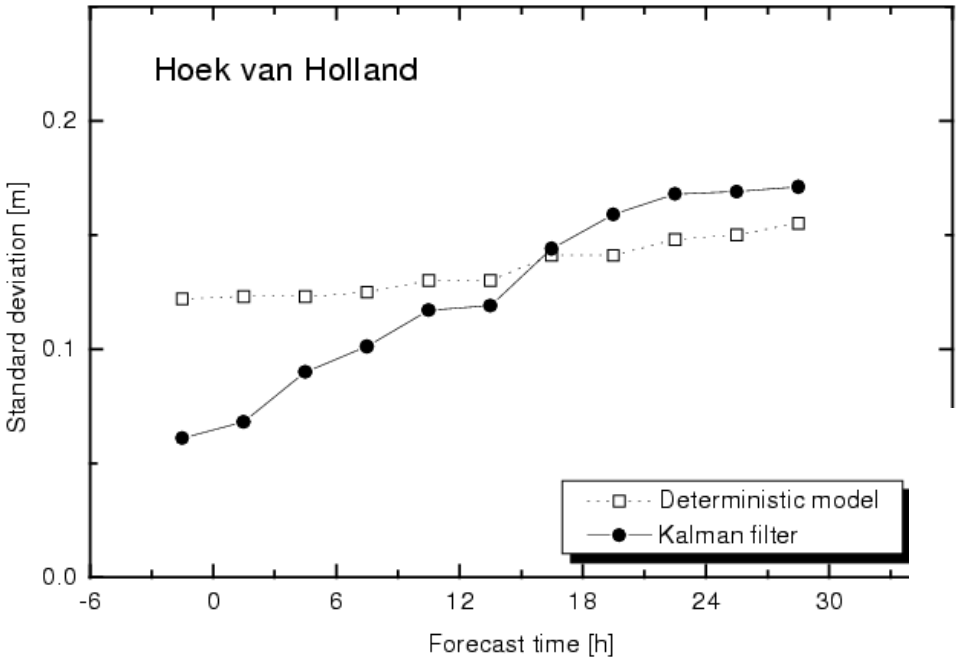
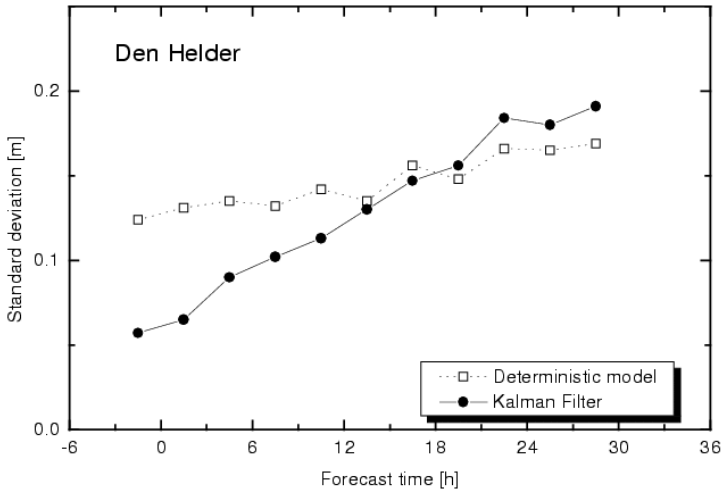
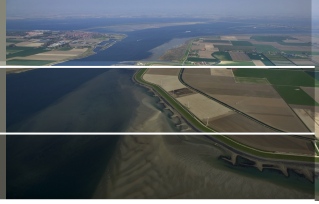
Average impact of assimilation







Kalman filtering concepts



- Statistics and probability
- Geostatistics
- Optimal control
- Behaviour of the Kalman filter
- Smoothing

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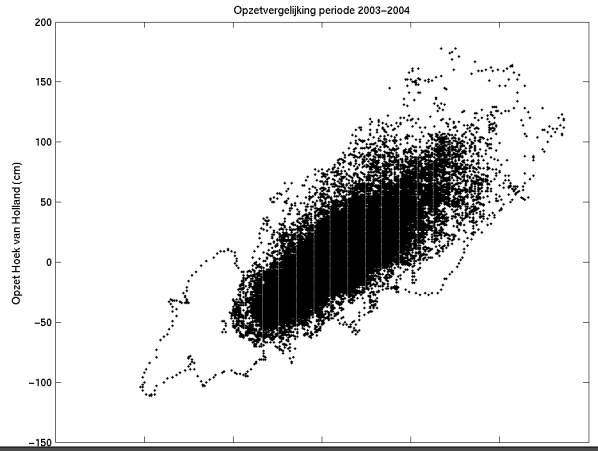
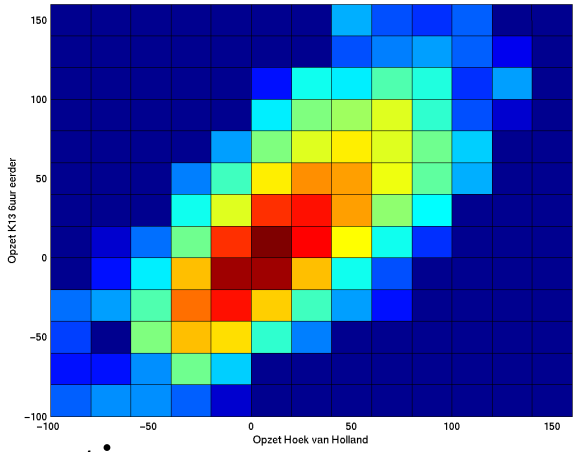
Statistics and probability





Compare 2 locations

K13 -6 uur

Hoekvhld \ opzet (cm)	-110	-90	-70	-50	-30	-10	10	30	50	70	90	110	130	150	170	
-110	5	9	3	0	0	0	0	0	0	0	0	0	0	0	0	**17**
-90	1	8	14	13	8	2	0	0	0	0	0	0	0	0	0	**46**
-70	0	4	4	14	130	27	0	0	0	0	0	0	0	0	0	**179**
-50	0	6	0	156	990	694	71	11	0	0	0	0	0	0	0	**1928**
-30	0	10	16	92	2240	5874	893	77	15	4	0	0	0	0	0	**9221**
-10	0	0	4	35	958	17225	18545	1044	48	7	0	0	0	0	0	**37866**
10	0	0	2	10	122	4069	25508	6730	554	48	5	0	0	0	0	**37048**
30	0	0	0	0	51	360	4049	5773	1392	172	43	0	0	0	0	**11840**
50	0	0	0	0	12	76	618	1580	1344	437	114	18	0	0	0	**4199**
70	0	0	0	0	1	17	139	361	610	402	136	27	0	0	0	**1693**
90	0	0	0	0	0	0	37	154	201	373	66	7	2	0	0	**840**
110	0	0	0	0	0	0	4	45	35	87	54	5	16	0	0	**246**
130	0	0	0	0	0	0	0	1	7	11	17	8	3	0	0	**47**
150	0	0	0	0	0	0	0	1	20	7	5	9	0	0	0	**42**
170	0	0	0	0	0	0	0	0	2	7	4	4	0	0	0	**17**
	6	**37**	**43**	**320**	**4512**	**28344**	**49864**	**15777**	**4228**	**1555**	**444**	**78**	**21**	**0**	**0**	**105229**

