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The concept of partial age, a generalisation of the notion of age: theory, idealised illustrations and realistic applications

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Age and residence time: a quick refresher

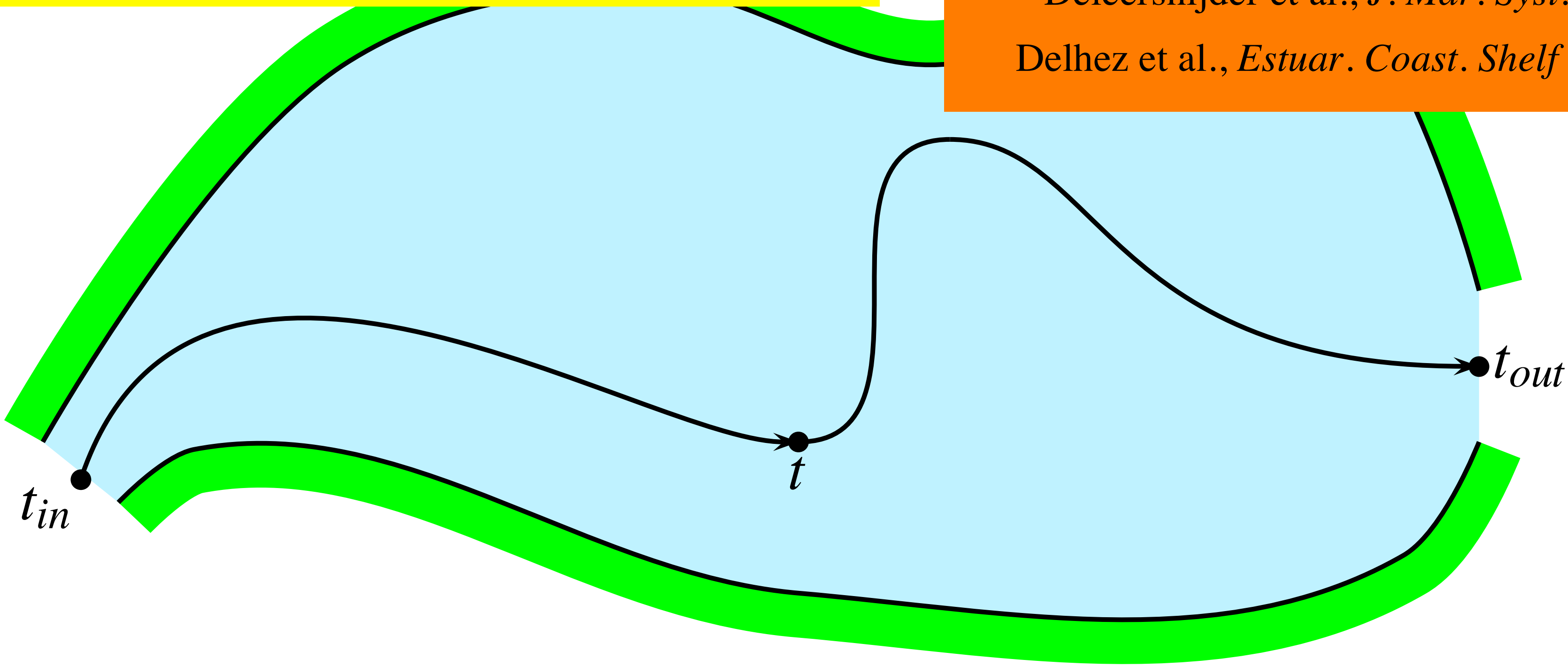
Age: forward/direct approach

Residence time: backward/adjoint approach

Constituent-oriented Age and Residence time Theory (CART, www.climate.be/cart)

Deleersnijder et al., *J. Mar. Syst.*, 2001

Delhez et al., *Estuar. Coast. Shelf S.*, 2004



$$age = t - t_{in} \ , \quad residence\ time = t_{out} - t$$

$$transit\ time = age + residence\ time$$

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- Partial ages in the World Ocean
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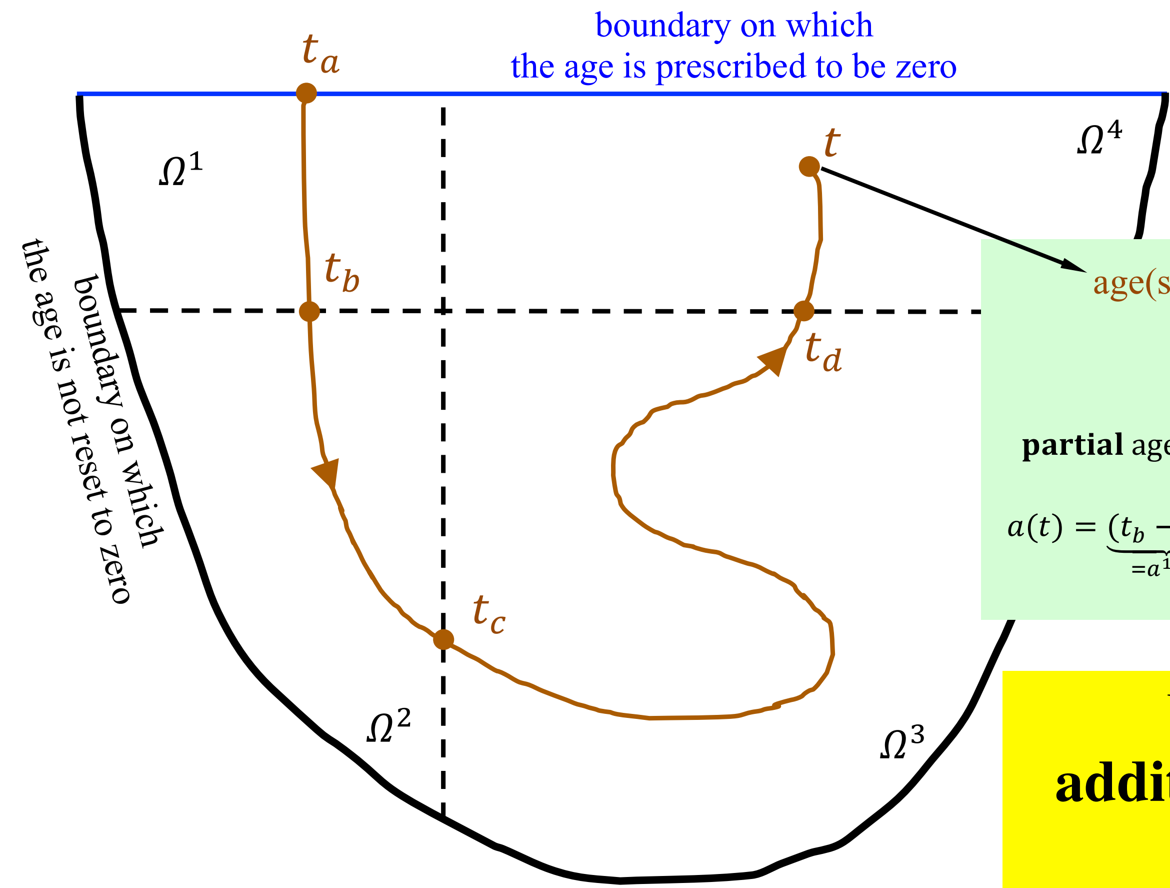
The concept of partial age (I)



