Eddy diffusivity in the Gulf of Finland: Dispersion model evaluation based on drifter data

T. Torsvik, J. Kalda & B. Viikmäe

Institute of Cybernetics at Tallinn University of Technology

JONSMOD, May 2014

Particle tracking by Lagrangian Stochastic models

Lagrangian Stochastic models are used to track the movement of individual particles, which may represent a fluid element, a solute or a distinct, insoluble substance.

- 0th-order LSM:

$$dX_i = (U_i + \partial K_i/\partial x_i)dt + \sqrt{2K_i}d\xi_i$$

X_i = particle position; U_i = mean Eulerian velocity
K_i = eddy diffusivity; $d\xi_i$ = random forcing

- *Wiener process:* $d\xi_i$ determined by a Gaussian distribution with zero mean and variance equal to time increment dt.
- The 0th-order LSM assumes that the time increment dt is large compared to the integral time scale of turbulence.

Q1 : What are appropriate values for K_i?

Particle tracking by Lagrangian Stochastic models

- 1st-order LSM:

$$dX_i = (U_i + u_i)dt$$
$$du_i = a_i(\vec{X}, \vec{U}, t)dt + b_{ij}(\vec{X}, \vec{U}, t)d\xi_j$$

  where $u_i =$ velocity fluctuations

- If the autocorrelation function decays exponentially: $R(t) = \exp(-t/T_v)$, then a_i and b_{ij} are determined as

$$a_i = -\frac{u_i}{T_v} \qquad b_{ij} = \sqrt{\frac{2\sigma_u^2}{T_v}}$$

- The 1st-order LSM retains memory of its initial velocity for a time T_v.

Q2 : Is the 0th-order LSM sufficient for simulations in the Gulf of Finland, or is a 1st-order LSM required?

Gulf of Finland

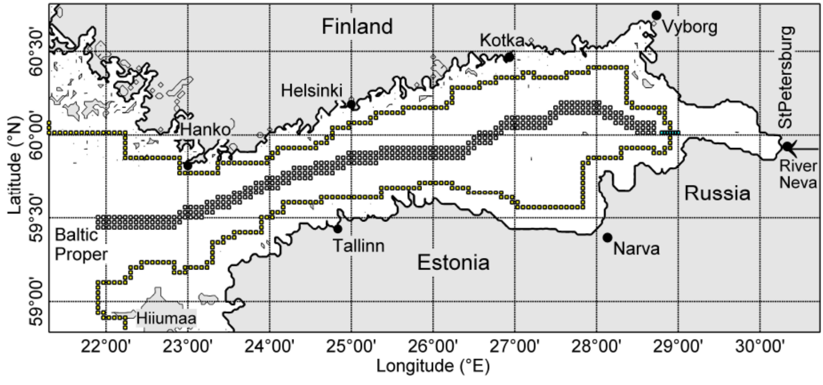
- Estuarine shallow sea
  (mean depth: 38 m)
- Surface currents dominated by wind forcing and river runoff.
- High risk area for pollution due to ship accidents
  - Intense ship traffic both along the main east-west axis of the GoF, crossing with traffic between Tallinn and Helsinki

BalticWay: Potential for coastal pollution from shipping

(Viikmäe et al. 2013, Ocean Dynamics, **63**(5), pp 589–597)

Potential for coastal pollution from shipping analyzed based on simulations (TRACMASS) of Lagrangian particle trajectories starting from the fairway.

- How does particle diffusion influence the simulation results?

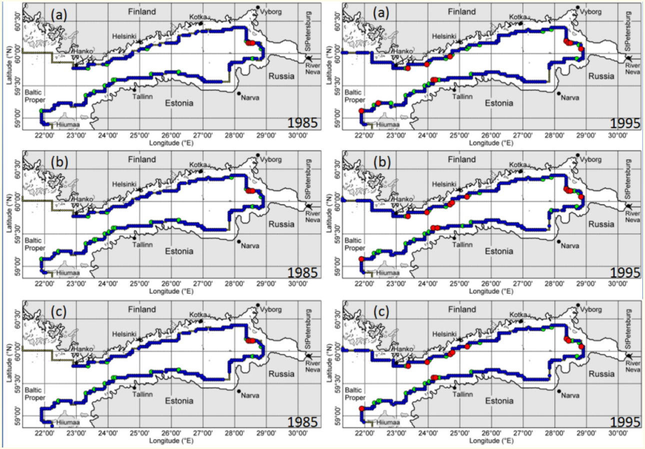


Map of the Gulf of Finland with near-shore points and major fairway.