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Linear regression – Gaussian distrbution

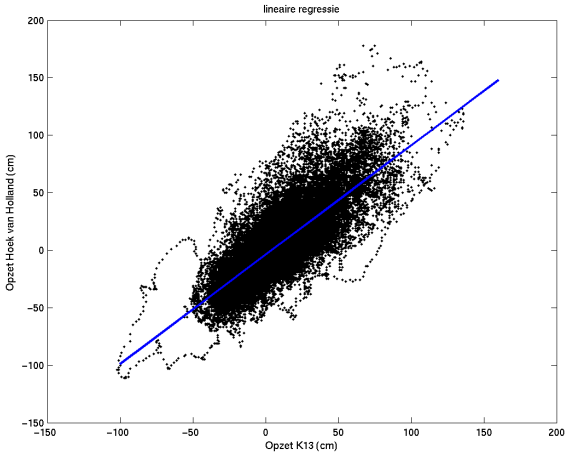
K13 -6 uur

Hoekvhld \ opzet (cm)	-110	-90	-70	-50	-30	-10	10	30	50	70	90	110	130	150	170	
-110	83%	24%	7%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%			0%
-90	17%	22%	33%	4%	0%	0%	0%	0%	0%	0%	0%	0%	0%			0%
-70	0%	11%	9%	4%	3%	0%	0%	0%	0%	0%	0%	0%	0%			0%
-50	0%	16%	0%	49%	22%	2%	0%	0%	0%	0%	0%	0%	0%			2%
-30	0%	27%	37%	29%	50%	21%	2%	0%	0%	0%	0%	0%	0%			9%
-10	0%	0%	9%	11%	21%	61%	37%	7%	1%	0%	0%	0%	0%			36%
10	0%	0%	5%	3%	3%	14%	51%	43%	13%	3%	1%	0%	0%			35%
30	0%	0%	0%	0%	1%	1%	8%	37%	33%	11%	10%	0%	0%			11%
50	0%	0%	0%	0%	0%	0%	1%	10%	32%	28%	26%	23%	0%			4%
70	0%	0%	0%	0%	0%	0%	0%	2%	14%	26%	31%	35%	0%			2%
90	0%	0%	0%	0%	0%	0%	0%	1%	5%	24%	15%	9%	10%			1%
110	0%	0%	0%	0%	0%	0%	0%	0%	1%	6%	12%	6%	76%			0%
130	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	4%	10%	14%			0%
150	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	12%	0%			0%
170	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	5%	0%			0%
	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%			

Conditional probability

Use Gaussian probability

Resulting in linear regression



Conditional probability 1

- 1) Linear combination based on spread/uncertainty

$$\hat{x} = (1-k)\,x + ky$$

$$k = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_y^2}$$

$$\frac{1}{\sigma_{\hat{x}}^2} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}$$

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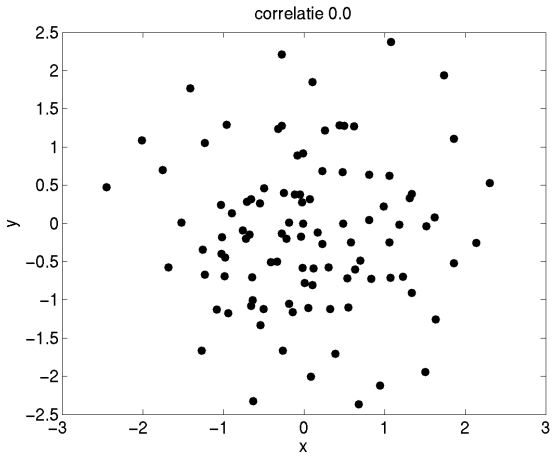
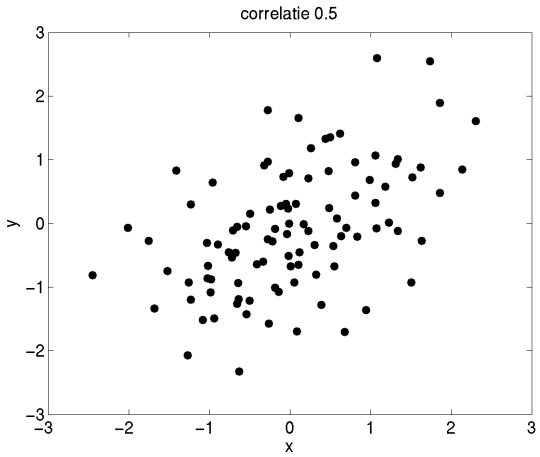
Spatial and multivariate correlation

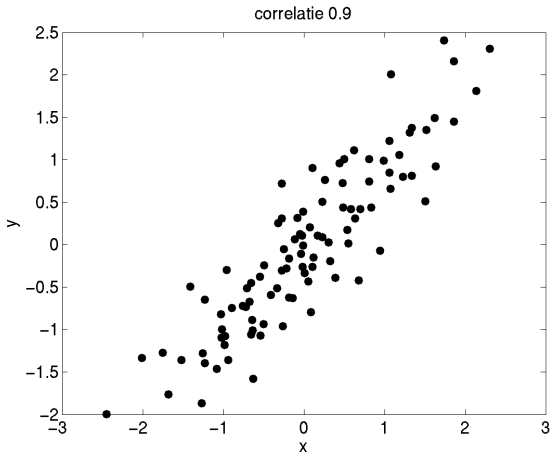
- 2) Spatial correlation

$$\hat{X}(p) = (1 - K(p))X + K(p)y$$

$$K(p) = \rho(p, p_y)$$





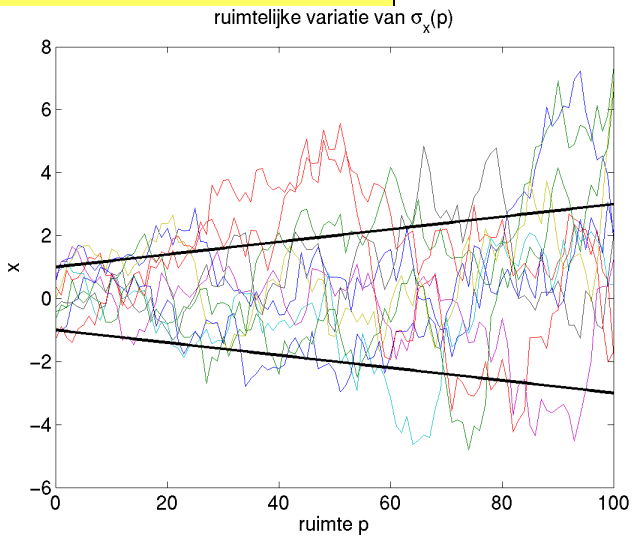


Spatially varying variance

- 3) Spatially varying variance/uncertainty

$$\hat{X}(p) = (1 - K(p))\,X + K(p)\,y$$

$$K(p) = \frac{\sigma_x(p)}{\sigma_x(p_y)}$$



Kalman analysis

- Combination of previous 3 steps

$$\hat{X}(p) = (1 - K(p))\, X + K(p)\, y$$

$$K(p) = \frac{\sigma_x(p)}{\sigma_x(p_y)}\, \rho(p, p_y)\, \frac{\sigma_x^2(p_y)}{\sigma_x^2(p_y) + \sigma_y^2}$$