age(s) of the **particle** under study at time *t*

classical age: $a(t) = t - t_a$

partial age: $a^n(t) \equiv$ time spent in the n— th subdomain

$$a(t) = \underbrace{(t_b - t_a)}_{=a^1(t)} + \underbrace{(t_c - t_b)}_{=a^2(t)} + \underbrace{(t_d - t_c)}_{=a^3(t)} + \underbrace{(t - t_d)}_{=a^4(t)} = t - t_a$$

domain of interest: $\Omega = \bigcup_{n=1}^{N} \Omega^n$

(here N=4)

Upon using partial ages,
additional pieces of **information**
are available about
the **past evolution** of the particles
of the constituent under study.

The concept of partial age (II)

• **Lagrangian** view of the **classical age**: every particle has **one watch** and the age is the time elapsed since the clock was (re-)set to zero.

• **Lagrangian** view of the **partial age**: every particle has ***N*** **watches** and the *n*-th watch is **ticking only in the *n*-th subdomain** (i.e. it is at rest in all ofther subdomains).

• The **classical age** of a one particle (or a set of particles) is the **sum** of the **partial ages**.

The concept of partial age (III)

• **Eulerian** calculation of the **classical age**: solve reactive transport equations for the constituent concentration $C(t,\mathbf{x})$ and the age concentration $\alpha(t,\mathbf{x})$

$$\frac{\partial C}{\partial t} = \overbrace{P - D}^{\text{source-sink}} - \nabla \bullet (\overbrace{C\mathbf{v} - \mathbf{K} \bullet \nabla C}^{\text{advect.+diff. flux}})$$

$$\frac{\partial \alpha}{\partial t} = \underbrace{C}_{\text{ageing}} + \underbrace{\Pi - \Delta}_{\text{source-sink}} - \nabla \bullet (\underbrace{\alpha\mathbf{v} - \mathbf{K} \bullet \nabla \alpha}_{\text{advect.+diff. flux}})$$

and, finally, estimate the mean age at time t and location $\mathbf{x}$ as

$$a(t,\mathbf{x}) = \frac{\alpha(t,\mathbf{x})}{C(t,\mathbf{x})}$$

The concept of partial age (IV)

- Introduce the characteristic function of the n-th subdomain:

$$\omega^n(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \in \Omega^n \\ 0 & \text{if } \mathbf{x} \notin \Omega^n \end{cases}$$

- The **partial age concentration** associated with the n-th subdomain (i.e. the n-th watch, which is ticking only in Ω^n), $\alpha^n(t,\mathbf{x})$, and the related **partial age**, $a^n(t,\mathbf{x})$, are obtained from

$$\frac{\partial \alpha^n}{\partial t} = \underbrace{\omega^n C}_{\substack{\text{ageing} \\ =0 \text{ if } \mathbf{x} \notin \Omega^n}} + \underbrace{\Pi - \Delta}_{\text{source-sink}} - \nabla \bullet (\underbrace{\alpha^n \mathbf{v} - \mathbf{K} \bullet \nabla \alpha^n}_{\text{advect.+diff. flux}})$$

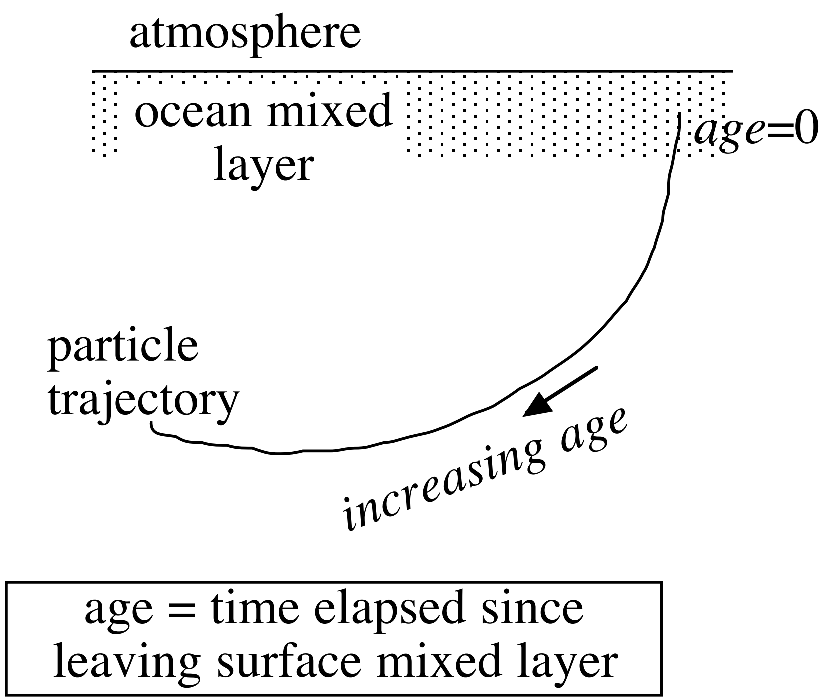
$$a^n(t,\mathbf{x}) = \frac{\alpha^n(t,\mathbf{x})}{C(t,\mathbf{x})} \quad , \quad \text{with} \quad \alpha = \sum_{n=1}^{N} \alpha^n \quad , \quad a = \sum_{n=1}^{N} a^n$$

Partial ages in the World Ocean (I)

According to England (*J. Phys. Oceanogr*., 1995), the "World Ocean circulation at its largest scale can be thought of as a gradual renewal or ventilation of the deep ocean by water that was once at the sea surface."

Therefore, the age, a measure of the time since leaving the ocean upper mixed layer, is a popular diagnostic tool in the World Ocean.