Annual mean distribution of particle hits along the coast.

Left panels: 1985
Right panels: 1995

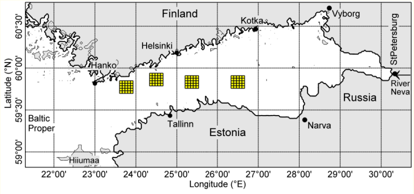
(a) No particle diffusion
(b) Constant particle diffusion coefficient
(c) Particle diffusion with Smagorinsky formula



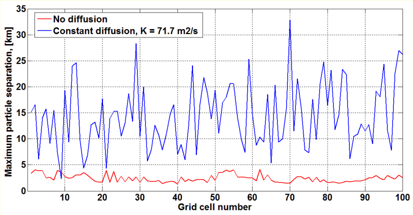
- Particle diffusion adds a small number of hits (about 3%) when compared with simulations without particle diffusion.

Effect of particle dispersion

Particle seeding bins



Maximum particle separation per grid cell



- Viikmäe et al. 2013: Constant eddy diffusivity coefficient $K = 5$ m^2/s, suggested by Döös and Engqvist for Baltic Sea (Estuar. Coast. Shelf. Sci. 74:585–597, 2007).
- De Dominicis et al. (Ocean Dynamics **62**(9), pp 1381–1398) found $K \sim 1 \times 10^3$ m^2/s for surface drifters in the Adriatic Sea and Ligurian Sea.

The appropriate value of K for Gulf of Finland should be determined by local surface drifter experiments.

Surface drifters

Design: GPS/GPRS tracker + battery pack + 2m long plastic tube
Battery life: approximately 4 weeks

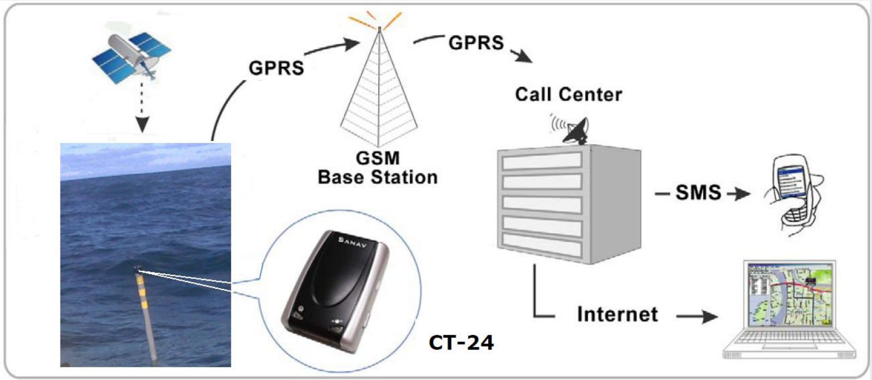
GPS tracker, communication by GPRS/GSM network

Sanav CT-24 (replaced by MU-201 in new design)

2013: Updated slender design above sea level, minimize wind drag

Surface drifters

Drifter data flow



Complexity of single deployment tracks

- Single drifter trajectories display chaotic behaviour.
- Pairs/clusters of drifters deployed close to each other (**r** $\sim$ 50m) typically stay together for several days.
- Drifters report position every 10 or 15 min.
- Connection errors may lead to gaps in drifter records. Short gaps ($\sim$3 hours) in drifter positions can be filled by linear data interpolation.