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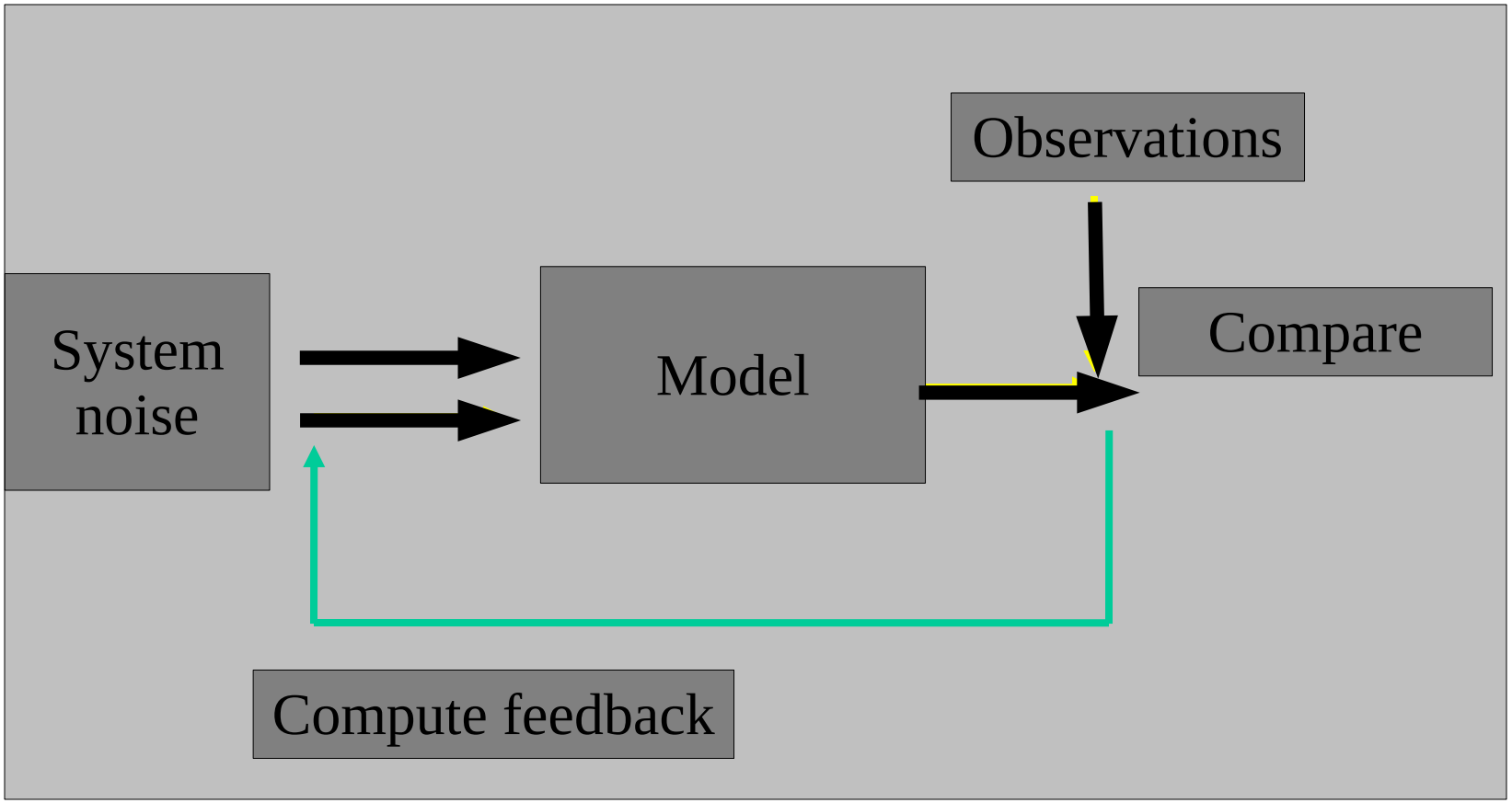
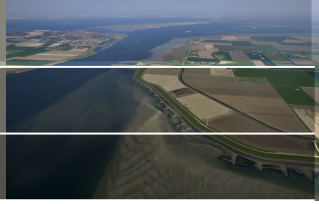
Optimal control



Behaviour of the Kalman filter



- Interpretation of the Kalman gain
  - Instantaneous correction of an observation
- Influence of the accuracy of observations and model
  - More accurate observations → larger corrections
  - More accurate model → smaller corrections
  - Signal-noise ratio

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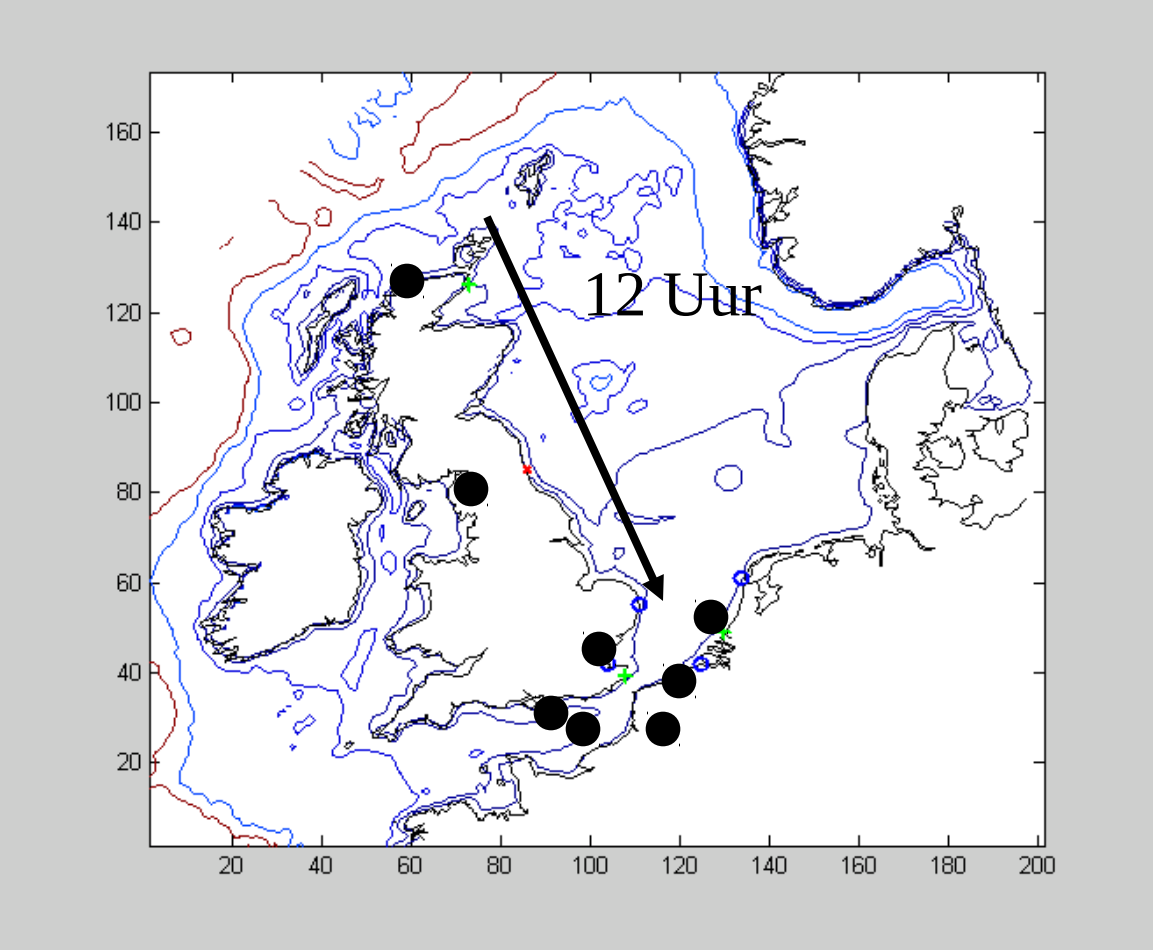
Operation Kalman filter for DCSM-v5