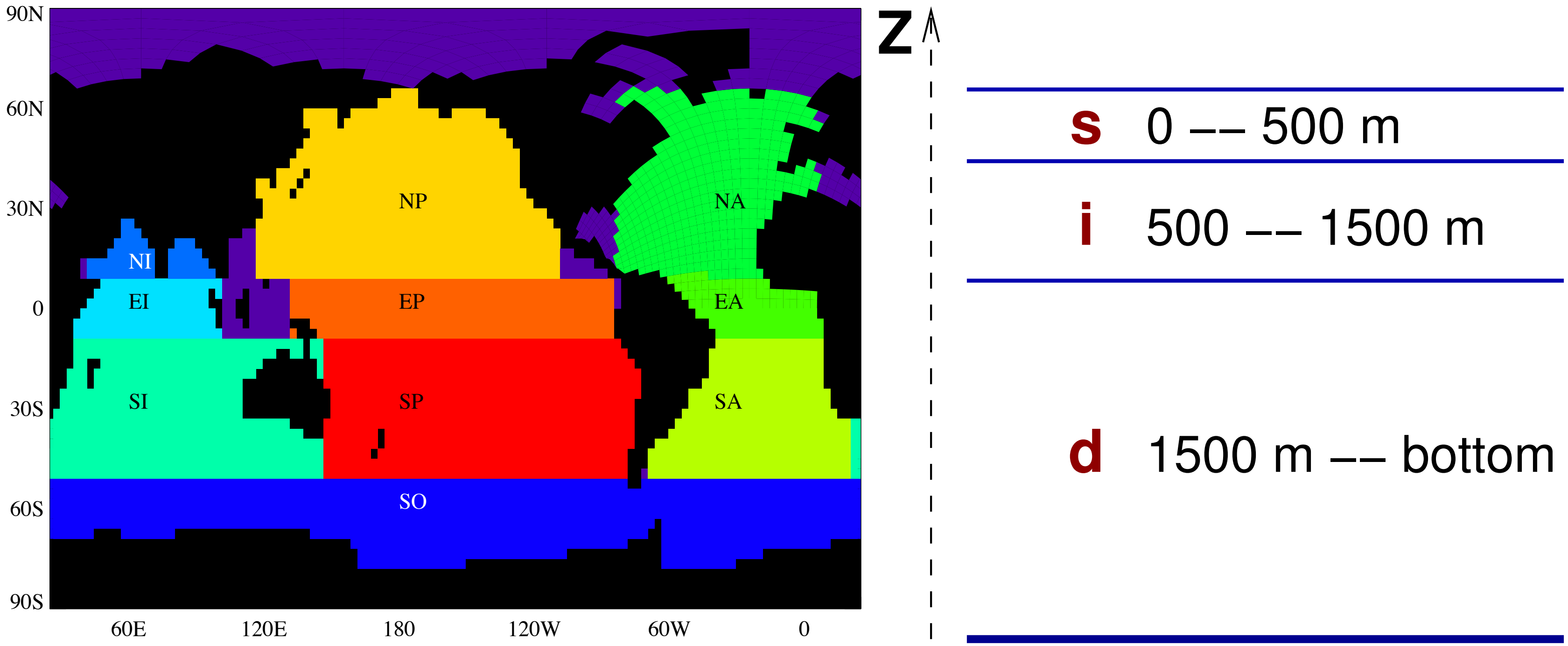
estimating ocean ventilation rate



Upon splitting the World Ocean into subdomains, a more detailed description of the ventilation processes could be obtained.

Partial ages in the World Ocean (II)

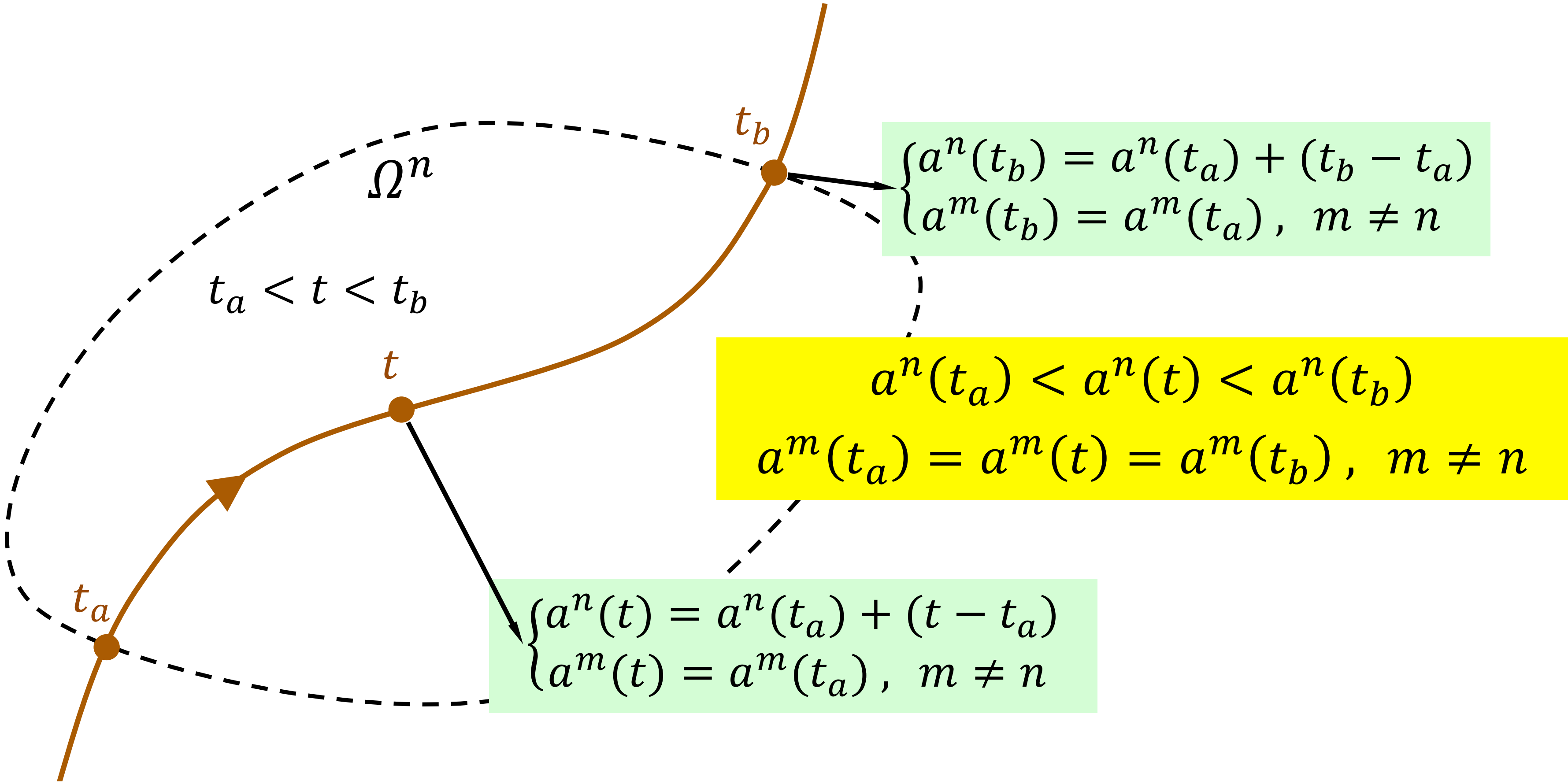
• Two coarse-grid (≈3.5°×3.5°) OGCMs (LSG-MPIM, OM-UCL) are used. There are N=31 subdomains. The age(s) of the **water** (i.e. a passive tracer) are examined at a **steady-state**.