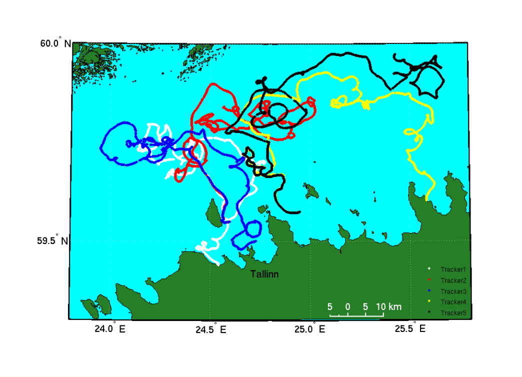


Figure: Deployment of 5 drifters, 10/4/2014

Surface drifter experiments: 2010 – 2014

YEAR	single	pair	triplet	quadruplet	total
2010	2	1	3	-	13
2011	-	1	8	-	26
2012	-	-	-	-	-
2013	-	3	5	1	25
2014	1	3	1	-	10

- Track duration: 1–35 days
- Drifter deployments within a few nautical miles from the Estonian coast.
  - Several short tracks due to drifters brought directly to land.
- Drifters deployed in the period April to October.
  - Most drifters deployed in summer and early autumn.
  - Season dominated by mild wind and wave climate.

Surface drifter experiments: 2010 – 2013

Study area: Western part of Gulf of Finland

Area of extensive ship traffic in crossing shipping lanes, where a potential oil spill is likely to reach either the Estonian or Finnish coast.

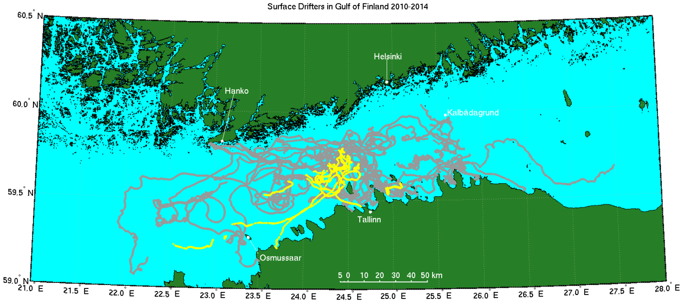


Figure: Surface drifter trajectories: Deployments in 2010

Surface drifter experiments: 2010 – 2013

Study area: Western part of Gulf of Finland

Area of extensive ship traffic in crossing shipping lanes, where a potential oil spill is likely to reach either the Estonian or Finnish coast.

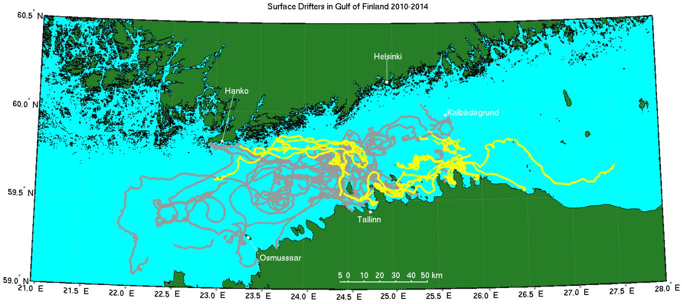


Figure: Surface drifter trajectories: Deployments in 2011

Surface drifter experiments: 2010 – 2013

Study area: Western part of Gulf of Finland

Area of extensive ship traffic in crossing shipping lanes, where a potential oil spill is likely to reach either the Estonian or Finnish coast.

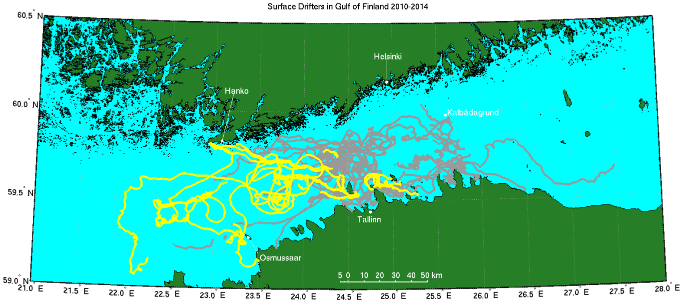


Figure: Surface drifter trajectories: Deployments in 2013

Study area: Western part of Gulf of Finland

Area of extensive ship traffic in crossing shipping lanes, where a potential oil spill is likely to reach either the Estonian or Finnish coast.