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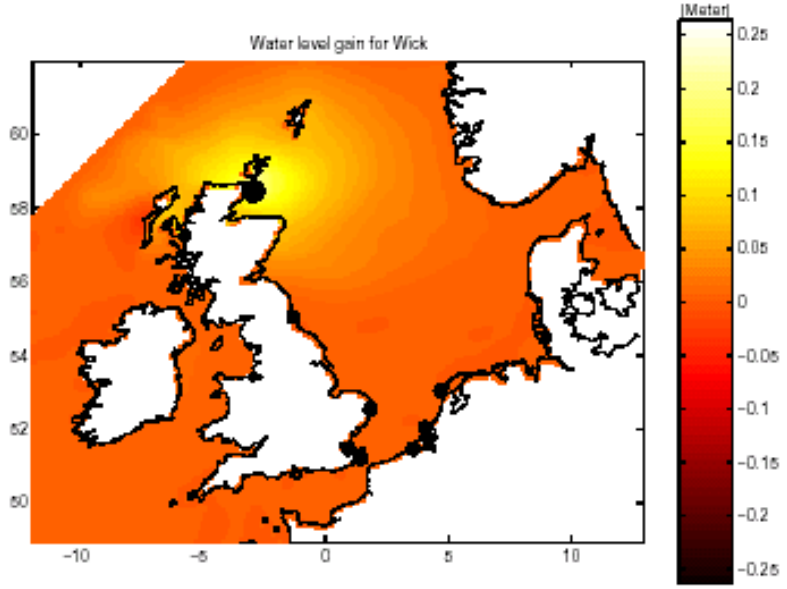
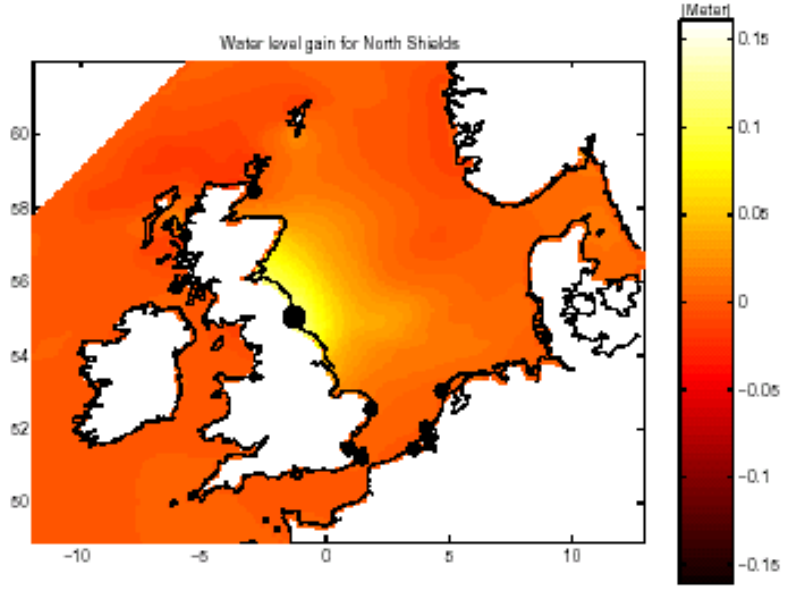
Instantaneous area of influence (Kalman gain)



(a) De Kalman gain van meetstation Wick.



(b) De Kalman gain van meetstation North Shields.

The model propagates these corrections in space

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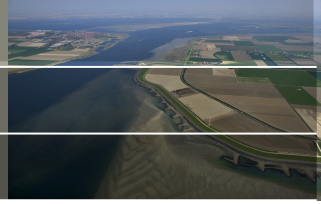
Properties of the Kalman filter



- Largest modification often near the boundary
- When observations are close together averaging occurs
- Observations close to area of interest give large improvements at small lead-times
- Locations further away give smaller improvement, but larger lead-time.
- When the model is good, then corrections will be small.