s: surface - **i**: intermediate - **d**: deep

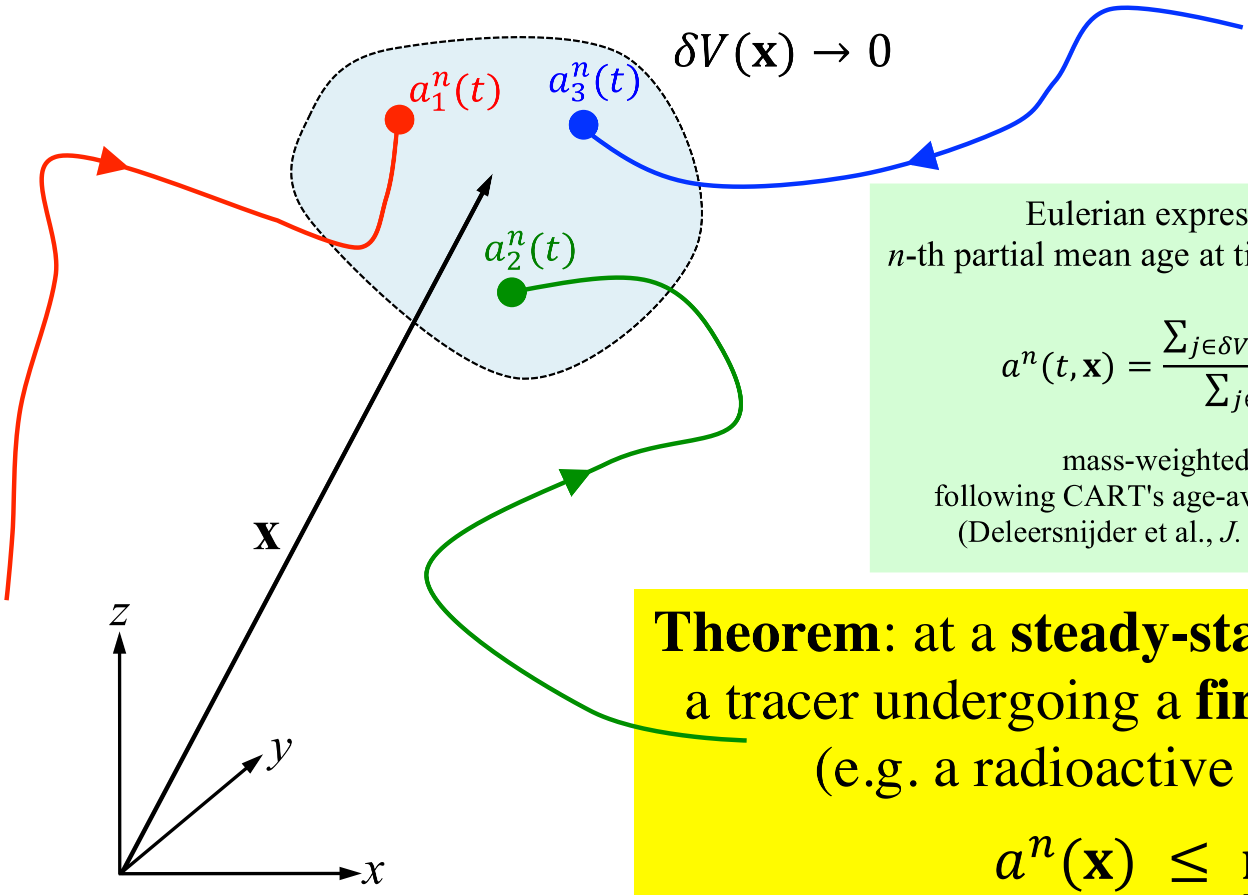
Partial ages in the World Ocean (III)

- Obviously, the n-th partial age of a particle reaches its maximum value at the moment it leaves the n-th subdomain.



Partial ages in the World Ocean (IV)

- Due to **diffusion**, the Eulerian partial age $a^n(t,\mathbf{x})$ does not necessarily reach its maximum on the n-th subdomain boundary.



Eulerian expression of the
n-th partial mean age at time t and location $\mathbf{x}$:

$$a^n(t,\mathbf{x}) = \frac{\sum_{j\in\delta V(\mathbf{x})} m_j a_j^n(t)}{\sum_{j\in\delta V(\mathbf{x})} m_j}$$

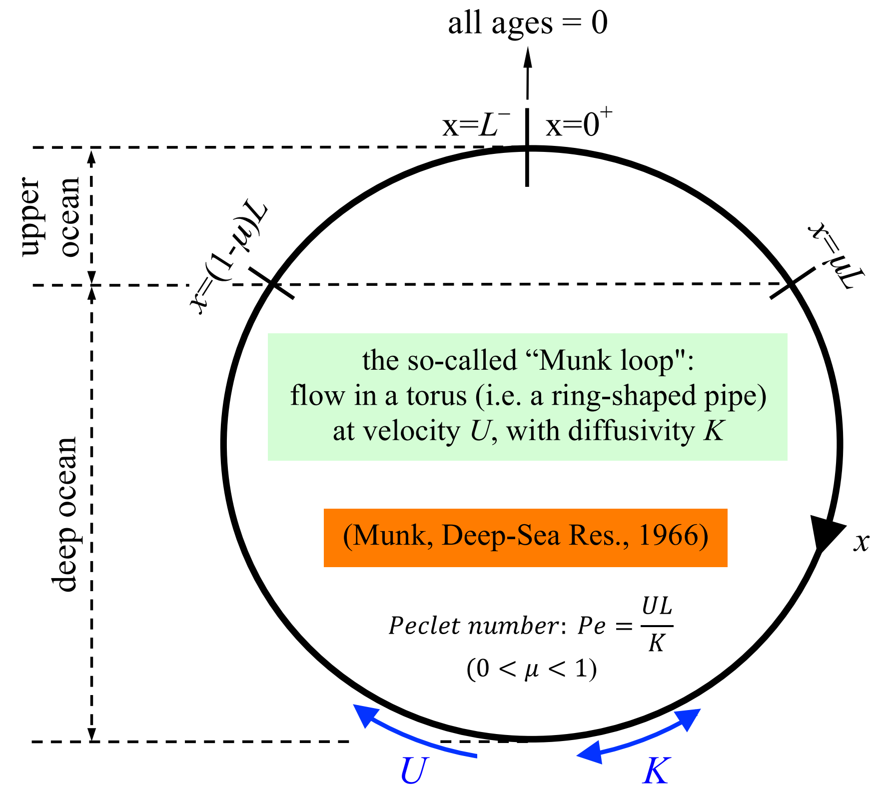
mass-weighted average,
following CART's age-averaging hypothesis
(Deleersnijder et al., *J. Mar. Syst.*, 2001)