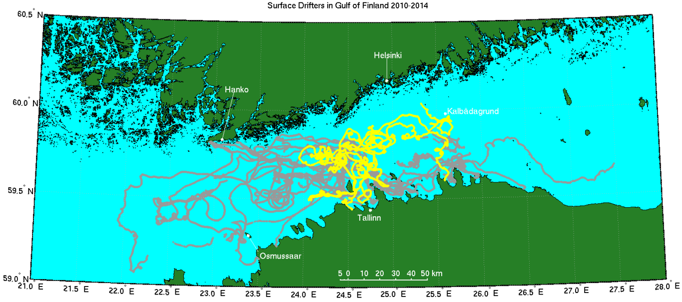


Figure: Surface drifter trajectories: Deployments in 2014

Drifter velocity distribution

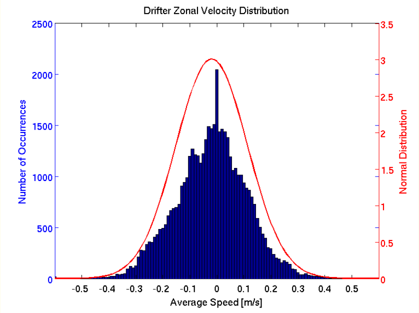


Figure: Zonal velocity distribution

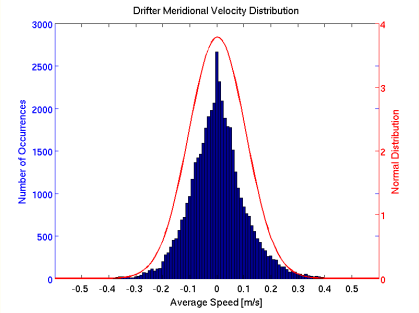


Figure: Meridional velocity distribution

Velocity	Mean [m/s]	SD [m/s]
Zonal	-0.019	0.133
Meridional	0.002	0.105

Table: Mean and Standard Deviation of normal distribution fit for the Zonal and Meridional velocity components.

Drifter velocity distribution

Sample: 46 080 records, 10 min sampling rate

U_{MEAN} $U_{RAYLEIGH}$	0.143 m/s 0.121 m/s
σ_U	0.092 m/s

Table: Comparison of drifter mean speed and maximum likelihood of Rayleigh distribution fit.

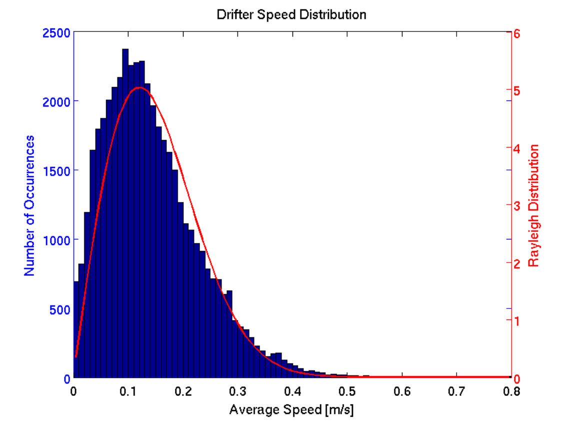


Figure: Drifter speed distribution

Lagrangian Integral Time

Lagrangian Integral Time

A key parameter for single particle analysis is the Lagrangian "integral time" T_L, which depend on the velocity auto-correlation for single trajectories, and provides a basic indicator of Lagrangian predictability.

Calculation of T_L:

- Decomposed velocity components as

$$U_i(t) = \overline{U_i} + u_i(t)\,, \quad i = \{1, 2\} \tag{1}$$

  where $\overline{U_i}$ is the time average of $U_i(t)$ and u_i is the time varying deviation from the mean.