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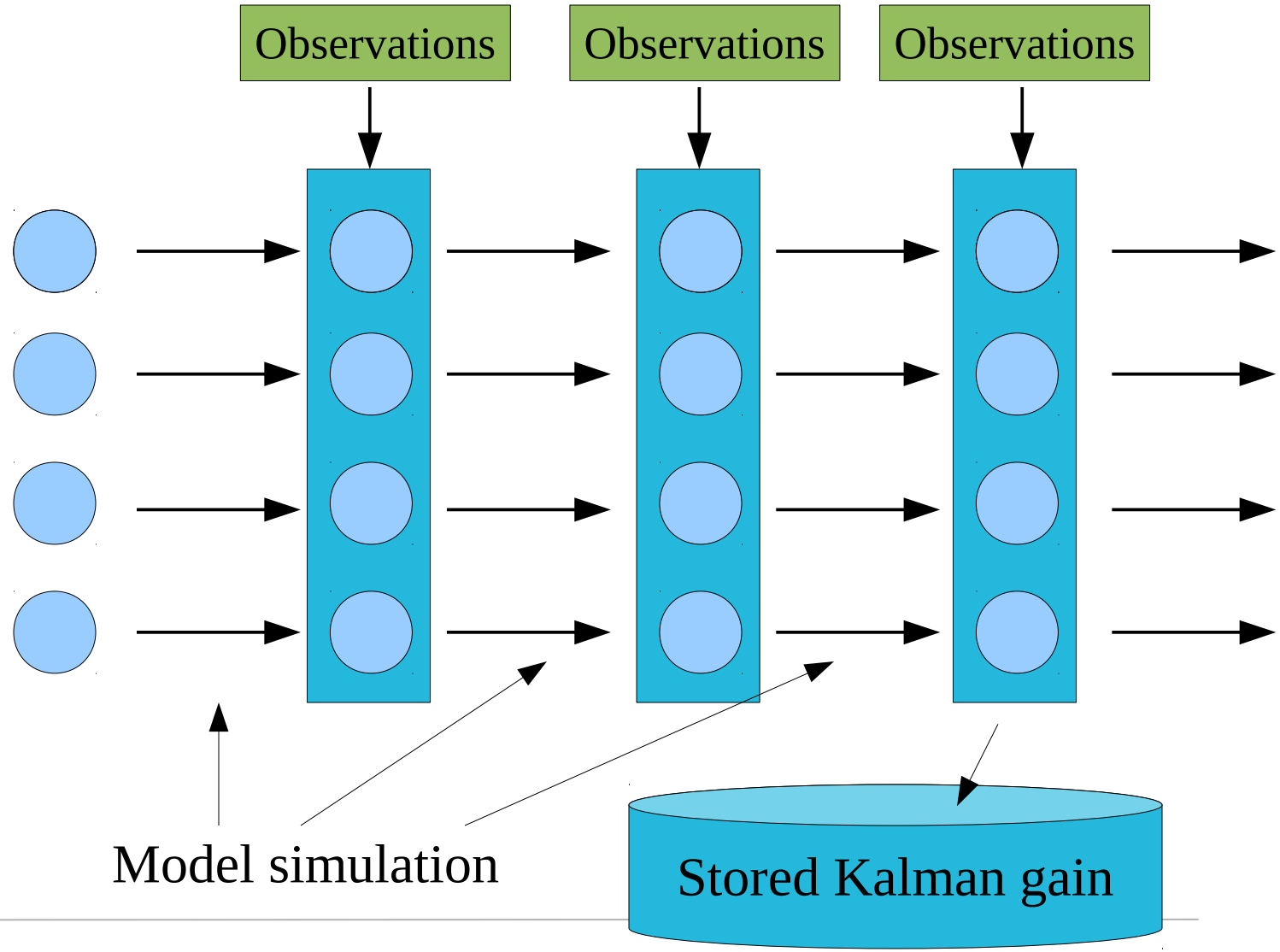
Ensemble Kalman filter



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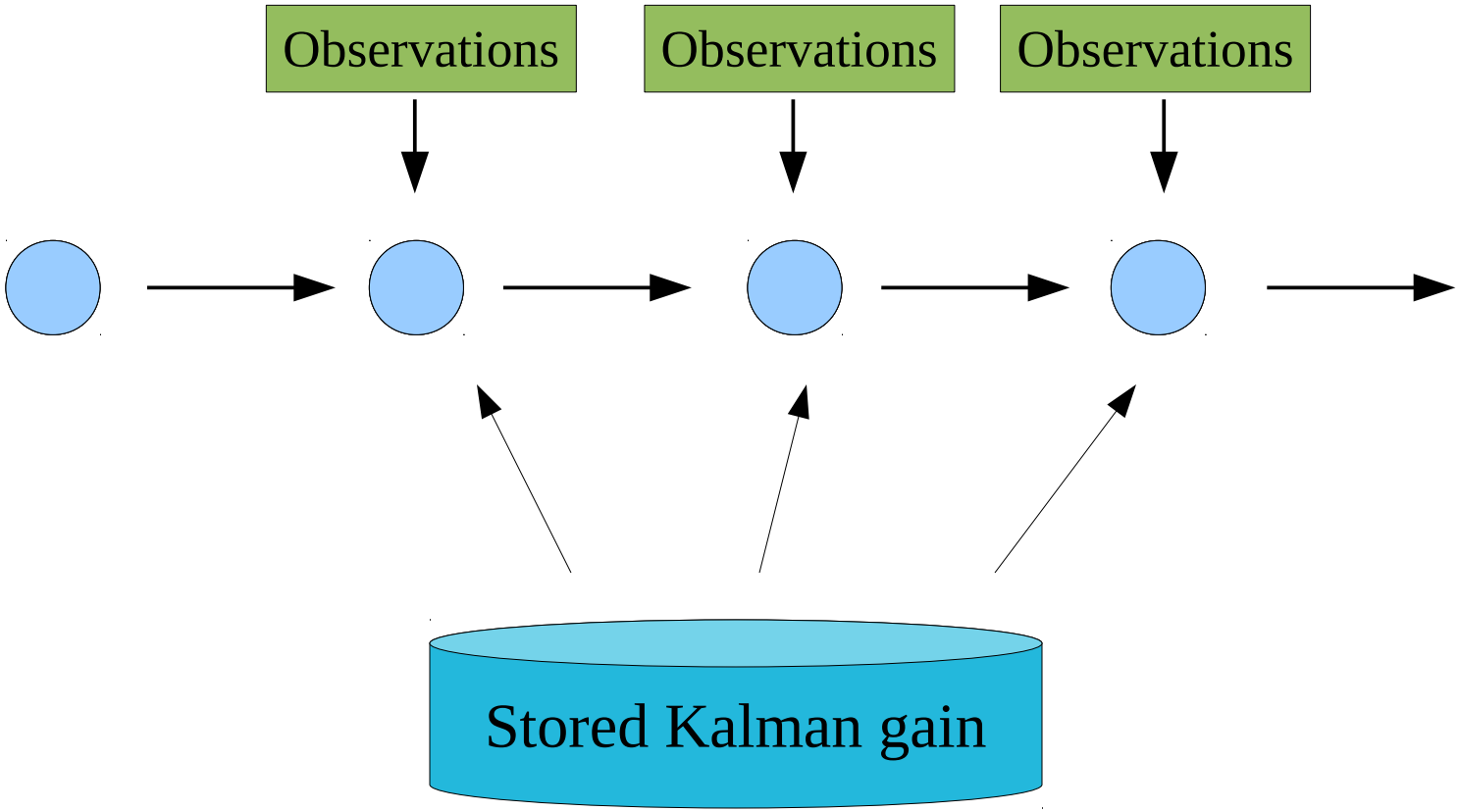
Steady-state Kalman filter