Theorem: at a **steady-state**, the n-th partial age of a tracer undergoing a **first-order** decay **process** (e.g. a radioactive tracer) is such that

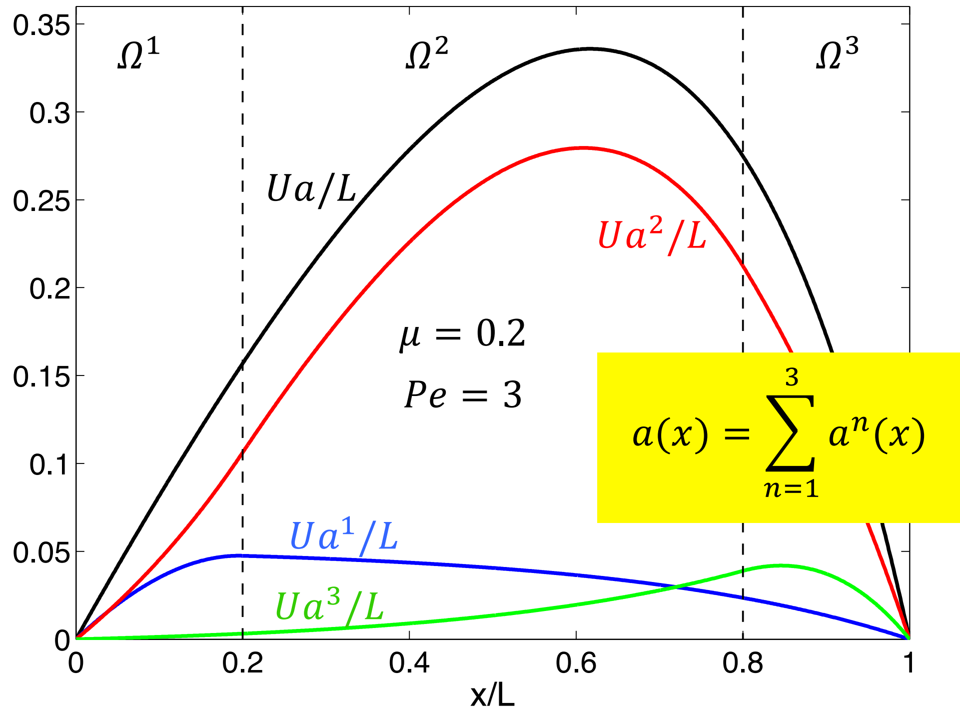
$$a^n(\mathbf{x}) \;\le\; \max_{\mathbf{x}\in\Omega^n} a^n(\mathbf{x})$$

Partial ages in the World Ocean (V)

• Illustration of the theorem about the maximum of the partial ages: ages of the water (i.e. a passive tracer) in the "Munk loop", a highly idealised model of the World Ocean's ventilation.



steady-state water ages in the "Munk-loop"

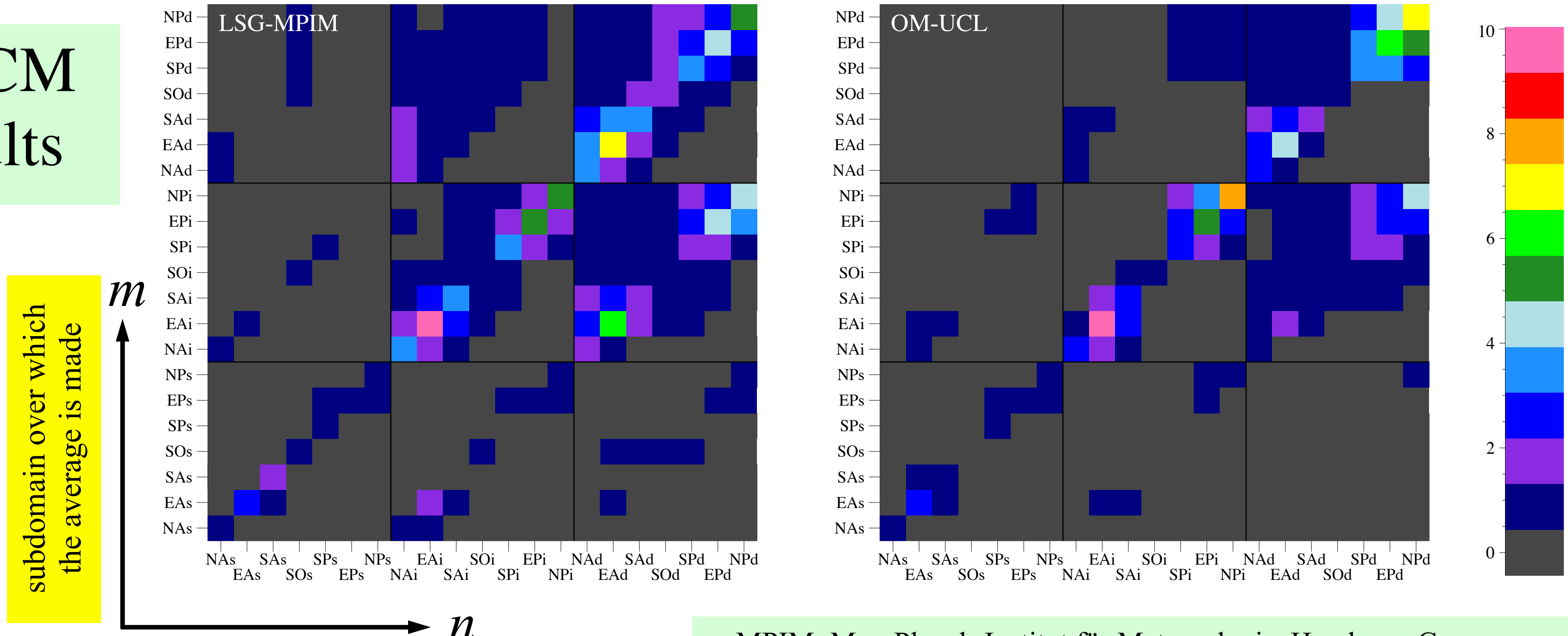


Partial ages in the World Ocean (VI)