- Calculate the auto-covariance of u_i, defined by

$$\sigma_i^2(\tau) = \lim_{T_m \to \infty} \frac{1}{T_m} \int_0^{T_m} u_i(t+\tau) \cdot u_i(t) dt \tag{2}$$

Lagrangian Integral Time

Calculation of T_L (cont.):

- The auto-correlation function

$$R_i(\tau) = \frac{\sigma_i^2(\tau)}{\sigma_i^2(0)} \qquad (3)$$

  is the auto-covariance normalized by $\sigma_i^2(\tau = 0)$.

- The zonal and meridional Lagrangian integral time is calculated as

$$T_{L,i} = \int_0^\infty R_i(\tau) d\tau \qquad (4)$$

- The total Lagrangian integral time T_L is calculated as the mean value of the zonal and meridional components

$$T_L = \frac{1}{2}\left[T_{L,1} + T_{L,2}\right] \qquad (5)$$

Lagrangian Integral Time : Problems

Problems with calculation of T_L for surface drifter data:

1. Finite drifter track duration

- Drifter trajectories have finite duration, which limits the auto-covariance calculation eq. (2) to use $T_m =$ *drifter track duration*.

$$\sigma_i^2(\tau) = \frac{1}{T_m} \int_0^{T_m} u_i(t+\tau) \cdot u_i(t) dt \tag{6}$$

- Splitting long trajectories into several short trajectories increases the pool of available tracks in the analysis. However, a small value of T_m changes the auto-covariance function σ_i^2.

Q1: How small can T_m be without changing the value of T_L?

Lagrangian Integral Time : Problems

Problems with calculation of T_L for surface drifter data:

2. Noise dominates the auto-correlation function $R(\tau)$

The auto-correlation function $R(\tau)$ becomes noisy for $\tau \gg 0$, hence it is customary to truncate the Lagrangian integral time at the de-correlation time scale. Sometimes alternative definitions of T_L are used:

standard formula	"squared auto-correlation"
$T_{L,i} = \int_0^{T_D} R_i(\tau) d\tau$	$T_{L,i} = \int_0^{T_D} R_i^2(\tau) d\tau$

Methods used for calculation of T_L:

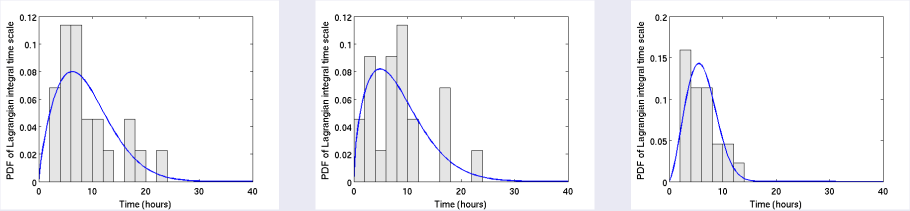
(A) T_D defined as the first time for which $R_i(\tau) = 0$.

(B) T_D defined as a constant value: $T_D = 48$ hours.

(C) Use the "squared auto-correlation" formula with with $T_D = 48$ hours.

Q2: What is the most reliable method for calculation of T_L?

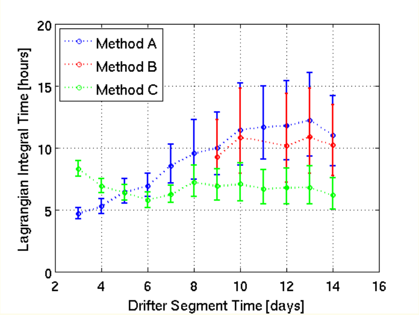
Lagrangian Integral Time : Example



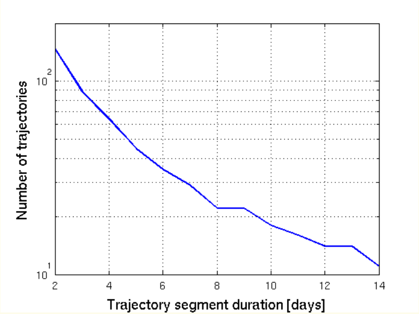
Method A | Method B | Method C

Lagrangian Integral Time T_L calculated for drifter trajectory segments with duration of 9 days. The blue curve represent data fitting using the Weibull distribution function.