Works well for nearly linear model with time-invariant equations

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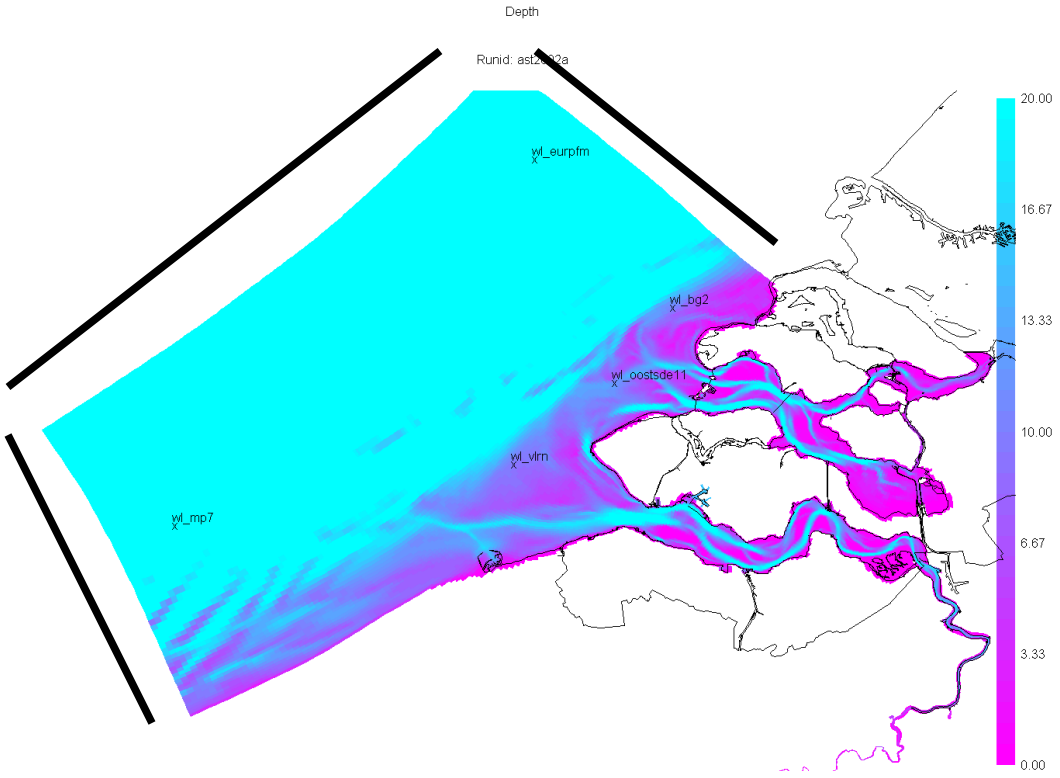
Example: Kustzuid shallow-water model

- Computation of Kalman gain (RRSQRT instead of EnKF)
- Use of steady-state gain
- Uncertainty for boundary conditions



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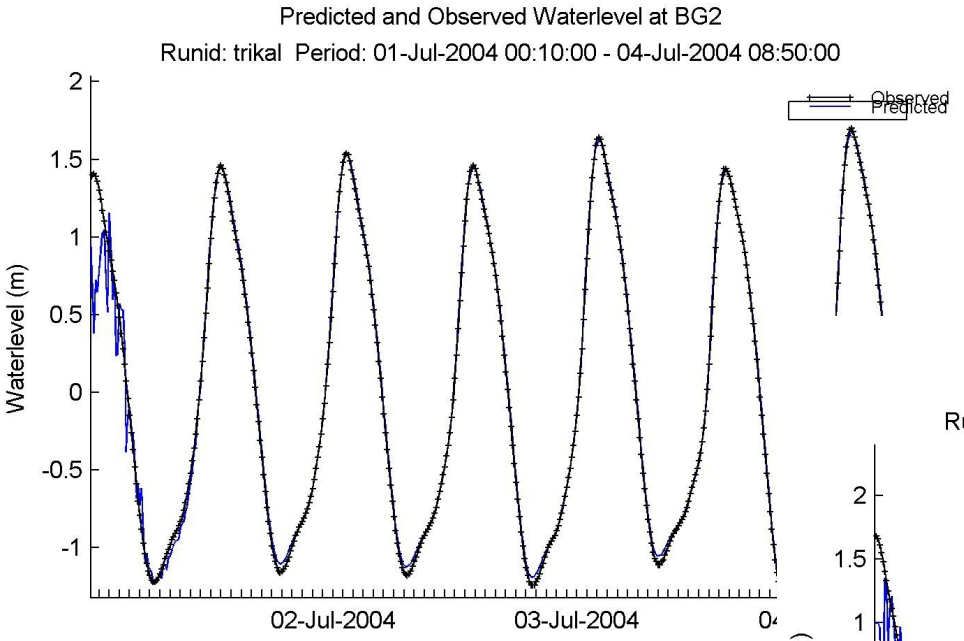
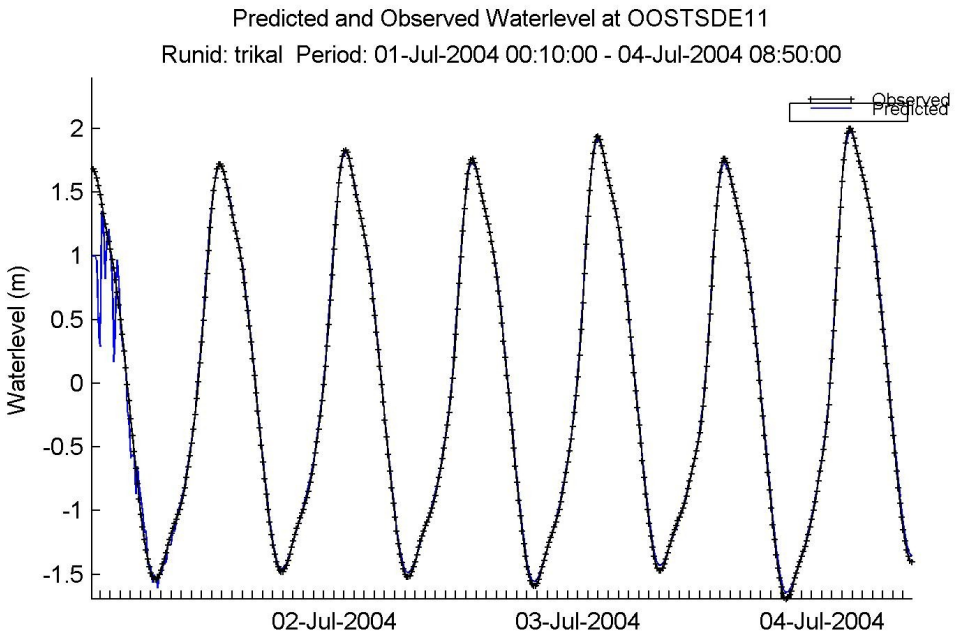
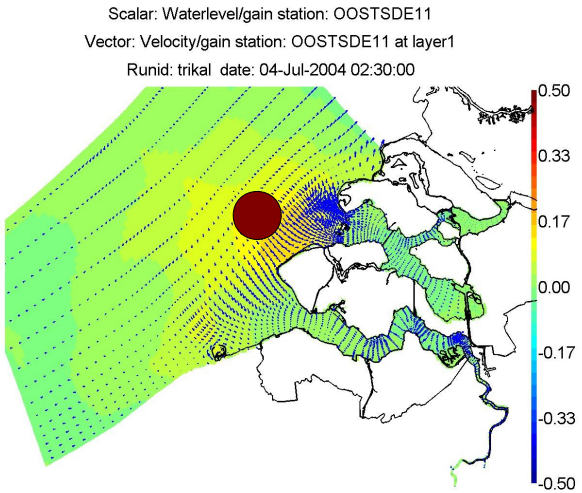
Check EnKF computation