- The (dimensionless) **connectivity matrix** is defined to be

$$\Lambda_{m,n} \equiv \frac{\Omega}{\bar{a}} \frac{\overline{a^n}^m}{\Omega^n} \quad \text{with} \quad \overline{a^n}^m = \text{average over } \Omega^m \text{ of } a^n$$

OGCM results



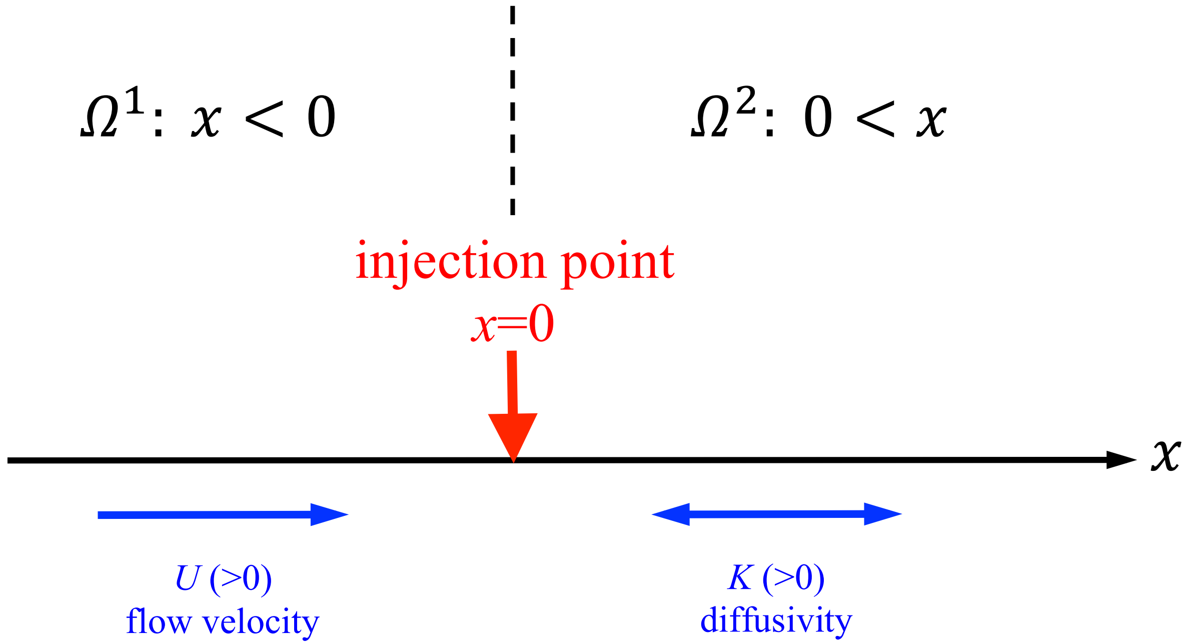
subdomain in which the watch is ticking

MPIM: Max-Planck-Institut für Meteorologie, Hamburg, Germany
UCL: Université catholique de Louvain, Louvain-la-Neuve, Belgium

Abrupt release and partial ages (I)

• Assume that a pollutant is abruptly released into the domain. If the age is defined to be the time elapsed since the injection, all the particles of the pollutant have the **same age**. Therefore, the **classical age** is **useless**, but the **partial ages** are **potentially of use**, since they measure the time spent in every subdomain.

• A one-dimensional illustration, leading to exact, analytical solutions.



Abrupt release and partial ages (II)

- For a **passive tracer**, the partial ages are

$$a^1(t,x) = \frac{t}{2} - \frac{x\sqrt{\pi t}\, e^{x^2/(4Kt)}}{4\sqrt{K}} \operatorname{erfc}\left(\frac{|x|}{\sqrt{4Kt}}\right)$$

$$a^2(t,x) = \frac{t}{2} + \frac{x\sqrt{\pi t}\, e^{x^2/(4Kt)}}{4\sqrt{K}} \operatorname{erfc}\left(\frac{|x|}{\sqrt{4Kt}}\right)$$

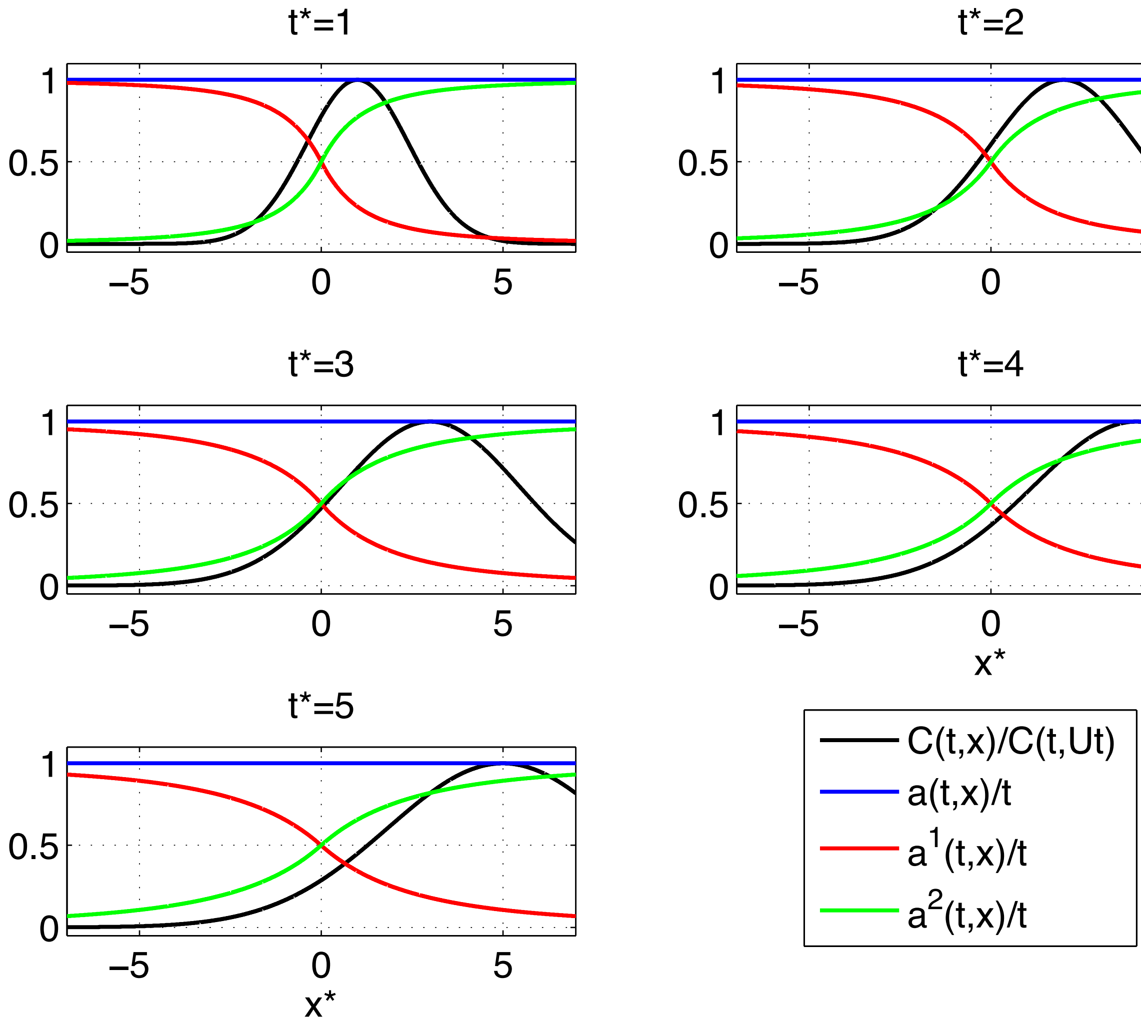
- The partials ages are **independent** of the **flow velocity** $\boldsymbol{U}$! They are also **symmetric**:

$$a^1(t,x) = a^2(t,-x)$$

He/she who will explain these surprising properties will be presented with a bottle of Champagne!

Abrupt release and partial ages (III)

- The 1D analytical solutions, with $t^* = U^2 t / K$ and $x^* = Ux / K$.

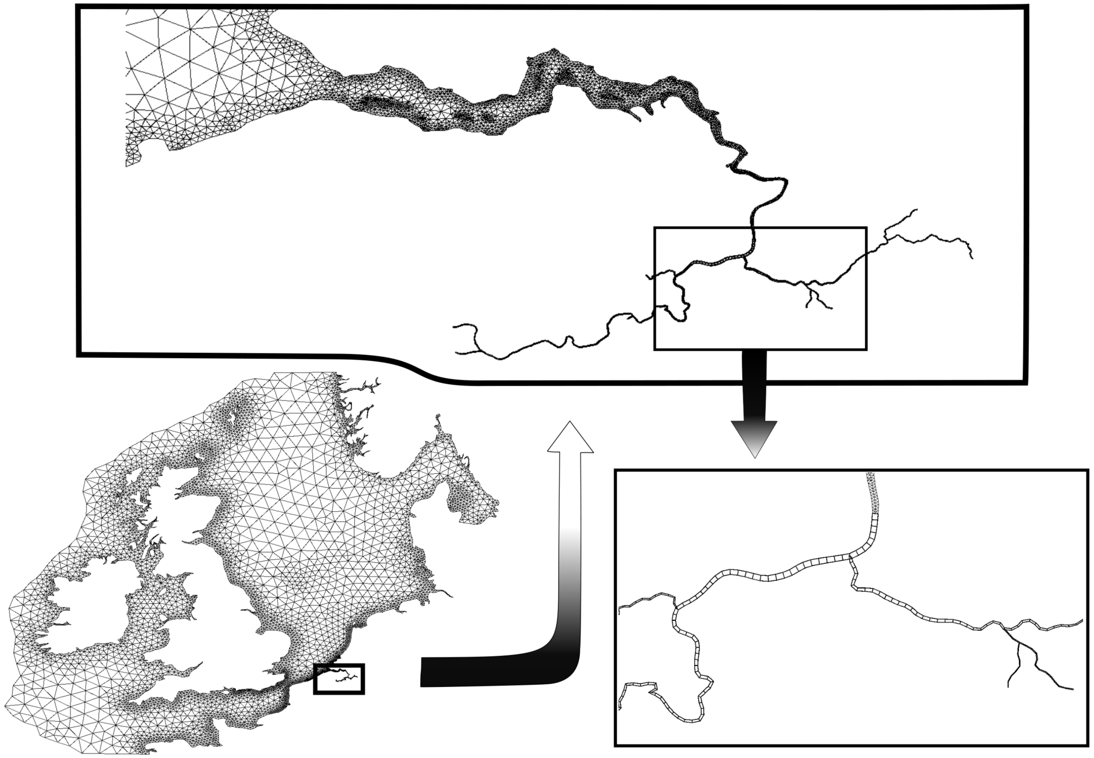


Abrupt release and partial ages (IV)

• Application to the Scheldt Estuary, using the Second-generation Louvain-la-Neuve Ice-ocean Model (SLIM, www.climate.be/slim), a Discontinuous Galerkin finite-element model.

• 40% of the meshes in the estuary, which represents 0.3% of the computational domain.

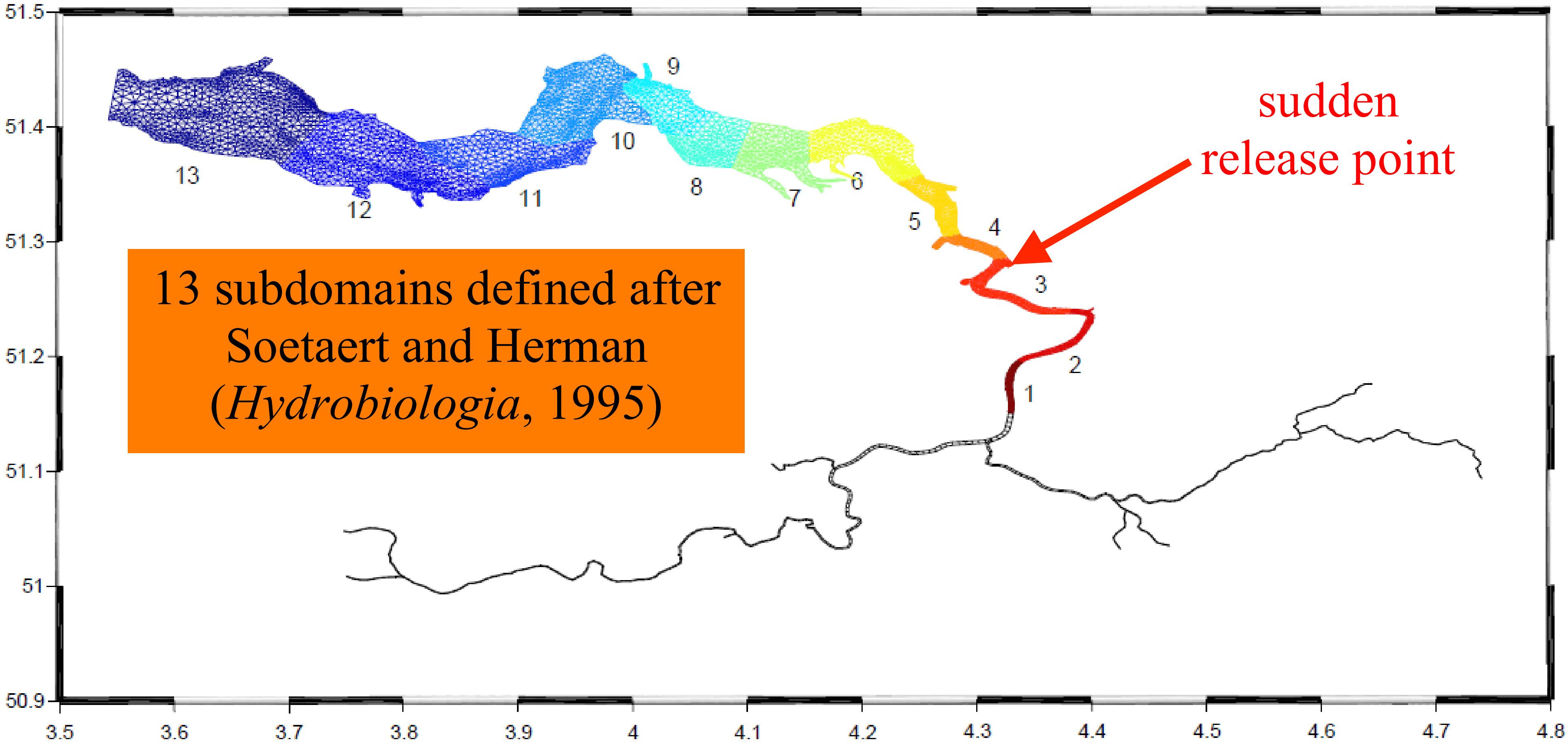
• No major problem with open boundary conditions (for tides, storms, river discharge).