Lagrangian Integral Time : Results



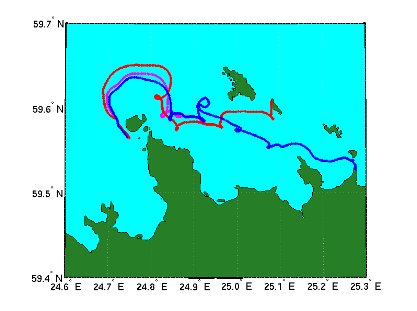
Maximum likelihood estimate for the Lagrangian Integral Time based on a Weibull data fit.

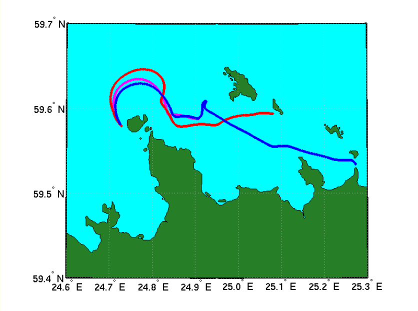


Number of drifter tracks available for analysis depending on track duration T_m.

Filtering of inertial oscillations

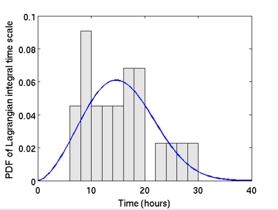
Surface drifters are subject to inertial and sub-inertial motions, which in the Gulf of Finland have periods $T_{osc} < 14$ hours. These motions are usually filtered out when analyzing the persistency of drifter trajectories, which can be accomplished by applying a running mean of $T = 14$ hours to the sequence of trajectory positions.



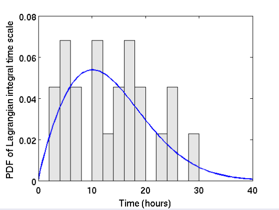


Comparison of original and filtered surface drifter tracks, for surface drifters deployed on 2nd July 2013.

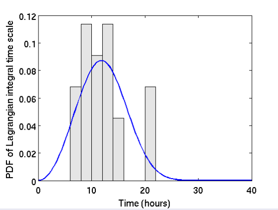
Filtering of inertial oscillations : Example



Method A



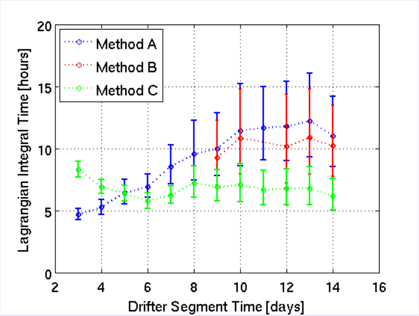
Method B



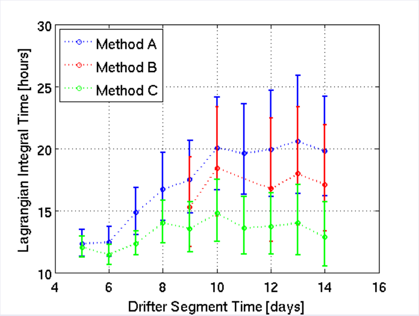
Method C

Lagrangian Integral Time calculated for drifter trajectory segments with duration of 9 days, filtered with a 14-hour running mean. The blue curve represent data fitting using the Weibull distribution function.

Filtering of inertial oscillations : Results



original data



filtered data

Comparison of T_L for original and filtered surface drifter data: Maximum likelihood estimate for the Lagrangian Integral Time based on a Weibull data fit.