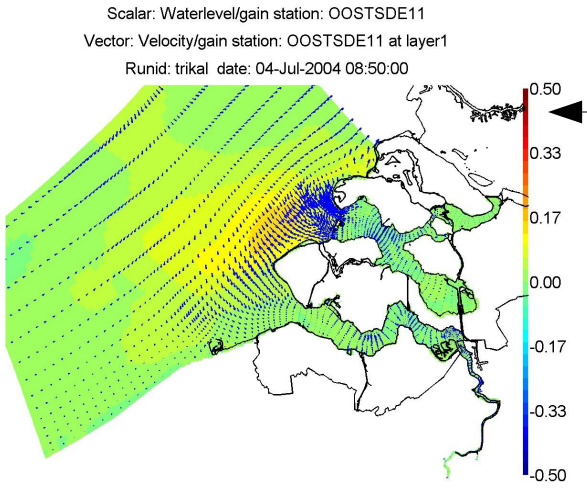
Here RRSQRT algorithm



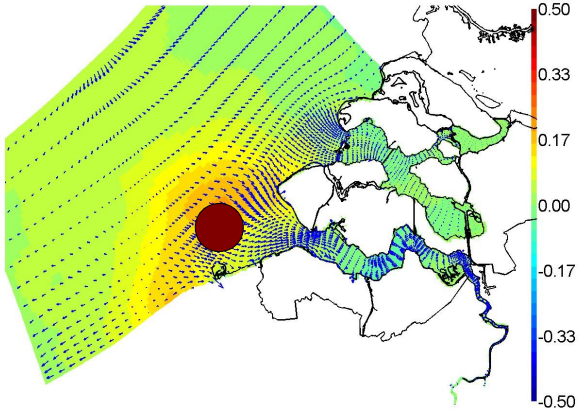


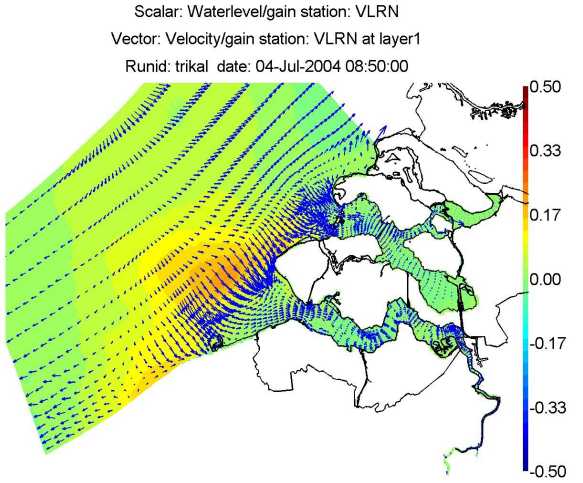
Computation of Kalman gain





Compare times





Check all locations

Similar?

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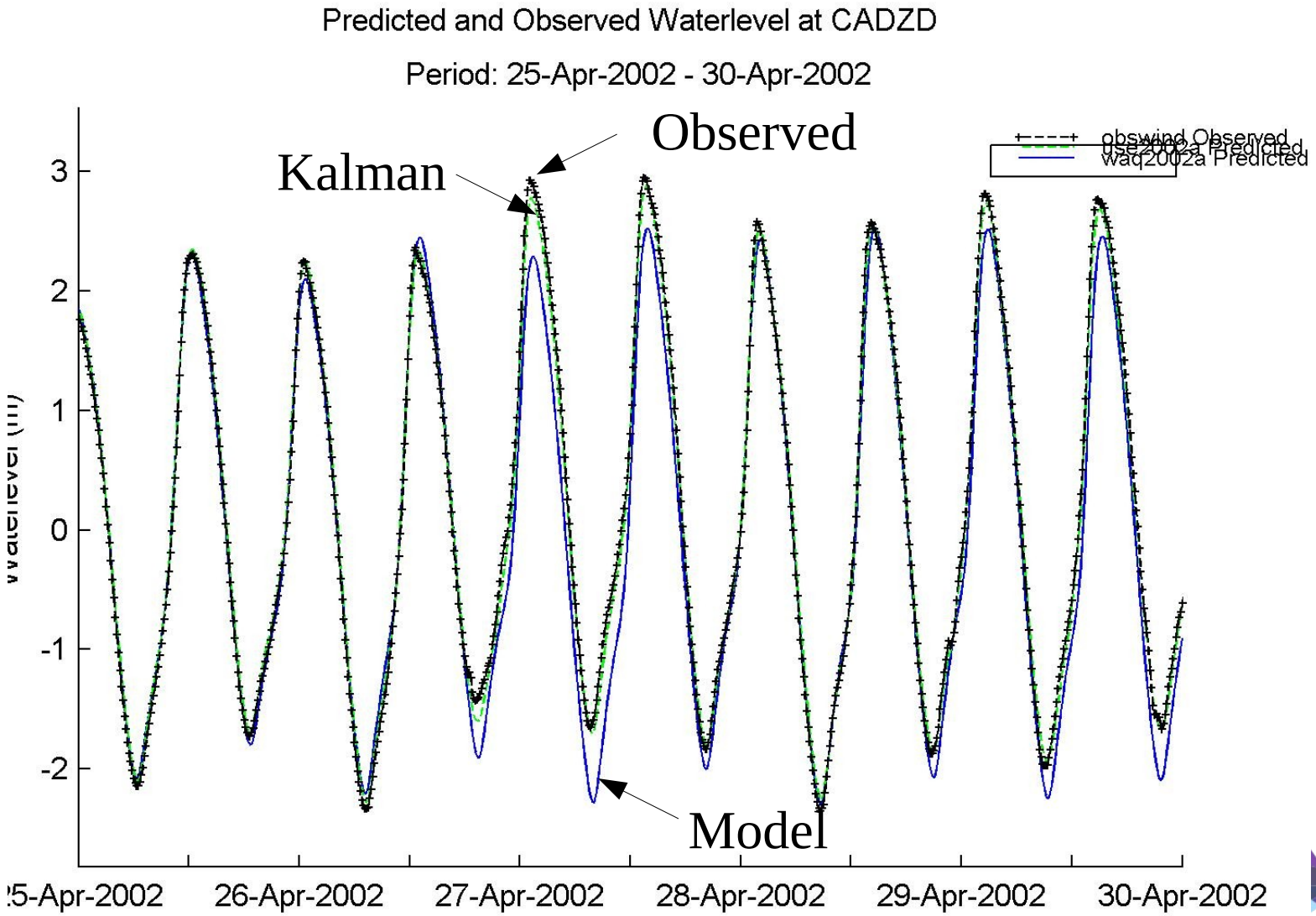
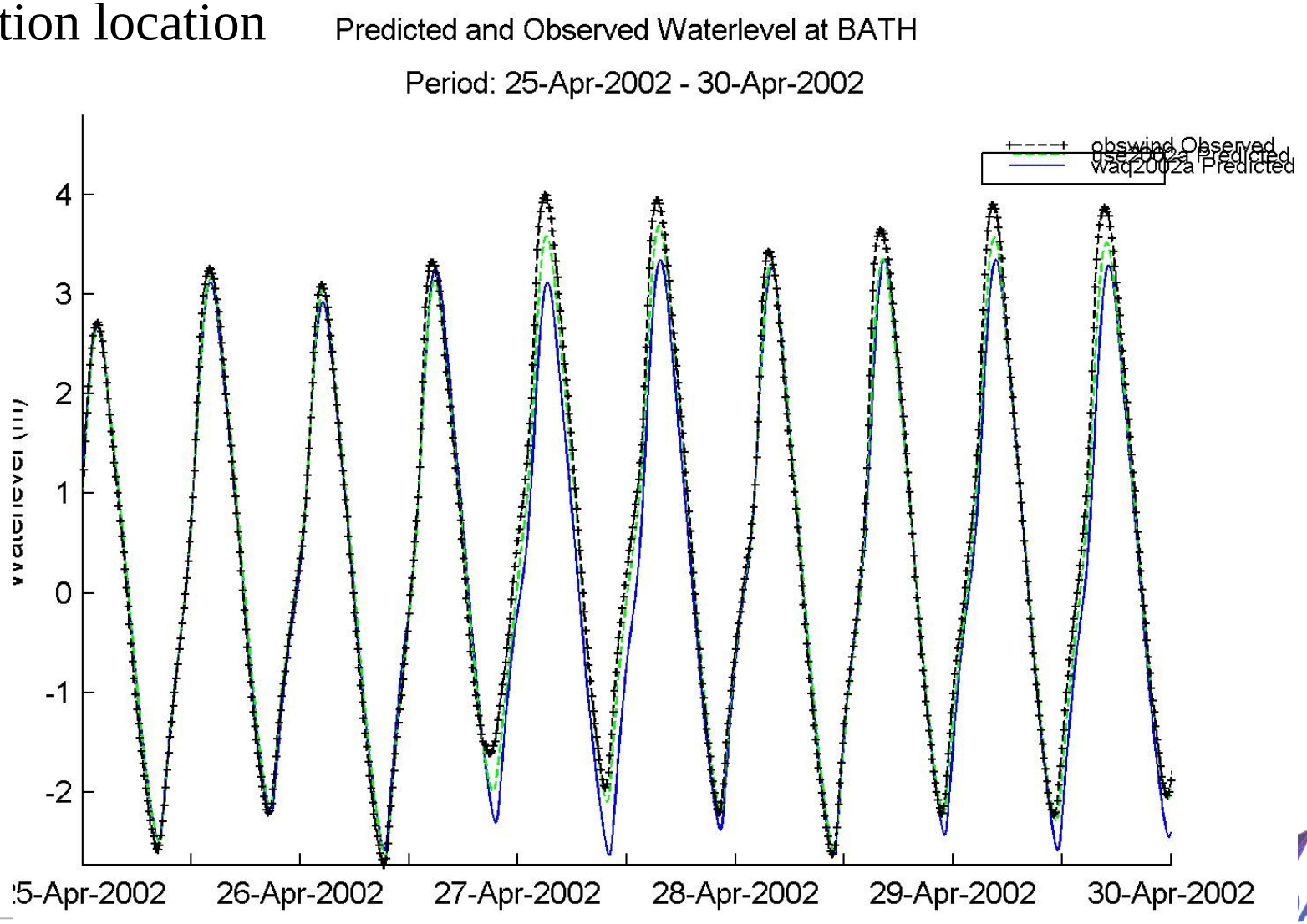
Steady-state computation