Gourgue et al. (*Advances in Water Research*, 2009)
de Brye et al. (*Coastal Engineering*, 2010)
Kärnä et al. (*Computer Methods in Applied Mechanics and Engineering*, 2011)

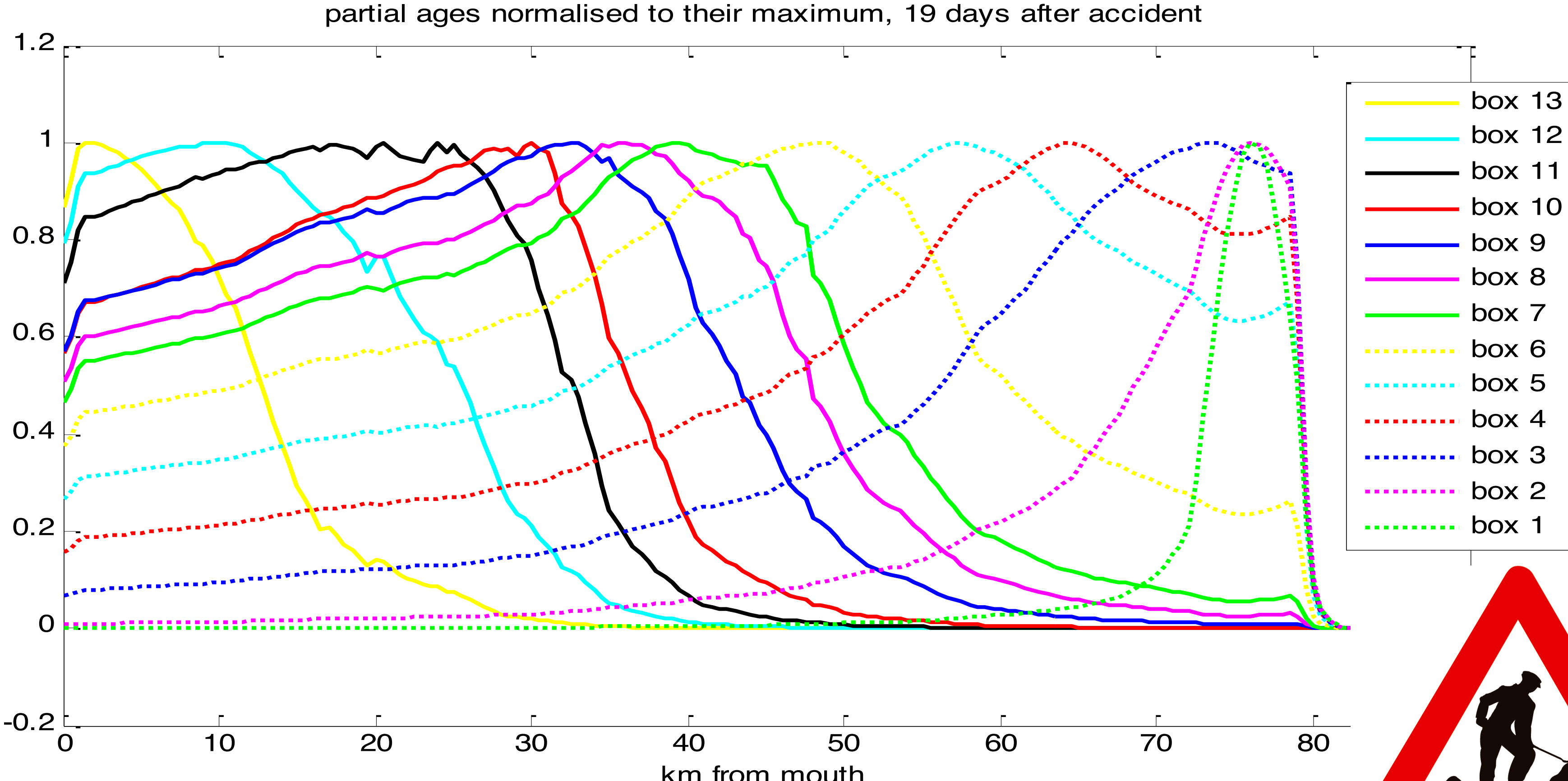
Abrupt release and partial ages (V)

• Partition into 13 subdomains of the Scheldt Estuary and location of the injection point of a passive tracer.



Abrupt release and partial ages (VI)

- The partial ages seem to behave in accordance with elementary physical intuition. These results are yet to be analysed in depth...



Conclusion and outlook

• Partial ages provide information about the **past evolution** of tracer particles, i.e. the **time spent** so far in **subdomains**.

• The matrix of subdomain-averaged partial ages is of use to assess the **connectivity** of subdomains and might offer invaluable pieces of information for model **intercomparison**.

• Partial ages may be of use in cases where the classical age is irrelevant (e.g. accidental release).

• In a box model, the inverse of the partial ages matrix is the **transport operator** (not shown herein). Would this help in building simple, **box models** on a **rational basis** (as opposed to the rule of thumb) from age results of 3D models?

• Much work has yet to be done...

Thank you!