Lagrangian Time and Length scales

Given the Lagrangian integral time scale T_L and the characteristic eddy speed defined by the standard deviation u_i' of the velocity components, we can define the Lagrangian eddy length scale

$$L_L = u' T_L \tag{7}$$

Lagrangian eddy diffusivity: $K = u'^2 T_L$

Method	un-filtered			filtered		
	T_L [h]	L_L [km]	K [m^2/s]	T_L [h]	L_L [km]	K [m^2/s]
A	12 ± 3	3.9 – 6.5	620	20 ± 4	6.9 – 10.4	580
B	11 ± 3	3.5 – 6.0	570	18 ± 4	6.0 – 9.5	520
C	7 ± 1	2.6 – 3.5	360	14 ± 2	5.2 – 6.9	410

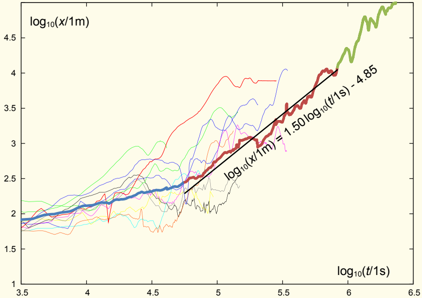
Table: Lagrangian integral time and Lagrangian eddy length scale calculated by methods A, B and C, with $u' \approx 0.12$ m/s ($u' \approx 0.09$ m/s filtered).

Drifter pair analysis: Relative dispersion

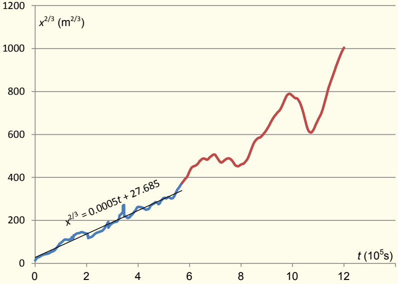
Relative dispersion

Analysis of relative dispersion of drifters is based on the displacement $\mathbf{r}(t)$ between drifter pairs. The relative distance $x(t) = |\mathbf{r}(t)|$ is determined as a function of time elapsed since the moment of release.

(a) Thin lines : Relative distance for deployments in 2011
Bold line: average for 2011 – 2013



(b) Drifter data 2011 – 2013, plotted in power-law scale $x^{2/3}$.



Relative distance x between drifters pairs. Bold curve depicts geometrical average of the pair distance using the data for all the 29 drifter pairs recorded during the years of 2011 and 2013.

Richardson's 4/3 power law

- Relative diffusivity for particle pairs: $K_R = \frac{1}{2}\frac{d}{dt}\langle x^2 \rangle$.
- Richardson (1926) proposed that $\langle x^2 \rangle \propto t^3$
  $\Rightarrow$ Scale dependent relative diffusivity: $K_R \propto x^{4/3}$.

Relative drifter velocity $\mathbf{v}(\mathbf{r}, t) = (v_1, v_2)$ is determined by

$$\mathbf{v}(\mathbf{r}, t) = \frac{\mathbf{r}(t_{i+1}) - \mathbf{r}(t_i)}{t_{i+1} - t_i}, \quad t = \frac{1}{2}(t_{i+1} + t_i) \tag{8}$$

- Energy in tubulence inertial range: $E(k) \propto k^{-5/3}$ (Kolmogorov, 1941).
- Richardson's law assumes that the velocity spectrum follows the Kolmogorov law: the fluctuations associated with a distance x should scale as $v = |\mathbf{v}(\mathbf{r}, t)| \propto x^{1/3}$.

Relative dispersion : Velocity spectrum

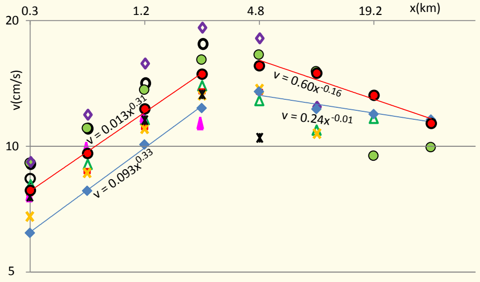


Figure: Drifter velocity fluctuations $v(x)$ are plotted against the associated scale x in log-log plot; Kolmogorov law corresponds to a linear curve with the slope of $1/3$. Different data points correspond to different experiments (with different winds and flow patterns). Linear regression lines are given for two data sets, separately for scales smaller than 4 km and for scales larger than 4 km.

The velocity spectrum calculation based only on data along trajectories becomes non-reliable for large scales due to the twisting of trajectories.