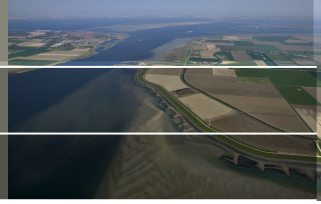


Steady-state computation

Validation location



Configuration EnKF



```
<?xml version="1.0" encoding="UTF-8"?>
<EnkfConfig>
        <!-- default <analysisTimes type="fromObservationTimes" ></analysisTimes>
        <analysisTimes type="fixed" timeFormat="dateTimeString" >
            201008241130,201008241140,...,201008242350
         </analysisTimes>

        <mainModel stochParameter="false"
                stochForcing="false" stochInit="false"/>

        <ensembleSize>50</ensembleSize>

        <ensembleModel stochParameter="false"
              stochForcing="true" stochInit="true" />

        <!-- config for saving kalman gain -->
         <saveGain>
            <times type="fixed" timeFormat="mjd" >
              48259.0,48259.125,48259.25
            </times>
         </saveGain>
</EnkfConfig>
```

Process observations

Model used for output

Number of model runs

Ensemble members

Store Kalman gain to file

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Configuration Steady-state filter

```
<?xml version="1.0" encoding="UTF-8"?>
<SteadyStateFilterConfig >
        <analysisTimes type="fixed" timeFormat="mjd">0.0,1.0,...,10.0</analysisTimes>
        <mainModel stochParameter="false" stochForcing="false" stochInit="false"/>
        <readGain>
                <dirPrefix>../enkf_oscillator_</dirPrefix>
                <!-- <time timeFormat="dateTimeString">201008240000</time> -->
                <time timeFormat="mjd">10.0</time>
                <file>enkf_oscillator_gain.xml</file>
        </readGain>
</SteadyStateFilterConfig>
```

Timesteps should be the same

File saved From eg EnKF

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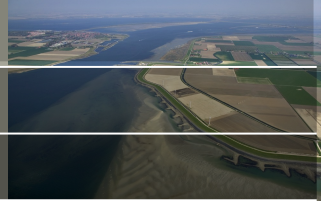
Configuration Toy-models



```
<?xml version="1.0" encoding="UTF-8"?>
<oscillatorConfig>
        <simulationTimespan>[0.0,0.05,10.0]</simulationTimespan>
        <parameters names="t_damp,omega">[8.0,1.5708]</parameters>
        <parameterUncertainty names="t_damp,omega">[1.0,0.1257]
         </parameterUncertainty>
        <systemNoise>{[0.0,0.0],[0.3,0.3]}</systemNoise>
        <initialState>[0.8,0.0]</initialState> <!-- start from zero -->
        <initialStateUncertainty>[0.8,0.8]</initialStateUncertainty>
</oscillatorConfig>
```

Time start,step,stop

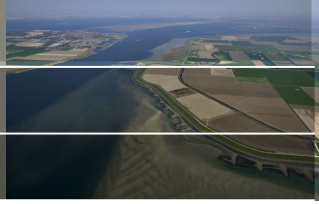
Calibration parameters

Stdev

Mean,stdev Noise for state

OpenDA

Various



- Localization
- Parallel computing

Future work

- Kalman smoothing
- Temporal gain averaging
- Covariance inflation

OpenDA