Spatial variability: Drifter velocity

Spatial variability: Drifter velocity

Average drifter velocity [m/s] for bin sizes if about 11 $\times$ 11 km^2. Calculation using all drifter data in period 2010–2014.

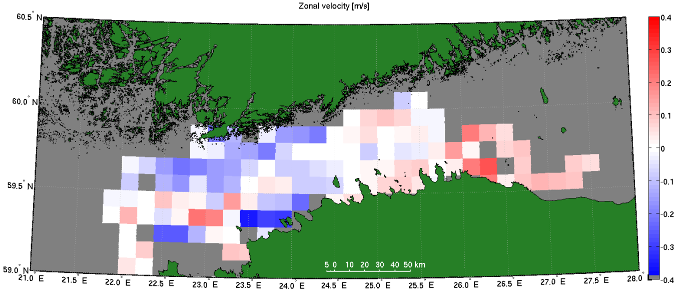


Figure: Zonal drifter velocity

Spatial variability: Drifter velocity

Average drifter velocity [m/s] for bin sizes if about 11 $\times$ 11 km^2. Calculation using all drifter data in period 2010–2014.

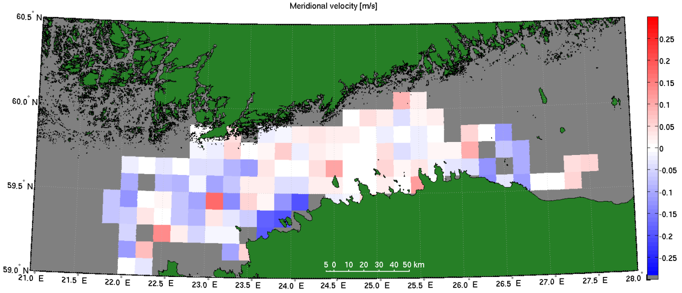


Figure: Meridional drifter velocity

Spatial variability: Drifter velocity

Average drifter velocity [m/s] for bin sizes if about 11 $\times$ 11 km^2. Calculation using all drifter data in period 2010–2014.

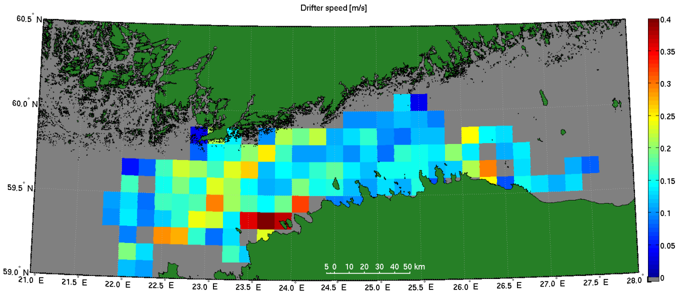


Figure: Drifter speed

Summary & Concluding remarks

- Surface drifters have an average speed $U_{MEAN} = 0.14$ m/s, with extremes up to $U \approx 0.5$ m/s, when calculated from data with 10 min sampling rate.
- Record durations of $T_m \sim 8$–10 days are required to obtain reliable results for calculation of the Lagrangian integral time T_L.
- The Lagrangian integral time is $T_L \in [7, 12]$ hours, extended to $T_L \in [14, 20]$ hours with filtering of inertial oscillations, depending on which method is used for the calculation.
- At large times $t \gg T_L$, the constant dispersion coefficient is in the range $K \in [360, 620]$ m^2/s.
- Relative drifter separation $x(t)$ follows a Richardson's law dispersion characterized by the time dependent relative distance increasing as $x(t) \propto t^{3/2}$. This behavior is a prominent feature for most surface drifter pairs separated by a distance from ~ 50 m up to ~ 5 km, and may extend even beyond this scale.

Summary & Concluding remarks

- Further analysis is required to determine spatial and seasonal variability for surface drifters. For such analysis the problem of insufficient data coverage is a significant factor.
- For a typical LSM simulation in the Gulf of Finland: $dt \ll T_L$. This suggest that a 1st-order LSM should be used for such simulations.

Thank you for your attention!